(189) $\bar{u} = \langle 3, -1, 2 \rangle$

$\bar{v} = \langle -2, 0, 1 \rangle$

$$\bar{u} \times \bar{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 2 \\ -2 & 0 & 1 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} - \vec{j} \underbrace{\begin{vmatrix} 3 & 2 \\ -2 & 1 \end{vmatrix}}_{\substack{3-(-4) \\ =7}} + \vec{k} \begin{vmatrix} 3 & -1 \\ -2 & 0 \end{vmatrix}$$

$$= -\vec{i} - 7\vec{j} - 2\vec{k}$$

$$|\bar{u} \times \bar{v}|^2 = 1^2 + 7^2 + 2^2$$

$$= 54$$

$$|\bar{u} \times \bar{v}| = \sqrt{54} = 3\sqrt{6}$$

$$\bar{w} = \frac{\bar{u} \times \bar{v}}{|\bar{u} \times \bar{v}|} = \frac{1}{3\sqrt{6}} \langle -1, -7, -2 \rangle$$

(191) $\vec{u} = \vec{AB} = \langle 0, -1, 2 \rangle$

$\bar{v} = \langle -1, 0, +4 \rangle$

$$\bar{u} \times \bar{v} = \left| \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -1 & 2 \\ -1 & 0 & 4 \end{matrix} \right|$$

$$= -4\vec{i} - 2\vec{j} - 1\vec{k}$$

$$|\bar{u} \times \bar{v}|^2 = 4^2 + 2^2 + 1^2$$
$$= 21$$
$$|\bar{u} \times \bar{v}| = \sqrt{21}$$

$$\bar{w} = \frac{\bar{u} \times \bar{v}}{|\bar{u} \times \bar{v}|} = \frac{1}{\sqrt{21}} \langle -4, -2, -1 \rangle$$

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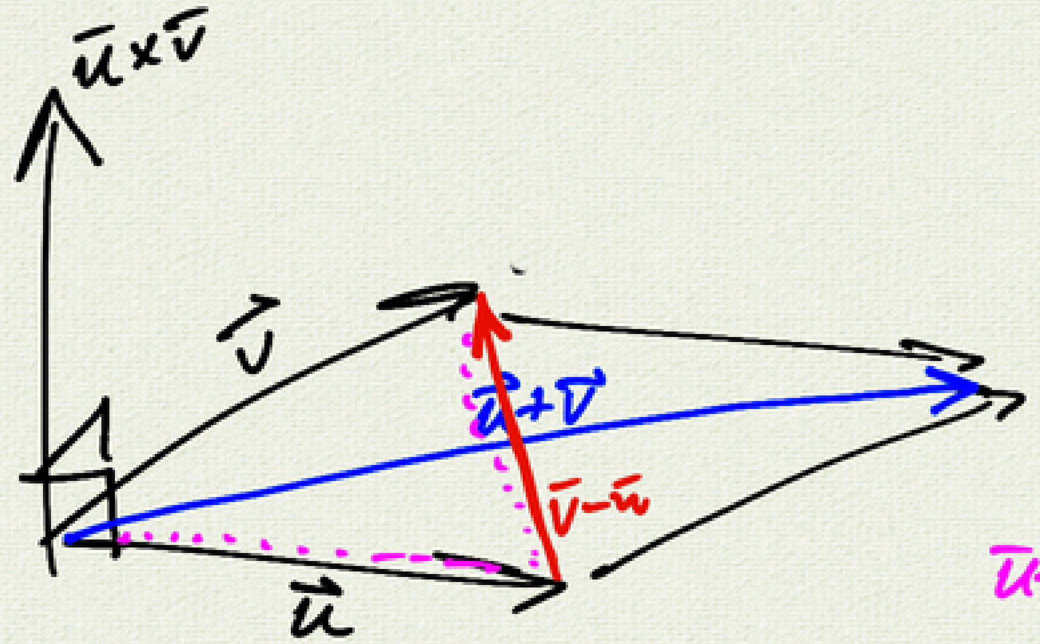
(195) $\bar{u} \times \bar{v} \perp \begin{matrix} \bar{u} + \bar{v} \\ \bar{u} - \bar{v} \end{matrix}$

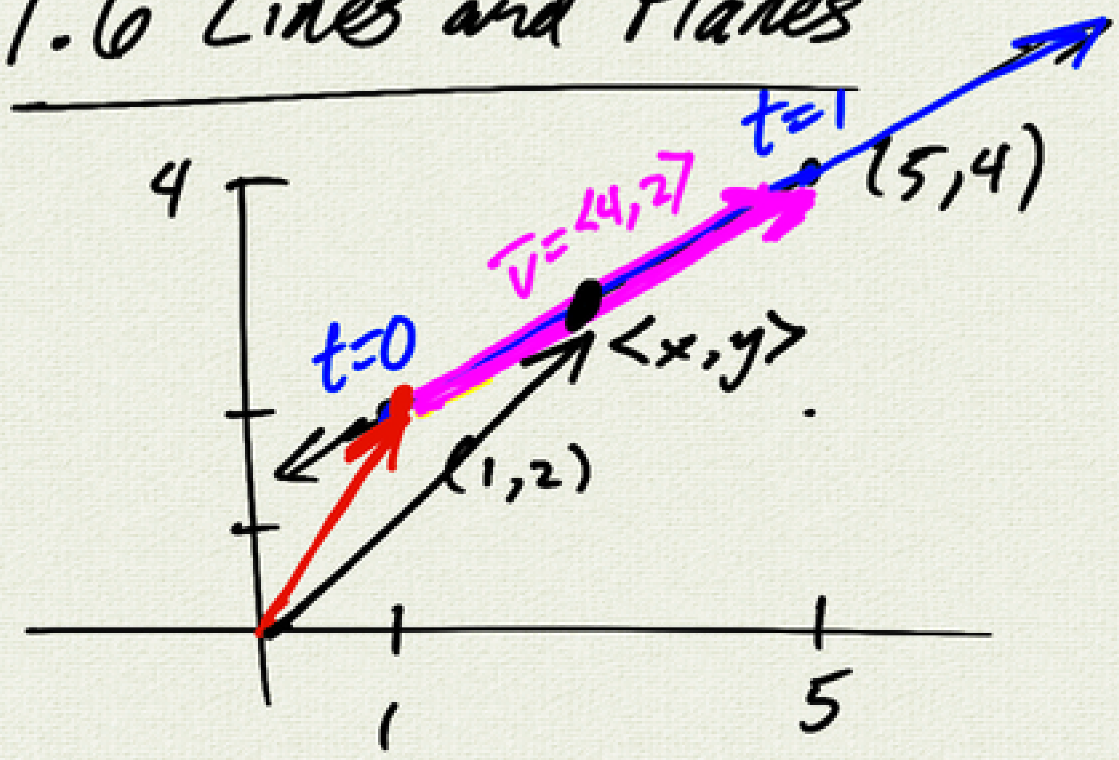


$\bar{u} + (\bar{v} - \bar{u}) = \bar{v}$

show $(\bar{u} \times \bar{v}) \cdot (\bar{u} + \bar{v}) = 0$

$$= \underbrace{(\bar{u} \times \bar{v}) \cdot \vec{u}}_{0} + \underbrace{(\bar{u} \times \bar{v}) \cdot \bar{v}}_{0}$$

1.6 Lines and Planes



parametri
find equations thru
$(1,2)$ $(5,4)$

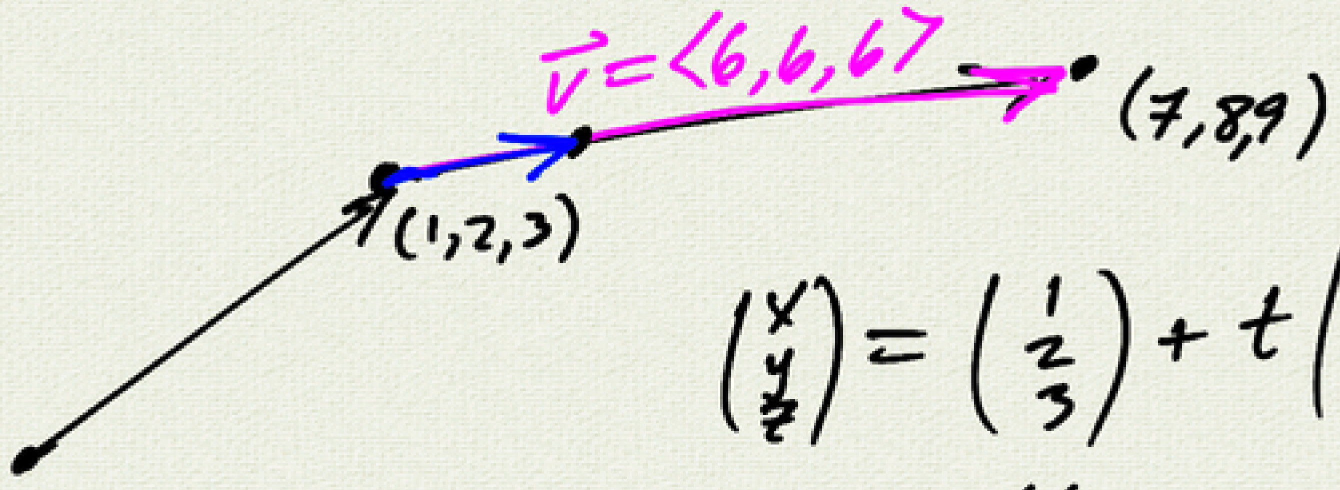
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \vec{v_0} + t\,\vec{v}$$

vector equation

$x(t) = 1 + 4t$
$y(t) = 2 + 2t$

<u>3D</u> find parametric equations of line from $(1,2,3)$ to $(7,8,9)$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$$ vector equation

$x(t) = 1 + 6t$
$y(t) = 2 + 6t$
$z(t) = 3 + 6t$

parametric equations

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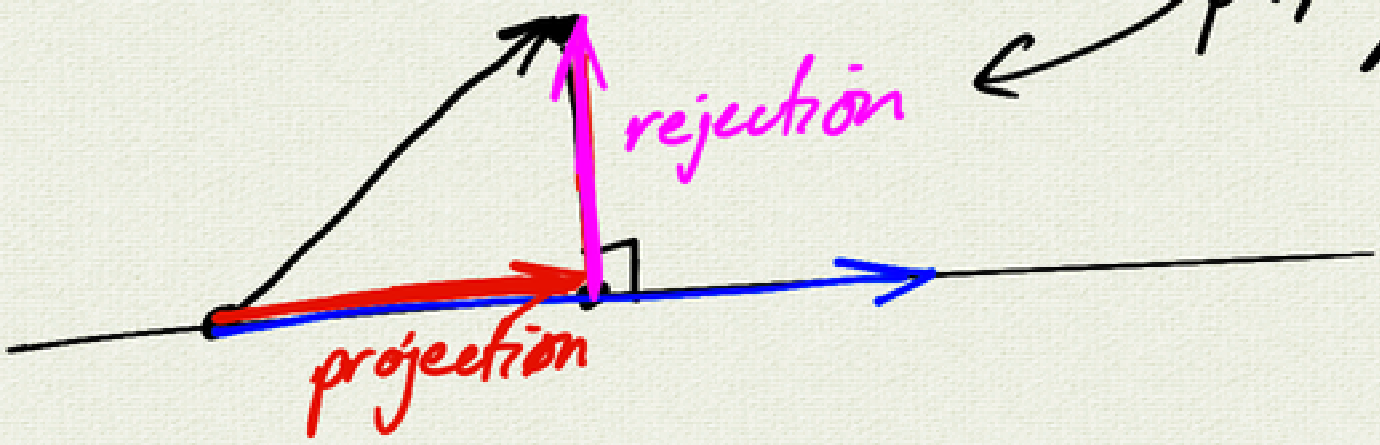
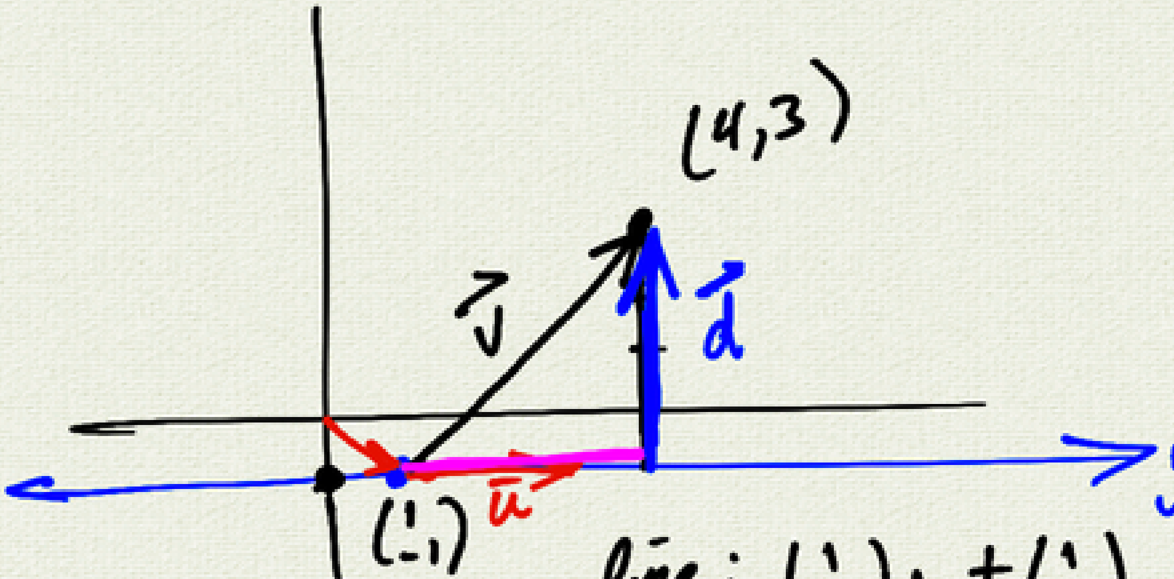
distance from point to line

perpendicular distance



example: find distance from (4,3) to $\boxed{y=-1}$



line: $\begin{pmatrix}1\\-1\end{pmatrix} + t\underbrace{\begin{pmatrix}1\\0\end{pmatrix}}_{\bar{u}}$

$\bar{v} = \langle 3,4 \rangle$

$\boxed{|proj_{\bar{u}}(\bar{v})|} = \bar{v} \cdot \frac{\bar{u}}{|\bar{u}|}$

$comp_{\bar{u}}(\bar{v})$

$= \begin{pmatrix}3\\4\end{pmatrix} \cdot \begin{pmatrix}1\\0\end{pmatrix}$

$= 3$

$\boxed{proj_{\bar{u}}(\bar{v}) = \begin{pmatrix}3\\0\end{pmatrix}}$

rejection $\vec{d} = \bar{v} - proj_{\bar{u}}(\bar{v})$

$= \begin{pmatrix}3\\4\end{pmatrix} - \begin{pmatrix}3\\0\end{pmatrix}$

$= \begin{pmatrix}0\\4\end{pmatrix}$

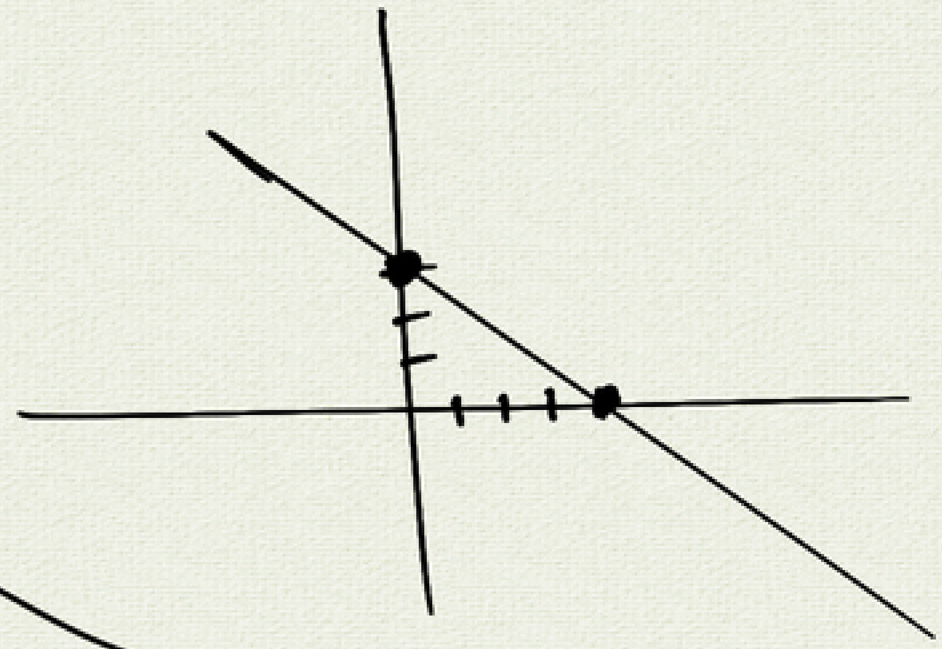
distance $d = |\vec{d}| = 4$

planes

$$3x+4y=12$$

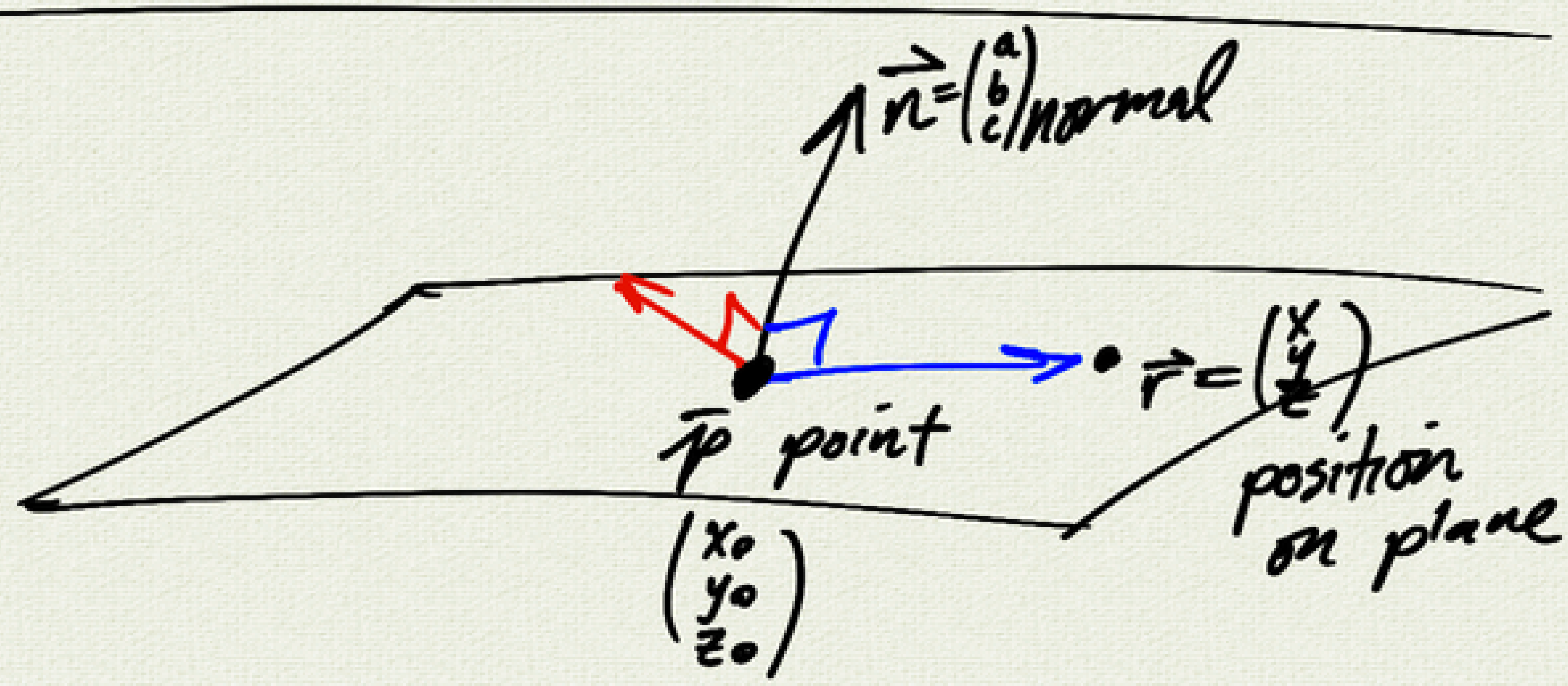
line

$$ax+by=c$$



guess:

plane $ax+by+cz=d$



points on plane: positions

$$\vec{n}\cdot(\vec{r}-\vec{p})=0$$

$$\begin{pmatrix}a\\b\\c\end{pmatrix}\cdot\begin{pmatrix}x-x_0\\y-y_0\\z-z_0\end{pmatrix}=0$$

point-normal form

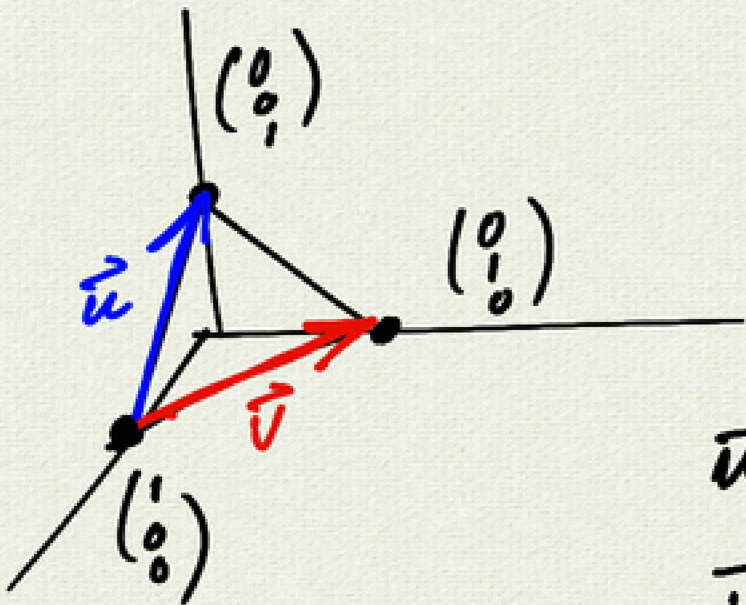
$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

"scalar equation"

$$ax+by+cz=d$$

$d=ax_0+by_0+cz_0$

standard form "general"



find equation of plane

find $\vec{n}$

$$\vec{u} = \langle -1, 0, 1 \rangle$$

$$\vec{v} = \langle -1, 1, 0 \rangle$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$

$$= -\vec{i} - \vec{j} - \vec{k}$$

$$= -\langle 1, 1, 1 \rangle$$

$$\vec{n} = \langle 1, 1, 1 \rangle$$

$$\vec{p} = \langle 1, 0, 0 \rangle$$

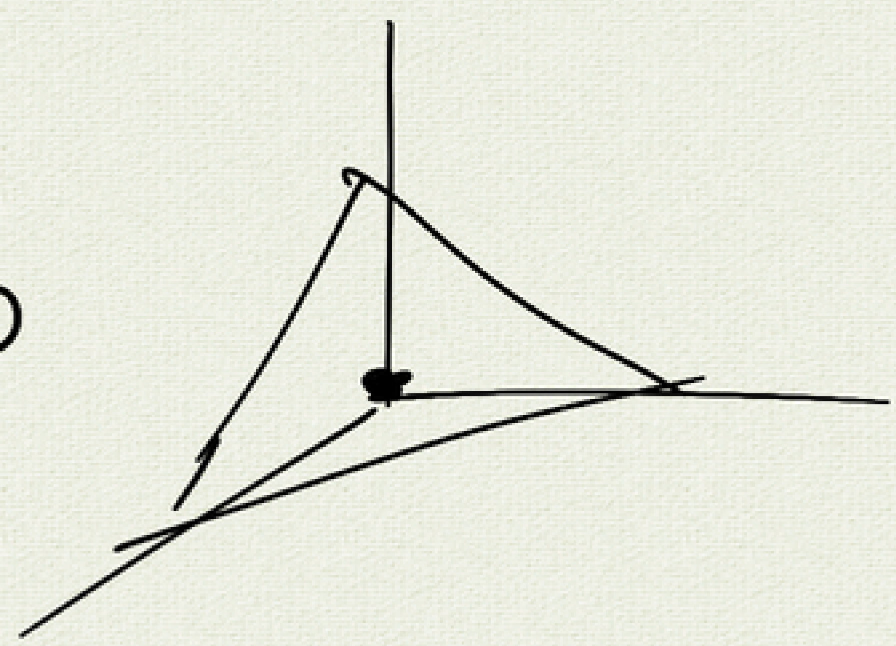
point-normal:

$$\vec{n} \cdot (\vec{r} - \vec{p}) = 0$$

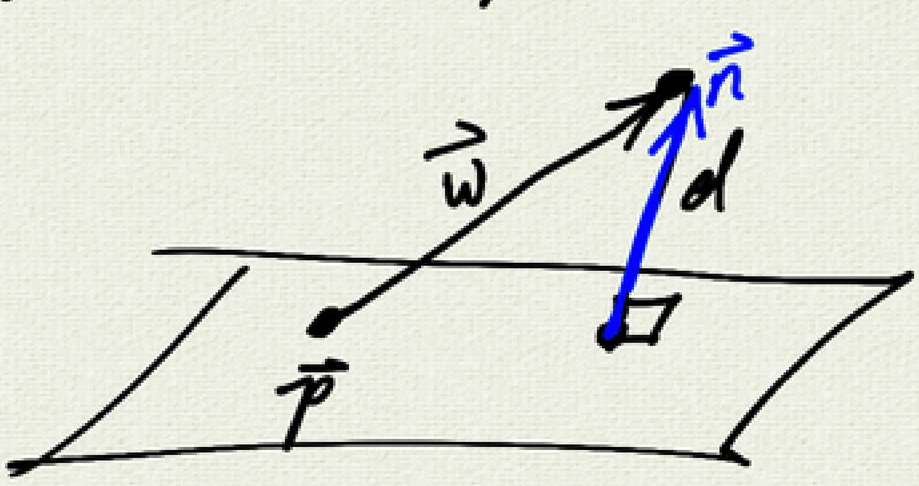
$$\begin{pmatrix}1\\1\\1\end{pmatrix} \cdot \begin{pmatrix}x-1\\y-0\\z-0\end{pmatrix} = 0$$

$$(x-1) + (y-0) + (z-0) = 0$$

$$x + y + z = 1$$



distance from plane to origin?



$$d = \left| \vec{w} \cdot \frac{\vec{n}}{|\vec{n}|} \right| \qquad \left\| \begin{pmatrix}1\\1\\1\end{pmatrix} \right\| = \sqrt{3}$$

$$d = \begin{pmatrix}1\\0\\0\end{pmatrix} \cdot \underbrace{\frac{1}{\sqrt{3}} \begin{pmatrix}1\\1\\1\end{pmatrix}}_{\text{unit normal } \frac{\vec{n}}{|\vec{n}|}}$$

$$d = \frac{1}{\sqrt{3}}$$