Problem Set 2

Econometrics II

Due: Tuesday, February 19, 2025 (10:00 a.m.)

Instructions

- PS2 will be submitted by mail to djsanchez@colmex.mx with copy to egameren@colmex.mx and fernando.garcia@colmex.mx.
- Your submission should include an R-script and a LaTeX compiled PDF document.
  - The R-script should be divided into subsections, each corresponding to a problem in the PS.
  - The PDF document should include answers to both theoretical and computer exercises. The document should be compiled in LaTeX, no other forms of submission will be allowed (I recommend using Overleaf).
- Computer excercises require the use of companion datasets to your textbook. You can access these datasets by installing the R-package **wooldridge** following the procedure seen in Lab Session 1.
- Any required visualization of data should be directly included in the PDF document. When the problem asks to include the output of a regression, you should include a table with those results (check R-package **stargazer**).
- Problems are based on **Chapters 11** and **12** of *Introductory Econometrics: A Modern Approach, 4th Edition* by Jeffrey Wooldridge.

1 Theoretical Excercises

1.1 W11.1

Let $\{x_t : t = 1, 2, ...\}$ be a covariance stationary process and define $\gamma_h = \text{Cov}(x_t, x_{t+h})$ for $h \geq 0$. Show that $\text{Corr}(x_t, x_{t+h}) = \gamma_h/\gamma_0$.

1.2 W11.2

Let $\{e_t : t = -1, 0, 1, \ldots\}$ be a sequence of independent, identically distributed random variables with mean zero and variance one. Define a stochastic process by:

$$x_t = e_t - \frac{1}{2}e_{t-1} + \frac{1}{2}e_{t-2}, t = 1, 2, ...$$

1. Find $\text{E}(x_t)$ and $\text{Var}(x_t)$. Do either depend on t?
2. Show that $\text{Corr}(x_t, x_{t-1}) = 1/2$ and $\text{Corr}(x_t, x_{t-2}) = 1/3$.
3. What is $\text{Corr}(x_t, x_{t-h})$ for $h > 2$?
4. Is $\{x_t\}$ an asymptotically uncorrelated process?

1.3 W11.4

Let $\{y_t : t = 1, 2, ...\}$ be a random walk with $y_0 = 0$. Show that $\text{Corr}(y_t, y_{t+h}) = \sqrt{t/(t+h)}$ for $t \geq 1$, $h > 0$.

1.4 K1

Consider the following time series regression model and errors:

$$y_t = \beta_0 + \beta_1 x_t + u_t, \quad t = 1, \ldots, T \tag{1}$$

$$u_t = \rho u_{t-1} + \varepsilon_t, \quad \text{with } |\rho| < 1, \quad \varepsilon_t \sim \text{i.i.d.}(0, \sigma^2) \tag{2}$$

1. Show that the variance of u_t is given by:

$$\text{Var}(u_t) = \frac{\sigma^2}{1 - \rho^2} \tag{3}$$

2. Derive the autocovariance function $\gamma_k = \text{Cov}(u_t, u_{t-k})$ for any lag k.

2 K2

The Durbin-Watson statistic is defined as:

$$DW = \frac{\sum_{t=2}^{T} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{T} \hat{u}_t^2} \tag{4}$$

Show that, under the assumption of AR(1) errors: $E[DW] \approx 2(1-\rho)$.

3 Computer Excercises

For the following exercises, use the **ps2.RData** dataset (attached to the email). None of the tests should be done using an pre-written package. All statistics should be computed using **only** the model object.

3.1 K1

Consider the following model:

$$y_t = \alpha_0 + \beta_1 x_{t,1} + \beta_2 x_{t,2} + \beta_3 x_{t,3} + e_t \tag{5}$$

1. Run a test for AR(1) serial autocorrelation assuming strict exogeneity on the regressors. (Page 413 of Wooldridge 4th Ed.; use *df1*).
2. Relax the assumption of strict exogeneity of regressors and perform the same test. (Page 416 of Wooldridge 4th Ed.; use *df1*).
3. Run a test for AR(3) serial autocorrelation on the residuals. (Page 418 of Wooldridge 4th Ed.; use *df2*).

3.2 K2

Consider the model presented on Equation (5). Using *df1*, estimate (5) correcting for serial autocorrelation assuming regressors are exogenous. Perform the Feasible GLS estimation manually. (Page 421 of Wooldridge 4th Ed.).

3.3 W11.1

Use the data in `HSEINV.RAW` for this exercise.

(i) Find the first order autocorrelation in $\log(invpc)$. Now, find the autocorrelation **after** linearly detrending $\log(invpc)$. Do the same for $\log(price)$. Which of the two series may have a unit root?

(ii) Based on your findings in part (i), estimate the equation

$$\log(invpc_t) = \beta_0 + \beta_1 \Delta \log(price_t) + \beta_2 t + u_t \tag{6}$$

and report the results in standard form. Interpret the coefficient $\hat{\beta}_1$ and determine whether it is statistically significant.

(iii) Linearly detrend $\log(invpc_t)$ and use the detrended version as the dependent variable in the regression from part (ii) (see Section 10.5). What happens to R^2?

(iv) Now use $\Delta \log(invpc_t)$ as the dependent variable. How do your results change from part (ii)? Is the time trend still significant? Why or why not?