Geometric Algebra HW 1 (Wedge Product)
MultiV 2021-22 / Dr. Kessner

1. For each of the following pairs of vectors $\mathbf{u}$ and $\mathbf{v}$, find the wedge product $\mathbf{u} \wedge \mathbf{v}$. Draw the vectors and make sure your answer makes sense geometrically.
   a. $\mathbf{u} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
   b. $\mathbf{u} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
   c. $\mathbf{u} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$
   d. $\mathbf{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$
2. Find the area of the triangle determined by the two vectors $\mathbf{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$.
   Find a general formula for the area of a triangle determined by two vectors $\mathbf{u}$ and $\mathbf{v}$.
3. Find the distance from the point $(2, 2)$ to the line $2x + 2y = 2$.
4. Find the distance from the point $(7, 7)$ to the line $6x + 8y = 48$.

Answers:
1a. $2\mathbf{e_1} \wedge \mathbf{e_2}$
1b. $-2\mathbf{e_1} \wedge \mathbf{e_2}$
1c. $9\mathbf{e_1} \wedge \mathbf{e_2}$
1d. $8\mathbf{e_1} \wedge \mathbf{e_2}$
2. $A = \frac{1}{2}|\mathbf{u} \wedge \mathbf{v}| = \frac{1}{2}(8) = 4$
3. $\sqrt{2}$
4. 5