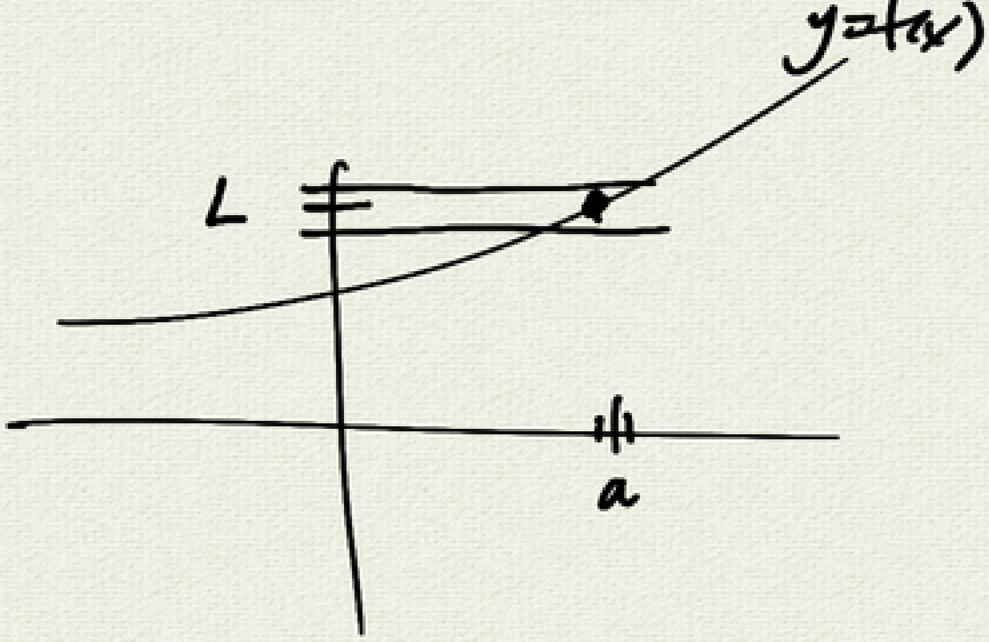
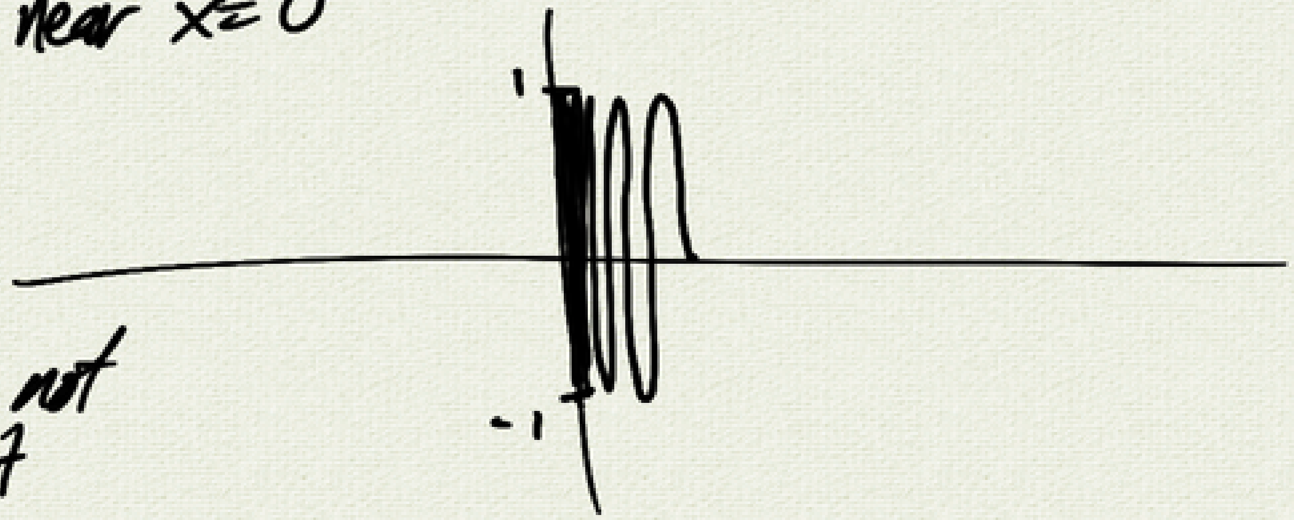
3.2 Limits and continuity



$$\lim_{x \to a} f(x) = L$$

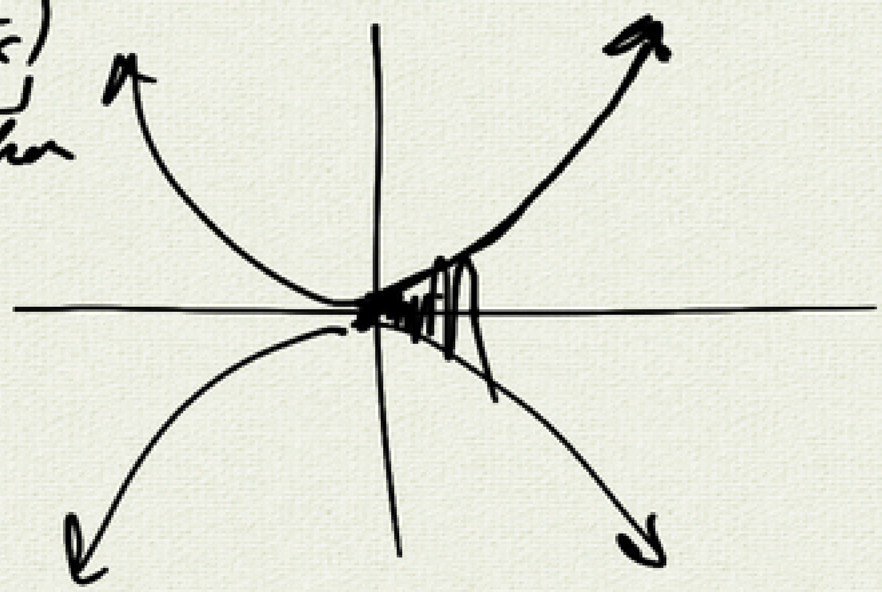
"by taking x close to a,
I can guarantee that
$f(x)$ is close to L"

$f(x) = \sin\left(\frac{1}{x}\right)$ near $x=0$



$\lim_{x \to 0^+} f(x)$ does not exist

$$g(x) = \underbrace{x^2}_{\text{near } 0} \underbrace{\sin\left(\tfrac{1}{x}\right)}_{\text{fraction } \pm}$$

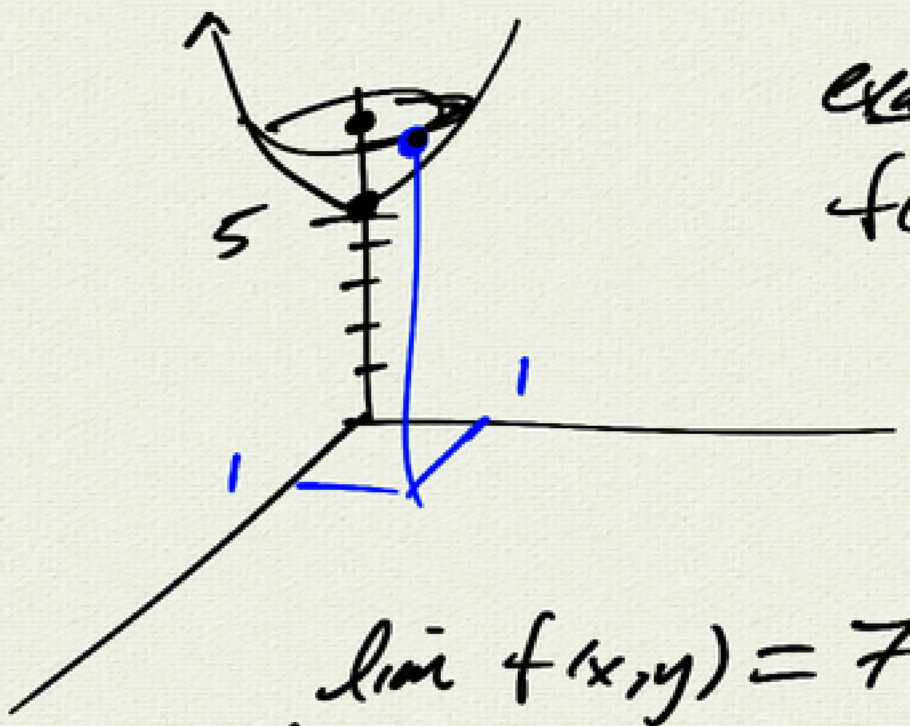


$$\lim_{x \to 0} g(x) = 0$$

$z = f(x,y)$

$f : \mathbb{R}^2 \to \mathbb{R}$

$(x,y) \mapsto z$



example:

$f(x,y) = x^2 + y^2 + 5$

paraboloid

$$\lim_{(x,y)\to(1,1)} f(x,y) = 7$$

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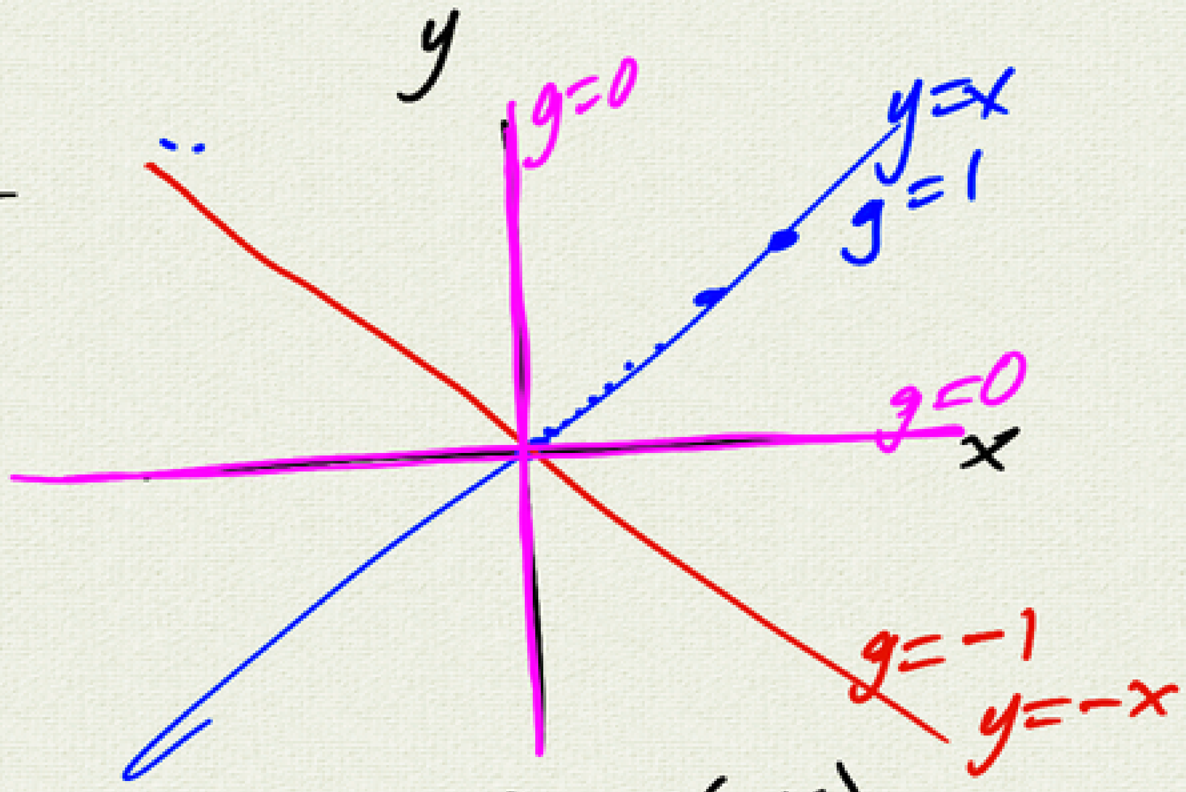
weird example

$$g(x,y) = \frac{2xy}{x^2+y^2}$$

not defined at $(0,0)$

$$\lim_{(x,y)\to(0,0)} g(x,y) = ?$$

limit does not exist



$\underline{y=x}$: $g(x,y) = g(x,x)$

$$= \frac{2x^2}{x^2+x^2} = 1$$

$\underline{y=-x}$: $g(x,-x) = -1$

$$g(x,y) = \frac{2xy}{x^2+y^2}$$

polar: $x = r\cos\theta$

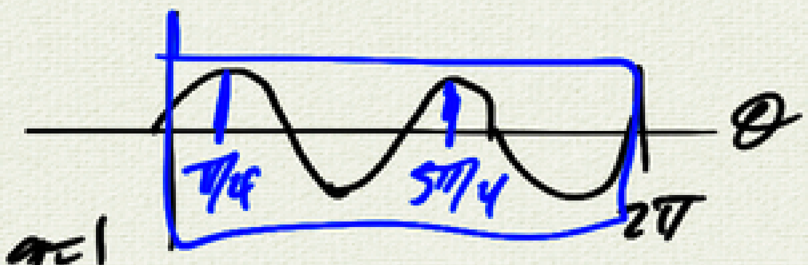
$y = r\sin\theta$

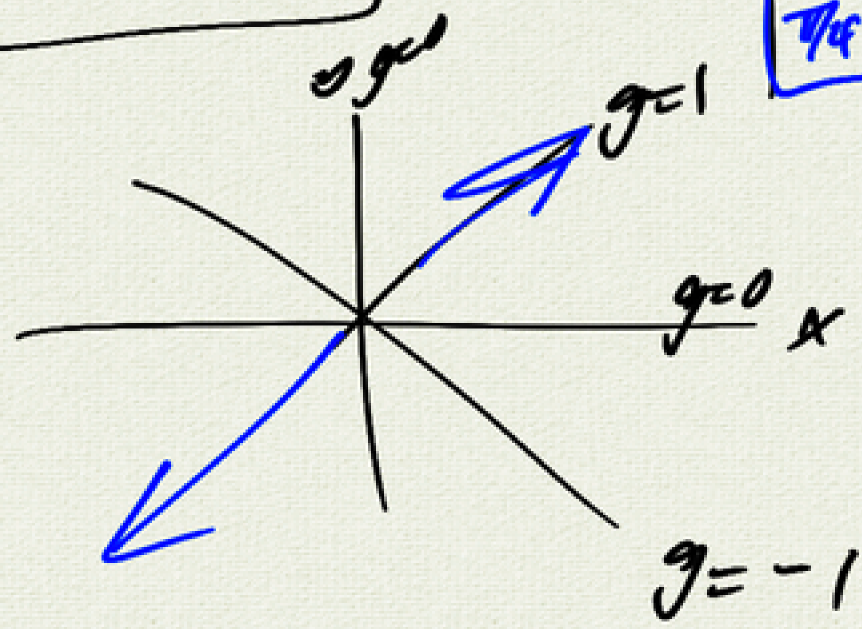
$x^2 + y^2 = r^2$

$$= \frac{2(r\cos\theta)(r\sin\theta)}{r^2}$$

$$= 2\sin\theta\cos\theta$$

$$\boxed{g(r,\theta) = \sin 2\theta}$$





infinite ruffled collar