Population Projection and Stable Age Structures

Economic Demography
Econ/Demog c175
Prof. Goldstein
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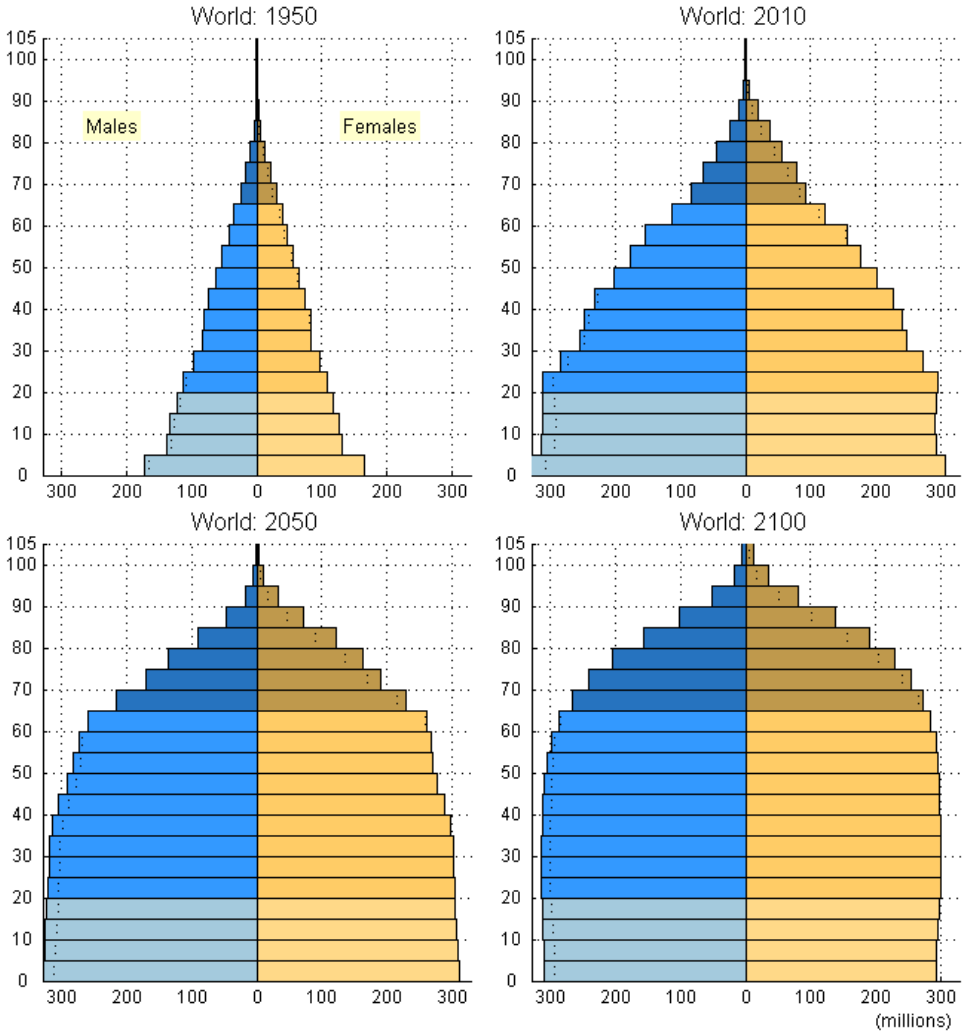
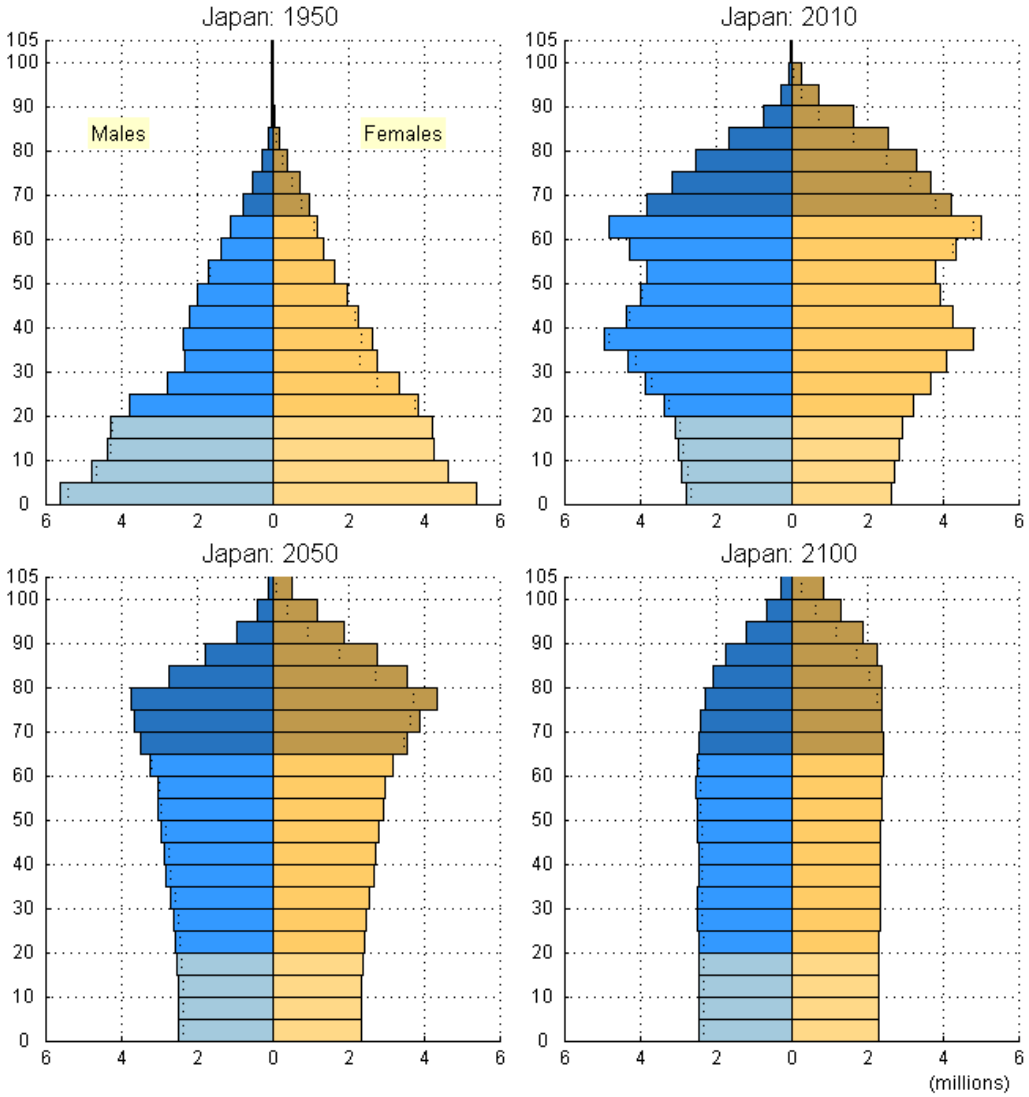
Week 5, Lecture B
UC Berkeley

Agenda

- Aging around the world
- A matrix model for age-structured populations (active in-class lab)
- Analytical derivation of stable age-structures
- Stable pyramid app(s)

UN, 2011; 2010 Revision




Japan: 1950
Japan: 2010
Japan: 2050
Japan: 2100
Males
Females
105
100
90
80
70
60
50
40
30
20
10
0
6
4
2
0
2
4
6
(millions)

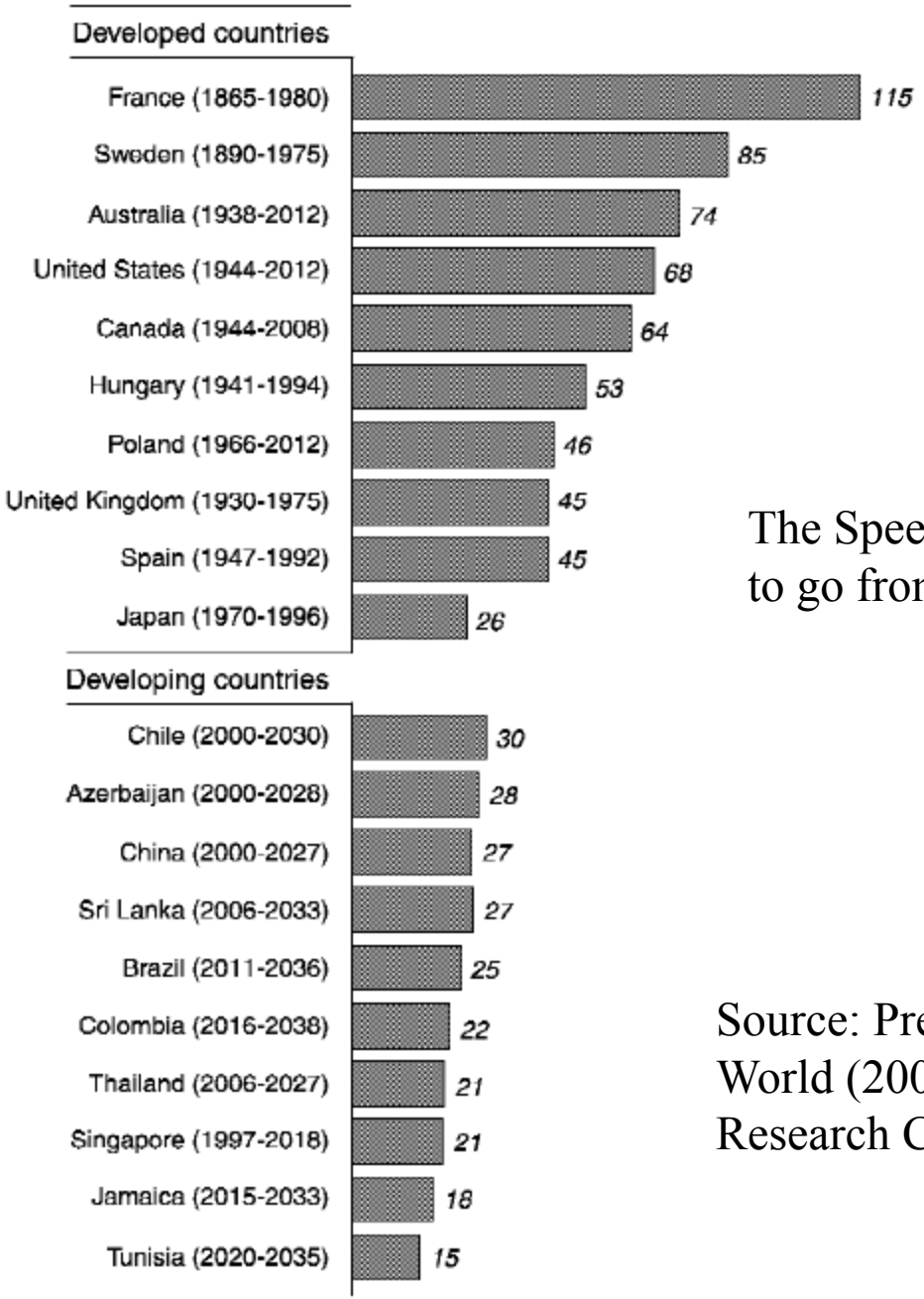


FIGURE 2-5 Speed of population aging (number of years required or expec for percent of population aged 65 and over to rise from 7% to 14%).
SOURCE: Kinsella and Gist (1995).

The Speed of Aging: Years to go from 7 to 14% 65+.

Source: Preparing for an Aging World (2001) National Research Council, NAP

Matrix model (in-class lab)

- Assumption 1: No migration (a “closed” population that changes only because of births and deaths)
- Assumption 2: No sex (everyone is female, no “two-sex” complications)
- Assumption 3: Age determines birth and death rates

Stable Age Structures

Analytical derivation

Stable populations

1. Grow exponentially at an annual rate “ *n* ”

2. Arise if age-specific fertility and mortality rates remain constant for a “long” time.

(We omit migration here)

The stable population age-structure

In counts, today's people are surviving babies born in the past

$$N(x, t) = B(t\text{-}x)\ \ l(x)$$

If births have been growing exponentially,

$$= B(t)\ e^{-nx}\ l(x)$$

Stable population age-structure (cont.)

In counts,

$$N(x, t) = B(t)\, e^{-nx}\, l(x)$$

In proportions (divide both sides by pop size)

$$n(x, t) = N(x, t) / N(t) = B(t)/N(t)\, e^{-nx}\, l(x)$$

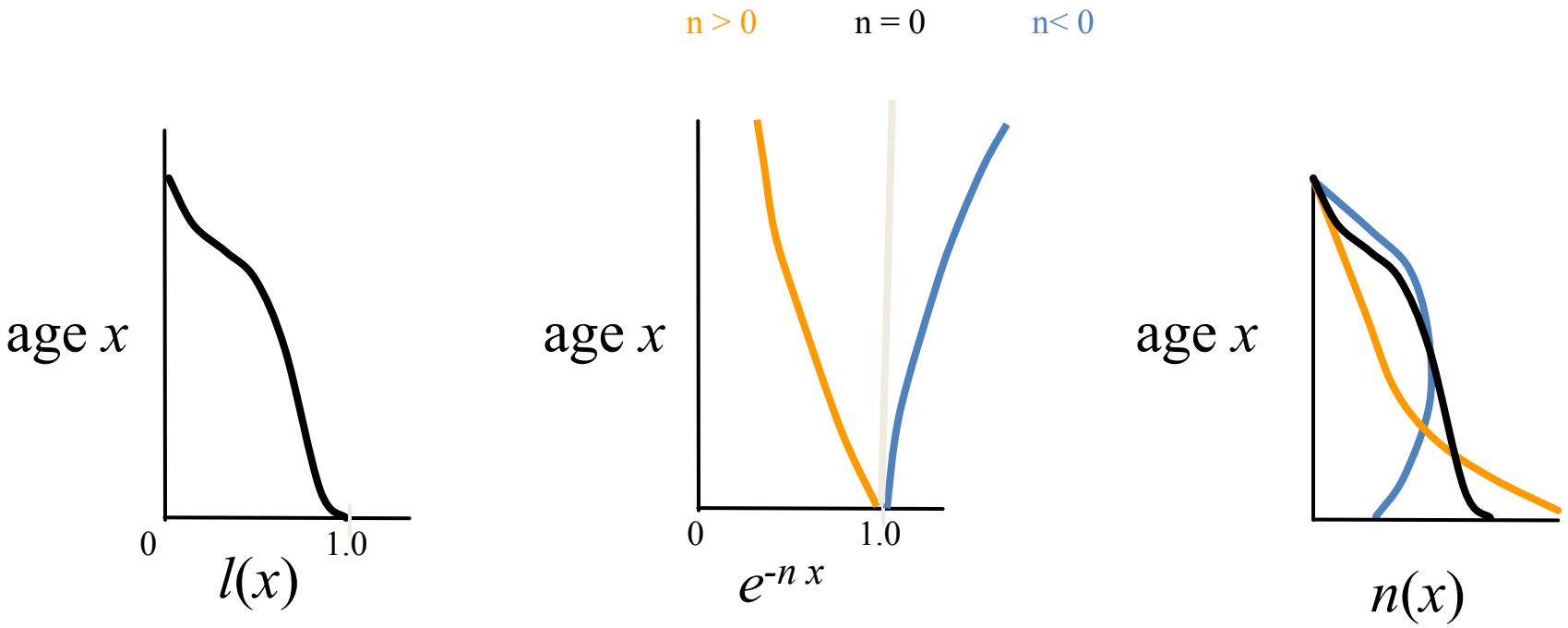
$$n(x) = b\, e^{-nx}\, l(x)$$

where b is crude birth rate

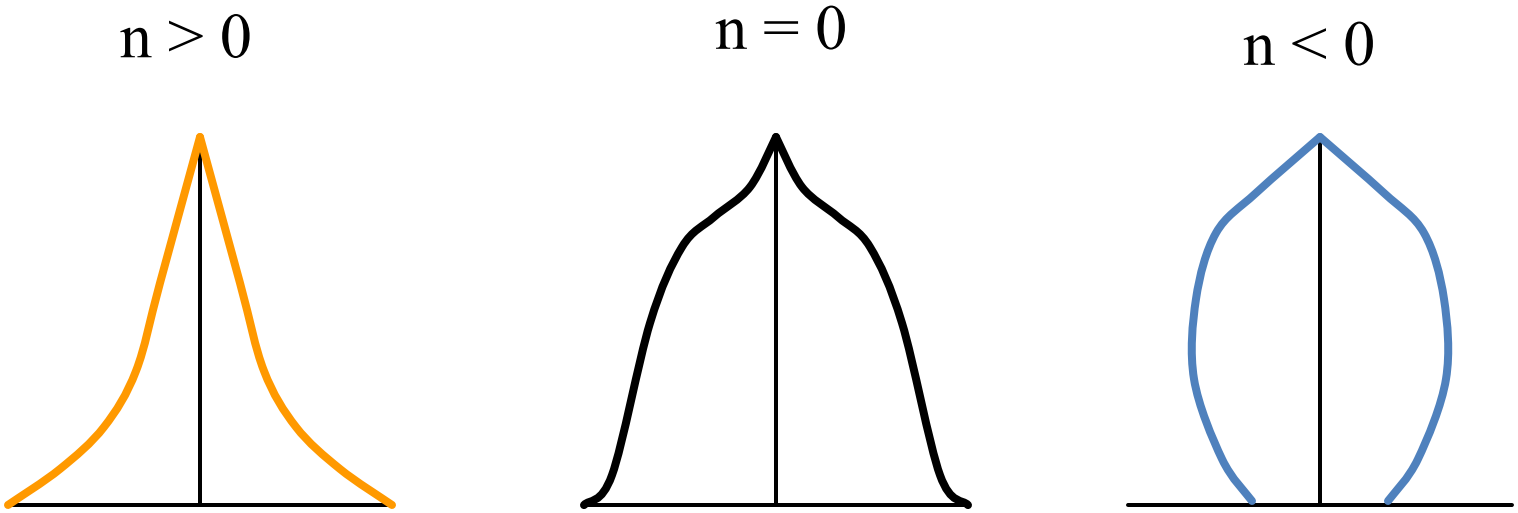
and shape of age structure is constant over time

Survival and history of births combine

$$l(x) \times b\, e^{-n\,x} = n(x)$$



Only three stable cases



What age structure is best?

Stable age structure game?