

EEC 289K

Ultrafast Optics

Recitation 1

Real vs. Complex Notation

$$\begin{aligned} E(t) &= \operatorname{Re} \{ A(t) e^{j\omega_0 t} \} \\ &\quad \begin{array}{l} \uparrow \\ \text{real} \end{array} \quad \begin{array}{l} \uparrow \\ \text{complex} \\ \text{envelope} \end{array} \\ &= \frac{1}{2} A(t) e^{j\omega_0 t} + \frac{1}{2} \underbrace{A^*(t)}_{a+ib, a-ib} e^{-j\omega_0 t} \\ &\quad \text{c.c.} \\ &= |A(t)| \cos(\omega_0 t + \varphi(t)) \\ A(t) &= |A(t)| e^{j\varphi(t)} \end{aligned}$$

Remember

- o Complex notation is book-keeping tool.
 - Keeps track of envelope and dispersion compactly.
 - Pull Out $e^{j\omega_0 t}$
- o In the lab \Rightarrow fields always real!

FT Review

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{F}(\omega) e^{j\omega t} d\omega$$

$$\hat{F}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$E(t) = \operatorname{Re} \left\{ A(t) e^{j\omega_0 t} \right\}$$

$$\hookrightarrow \bar{E}(t) = \left[\int \hat{A}(\omega) e^{j\omega t} d\omega \right] e^{j\omega_0 t}$$

$\mathcal{F}\left\{ A(t) e^{j\omega_0 t} \right\}$

Convolution Property

$$f(t) g(t) \Rightarrow \frac{1}{2\pi} (f * g)(\omega)$$
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega') g(\omega - \omega') d\omega'$$

$$(f * g)(t) \Rightarrow \hat{f}(\omega) \cdot \hat{g}(\omega)$$

$$A(t) \Rightarrow \hat{A}(\omega)$$

$$e^{j\omega_0 t} \Rightarrow 2\pi \delta(\omega - \omega_0)$$

$$\hat{E}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{A}(\omega') 2\pi \delta(\omega - \omega' - \omega_0) d\omega'$$

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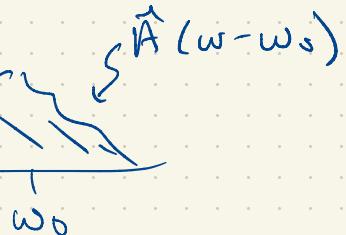
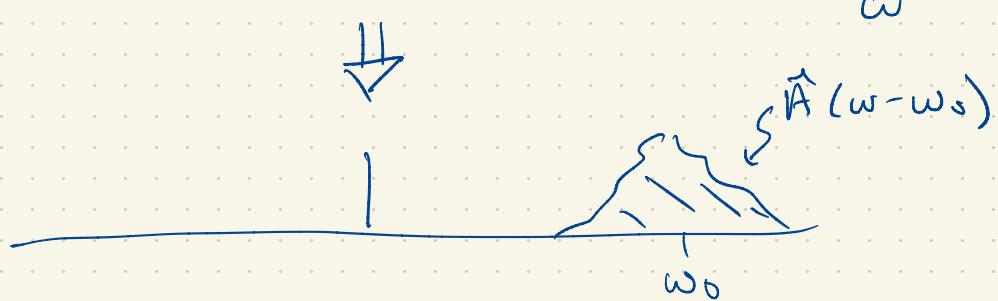
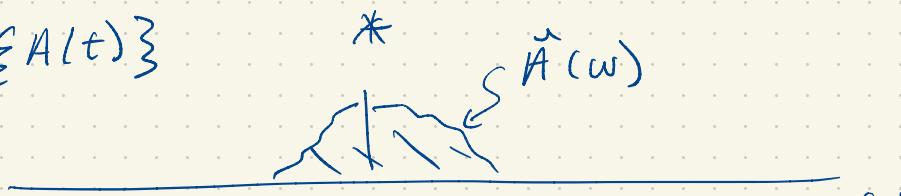
$$\int_{-\infty}^{\infty} F(x) \delta(x - \alpha) dx = F(\alpha)$$

$$\hat{E}(\omega) = \hat{A}(\omega - \omega_0)$$

$\Re \{ e^{i\omega_0 t} \}$



$\Re \{ A(t) \}$



Power and Intensity

$$\vec{E} = E_x \cos(\omega t - kz) \hat{z}$$

$$\vec{H} = H_y \cos(\omega t - kz) \hat{y}$$

$$\vec{S} = \vec{E} \times \vec{H} \quad [\text{W/m}^2]$$

$$\vec{S} = E_x H_y \hat{z} \quad [\text{W/m}^2]$$

$$H_y = \frac{E_x}{\eta_0}$$

@ $z=0$

$$\vec{S} = \frac{E_x^2}{\eta_0} \cos^2(\omega t) \hat{z}$$

↑
time-dep.

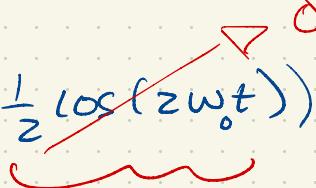
$$\langle \vec{S} \rangle = \frac{1}{T} \int_0^T \vec{S} \cdot dt$$

$$= \frac{\pi^2}{T} \frac{E_x^2}{\eta_0} \int_0^T \cos^2(\omega t) dt$$

$$= \frac{\sum}{T} \frac{E_x^2}{\eta_0} \int_0^T \cos^2(\omega_0 t) dt$$

$$\Rightarrow \cos^2(\omega t) = \frac{1}{2} + \frac{1}{2} \cos(2\omega_0 t)$$

$$= \int_0^T \left(\frac{1}{2} + \frac{1}{2} \cos(2\omega_0 t) \right) dt$$

integral over nT is zero!


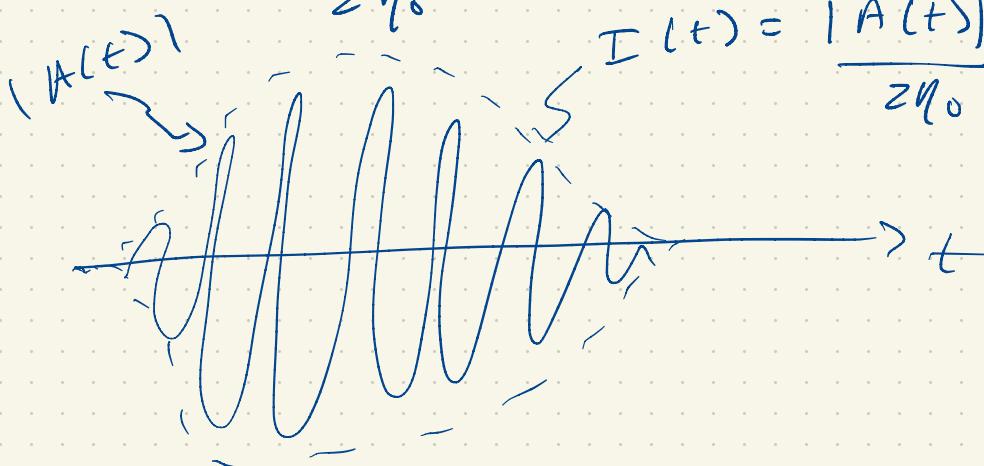
$$= \frac{1}{2} T$$

$$\langle \vec{s} \rangle = \frac{E_x^2 \hat{z}}{2\eta_0}$$

$[W/m^2]$

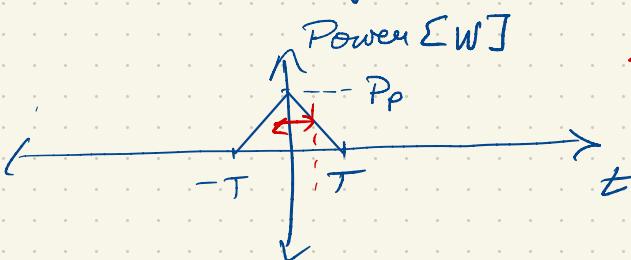


$$I(t) = \frac{|A(t)|^2}{2\eta_0}$$



Practice: Energy, Intensity, Fields

Say we have a triangular pulse:



a) What is the FWHM?

$$\boxed{\text{FWHM} = T}$$

b) Given energy W , what is P_p (peak power)?

$$W = T \cdot P_p$$

$$\boxed{P_p = W/T}$$

c) If the beam has a circular tophat profile in space with radius R_1 , what is the peak intensity, I_p ?

$$I(r) = \begin{cases} I_p, & r \leq R \\ 0, & r > R \end{cases}$$

$$P_p = \iint_A I(r) dA = I_p \cdot \pi \cdot R^2$$

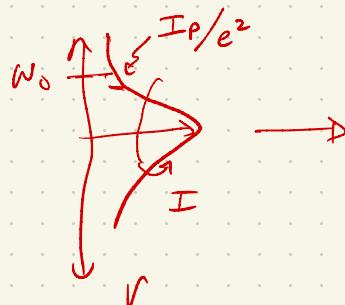
$$\boxed{I_p = \frac{P_p}{\pi R^2}}$$

But what about something more realistic?

d) If the beam is Gaussian in space, that is:

$$I(r) = I_p e^{-2r^2/w_0^2}$$

What is I_p ?



$$P_p = \iint I(r) dA$$

$$= \int_0^{2\pi} \int_0^\infty I_p e^{-2r^2/w_0^2} r dr d\theta$$

$$= 2\pi \int_0^\infty I_p e^{-2r^2/w_0^2} r dr$$

$$u = \frac{2r^2}{w_0^2}$$

$$du = \frac{4r}{w_0^2} dr$$

$$\frac{w_0^2 \cdot du}{4} = r \cdot dr$$

$$P_p = \frac{2\pi \cdot I_p \cdot w_0^2}{4} \int_0^\infty e^{-u} du$$

$$P_p = \frac{\cancel{z\pi} \cdot I_p \cdot w_0^2}{2} \int_0^\infty e^{-u} du$$

$$\left. \infty \right| -e^{-u} = \left. \infty \right| e^{-u} = 1$$

$$P_p = \frac{\pi I_p \cdot w_0^2}{2}$$

$$\boxed{I_p = \frac{2 \cdot P_p}{\pi w_0^2}}$$

e) What is the peak field in vacuum?

$$E_p \Rightarrow \max \text{ of } |\hat{A}(t)|$$

$$I_p = \frac{2 \cdot P_p}{\pi w_0^2}$$

$$\frac{E_p^2}{2\eta_0} = I_p$$

$$E_p^2 = 2\eta_0 \cdot I_p$$

$$\boxed{E_p = \sqrt{2\eta_0 I_p}}$$

F) Let's say these pulses are coming from a laser system with:

$$\circ P_{avg} = 1 \text{ mW}$$

$$\circ \text{ freq} = 1 \text{ MHz}$$

$$\circ \text{ Gaussian profile, } w_0 = 5 \mu\text{m}$$

$$\circ T = 10 \text{ fs}$$

What is the peak field, E_p ?

$$P_{\text{Avg}} = W \cdot f_{\text{rep}}$$

$$t_{\text{rep}} = \frac{1}{f_{\text{rep}}}$$

$$W = P_{\text{Avg}} / f_{\text{rep}}$$

$$= \frac{1 \times 10^{-3} \text{ [J]}}{1 \times 10^6 \text{ [1/s]}}$$

$$= 1 \times 10^{-9} \text{ J} = 1 \text{ nJ}$$

$$P_p = \frac{W}{t} = \frac{1 \times 10^{-9} \text{ J}}{10 \times 10^{15} \text{ s}} = 10^5 \text{ W}$$
$$= 100 \text{ kW}$$

$$E_p = \frac{2}{W_0} \sqrt{\frac{\rho_0 P_p}{\pi}}$$

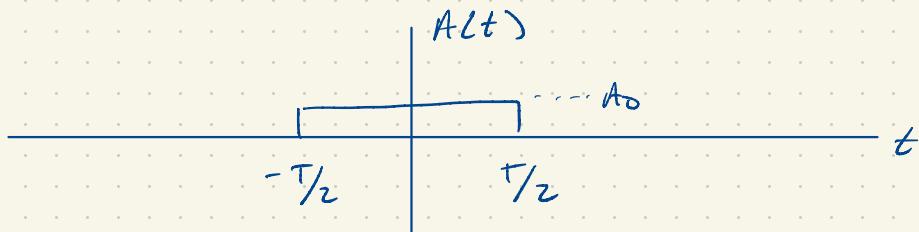
$$\approx 1.4 \times 10^9 \text{ V/m} = \boxed{1.4 \text{ GV/m}}$$

Breakdown of air $\approx 10^6 \text{ V/m}$.

Note: this is for DC Voltage

Practice: Time-Frequency Relations

Imagine a pulse like the following



a) What is $\hat{A}(\omega)$

We stopped here during recitation.

Work through this exercise and interact with the code/
Jupyter notebook for next recitation.

b) What is the time-bandwidth product?

