Geometric Algebra Classwork (Reflection)
MultiV 2021-22 / Dr. Kessner

Let $u = e_1$ and $v = \frac{1}{\sqrt{2}}(e_1 + e_2)$.

Define the transformations $R_u(w) = uwu$, $R_v(w) = vwv$, and $R_{uv}(w) = (vu)w(uv)$.

1. Show that $R_u(e_1 + e_3) = e_1 - e_3$ and $R(u) = u$.

2. For a general $w = w_x e_1 + w_y e_2 + w_z e_3$, show that $R(w) = w_x e_1 - w_y e_2 - w_z e_3$. In other words, the y and z coordinates are negated. What is the transformation R_u?

3. Show that $R_v(e_1) = e_2$, $R(v) = v$ and $R_v(e_1 + e_3) = e_2 - e_3$. What is the transformation R_v?

4. Show that $R_{uv}(e_1) = e_2$, $R_{uv}(e_3) = e_3$, and $R_{uv}(e_1 + e_3) = e_2 + e_3$. What is the transformation R_{uv}. Note that $R_{uv} = R_v R_u$.

5. Define the transformation $M_x(w) = -R_u(w)$. Calculate $M_x(w)$ for a general $w = w_x e_1 + w_y e_2 + w_z e_3$. What is this transformation? Describe how the transformation changes the coordinates.

6. Define transformations M_y and M_z and show that they act as expected on a general $w = w_x e_1 + w_y e_2 + w_z e_3$.