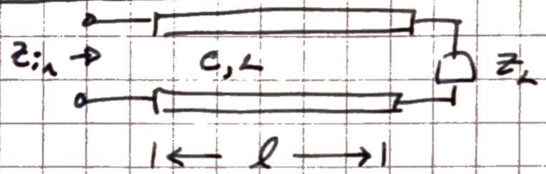
Continuing from 220328b

T-line Approach



- Cable of length l.

- Intrinsic Cap. / unit length : C
- Intrinsic Ind. / unit length : L

$$Z_{in} = \frac{Z_0 \left[Z_L + i Z_0 \tan(\beta l) \right]}{Z_0 + i Z_L \tan(\beta l)}$$

$$\beta = \omega\sqrt{LC}$$

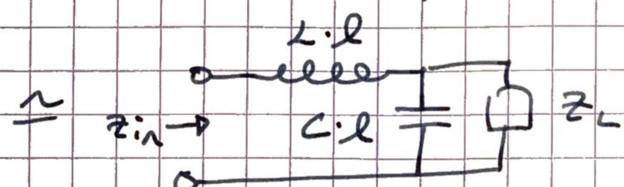
$$Z_0 = \sqrt{L/C}$$

For short cable: $\boxed{\beta \cdot l = \omega\sqrt{LC} \cdot l \ll 1}$

$$Z_{in} \simeq \frac{\cancel{\sqrt{L/C}} \left[Z_L + i \sqrt{L/C} \cdot \omega\sqrt{LC}\, l \right]}{1 \cancel{\sqrt{L/C}} + i Z_L \cdot \omega\sqrt{LC} \cdot \sqrt{\frac{C}{L}} \cdot l}$$

$$\boxed{Z_{in} \simeq \frac{Z_L + j\omega L l}{1 + j\omega Z_L \cdot C l}}$$

We can also use a lumped-element approximation:



$$Z_{in} = j\omega L l + \frac{Z_L \cdot (1/j\omega C l)}{Z_L + \frac{1}{j\omega C l}}$$

$$= j\omega L l + \frac{Z_L}{j\omega Z_L C l + 1}$$

$$= \frac{j(\omega L C l)^2 \cdot Z_L + j\omega L l + Z_L}{j\omega Z_L C l + 1}$$

Note, we assumed $\omega L C l \ll 1$, so $j(\omega L C l)^2$ very, very small and can be ignored.

$$\boxed{Z_{in} \simeq \frac{j\omega L l + Z_L}{1 + j\omega Z_L C l}}$$

$\Rightarrow$ Same result!

(But T-line approach more general).