

**Geometric Algebra HW 4 (Geometric Product in  $\mathbb{R}^3$ )**  
**MultiV 2021-22 / Dr. Kessner**

1. Let  $w = e_1 + e_3$ . Let  $w' = (e_2 e_1)w(e_1 e_2)$

Show that  $w' = -e_1 + e_3$ .

Draw  $w$  and  $w'$ . Verify that  $w'$  is the result of reflecting  $w$  in  $e_1$ , and then  $e_2$ . Also verify that this equivalent to rotation by  $\pi$  in the  $e_1 e_2$  plane.

2. Let  $w = e_1 + e_3$ . Let  $u = \frac{w}{\sqrt{2}}$  and  $v = e_1$ . Note that  $u$  and  $v$  are unit vectors in the  $e_1 e_3$  plane, and the angle between the two vectors is  $\frac{\pi}{4}$ .

Let  $w' = (vu)w(uv)$ .

Show that  $w' = e_1 - e_3$ .

Draw  $w$  and  $w'$ . Verify that  $w'$  is the result of rotating  $w$  by  $\frac{\pi}{2}$  in the  $e_1 e_3$  plane .

3. Let  $w = e_1 + e_3$ . Find two vectors  $u$  and  $v$  to represent rotation by  $-\frac{\pi}{4}$  in the  $e_2 e_3$  plane. (Clockwise  $45^\circ$  if you're on the positive  $e_1$  axis looking at the origin). Let  $w' = (vu)w(uv)$ .

Show that  $w' = e_1 + e_2$ .