

**Geometric Algebra Classwork (Reflection)**  
**MultiV 2021-22 / Dr. Kessner**

Let  $u = e_1$  and  $v = \frac{1}{\sqrt{2}}(e_1 + e_2)$ .

Define the transformations  $R_u(w) = u w u$ ,  $R_v(w) = v w v$ , and  $R_{uv}(w) = (v u) w (u v)$ .

1. Show that  $R_u(e_1 + e_3) = e_1 - e_3$  and  $R(u) = u$ .
2. For a general  $w = w_x e_1 + w_y e_2 + w_z e_3$ , show that  $R_u(w) = w_x e_1 - w_y e_2 - w_z e_3$ . In other words, the  $y$  and  $z$  coordinates are negated. What is the transformation  $R_u$ ?
3. Show that  $R_v(e_1) = e_2$ ,  $R(v) = v$  and  $R_v(e_1 + e_3) = e_2 - e_3$ . What is the transformation  $R_v$ ?
4. Show that  $R_{uv}(e_1) = e_2$ ,  $R_{uv}(e_3) = e_3$ , and  $R_{uv}(e_1 + e_3) = e_2 + e_3$ . What is the transformation  $R_{uv}$ . Note that  $R_{uv} = R_v R_u$ .
5. Define the transformation  $M_x(w) = -R_u(w)$ . Calculate  $M_x(w)$  for a general  $w = w_x e_1 + w_y e_2 + w_z e_3$ . What is this transformation? Describe how the transformation changes the coordinates.
6. Define transformations  $M_y$  and  $M_z$  and show that they act as expected on a general  $w = w_x e_1 + w_y e_2 + w_z e_3$ .