EEC 289K

Ultrafast Optics

Recitation 1

Real vs. Complex Notation

$$\underset{\text{real}}{E(t)} = \mathrm{Re}\{ \underset{\text{complex envelope}}{A(t)} e^{j\omega_0 t} \}$$

$$= \frac{1}{2} A(t) e^{j\omega_0 t} + \underbrace{\tfrac{1}{2} A^*(t) e^{-j\omega_0 t}}_{c.c.}$$

$a + jb \quad , \quad a - jb$

$$= |A(t)| \cos(\omega_0 t + \varphi(t))$$

$$A(t) = |A(t)| e^{j\varphi(t)}$$

Remember

- Complex notation is book-keeping tool.
  - Keeps track of envelope and dispersion compactly.
  - Pull out $e^{j\omega_0 t}$
- In the lab $\Longrightarrow$ fields always real!

FT Review

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega t}\, d\omega$$

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt$$

$$E(t) = \mathrm{Re}\left\{ \underbrace{A(t) e^{i\omega_0 t}} \right\}$$

$$\tilde{E}(t) = \left[\int \hat{A}(\omega) e^{i\omega t}\, d\omega \right] e^{i\omega_0 t}$$

$$\mathcal{F}\left\{ A(t) e^{i\omega_0 t} \right\}$$

$\boxed{\begin{array}{l} \text{Convolution Property} \\ f(t)g(t) \Rightarrow \frac{1}{2\pi}(f * g)(\omega) \\ \qquad = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega') g(\omega-\omega')\, d\omega' \\ (f * g)(t) \Rightarrow \hat{f}(\omega)\cdot\hat{g}(\omega) \end{array}}$

$$A(t) \Rightarrow \hat{A}(\omega)$$

$$e^{i\omega_0 t} \Rightarrow 2\pi\,\delta(\omega-\omega_0)$$

$$\hat{E}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{A}(\omega')\, 2\pi\,\delta(\omega-\omega'-\omega_0)\, d\omega'$$

$$\hat{E}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{A}(\omega')\, 2\pi\,\delta(\omega-\omega'-\omega_0)\,d\omega'$$

$$\int_{-\infty}^{\infty} f(x)\,\delta(x-\alpha)\,dx = f(\alpha)$$

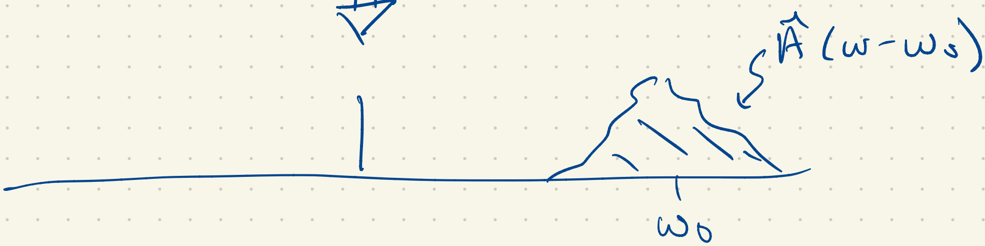
$$\hat{E}(\omega) = \hat{A}(\omega-\omega_0)$$

$\mathcal{F}\{e^{i\omega_0 t}\}$

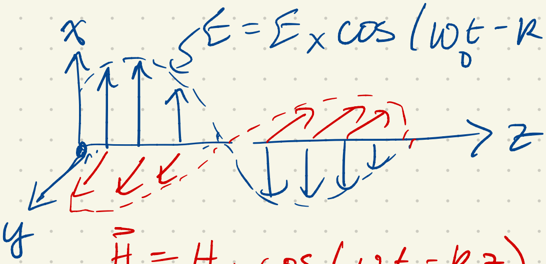
$\mathcal{F}\{A(t)\}$

$*$

$\Downarrow$



Power and Intensity



$$\vec{E} = E_x \cos(\omega_0 t - kz)\,\hat{x}$$

$$\vec{H} = H_y \cos(\omega_0 t - kz)\,\hat{y}$$

$$\vec{S} = \vec{E} \times \vec{H} \quad [W/m^2]$$

$$\vec{S} = E_x H_y \,\hat{z} \; [W/m^2]$$

$$H_y = \frac{E_x}{\eta_0}$$

@ $z = 0$

$$\vec{S} = \frac{E_x^2}{\eta_0} \cos^2(\omega t)\,\hat{z}$$

time-dep.

$$\langle \vec{S} \rangle = \frac{1}{T} \int_0^T \vec{S} \cdot dt$$

$$= \frac{\hat{z}}{T} \frac{E_x^2}{\eta_0} \int_0^T \cos^2(\omega t)\,dt$$

$$= \frac{\hat{z}}{T}\frac{E_x^2}{\eta_0}\int_0^T \cos^2(\omega_0 t)\,dt$$

$$\Rightarrow \cos^2(\omega t) = \frac{1}{2} + \frac{1}{2}\cos(2\omega_0 t)$$

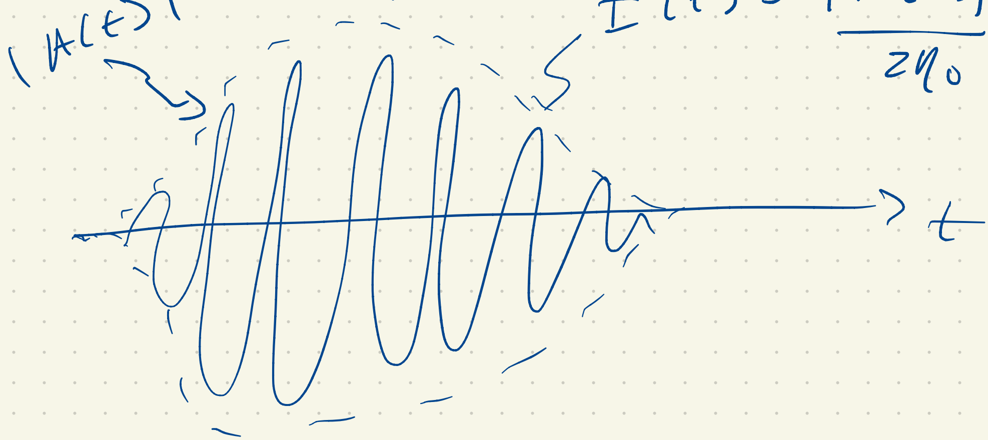
$$\int_0^T \cos^2(\omega t)\,dt$$

$$= \int_0^T \Big(\frac{1}{2} + \underbrace{\cancel{\frac{1}{2}\cos(2\omega_0 t)}}_{\text{integral over } nT \text{ is zero!}}\Big)\,dt \quad \to 0$$

$$= \frac{1}{2}T$$

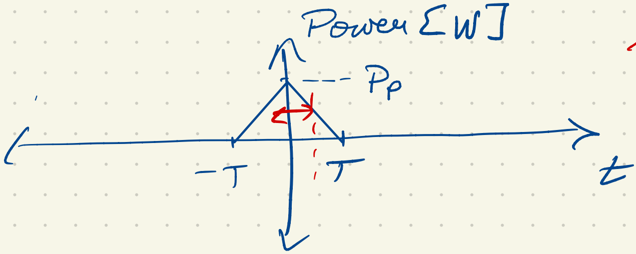
$$\langle \vec{S} \rangle = \frac{E_x^2\,\hat{z}}{2\eta_0} \qquad [W/m^2]$$

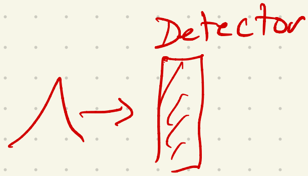
$$I(t) = \frac{|A(t)|^2}{2\eta_0}$$



Practice: Energy, Intensity, Fields

Say we have a triangular pulse:





a) What is the FWHM?

$$\boxed{FWHM = T}$$

b) Given energy W, what is P_p (peak power)?

$$W = T \cdot P_p$$

$$\boxed{P_p = W/T}$$

c) If the beam has a circular tophat profile in space with radius R, what is the peak intensity, I_p?

$$I(r) = \begin{cases} I_p, & r \le R \\ 0, & r > R \end{cases}$$

$$P_p = \iint_A I(r)\, dA = I_p \cdot \pi \cdot R^2$$

$$\boxed{I_p = \frac{P_p}{\pi R^2}}$$

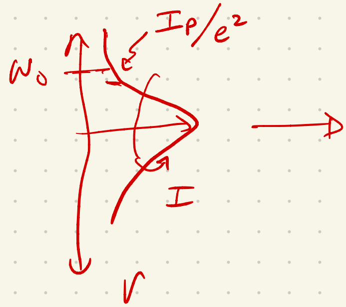
But what about something more realistic?

d) If the beam is Gaussian in space, that is:

$$I(r) = I_p e^{-2r^2/w_0^2}$$

waist

What is I_p?



$$P_p = \iint_A I(r)\,dA$$

$$= \int_0^{2\pi}\int_0^{\infty} I_p e^{-2r^2/w_0^2}\, r\,dr\,d\theta$$

$$= 2\pi \int_0^{\infty} I_p e^{-2r^2/w_0^2}\, r\,dr$$

$$u = \frac{2r^2}{w_0^2}$$

$$du = \frac{4r}{w_0^2} \cdot dr$$

$$\frac{w_0^2 \cdot du}{4} = r \cdot dr$$

$$P_p = \frac{2\pi \cdot I_p \cdot w_0^2}{4} \int_0^{\infty} e^{-u}\, du$$

$$P_p = \frac{\cancel{2}\pi \cdot I_p \cdot w_0^2}{2\cancel{4}} \underbrace{\int_0^\infty e^{-u}\,du}$$

$$\Big|_0^\infty -e^{-u} = \Big|_\infty^0 e^{-u} = 1$$

$$P_p = \frac{\pi I_p \cdot w_0^2}{2}$$

$$\boxed{I_p = \frac{2 \cdot P_p}{\pi w_0^2}}$$

e) What is the peak field in vacuum?

$E_p \Rightarrow$ max of $|\hat{A}(t)|$

$$I_p = \frac{2 \cdot P_p}{\pi w_0^2}$$

$$\frac{E_p^2}{2\eta_0} = I_p$$

$$E_p^2 = 2\eta_0 \cdot I_p$$

$$\boxed{E_p = \sqrt{2\eta_0 I_p}}$$

f) Let's say these pulses are coming from a laser system with:

- $P_{AVG} = 1\,mW$
- $f_{rep} = 1\,MHz$
- Gaussian profile, $w_0 = 5\,\mu m$
- $T = 10\,fs$

What is the peak field, E_p?

$$P_{Avg} = W \cdot f_{rep}$$

$$t_{rep} = \frac{1}{f_{rep}}$$

$$W = P_{Avg} / f_{rep}$$

$$= \frac{1\times10^{-3}\left[\overset{J}{W}\right]}{1\times10^{6}\left[\frac{1}{\cancel{s}}\right]}$$

$$= 1\times10^{-9}\ J = 1\ nJ$$

$$P_p = \frac{W}{T} = \frac{1\times10^{-9}\ J}{\cancel{10}\times10^{\cancel{-15}\,-14}\ s} = 10^{5}\ W$$

$$= 100\ kW$$

$$E_p = \frac{2}{w_0}\sqrt{\frac{\eta_0 P_p}{\pi}}$$

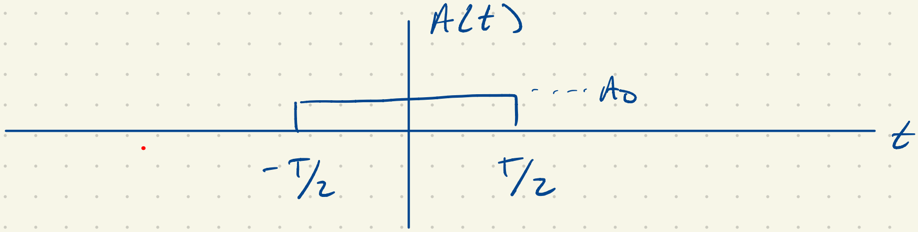
$$\simeq 1.4\times10^{9}\ V/m = \boxed{1.4\ GV/m}$$

Breakdown of air ~ 10^{6} V/m.

Note: this is for DC voltage

Practice: Time-Frequency Relations

Imagine a pulse like the following



a) What is $\hat{A}(\omega)$

b) What is the time-bandwidth product?