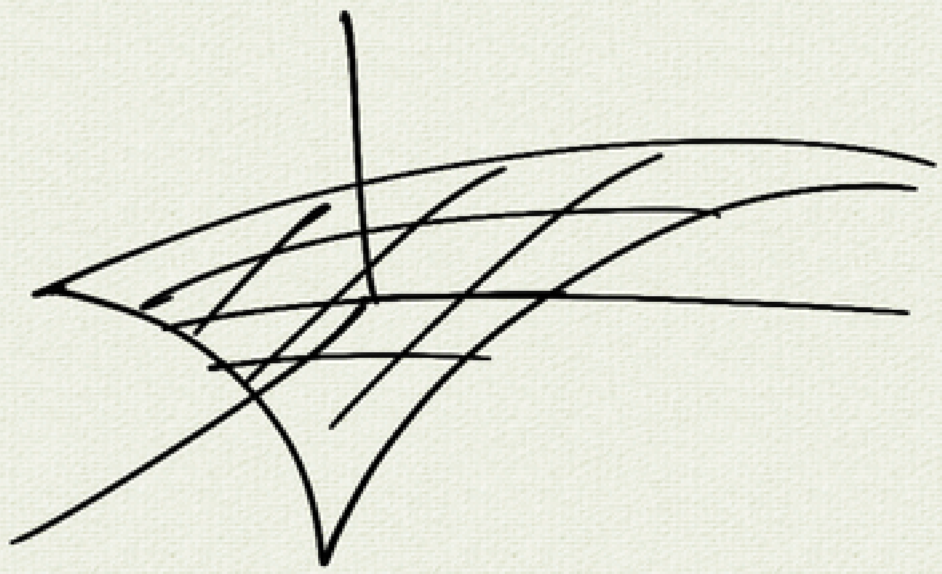
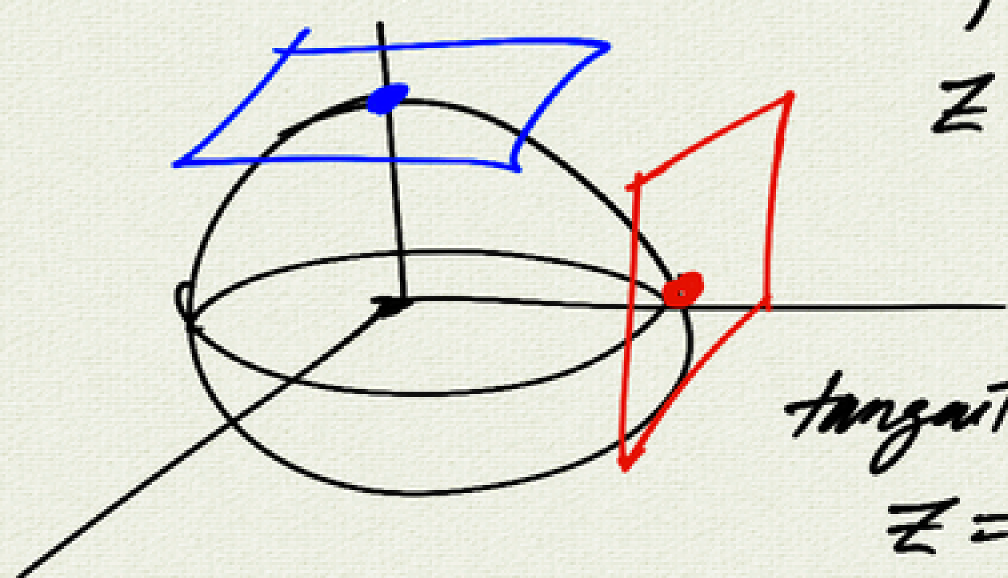
3.8 Level Surfaces

$z = f(x,y)$

surface



<u>example:</u> unit sphere



$x^2+y^2+z^2=1$

$z = \pm\sqrt{1-x^2-y^2}$

$f(x,y) = \sqrt{1-x^2-y^2} = (1-x^2-y^2)^{1/2}$

tangent plane

$z = z_0 + f_x(x-x_0) + f_y(y-y_0)$

at $(x_0,y_0) = (0,0)$

$(x_0,y_0,z_0) = (0,0,1)$

tangent plane

$z = 1 + 0(x-0) + 0(y-0)$

$z = 1$

$f_x = \frac{1}{2}\frac{(-2x)}{\sqrt{1-x^2-y^2}} = \frac{-x}{z}$

$f_y = -y/z$

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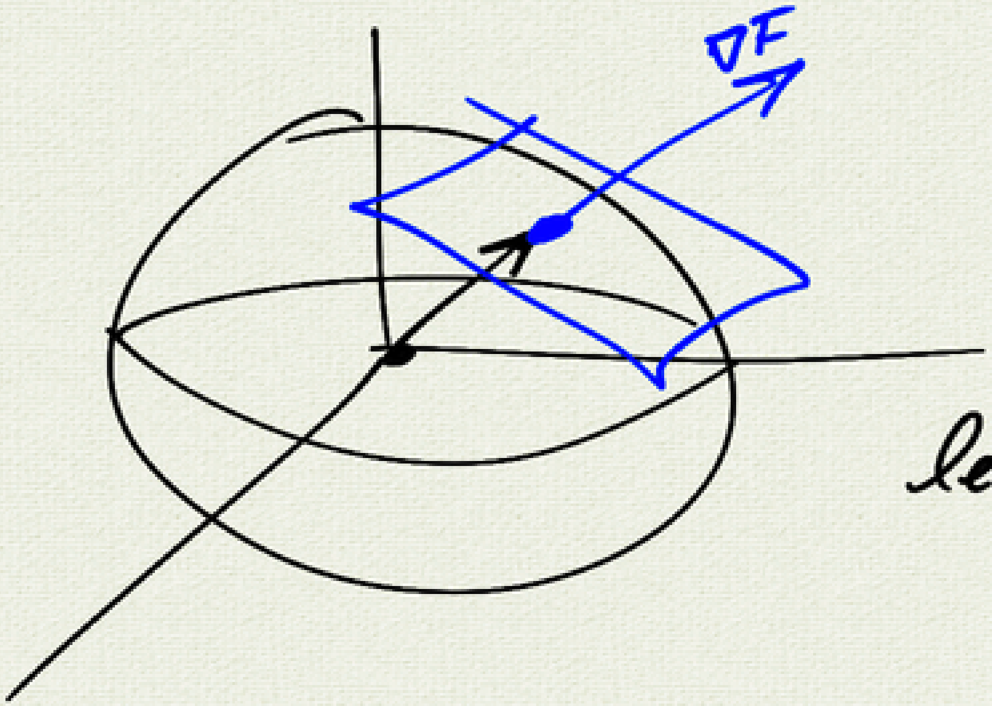
at $(x_0,y_0) = (0,1)$

$(x_0,y_0,z_0) = (0,1,0)$

$z = z_0 + \underline{\underline{f_x}}(x-x_0) + \underline{\underline{f_y}}(y-y_0)$

undefined



another view:

$$F(x,y,z) = x^2+y^2+z^2$$

level set (level surface)

$$F(x,y,z) = 1$$

$$\nabla F = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

$\Rightarrow$ ∇F gradient is normal to tangent plane

$\Rightarrow$ tangent plane

$$\nabla F \cdot (\bar{r} - \bar{r}_0) = 0$$

$$(x_0, y_0, z_0) = (0, 0, 1)$$

$$\nabla F = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \qquad \nabla F \cdot (\bar{r} - \bar{r}_0) = 0$$

tangent plane: $0(x-0) + 0(y-0) + 2(z-1) = 0$

$$z = 1$$

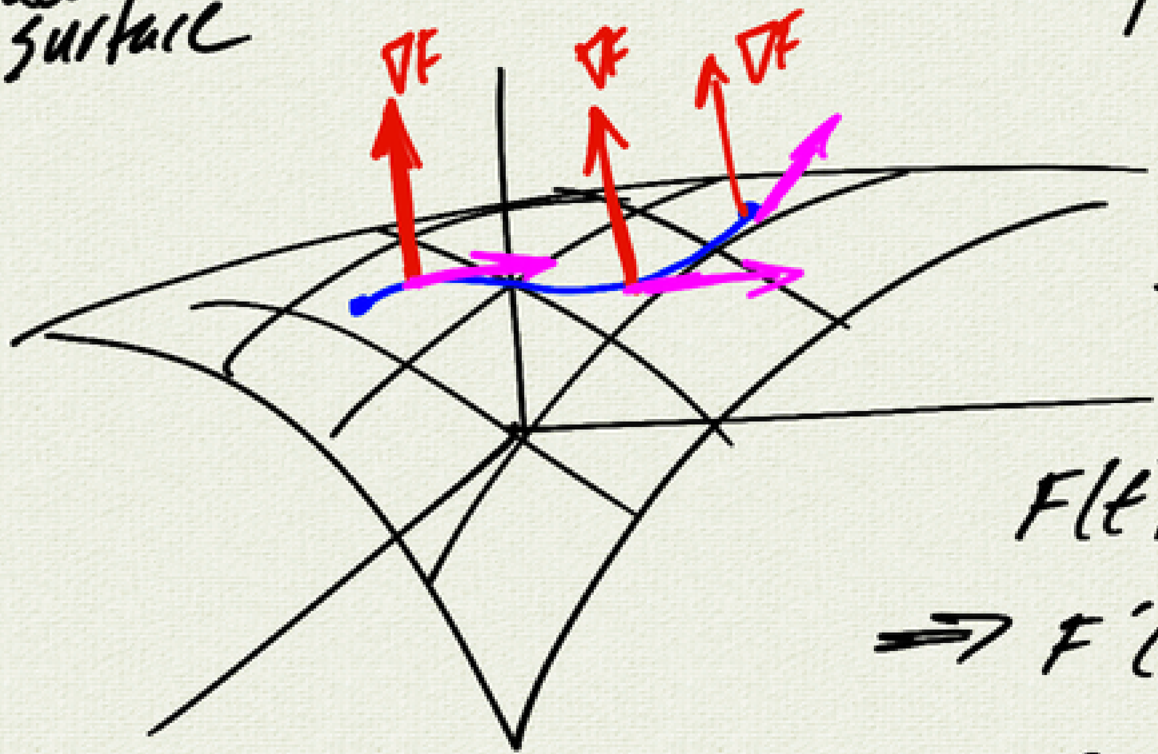
$$(x_0, y_0, z_0) = (0, 1, 0)$$

$$\nabla F = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \qquad \nabla F \cdot (\bar{r} - \bar{r}_0) = 0$$

$$0(x-0) + 2(y-1) + 0(z-0) = 0$$

$$y = 1$$

level surface



$F(x,y,z) = \text{const.}$

curve $\bar{r}(t)$ stays on level surface

$$F(t) = F(\bar{r}(t)) = \text{const}$$

$$\Rightarrow F'(t) = 0$$

$$F'(t) = \frac{\partial F}{\partial x}\frac{dx}{dt} + \frac{\partial F}{\partial y}\frac{dy}{dt} + \frac{\partial F}{\partial z}\frac{dz}{dt}$$

$$= \underbrace{\begin{pmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \end{pmatrix}}_{dF} \underbrace{\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{pmatrix}}_{\bar{r}'(t)}$$

$$F'(t) = \underbrace{\underset{\text{gradient}}{\nabla F} \cdot \underset{\text{tangent vector to curve}}{\bar{r}'(t)}}_{\text{orthogonal}} = 0$$