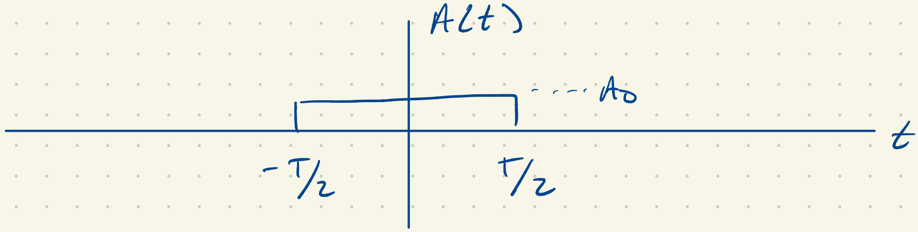
Top Hat Pulse

Imagine a pulse like the following



a) What is $\hat{A}(\omega)$

$$\hat{A}(\omega) = \int_{-\infty}^{\infty} A(t) e^{-j\omega t} dt$$

$$= \int_{-T/2}^{T/2} A_0 e^{-j\omega t} dt$$

$$= \frac{-A_0}{j\omega} \Big]_{-T/2}^{T/2} e^{-j\omega t}$$

$$= \frac{-A_0}{j\omega} \left[e^{-j\omega T/2} - e^{j\omega T/2} \right]$$

$$= \frac{-A_0}{j\omega} \left[\cancel{\cos(+\omega T/2)} + j\sin(+\omega T/2) - \cancel{\cos(\omega T/2)} - j\sin(\omega T/2) \right]$$

$$= \frac{+A_0}{\cancel{j}\omega} \cdot +2\cancel{j}\sin(\omega T/2)$$

$$= \frac{2A_0 \sin(\omega T/2)}{\omega} = \left(\frac{2A_0 T}{\cancel{2}}\right) \frac{\sin(\omega T/2)}{(\omega T/2)}$$

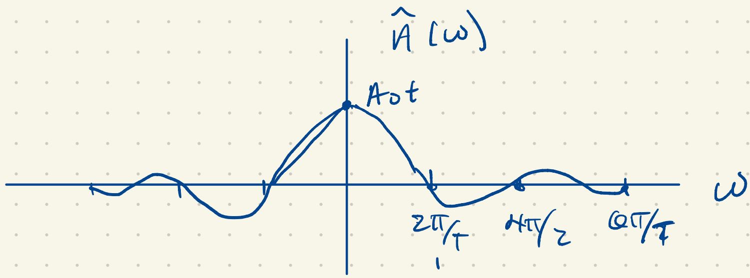
$$\boxed{\hat{A}(\omega) = A_0 T \, \underbrace{\frac{\sin(\omega T/2)}{(\omega T/2)}}_{\text{like sinc function.}}}$$

$$\text{sinc}(x) = \frac{\sin(\pi \cdot x)}{\pi x}$$

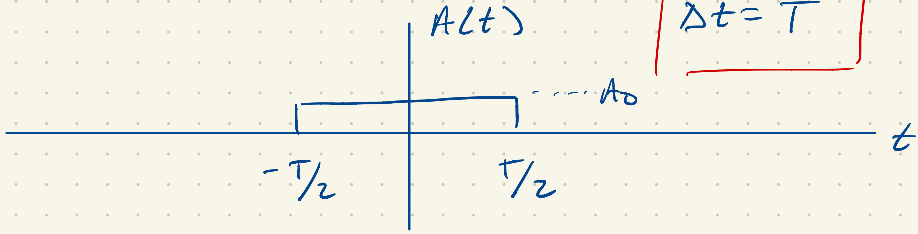
$$\text{sinc}(0) = 1$$

zero points when: $\omega T/2 = \pm n \cdot \pi$, $n > 0$

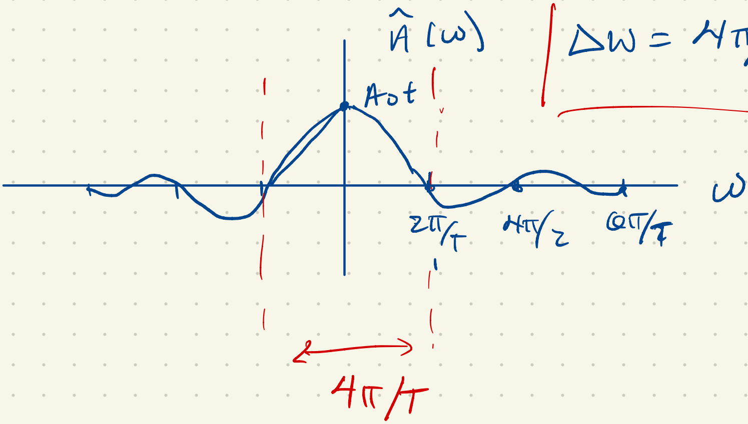
$$\omega = \pm \frac{2n\pi}{T}$$



b) What is the time-bandwidth product?



$$\boxed{\Delta t = T}$$



$$\boxed{\Delta W = 4\pi/T}$$

$$\boxed{\Delta t \Delta W = 4\pi}$$

Key Points

- $\Delta t \Delta \omega$ not dependent on T
- Broader bandwidth → shorter time
- Longer time → narrower bandwidth
- However, equality only applies to case of "transform-limit"
  - Know that you can add GDD, which broadens pulse in time without increasing bandwidth.
  - $\Delta t \Delta \omega \geq 4\pi$