Lesson 3-7: Absolute Value Equations   Name: _______________

In this activity, we will learn to solve **absolute value equations**. An absolute value equation is any equation that contains an absolute value symbol. To start, let's review a little of what we know about the absolute value function.

1. a. $|2| =$ b. $|-2| =$ c. $|-3| =$ d. $|3| =$

2. Circle the numbers in the set S that can be substituted for x to make the equation $|x| = 2$ true.

$$S = \{-3, -2, -1, 0, 1, 2, 3\}$$

3. Circle the numbers in the set S that can be substituted for y to make the equation $|y| = 1$ true.

$$S = \{-3, -2, -1, 0, 1, 2, 3\}$$

Almost every absolute value equation has two answers:
In the equation $|x| = 6$, x could be equal to 6, or x could be equal to -6.
Either value will make the equation true: $|6| = 6$, and $|-6| = 6$

Use the principle above to fill in the blanks for each question below:

4. $|x| = 5$ means that x could be equal to _____ or x could be equal to _____ .

5. $|x| = 13$ means that x could be equal to _____ or x could be equal to _____ .

6. $|x| = 250$ means that x could be equal to _____ or x could be equal to _____ .

Every absolute value equation represents *two* equations combined into one:
$|x| = 10$ means that $x = 10$ or $x = -10$.
$|x-1| = 7$ means that $x - 1 = 7$ or $x - 1 = -7$

Use this principle to fill in the blanks for each question below:

7. $|x+3| = 5$ means $x + 3 = 5$ or _________________

8. $|2x-1| = 9$ means _______________ or _________________

9. $|5-3x| = 25$ means _______________ or _________________

10. $2|5x| = 10$ changes to ___________ which means ____________ or _____________

11. $|3e| - 8 = 7$ changes to ___________ which means ____________ or _____________

Skill 3: Solve Absolute Value Equations

To solve absolute value equations, solve the two equations each represents:

$$|y-2|=4 \quad \text{means} \quad y-2=4 \quad \text{or} \quad y-2=-4$$

$$\begin{array}{rcl} \underline{+2 \quad +2} & & \underline{+2 \quad +2} \\ y \quad = 6 & \text{or} & y \quad = -2 \end{array}$$

The solution set is $y = 6$ or $y = -2$

Check: Does $|6-2|=4$? Does $|-2-2|=4$?

$|4|=4$ Yes. $|-4|=4$ Yes.

Use this principle to solve each absolute value equation below and check the solutions.

12. $|x+3|=5$

13. $|2x-1|=9$

14. $|5-3x|=25$

15. $2|5x|=10$

16. $|3e|-8=7$

Scrambled answers for #12-16: -8, $-\frac{20}{3}$, -5, -4, -1, 1, 2, 5, 5, 10

More 3-7: Absolutely Less

Answer each question below:

1. True or False?
   a. $|1| < 2$ b. $|-3| < 2$ c. $|-1| < 2$ d. $|0| < 2$
2. Circle numbers in the set S that can be substituted for x to make the inequality $|x| < 2$ true.
   $S = \{-3, -2.75, -2, -1.33, -1, 0, 0.5, 1, 1.99, 2, 2.66, 3\}$
3. Circle numbers in the set S that can be substituted for y to make the equation $|y| \leq 2$ true.
   $S = \{-3, -2.75, -2, -1.33, -1, 0, 0.5, 1, 1.99, 2, 2.66, 3\}$

Every absolute value inequality with < or $\leq$ represents *two* inequalities combined with "<u>and</u>":

$|x| < 2$ means that $x < 2$ <u>and</u> $x > -2$.

It means $x < 2$ because for numbers less than 2, the absolute value will be less than 2: $|1| < 2$

It means $x > -2$ because for numbers greater than –2, the absolute value will be less than 2: $|-1| < 2$

The numbers that work in $|x| < 2$ must meet <u>both</u> of these requirements:

-1.7 works because it is less than 2 <u>and</u> greater than –2, so $|-1.7| < 2$ is a true statement.

-3 does not work, even though it is less than 2, because it is not greater than –2, so $|-3| < 2$ is not true.

Use the principle above to fill in the blanks for each question.

4. $|x| \leq 5$ means $x \leq 5$ and ________________
5. $|x| < 250$ means ________________ and ________________
6. $|x+3| < 5$ means ________________ and ________________
7. $|2x| - 1 \leq 9$ changes to ___________ which means __________ and ______________

Skill 4: Solve absolute value inequalities

To solve absolute value inequalities, solve the two inequalities that each represents.

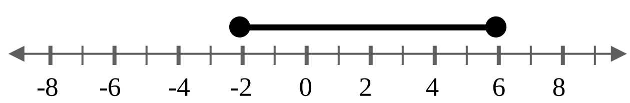
For example, to solve $|y - 2| \leq 4$:

$$|y - 2| \leq 4 \text{ means } y - 2 \leq 4 \text{ and } y - 2 \geq -4$$

$$\begin{array}{ccc} \underline{+2 \quad +2} & & \underline{+2 \quad +2} \\ y \quad \leq 6 & \text{and} & y \quad \geq -2 \end{array}$$

The solution set is $y \leq 6$ *and* $y \geq -2$, which is the same as $-2 \leq y \leq 6$.

The graph of this solution set looks like:



Check $x = 5$: Is $|5 - 2| \leq 4$?
$|3| \leq 4$ Yes.
($x = 5$ is in solution set.)

Check $x = -3$: Is $|-3 - 2| \leq 4$?
$|-5| \leq 4$ No.
($x = -3$ is not in solution set.)

Use this principle to solve each absolute value inequality, graph the solution set, and check two values – one in the solution set and one not in the solution set.

8. $|2x + 5| \leq 5$

9. $|4y - 8| < 0$

10. $|2 - y| \leq 1$

11. $3|e - 2| < 9$

12. $|3x| + 4 < -8$

13. $|8 - (w - 1)| \leq 9$

Even More 3-7: More Absolutely

Every absolute value inequality with > or $\geq$ represents *two* inequalities combined with "or":

$|x| \geq 4$ means that $x \geq 4$ or $x \leq -4$.

It means $x \geq 4$ because for numbers greater than 4, the absolute value will be greater than 4: $|6.3| > 4$

It means $x \leq -4$ because for numbers less than –4, the absolute value will be greater than 4: $|-8| > 4$

The numbers that work in $|x| \geq 4$ can meet either of these requirements, but don't have to meet both:

9 works because it is greater than 4, even though it is not less than –4. $|9| \geq 4$ is a true statement.

-7.3 works, even though it is not greater than 4, because it is less than –4. $|-7.3| \geq 4$ is a true statement.

Use the principle above to fill in the blanks for each question below:

1. $|x - 1| > 7$ means $x - 1 > 7$ or ________________

2. $|5 - 3x| > 25$ means ______________ or ________________

3. $|3e - 8| > 7$ means ______________ or ________________

4. $|4y| - 8 > 16$ changes to ___________ which means __________ or ______________

Skill 4: Solve absolute value inequalities

To solve absolute value inequalities, solve the two inequalities that each represents.

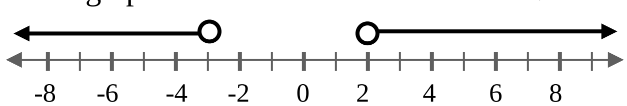
For example, to solve $|2x + 1| > 5$:

$|2x + 1| > 5$ means

$$\begin{array}{lcl} 2x + 1 > 5 & \text{or} & 2x + 1 < -5 \\ \quad\;\; -1 \quad -1 & & \quad\;\; -1 \quad -1 \\ \hline 2x \quad > 4 & & 2x \quad < -6 \\ x \quad > 2 & \text{or} & x \quad < -3 \end{array}$$

The solution set is $x > 2$ *or* $x < -3$.

The graph of this solution set looks like:



Check $x = -4$: Is $|2(-4) + 1| > 5$?

$|-7| > 5$ Yes.

(–4 is in solution set.)

Check $x = 0$: Is $|2(0) + 1| > 5$?

$|1| > 5$ No.

(0 is not in solution set.)

Use the principle above to solve each absolute value equation, graph the solution set, and check two values – one in the solution set and one not in the solution set.

5. $|w + 8| \geq 1$

6. $|t + 4| > 3$

7. $|2y - 5| \geq 0$

8. $|6 - 3x| > 9$

9. $8 + |2z| \geq 40$

10. $1 - 3|y| > -2$

11. Compare the process of solving inequalities with "<" or "≤" to solving inequalities with ">" or "≥". What is the same? What's different?