3. Overlapping Generations Model (OLG)

Advanced Macroeconomics

Daniel Kelly
El Colegio de México

Winter 2024

Outline

Reading: Romer (2012), *Advanced Macroeconomics*, Chapter 2: Part B.

Introduction

Introduction

The Diamond Overlaping Generations Model

Household Preferences

Households that belong to cohort t (born on t) have an additively separable utility function between consumption in both periods given by:

$$U(c_t, c_{t+1}) = u(c_t) + \beta u(c_{t+1}) \tag{1}$$

The preferences for consumption on each period, summarized by the utility function $u(\cdot)$ (that is twice differentiable) and:

- **Risk Aversity**, so that $u(\cdot)$ is concave and, in particular $u''(x) < 0$, $\forall x$.
- **Strictly Monotonic**, so that $u(\cdot)$ is strictly increasing, and in particular $u'(x) > 0$, $\forall x$.

Household Preferences

Households that belong to cohort t (born on t) have an additively separable utility function between consumption in both periods given by:

$$U(c_t, c_{t+1}) = u(c_t) + \beta u(c_{t+1}) \tag{1}$$

The preferences for consumption on each period, summarized by the utility function $u(\cdot)$ (that is twice differentiable) and:

- **Risk Aversity**, so that $u(\cdot)$ is concave and, in particular $u''(x) < 0$, $\forall x$.
- **Strictly Monotonic**, so that $u(\cdot)$ is strictly increasing, and in particular $u'(x) > 0$, $\forall x$.

The discount factor β, as usual, is defined as:

$$\beta \equiv \frac{1}{1+\rho} \tag{2}$$

Where ρ is the discount rate, that shows the differences in valuation of consumption for the individual across different periods:

- If $\rho > 0$, the individual places a greater weight on first period consumption $\Rightarrow \beta < 1$.
- If $\rho < 0$, the individual places a greater weight on second period consumption $\Rightarrow \beta > 1$.

Furthermore, we assume that $\rho > -1$, so that the weight put on second period consumption is positive.

The discount factor β, as usual, is defined as:

$$\beta \equiv \frac{1}{1+\rho} \tag{2}$$

Where ρ is the discount rate, that shows the differences in valuation of consumption for the individual across different periods:

- If $\rho > 0$, the individual places a greater weight on first period consumption $\Rightarrow \beta < 1$.
- If $\rho < 0$, the individual places a greater weight on second period consumption $\Rightarrow \beta > 1$.

Furthermore, we assume that $\rho > -1$, so that the weight put on second period consumption is positive.

The discount factor β, as usual, is defined as:

$$\beta \equiv \frac{1}{1+\rho} \tag{2}$$

Where ρ is the discount rate, that shows the differences in valuation of consumption for the individual across different periods:

- If $\rho > 0$, the individual places a greater weight on first period consumption $\Rightarrow \beta < 1$.
- If $\rho < 0$, the individual places a greater weight on second period consumption $\Rightarrow \beta > 1$.

Furthermore, we assume that $\rho > -1$, so that the weight put on second period consumption is positive.

A balanced growth path (Backward Induction) requires the assumption that the individual period utility function $u(\cdot)$ takes the CRRA (*Constant Relative Risk Aversion*) form:

$$U(c_t, c_{t+1}) = \frac{c_t^{1-\theta}}{1-\theta} + \beta \cdot \frac{c_{t+1}^{1-\theta}}{1-\theta} \tag{3}$$

In regards to population dynamics, the number of young people (this is, those who are born) at period t is denoted L_t, and we assume that population grows at a constant rate n:

$$L_t = (1+n)L_{t-1} \tag{4}$$

A balanced growth path (Backward Induction) requires the assumption that the individual period utility function $u(\cdot)$ takes the CRRA (*Constant Relative Risk Aversion*) form:

$$U(c_t, c_{t+1}) = \frac{c_t^{1-\theta}}{1-\theta} + \beta \cdot \frac{c_{t+1}^{1-\theta}}{1-\theta} \tag{3}$$

In regards to population dynamics, the number of young people (this is, those who are born) at period t is denoted L_t, and we assume that population grows at a constant rate n:

$$L_t = (1+n)L_{t-1} \tag{4}$$

Capital and Production

Output is homogeneous, and can be used either for consumption or investment. Capital is owned by households and is rented out to firms.

The whole economy is characterized by three perfectly competitive markets:

1. Market for Output (in which there are zero profits).
2. Labor Market (in which labor earns exactly its marginal product).
3. Capital Market (in which the rental price of capital is equalized to its marginal product).

We assume that there is full capital depreciation to make the calculations easier (so that $\delta = 1$).

Capital and Production

Output is homogeneous, and can be used either for consumption or investment. Capital is owned by households and is rented out to firms.

The whole economy is characterized by three perfectly competitive markets:

1. Market for Output (in which there are zero profits).
2. Labor Market (in which labor earns exactly its marginal product).
3. Capital Market (in which the rental price of capital is equalized to its marginal product).

We assume that there is full capital depreciation to make the calculations easier (so that $\delta = 1$).

Production is characterized by an aggregate neoclassical production function with labor augmenting technological progress:

$$Y_t = F(K_t, A_t L_t) \tag{5}$$

Technological progress, grows at a constant rate g and its dynamic is described by the difference equation:

$$A_t = (1+g)A_{t-1} \tag{6}$$

Production is characterized by an aggregate neoclassical production function with labor augmenting technological progress:

$$Y_t = F(K_t, A_t L_t) \tag{5}$$

Technological progress, grows at a constant rate g and its dynamic is described by the difference equation:

$$A_t = (1+g)A_{t-1} \tag{6}$$

Each household only works when they are young, and they provide, inelastically, 1 unit of labor. The wage in the labor market is equal to the marginal product of effective labor:

$$W_t = \frac{\partial F(K_t, A_t L_t)}{\partial A_t L_t} = \frac{\partial F(K_t, A_t L_t)}{\partial L_t} \cdot A_t = w_t A_t \tag{7}$$

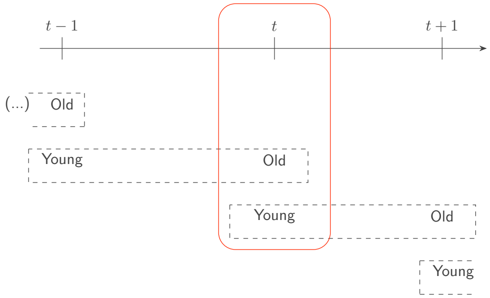
Summary of the Setup

- In period t, capital supplied by the **old** and labor from the **young** are combined to produce output.
- The **old** consume the gains from capital and their existing wealth, then die and exit the model.
- The **young** divide their capital between consumption and savings for the next period (when they become *old*).
- Capital stock in period $t+1$ is equal to the savings by the young on period t times the amount of young households:

$$K_{t+1} = L_t(A_t w_t - c_t) \tag{8}$$

- Capital is then combined with the labor supply of the following generation to produce output in period $t+1$.

Figure 3.1: Structure of the Diamond Overlapping Generations Model



Young

- Income: $A_t w_t$
- Consumption: c_t
- Savings: $A_t w_t - c_t$

Old

- Consumption:

$$R_{t+1} \cdot K_{t+1} = R_t \cdot L_t(A_t - c_t)$$

Problem of Firms

Model with Log Utility and Cobb-Douglas Tech.

Policy Experiments

Capital Accumulation and Dynamic Inefficiency

Macroeconomics of Pensions

sadas

EL COLEGIO
DE MÉXICO