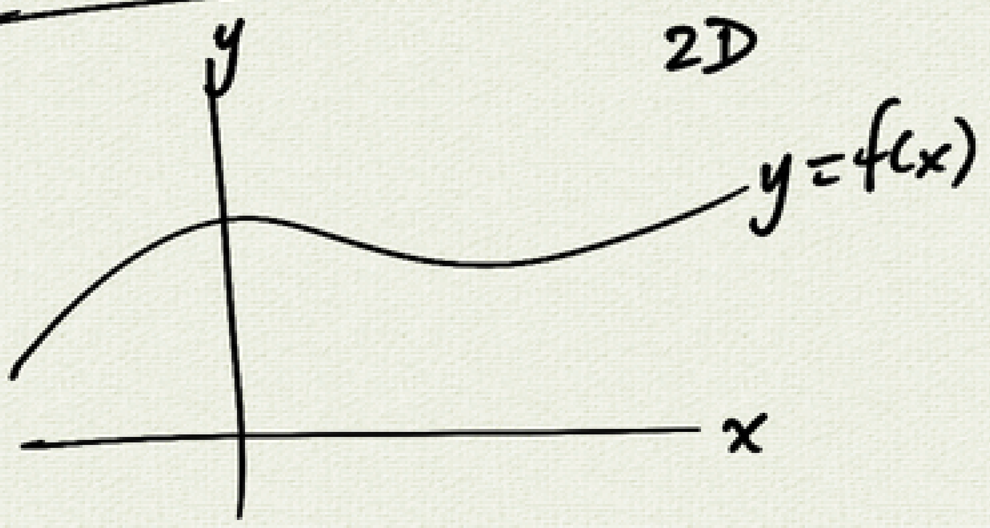
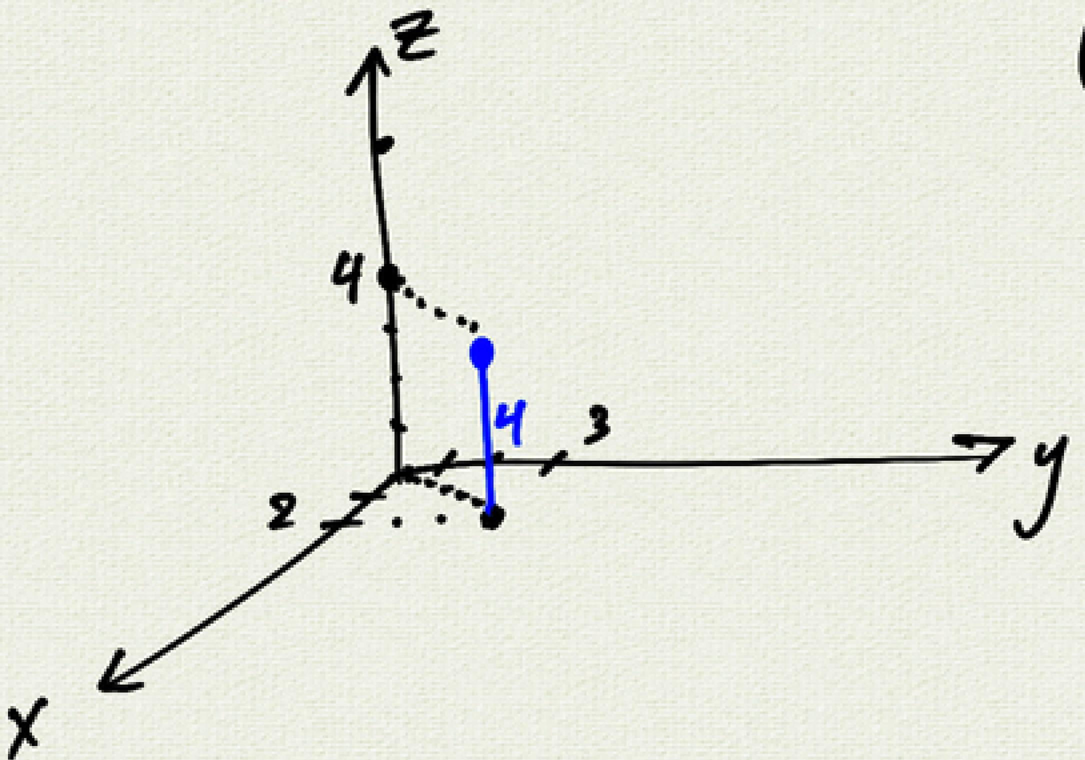
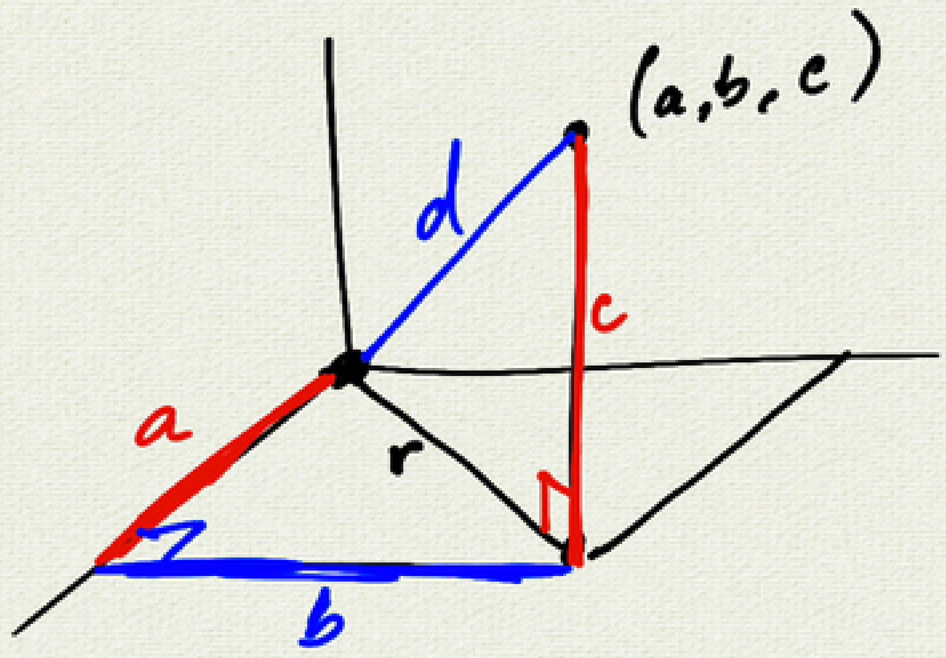
1.1 3D space





$(2, 3, 4)$
x y z

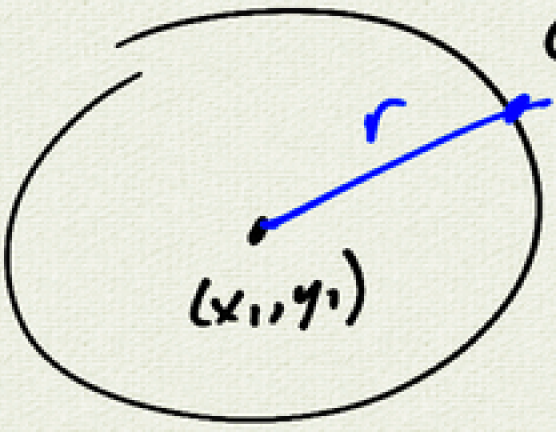


$$r^2 = a^2 + b^2$$
$$d^2 = r^2 + c^2$$
$$= (a^2+b^2)+c^2$$
$$d^2 = a^2+b^2+c^2$$

distance formula: (x_1, y_1, z_1)
(x_2, y_2, z_2)

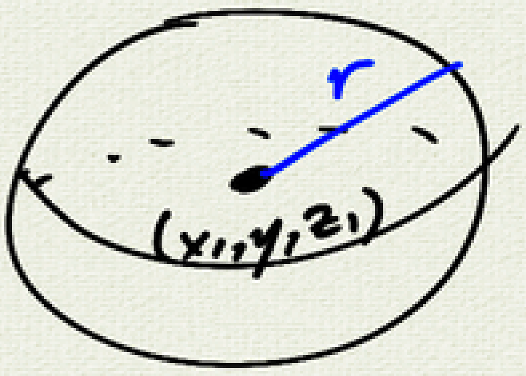
$$d^2 = (x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2$$

circle

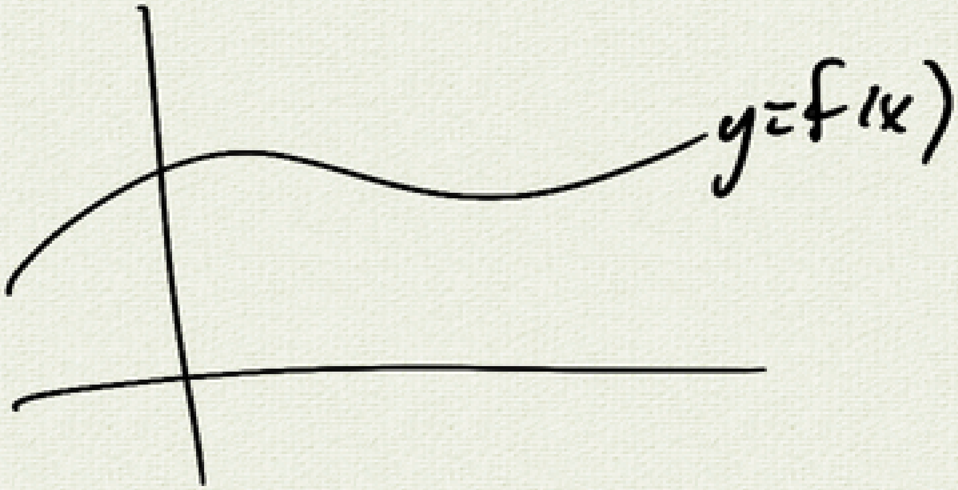


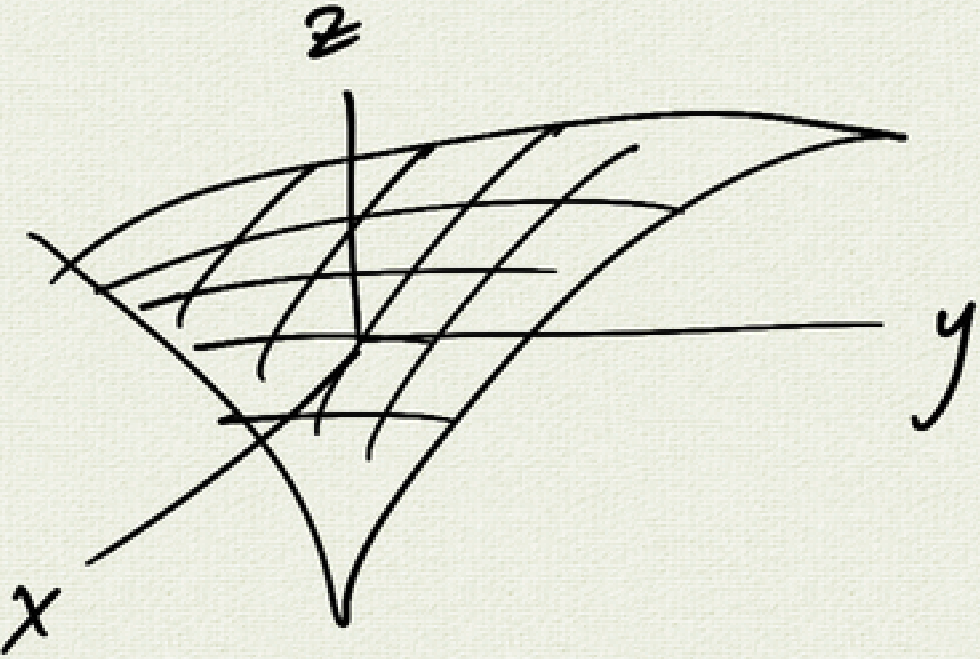
$$(x-x_1)^2 + (y-y_1)^2 = r^2$$

Sphere

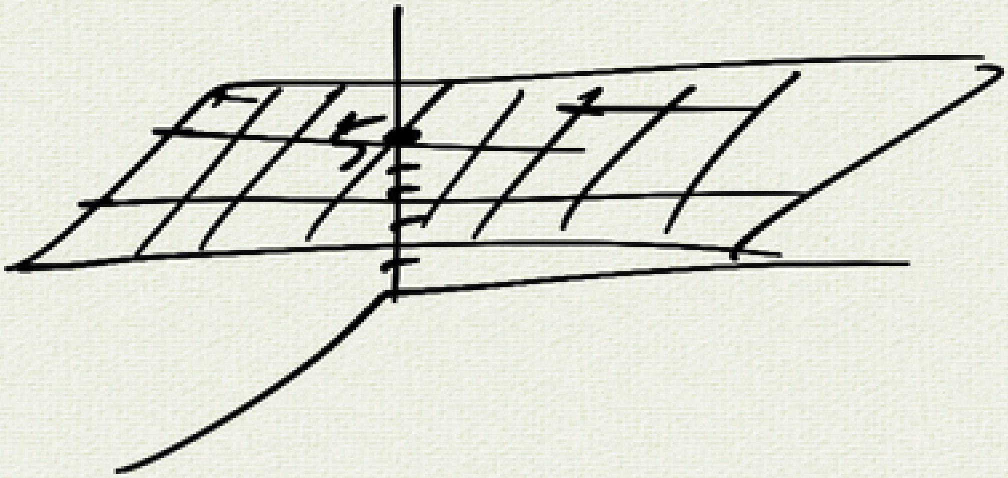


$$(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 = r^2$$





$z = f(x,y)$



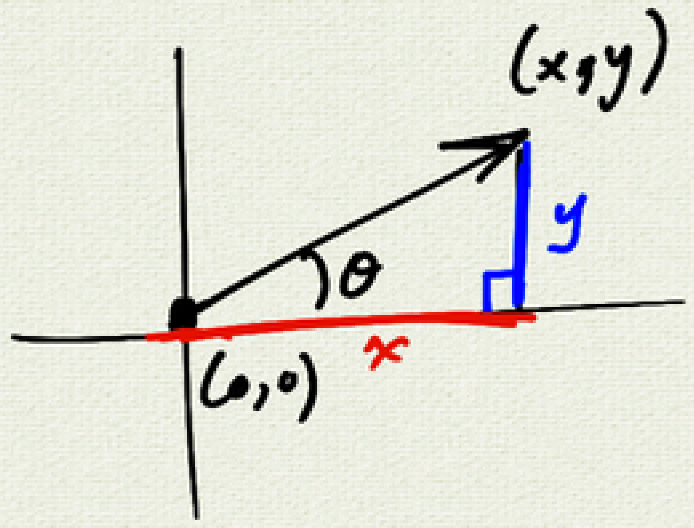
$z = 5$

plane x, y can be anything

1.2 Vectors
1.3 Dot Product

$\vec{v} = \langle x, y \rangle$

vector — component form



magnitude $|\vec{v}| = \sqrt{x^2+y^2}$

direction $\tan\theta = \dfrac{y}{x}$

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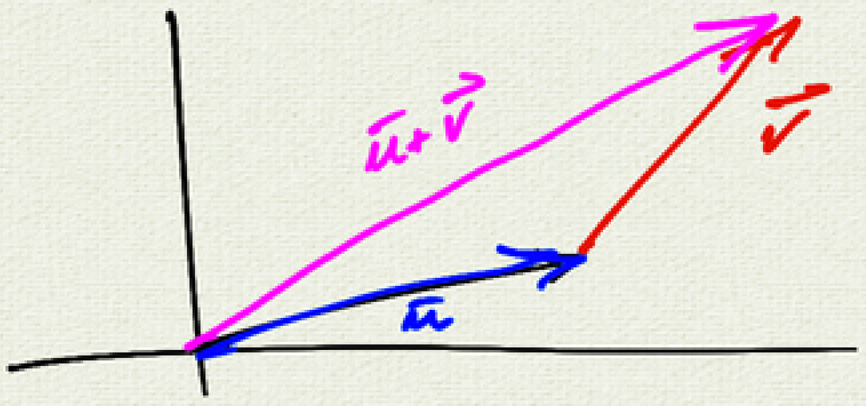
2 basic operations

① add $\bar{u} = \langle x_1, y_1 \rangle$ $\quad \bar{u}+\bar{v} = \langle x_1+x_2, y_1+y_2 \rangle$

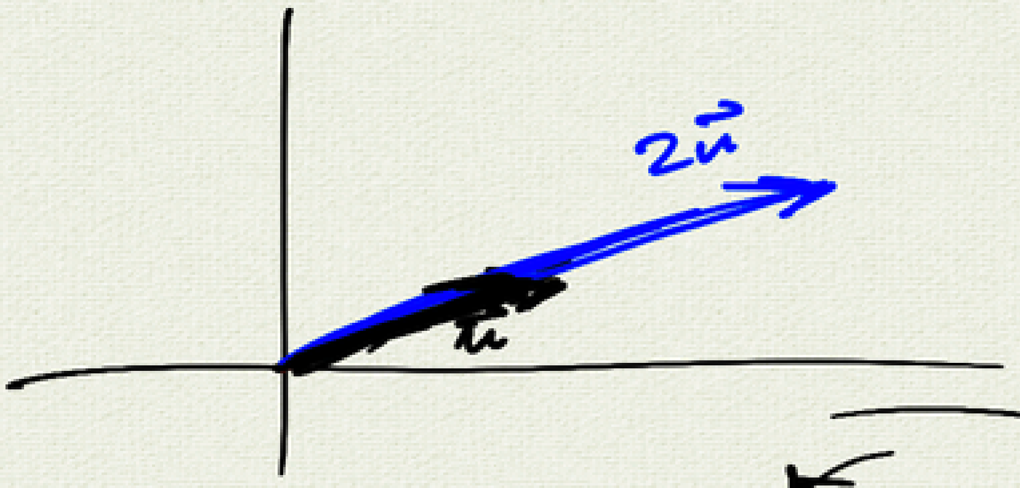
$\bar{v} = \langle x_2, y_2 \rangle$



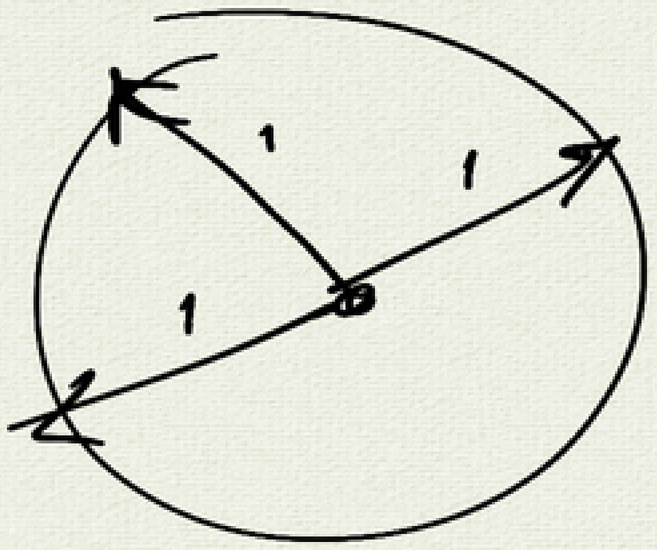
② Scalar multiplication

$\vec{u} = \langle x, y \rangle \quad , k \in \mathbb{R}$

$k\vec{u} = \langle kx, ky \rangle$



unit vector: $|\bar{u}| = 1$



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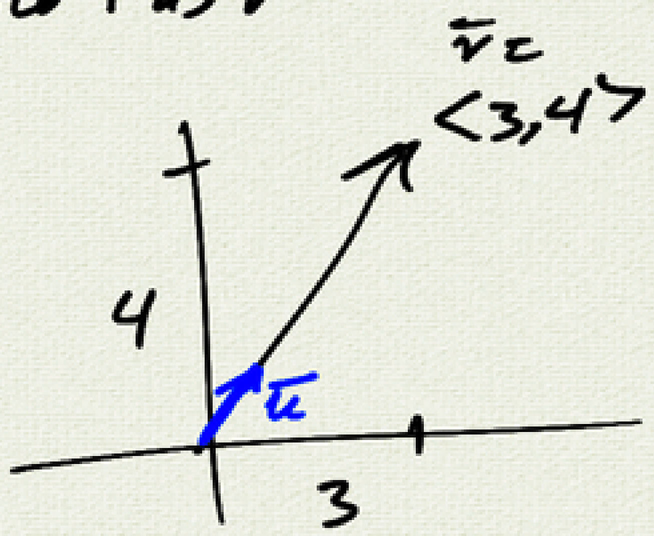
given $\bar{v} = \langle 3, 4 \rangle$

find $\bar{u}$ unit vector in same direction as $\bar{v}$

$\bar{u} = \dfrac{\bar{v}}{|\bar{v}|} = \dfrac{1}{5}\langle 3, 4 \rangle$

$= \langle \frac{3}{5}, \frac{4}{5} \rangle$



dot product: $\vec{u} = \langle x_1, y_1 \rangle$

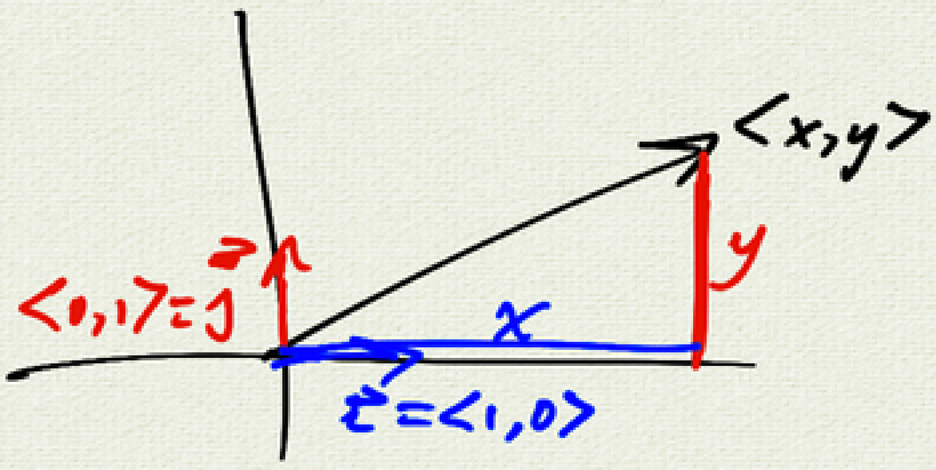
$\vec{v} = \langle x_2, y_2 \rangle$

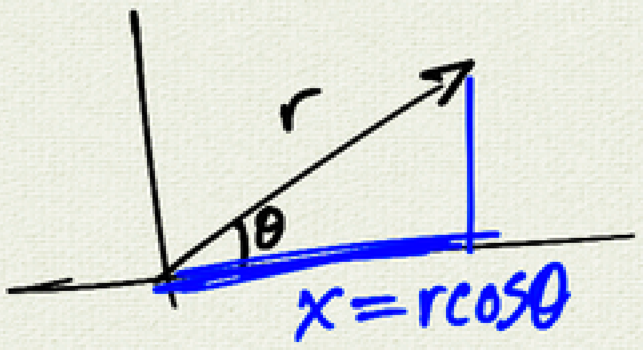
define $\vec{u} \cdot \vec{v} = x_1 x_2 + y_1 y_2$

$\vec{u} = \langle x, y \rangle$

$\vec{u} \cdot \vec{i} = x$

$\vec{u} \cdot \vec{j} = y$





$\vec{u} = \langle x, y \rangle$

$\vec{u} \cdot \vec{0} = 0$

$\vec{u} \cdot \vec{u} = x^2 + y^2$

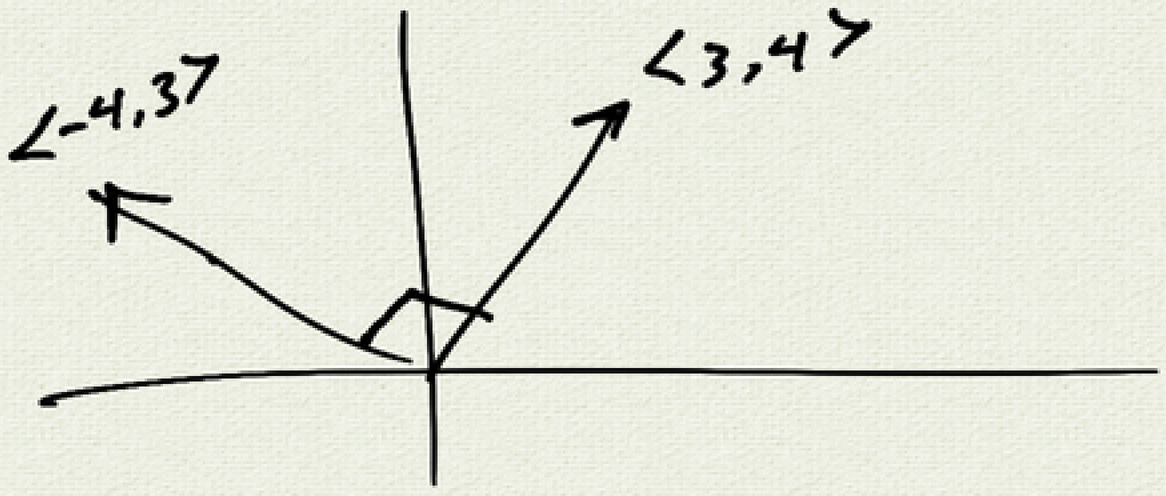
$\vec{u} \cdot \vec{u} = |\vec{u}|^2$

$\vec{i} \cdot \vec{i} = 1$

$\vec{j} \cdot \vec{j} = 1$

$\vec{i} \cdot \vec{j} = 0$

$\langle 3, 4 \rangle \cdot \langle -4, 3 \rangle = 0$



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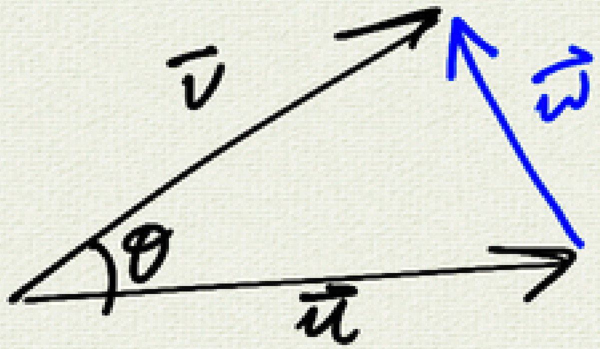
properties: $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$ commutative

$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$ distributive

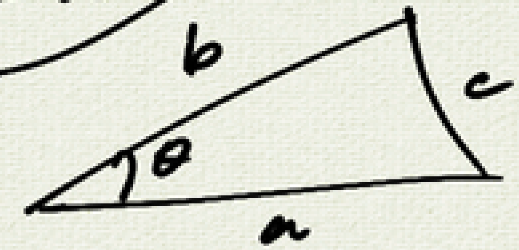
FOIL: $(\vec{u} + \vec{v}) \cdot (\vec{w} + \vec{q})$

$= \vec{u} \cdot \vec{w} + \vec{u} \cdot \vec{q} + \vec{v} \cdot \vec{w} + \vec{v} \cdot \vec{q}$



$$\bar u + \bar w = \vec v$$

$$\vec w = \bar v - \vec u$$



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Law of Cosines

$$|\bar v - \bar u|^2 = (\bar v - \bar u)\cdot(\bar v - \bar u)$$

$$= \bar v\cdot\bar v - \bar v\cdot\bar u - \bar u\cdot\bar v + \bar u\cdot\bar u$$

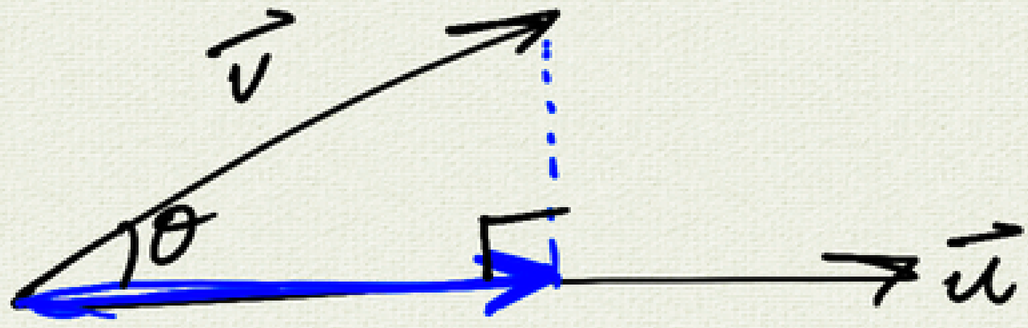
$$|v-u|^2 = |\bar v|^2 + |u|^2 - 2\boxed{\bar u\cdot\bar v}$$

$$|\bar v - u|^2 = |u|^2 + |v|^2 - 2\boxed{|\bar u||\bar v|\cos\theta}$$

$$\boxed{\bar u\cdot\bar v = |\bar u||\bar v|\cos\theta}$$

$$\Rightarrow \cos\theta = \frac{\bar u\cdot\bar v}{|\bar u||\bar v|}$$

find angle between vectors



projection of $\bar{v}$ on $\bar{u}$

length of projection $= |\bar{v}|\cos\theta$

remember: $\bar{u}\cdot\bar{v} = |\bar{u}||\bar{v}|\cos\theta$

$\Rightarrow$ length of projection $= \bar{v}\cdot\dfrac{\bar{u}}{|\bar{u}|}$

projection $= \underbrace{\left(\bar{v}\cdot\dfrac{\bar{u}}{|\bar{u}|}\right)}_{\text{length}} \underbrace{\dfrac{\bar{u}}{|\bar{u}|}}_{\text{unit vector in direction of } \bar{u}}$

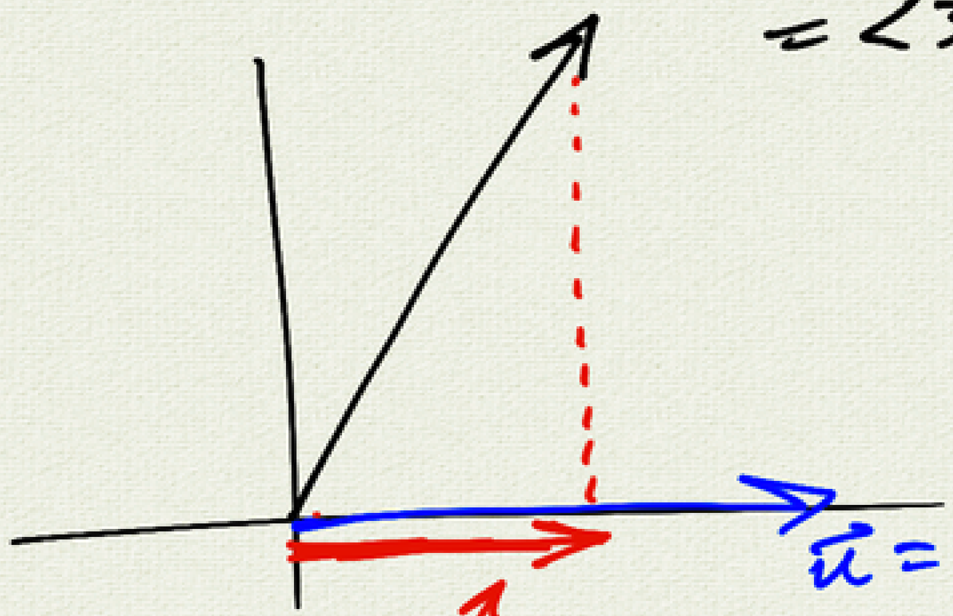
$\mathrm{proj}_{\bar{u}}(\bar{v})$

$\mathrm{comp}_{\bar{u}}(\bar{v})$

component

Example:

$\bar{v} = 6\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$
$= \langle 3, 3\sqrt{3} \rangle$



find $proj_{\bar{u}}(\bar{v})$

$$\text{length} = comp_{\bar{u}}(\bar{v}) = \bar{v} \cdot \boxed{\frac{\bar{u}}{|\bar{u}|}}$$

$\frac{\langle 4, 0 \rangle}{4}$

$$= \langle 3, 3\sqrt{3} \rangle \cdot \langle 1, 0 \rangle$$
$$= 3$$

$$proj_{\bar{u}}(\bar{v}) = 3 \left(\frac{\bar{u}}{|\bar{u}|} \right)$$
$$= 3\langle 1, 0 \rangle$$
$$= \langle 3, 0 \rangle$$

$$\langle x_1, y_1, z_1 \rangle \cdot \langle x_2, y_2, z_2 \rangle$$
$$= x_1x_2 + y_1y_2 + z_1z_2$$

we still have:

$$\bar{u} \cdot \bar{v} = |\bar{u}||\bar{v}|\cos\theta$$