Geometric Algebra HW 3 (Wedge Product in $\mathbb{R}^3$)
MultiV 2021-22 / Dr. Kessner

1. For each of the following sets of vectors, find the following: $u \wedge v$, $u \times v$, $u \wedge v \wedge w$, and $(u \times v) \cdot w$.

   a. $u = 3e_1$, $v = 2e_2$, $w = 5e_3$

   b. $u = 3e_1 + e_2$, $v = 2e_1 + 2e_2$, $w = 5e_3$

   c. $u = 3e_1 + e_2$, $v = 2e_1 + 2e_2$, $w = 7e_1 + 11e_2 + 5e_3$

2. Use the wedge product representation of the plane

$$(r - r_0) \wedge u \wedge v = 0$$

   to solve the following problems.

   a. Find the standard equation of the plane through the points $\begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 10 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix}$.

   Also find the distance from the plane to the origin.

   b. Find the standard equation of the plane through the points $\begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$.

   Also find the distance from the plane to $\begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$.

Answers:

- 1a. $u \wedge v = 6e_1e_2$, $u \times v = 6e_3$, $u \wedge v \wedge w = 30\,e_1e_2e_3$, $(u \times v) \cdot w = 30$.
- 1b. $u \wedge v = 4e_1e_2$, $u \times v = 4e_3$, $u \wedge v \wedge w = 20\,e_1e_2e_3$, $(u \times v) \cdot w = 20$.
- 1c. $u \wedge v = 4e_1e_2$, $u \times v = 4e_3$, $u \wedge v \wedge w = 20\,e_1e_2e_3$, $(u \times v) \cdot w = 20$.
- 2a. $x + y + z = 10$, $d = \frac{10}{\sqrt{3}}$
- 2b. $x + z = 4$, $d = 2\sqrt{2}$