Basic Regression Analysis with Time Series Data

Chapter 10

Wooldridge: Introductory Econometrics:
A Modern Approach, 5e

with some adjustments by me

Analyzing Time Series: Basic Regression Analysis

- Time series data have certain aspects that cross-section data do not have, and that require special attention when applying OLS

- For example, stock prices, money supply, consumer price index, gross domestic product, annual homicide rates, automobile sales, …
- Typical applications: applied macroeconomics and finance

Analyzing Time Series: Basic Regression Analysis

- *10.1* **The nature of time series data**
  - Temporal ordering of observations; may not be arbitrarily reordered
  - Typical features: serial correlation and non-independence of observations

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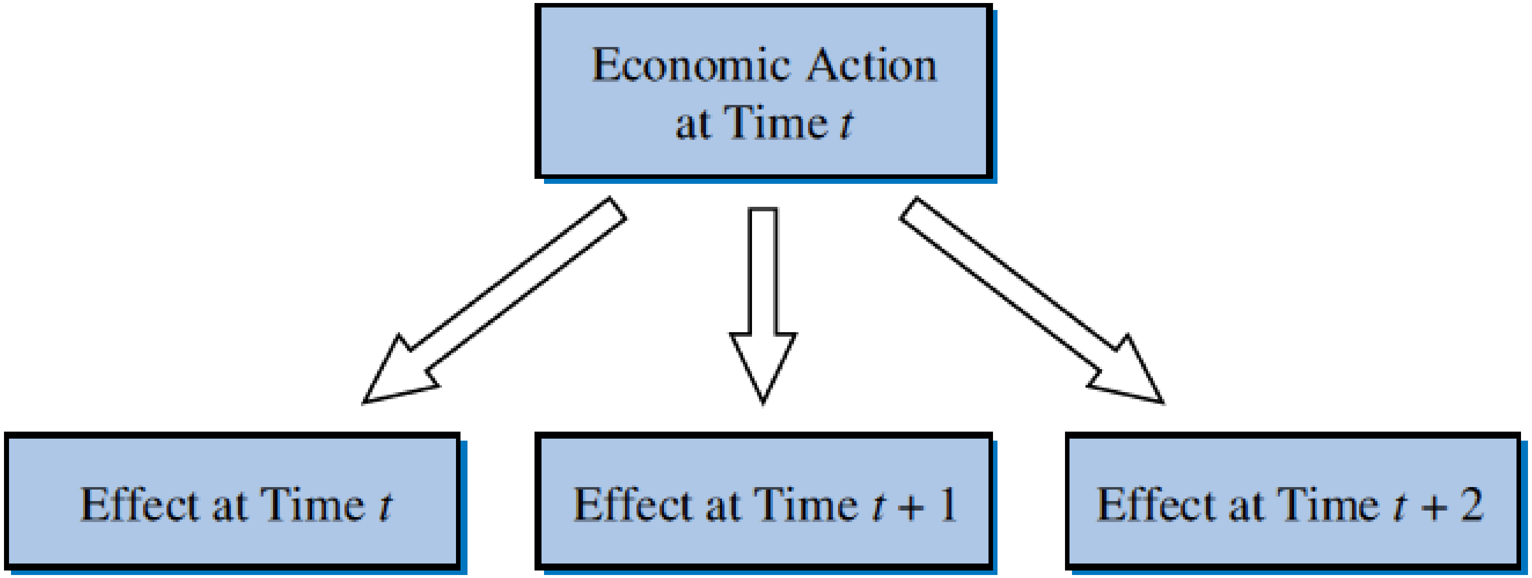
- How should we think about the randomness in time series data?
  - The outcome of economic variables (e.g. GNP, Dow Jones) is uncertain; they should therefore be modeled as random variables
  - Time series are sequences of r.v. (= stochastic processes)
  - Randomness does not come from sampling from a population

→ „Sample" = the one realized path of the time series out of the many possible paths the stochastic process could have taken

- A dynamic relationship between variables is one in which the change in a variable now has an impact on that same variable, or other variables, in one or more future time periods



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- **Example: US inflation and unemployment rates 1948-2003**

TABLE 10.1 Partial Listing of Data on U.S. Inflation and Unemployment Rates, 1948–2003

Year	Inflation	Unemployment
1948	8.1	3.8
1949	−1.2	5.9
1950	1.3	5.3
1951	7.9	3.3
.	.	.
.	.	.
.	.	.
1998	1.6	4.5
1999	2.2	4.2
2000	3.4	4.0
2001	2.8	4.7
2002	1.6	5.8
2003	2.3	6.0

Here, there are only two time series. There may be many more variables whose paths over time are observed simultaneously.

Time series analysis focuses ...
→ ... on modeling the dependency of a variable on its <u>own past</u>, and
→ ... on the <u>present and past values of other variables</u>.

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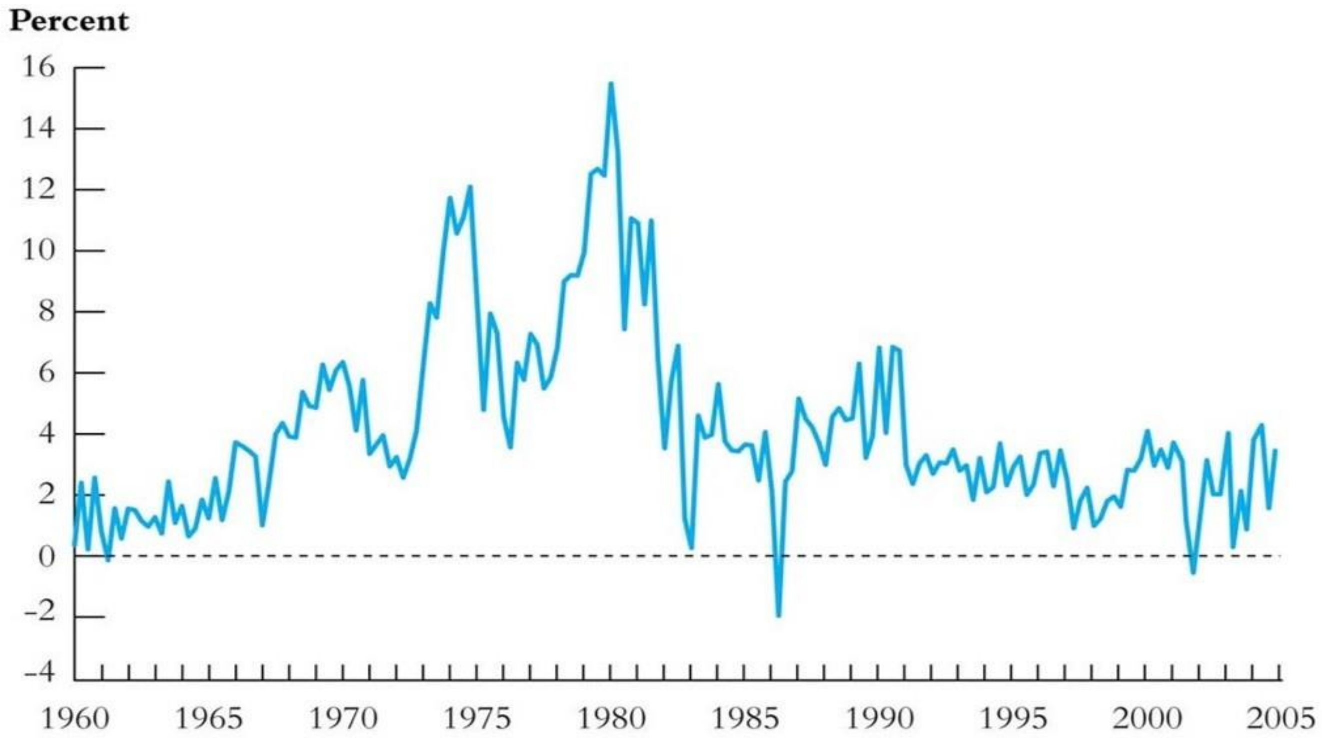
- **Example: US Quarterly CPI Inflation Rate: 1960:I-2004:IV**



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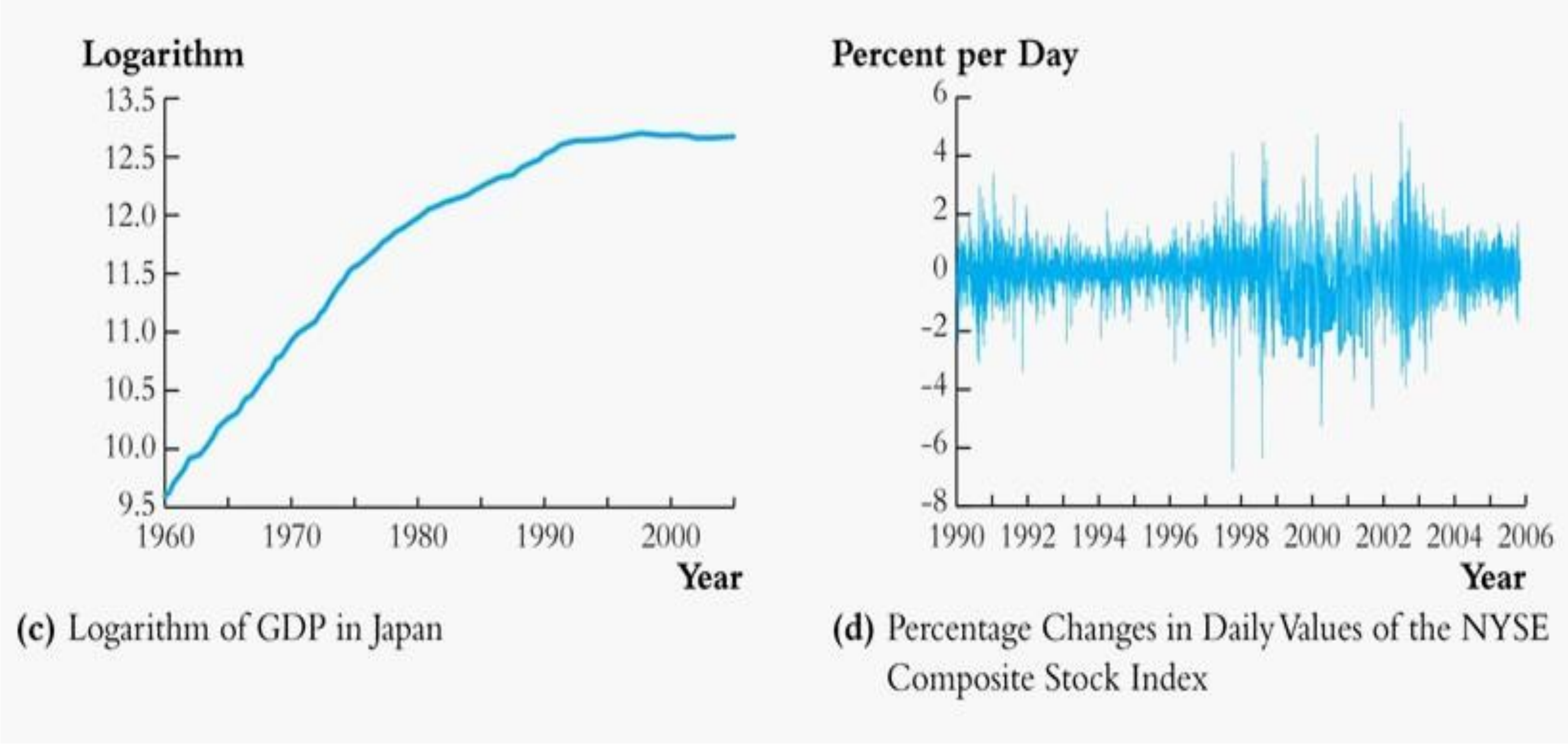
- **More examples: annual log GDP, daily stock prices, …**



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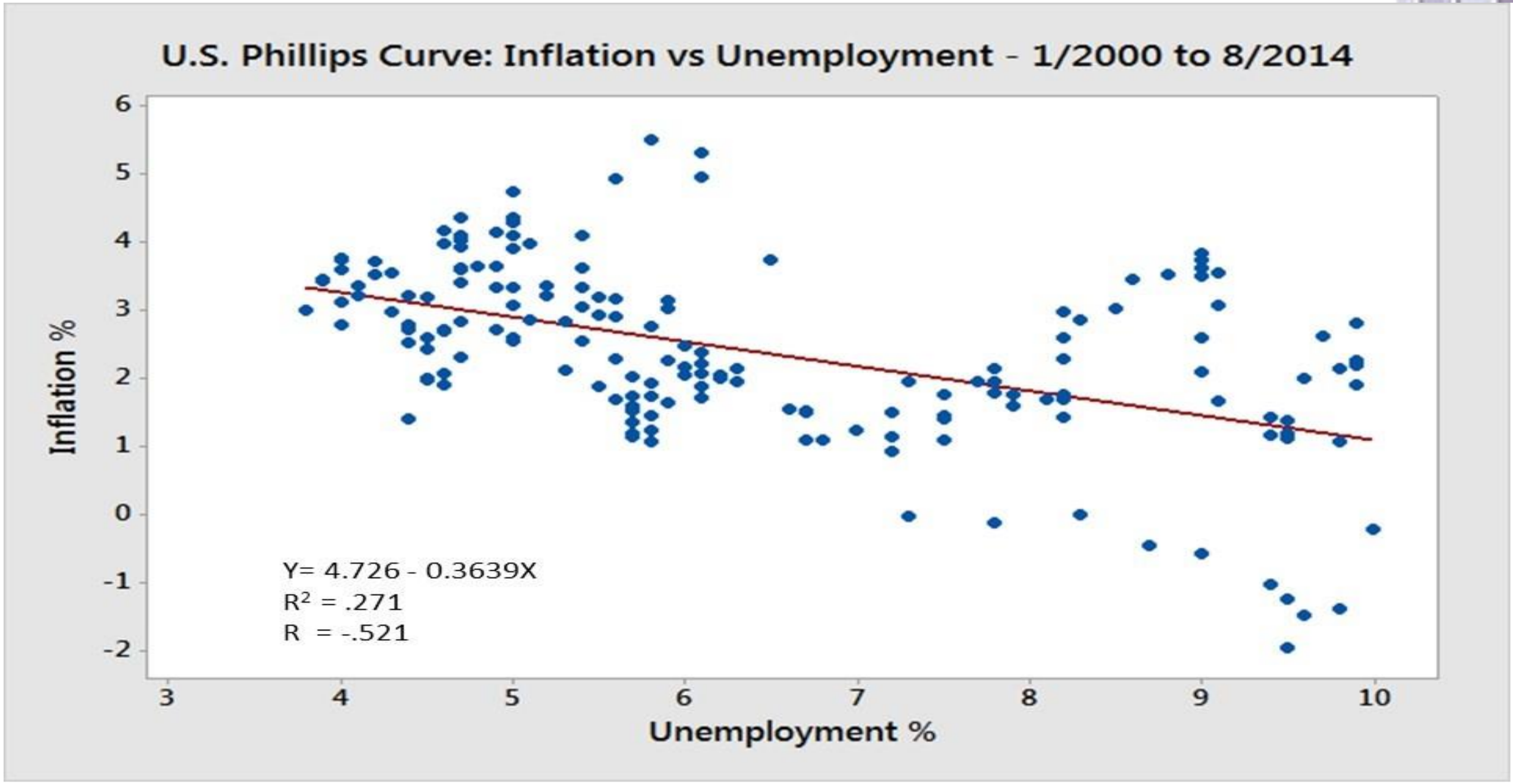

Source Data: FRED Database
Inflation: CPI for All Urban Consumers

- Ways to model the dynamic relationship:

1. Specify that a dependent variable *y* is a function of current and past values of an explanatory variable *x*

$$y_t = f(x_t, x_{t-1}, x_{t-2}, \ldots)$$

- Because of the existence of these lagged effects, Eq. 9.1 is called a distributed lag model

- Ways to model the dynamic relationship (Continued):

2. Capturing the dynamic characteristics of time-series by specifying a model with a lagged dependent variable as one of the explanatory variables

$$y_t = f(y_{t-1}, x_t)$$

- Or have:

$$y_t = f(y_{t-1}, x_t, x_{t-1}, x_{t-2})$$

- Such models are called **autoregressive distributed lag** (**ARDL**) models, with "autoregressive" meaning a regression of y_t on its own lag or lags

- Ways to model the dynamic relationship (Continued):

3. Model the continuing impact of change over several periods via the error term

$$y_t = f(x_t) + e_t \qquad e_t = f_e(e_{t-1})$$

- In this case e_t is correlated with e_{t-1}
- We say the errors are **serially correlated** or **autocorrelated**

- Summarizing:
    - Lagged explanatory variables,
    - Lagged dependent variables,
    - Lagged error-terms

  or combinations of those

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- *10.2* **<u>Examples of time series regression models</u>**
- **Static models**
  - In static time series models, the current value of one variable is modeled as the result of the current values of explanatory variables
- **Examples for static models**

$$inf_t = \beta_0 + \beta_1 unem_t + u_t$$

There is a contemporaneous relationship between unemployment and inflation (= Phillips-Curve).

$$mrdrte_t = \beta_0 + \beta_1 convrte_t + \beta_2 unem_t + \beta_3 yngmle_t + u_t$$

The <u>current</u> murder rate is determined by the <u>current</u> conviction rate, unemployment rate, and fraction of young males in the population.

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- **Finite distributed lag models (FDL)**
  - In finite distributed lag models, the *explanatory variables* are allowed to influence the dependent variable *with a time lag*.
  - Mathematically, $y_t = \alpha_0 + \delta_0 z_t + \delta_1 z_{t-1} + \cdots + \delta_q z_{t-q} + u_t$ is an FDL of order *q*, where *q* is finite.

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- **Example for a finite distributed lag model**
  - The fertility rate may depend on the tax value of a child, but for biological and behavioral reasons, the effect may have a lag

$$gfr_t = \alpha_0 + \delta_0 \boxed{pe_t} + \delta_1 \boxed{pe_{t-1}} + \delta_2 \boxed{pe_{t-2}} + u_t$$

Children born per 1,000 women in year t

Tax exemption in year t

Tax exemption in year t–1

Tax exemption in year t–2

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- **Effect of a Transitory Shock in a FDL**
  - If there is a one time shock in a past period, the dependent variable will change temporarily by the amount indicated by the coefficient of the corresponding lag.
  - Suppose, that at time t, z increases by one unit from c to c+1 and then reverts to its previous level at time t+1:

$$\cdots, z_{t-2} = c,\ z_{t-1} = c,\ z_t = c + 1,\ z_{t+1} = c,\ z_{t+2} = c,\ \cdots$$

  - Suppose an FDL of order 2,

$$y_t = \alpha_0 + \delta_0 z_t + \delta_1 z_{t-1} + \delta_2 z_{t-2} + u_t .$$

Analyzing Time Series: Basic Regression Analysis

- **Effect of a Transitory Shock in a FDL**

  - Then, by setting the errors to be zero, this implies :

$$
\begin{aligned}
y_{t-1} &= \alpha_0 + \delta_0 c + \delta_1 c + \delta_2 c, \\
y_t &= \alpha_0 + \delta_0 (c+1) + \delta_1 c + \delta_2 c,, \\
y_{t+1} &= \alpha_0 + \delta_0 c + \delta_1 (c+1) + \delta_2 c, \\
y_{t+2} &= \alpha_0 + \delta_0 c + \delta_1 c + \delta_2 (c+1), \\
y_{t+3} &= \alpha_0 + \delta_0 c + \delta_1 c + \delta_2 c.
\end{aligned}
$$

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- **Effect of a Transitory Shock in a FDL**

Thus, $$y_t - y_{t-1} = \delta_0,$$

i.e., δ_0 is the immediate change in y due to the one-unit increase in z at time t, and is usually called impact propensity or impact multiplier.

Similarly,

$$\delta_1 = y_{t+1} - y_{t-1},$$

is the change in y one period after the temporary change, and

$$\delta_2 = y_{t+2} - y_{t-1},$$

is the change in y two periods after the temporary change.

Note that at time $t+3$, y has reverted back to its initial level: $y_{t+3} = y_{t-1}$ because only two lags of z appears in the FDL model.

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- **Effect of a Transitory Shock in a FDL**

  In summary

  - Lag Distribution: δ_j as a function of j, which summarizes the dynamic effect that a temporary increase in z has on y.

$$\boxed{\frac{\partial y_t}{\partial z_{t-s}} = \delta_s}$$

Effect of a transitory shock:
If there is a one time shock in a past period, the dep. variable will change temporarily by the amount indicated by the coefficient of the corresponding lag.

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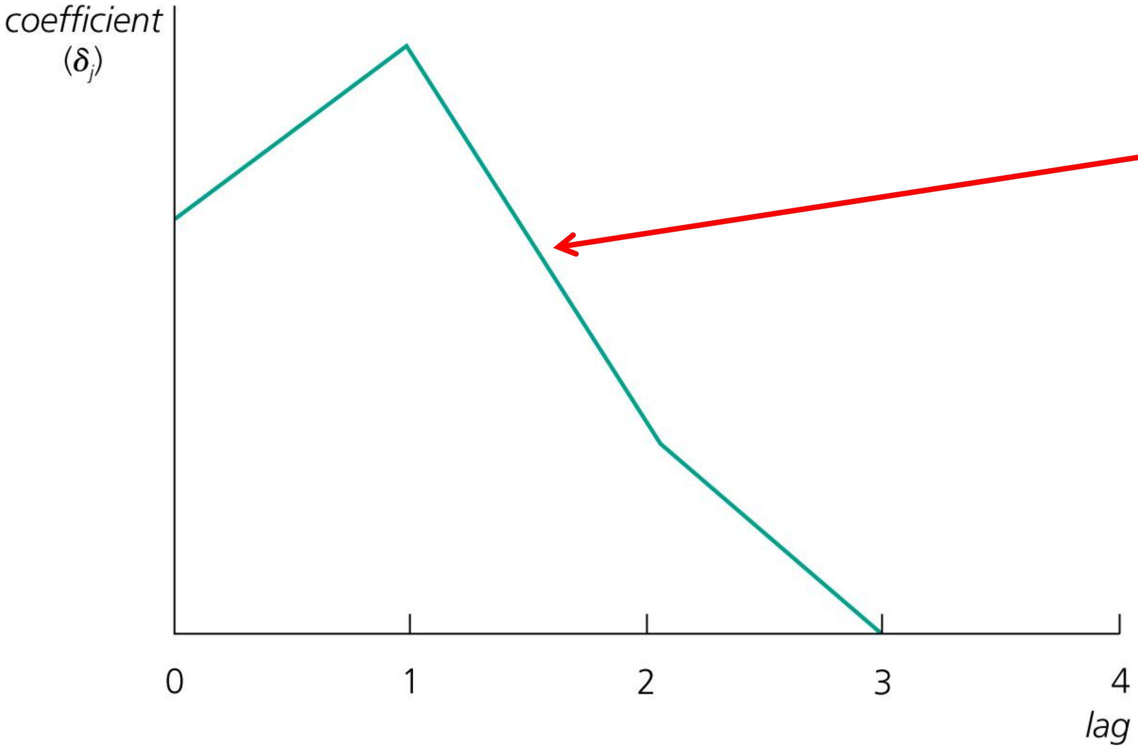
- **Effect of a Transitory Shock in a FDL**

  I *Graphical illustration of lagged effects*



For example, the effect is biggest after a lag of one period. After that, the effect vanishes (if the initial shock was transitory).

Analyzing Time Series: Basic Regression Analysis

- **Effect of Permanent Shock in a FDL**
  - If there is a permanent shock in a past period, i.e., the explanatory variable permanently increases by one unit, the effect on the dependent variable will be the cumulated effect of all relevant lags. This is a long-run effect on the dependent variable.
  - Suppose same FDL, $y_t = \alpha_0 + \delta_0 z_t + \delta_1 z_{t-1} + \delta_2 z_{t-2} + u_t$.
  - Now, at time t, z *permanently* increases by one unit from c to $c+1$:

$$z_s = c \qquad \text{for } s < t \text{, and}$$

$$z_s = c+1 \qquad \text{for } s \geq t$$

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- **Effect of Permanent Shock in a FDL**

  - Then, while again assuming errors equal to zero,

$$
\begin{aligned}
y_{t-1} &= \alpha_0 + \delta_0 c + \delta_1 c + \delta_2 c, \\
y_t &= \alpha_0 + \delta_0 (c+1) + \delta_1 c + \delta_2 c,, \\
y_{t+1} &= \alpha_0 + \delta_0 (c+1) + \delta_1 (c+1) + \delta_2 c, \\
y_{t+2} &= \alpha_0 + \delta_0 (c+1) + \delta_1 (c+1) + \delta_2 (c+1), \\
y_{t+3} &= y_{t+2}.
\end{aligned}
$$

  - Hence, a permanent change in the explanatory variable leads to a permanente change in the dependent variable

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- **Effect of Permanent Shock in a FDL**
  - With a permanent increase in *z*, after one period, *y* will increase by $\delta_0 + \delta_1$, and after two periods, *y* will increase by $\delta_0 + \delta_1 + \delta_2$ and then stay there.
  - The sum of the coefficients on current and lagged *z*, $\delta_0 + \delta_1 + \delta_2$, is the long-run change in *y* given a permanent increase in z, and is called the long-run propensity (LRP) or long-run multiplier.
  - In summary, in an FDL of order *q*,

$$\boxed{\frac{\partial y_t}{\partial z_{t-q}} + \cdots + \frac{\partial y_t}{\partial z_t} = \delta_1 + \cdots + \delta_q}$$

Effect of permanent shock:
If there is a permanent shock in a past period, i.e. the explanatory variable permanently increases by one unit, the effect on the dependent variable will be the cumulated effect of all relevant lags. This is a long-run effect on the dependent variable.

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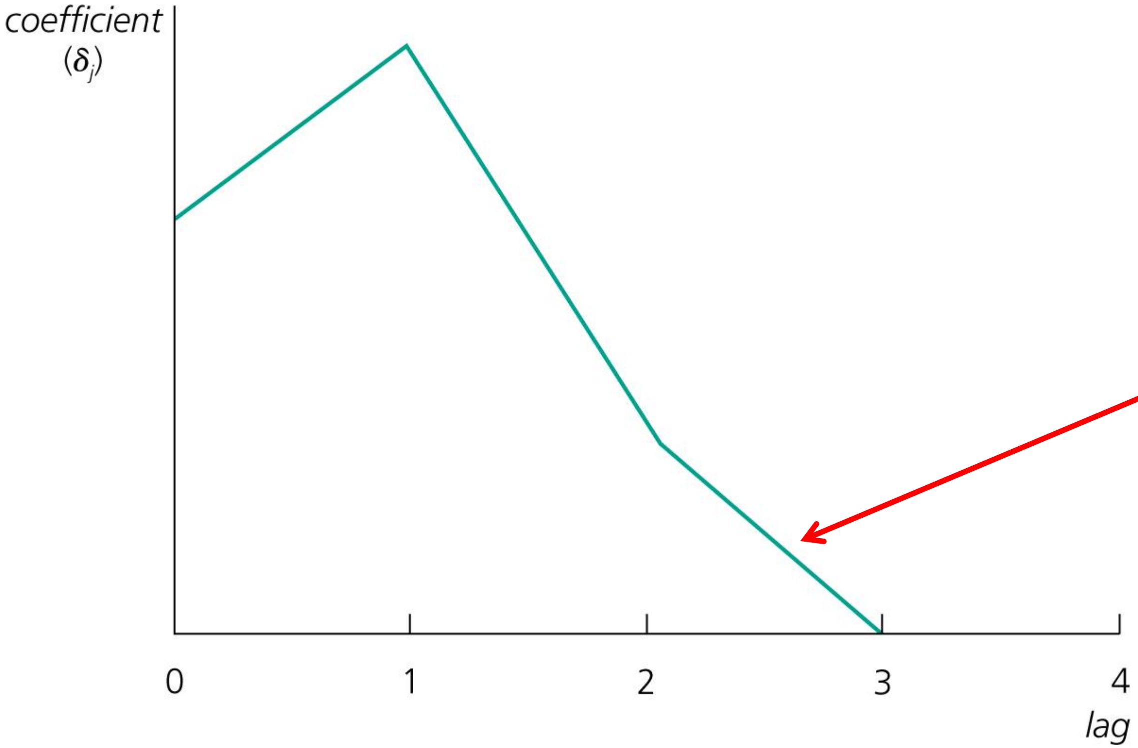
Graphical illustration of lagged effects



The long run effect of a permanent shock is the cumulated effect of all relevant lagged effects. It does not vanish (if the initial shock is a permanent one).

Analyzing Time Series: Basic Regression Analysis

- **Effect of a Transitory Shock in a FDL**
  - A one time shock to the explanatory variable in a past period, will change the dependent variable temporarily by the amount indicated by the coefficient of the corresponding lag.
  - The immediate change in y due to the one-unit increase in z at time t, is usually called impact propensity or impact multiplier.
- **Effect of Permanent Shock in a FDL**
  - For a permanent shock in a past period, *i.e.*, the explanatory variable permanently increases by one unit, the effect on the dependent variable will be the cumulated effect of all relevant lags.
  - The sum of the coefficients on current and lagged z is the long-run change in y given a permanent increase in z, and is called the long-run propensity (LRP) or long-run multiplier.