# EEC 289K

# Ultrafast Optics

# Recitation 2

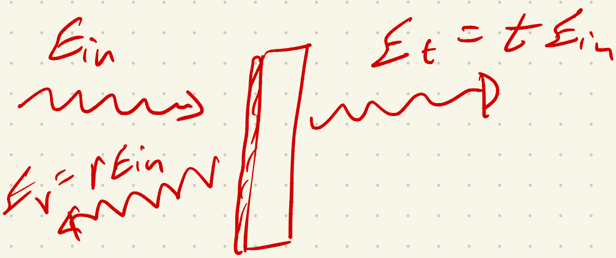
Imagine you have an optical element:

(1) Partially reflects light

(2) Partially transmits light

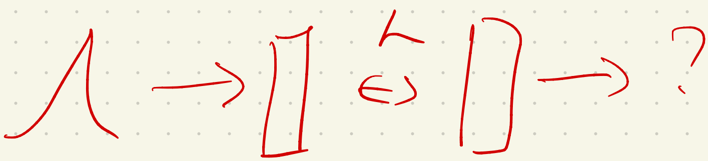
(3) lossless

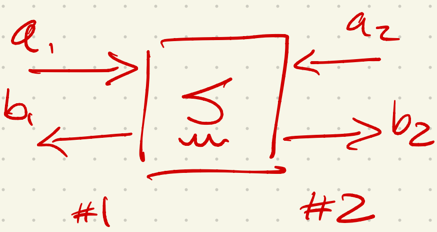
(4) linear system




$$|E_{in}|^2$$

$$|t|^2 + |r|^2 = 1$$ $\Rightarrow$ conservation of energy.






$a_n \Rightarrow$ input ~~from~~ to port $n$

$b_n \Rightarrow$ output from port $n$

$$S_{mn} = \frac{b_m}{a_n}$$

output at m        input at n

$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

$$\vec{b} = \underset{\sim}{S}\, \vec{a}$$

$$\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \quad \vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\underbrace{|a_1|^2 + |a_2|^2}_{\vec{a}^{\dagger}\vec{a}} = \underbrace{|b_1|^2 + |b_2|^2}_{\vec{b}^{\dagger} b} = P$$

$$(a_1^*, a_2^*)$$

$$b^\dagger b = (\vec{a}^\dagger \underset{\sim}{S}^\dagger)(\underset{\sim}{S}\,\vec{a}) = \underline{P}$$

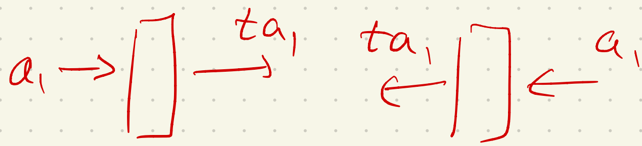
$$= a^\dagger a$$

$$= \vec{a}^\dagger (\underset{\sim}{S}^\dagger \underset{\sim}{S})\,\vec{a}$$

$$\underset{\sim}{S}^\dagger \underset{\sim}{S} = \underset{\sim}{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{pmatrix} \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\boxed{\begin{aligned} &|S_{11}|^2 + |S_{21}|^2 = 1 \\ &|S_{12}|^2 + |S_{22}|^2 = 1 \\ &S_{11}^* S_{12} + S_{21}^* S_{22} = 0 \end{aligned}}$$




$$\boxed{S_{21} = S_{12}}$$

trans at 2 from 1

trans at 1 from 2

$$\boxed{\begin{aligned} |S_{11}| &= |S_{22}| \\ S_{12} &= S_{21} \end{aligned}}$$

$$S_{11} = S_{22} = r$$

$$S_{12} = S_{21} = t$$

$$\begin{cases} r = |r| e^{j\varphi_r} \\ t = |t| e^{j\varphi_t} \end{cases}$$

$$|r|^2 + |t|^2 = 1$$

$$|t| = \sqrt{1 - |r|^2}$$

$$S_{11}^* S_{12} + S_{21}^* S_{22} = 0$$

$$r^* t + t^* r = 0$$

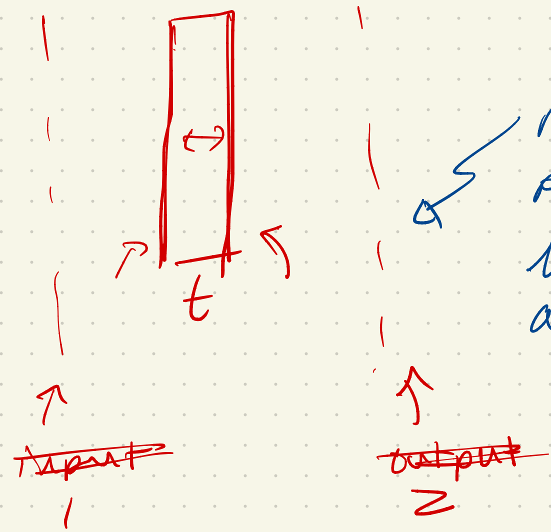
$$\cancel{|t||r|} \left\{ e^{-j\varphi_r} e^{j\varphi_t} + e^{j\varphi_r} e^{-j\varphi_t} \right\} = 0$$

$$\varphi = \varphi_t - \varphi_r$$

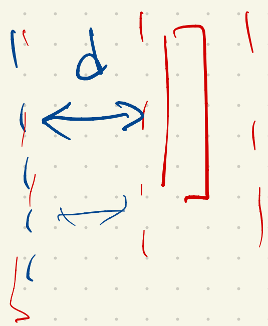
$$e^{j\varphi} + e^{-j\varphi} = 0$$

$$2\cos(\varphi) = 0, \quad \underline{\underline{\varphi = \pm \pi/2}}$$

$$\underset{\sim}{S} = \begin{pmatrix} |r| & j|t| \\ j|t| & |r| \end{pmatrix}$$



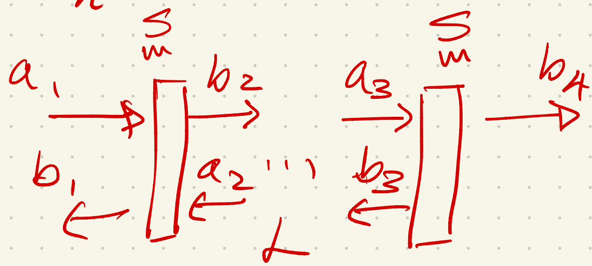

reference planes don't have to be at interfaces!




You can adust reference plane! Move out by d to give $\pi$ phase-shift on r. ($\pi/2$ phase-shift on t).

$$\underset{\sim}{S} = \begin{pmatrix} |r| & |t| \\ |t| & -|r| \end{pmatrix}$$

# Fabry Perot Cavity




$$a_3 = b_2 e^{j\varphi}$$

$$\varphi = -kL$$

$$e^{j\omega t} e^{-jkz}$$

$$a_2 = b_3 e^{j\varphi}$$

$$\begin{pmatrix} b_3 \\ b_4 \end{pmatrix} = \underset{\sim}{S} \begin{pmatrix} a_3 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} |r| & j|t| \\ j|t| & |r| \end{pmatrix} \begin{pmatrix} a_3 \\ 0 \end{pmatrix}$$

$$b_3 = r a_3$$

$$b_4 = jt a_3$$

$$a_3 = b_4 / jt$$

$$b_3 = \frac{|r| b_4}{j|t|}$$

$$a_3 = b_4 / it$$

$$b_3 = \frac{r\, b_4}{it}$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} r & it \\ it & r \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

⋮ steps to fill

$$b_4 = \frac{-t^2 a_1 e^{i\varphi}}{1 - r^2 e^{i2\varphi}}$$

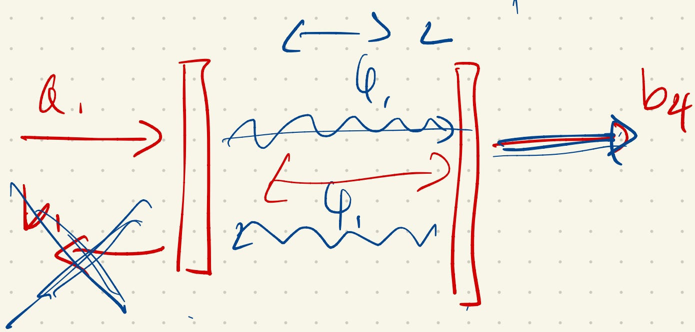
$$H = \frac{b_4}{a_1} = \frac{-t^2 e^{i\varphi}}{1 - r^2 e^{i2\varphi}}$$

$$\varphi = \frac{-kL}{k(\omega)}$$

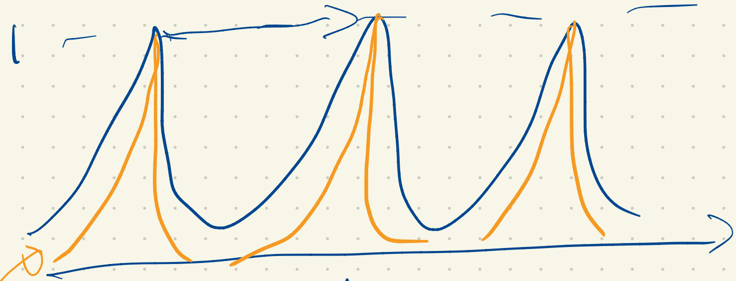
$$R = r^2$$

$$T = t^2$$

$$\boxed{H = \frac{-T e^{i\varphi}\left(1 - R e^{-i2\varphi}\right)}{1 + R^2 - 2R\cos(2\varphi)}}$$




R=0.5 ↑ R=0.5 ↑




$|H|^2$

$$\varphi = -kL = -\frac{\omega L}{c}$$

Increased $\frac{d\varphi}{d\omega}$ as $L\uparrow$.

Increasing R → increases time in cavity → decreases bandwidth

(Higher "finesse")