Geometric Algebra HW 2 (Geometric Product)
MultiV 2021-22 / Dr. Kessner

1. For each of the following vectors, find the inverse. Draw the unit circle on the plane, and draw each vector and its inverse.

   a. $u = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

   b. $v = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

   c. $w = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$

   d. $x = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$

Answers: $u^{-1} = \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}$, $v^{-1} = \begin{pmatrix} 0 \\ \frac{1}{3} \end{pmatrix}$, $w^{-1} = w = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$, $x^{-1} = \begin{pmatrix} \frac{\sqrt{3}}{4} \\ \frac{1}{4} \end{pmatrix}$

2. Let $u = e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

   Let $v = (\cos\frac{\pi}{6})e_1 + (\sin\frac{\pi}{6})e_2 = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$.

   Show the following:

   a. $uv = (\cos\frac{\pi}{6}) + (\sin\frac{\pi}{6})e_1e_2 = \frac{\sqrt{3}}{2} + \frac{1}{2}e_1e_2$

      (uv is a rotor representing a rotation by $\frac{\pi}{6}$.)

   b. $vu = \frac{\sqrt{3}}{2} - \frac{1}{2}e_1e_2$

      (vu is a rotor representing a rotation by $-\frac{\pi}{6}$.)

   c. $vuv = v(uv) = (vu)v = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$

      (applying uv on the right (or vu on the left) rotates v by $\frac{\pi}{6}$)

d. $vvu = v(vu) = (uv)v = e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(applying vu on the right (or uv on the left) rotates v by $-\frac{\pi}{6}$)

e. $uvuv = \frac{1}{2} + \frac{\sqrt{3}}{2} e_1 e_2$

($uvuv = (uv)^2$ is a rotor representing rotation by $\frac{\pi}{3}$)

f. $(vu)(uv) = 1$

g. $vuvuv = e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

h. $uvuvuv = e_1 e_2$

($uvuvuv = (uv)^3$ a rotor representing rotation by $\frac{\pi}{2}$ (the unit bivector))