Semester 2 Warmup
MultiV 2021-22 / Dr. Kessner

No calculator! Have fun!

1. Consider the function $f(x, y) = 2x^2 - 4x + 3y^2 + 12y + 20$.

a. Find the equation of the tangent plane to the surface $z = f(x, y)$ at $(x, y) = (0, 0)$.

b. A *critical point* of f is a point (x, y) where both f_x and f_y are either zero or undefined. Find all critical points of f (there is only one for this example).

c. At the critical point, find the linear approximation of f.

d. Let d^2f be the matrix of 2nd partial derivatives:

$$d^2f = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$$

Find d^2f and $\det d^2f$ (at the critical point). What does d^2f tell you about the shape of the surface at the critical point?

e. Complete the square to write the function in the form $f(x, y) = a(x - h)^2 + b(y - k)^2 + c$. What does this tell you about the surface?

2. Do the same calculations for the function $g(x, y) = -4x^2 + 16x - 5y^2 - 11$.

3. Do the same calculations for the function $h(x, y) = x^2 - 2x - 2y^2 + 8y - 9$.