

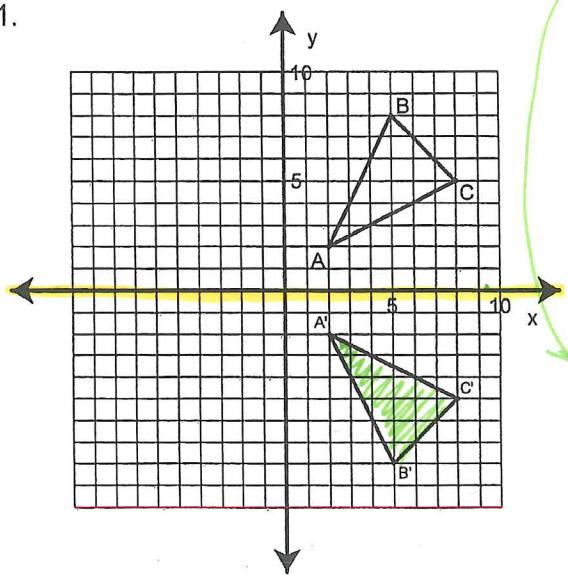
Geometry

Composite of Reflections over Two Intersecting Lines

Name Key
 Hour _____

1 – 4 Find the line of reflection and highlight it with a colored pencil. Write the equation of the line of reflection. Find the coordinates of the reflected image and use them to write the image formula that would reflect any point (x,y) over the given reflection line.

1.



| | | | | | |
|----|--------|----|--------|----|--------|
| A | (2,2) | B | (5,8) | C | (8,5) |
| A' | (2,-2) | B' | (5,-8) | C' | (8,-5) |

$(x, -y)$

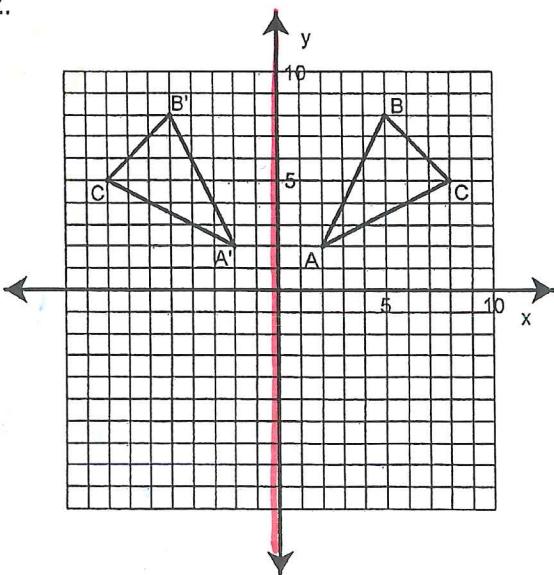
Equation of the line of reflection:

$$y = 0 \text{ (highlighted)}$$

Image formula for this reflection:

$$(x, y) \rightarrow (x, -y)$$

2.



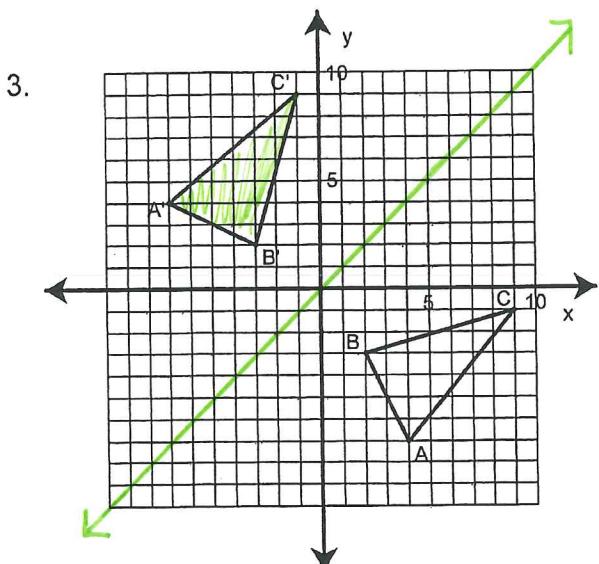
| | | | | | |
|----|--------|----|--------|----|--------|
| A | (2,2) | B | (5,8) | C | (8,5) |
| A' | (-2,2) | B' | (-5,8) | C' | (-8,5) |

Equation of the line of reflection:

$$x = 0$$

Image formula for this reflection:

$$(x, y) \rightarrow (-x, y)$$



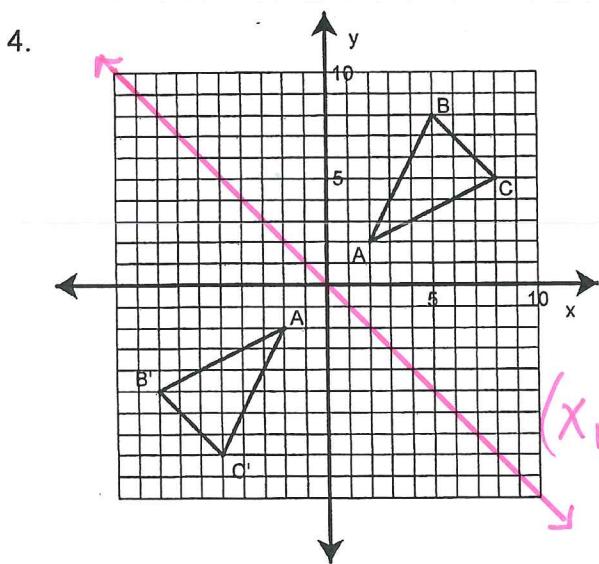
| | | | | | |
|----|---------|----|---------|----|---------|
| A | (4, -7) | B | (2, -3) | C | (9, -1) |
| A' | (-7, 4) | B' | (-3, 2) | C' | (-1, 9) |

Equation of the line of reflection:

$$x = y$$

Image formula for this reflection:

$$(x, y) \rightarrow (y, x)$$



| | | | | | |
|----|----------|----|----------|----|----------|
| A | (2, 2) | B | (5, 8) | C | (8, 5) |
| A' | (-2, -2) | B' | (-8, -5) | C' | (-5, -8) |

Equation of the line of reflection:

$$y = -x$$

Image formula for this reflection:

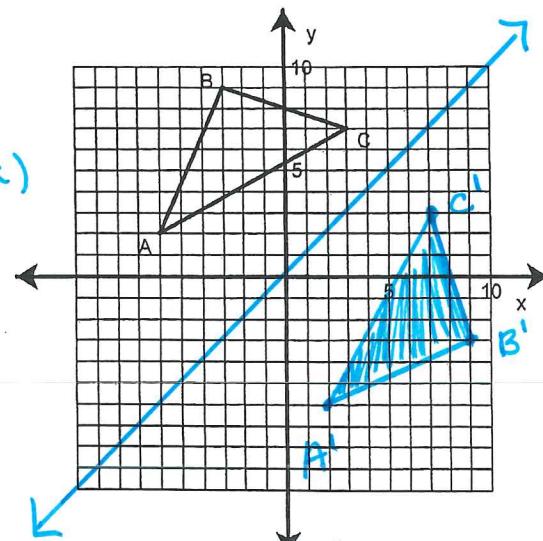
$$(x, y) \rightarrow (-y, -x)$$

5 – 6 Use the image formulas written in numbers 1 – 4 to find the new coordinates of $\triangle ABC$. Then graph the new triangle ($\triangle A'B'C'$).

5. Equation of the line of reflection: $y = x$

Image formula for this reflection: $(x, y) \rightarrow (y, x)$

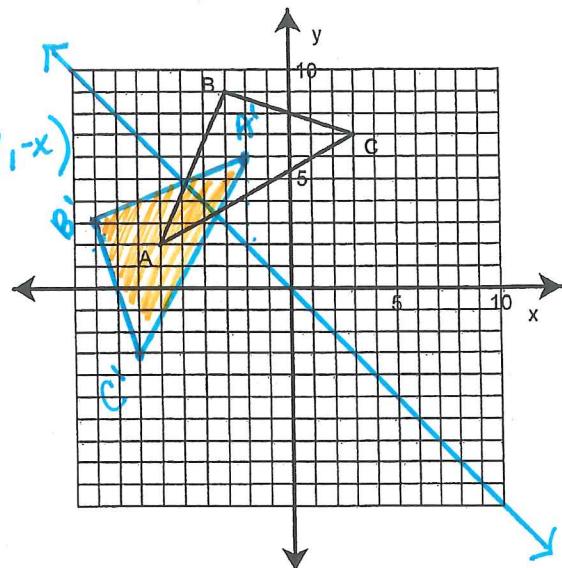
| | | | | | |
|----|---------|----|---------|----|--------|
| A | (-6, 2) | B | (-3, 9) | C | (3, 7) |
| A' | (2, -6) | B' | (9, -3) | C' | (7, 3) |



6. Equation of the line of reflection: $y = -x$

Image formula for this reflection: $(x, y) \rightarrow (-y, -x)$

| | | | | | |
|----|---------|----|---------|----|----------|
| A | (-6, 2) | B | (-3, 9) | C | (3, 7) |
| A' | (-2, 6) | B' | (-9, 3) | C' | (-7, -3) |



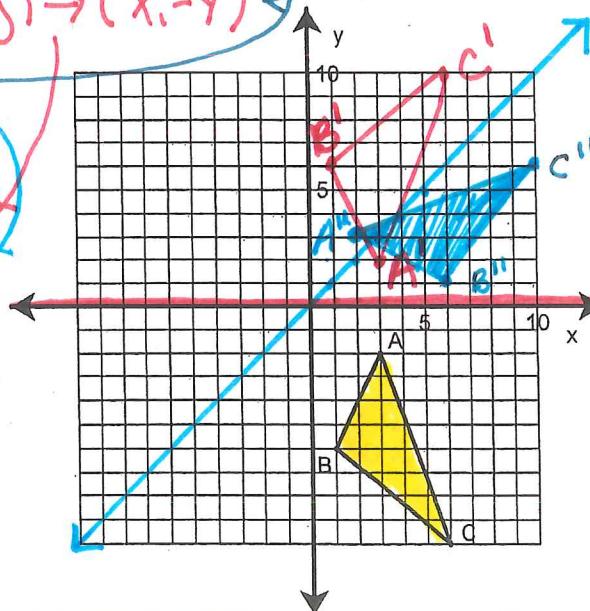
7 – 8 Use the image formulas written in numbers 1 – 4 to first find the coordinates of $\Delta A'B'C'$ and then to find the coordinates of $\Delta A''B''C''$ for the given composites. Then graph ONLY $\Delta A''B''C''$. Also graph the two lines of reflection.

Algebra
 2nd 1st
 ↓ ↓
 Composite: $(r_{y=x} \circ r_{y=0})(\Delta ABC) = R_{y=x}(R_{y=0})(\Delta ABC)$ Just follow directions below

Equation of the first line of reflection: $y = 0$ $(x, y) \rightarrow (x, -y)$

Equation of the second line of reflection: $y = x$ $(x, y) \rightarrow (y, x)$

| | | | | | |
|-----|---------|-----|---------|-----|----------|
| A | (3, -2) | B | (1, -6) | C | (6, -10) |
| A' | (3, 2) | B' | (1, 6) | C' | (6, 10) |
| A'' | (2, 3) | B'' | (6, 1) | C'' | (10, 6) |



What transformation occurred from this composite? In other words, what transformation would transform ΔABC to $\Delta A''B''C''$ without using any reflections?

- * Rotation
- * 90° counter clockwise
- * about the origin

Give an image formula for this transformation.

$$A(3, -2) \quad B(1, -6) \quad C(6, -10)$$

$$A''(2, 3) \quad B''(6, 1) \quad C''(10, 6)$$

$$(x, y) \rightarrow (-y, x)$$

$$(-y, x)$$

↓
 2nd
 do 1st

8. Composite: $r_{y=0}(r_{y=x}(\Delta ABC))$

Equation of the first line of reflection: $y = x$ $(x, y) \rightarrow (y, x)$

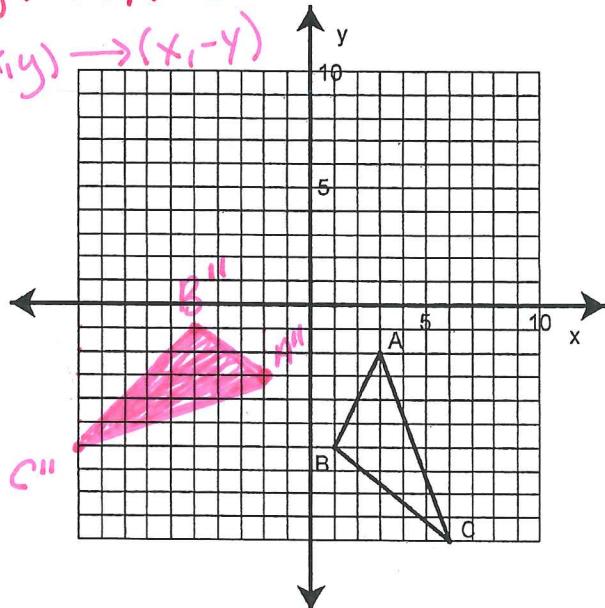
Equation of the second line of reflection: $y = 0$ $(x, y) \rightarrow (x, -y)$

| | | | | | |
|-----|----------|-----|----------|-----|-----------|
| A | (3, -2) | B | (1, -6) | C | (6, -10) |
| A' | (-2, 3) | B' | (-6, 1) | C' | (-10, 6) |
| A'' | (-2, -3) | B'' | (-6, -1) | C'' | (-10, -6) |

What transformation occurred from this composite? In other words, what transformation would transform ΔABC to $\Delta A''B''C''$ without using any reflections?

- * Rotation
- * 90° clockwise
- * around origin

Give an image formula for this transformation.



$$(x, y) \rightarrow (y, -x)$$

Based on $A(3, -2)$
 $A''(-2, -3)$

9. Notice in numbers 7 and 8, the same two lines of reflection were used, however the image triangle is located in a different position. Make a conjecture on what you think makes the difference.

Changing the order in which you reflect, changes the direction of the rotation.

11 Composite: $r_{x=0}(r_{y=0}(\Delta ABC))$

Equation of the first line of reflection: $y=0 \quad (x, y) \rightarrow (x, -y)$

Equation of the second line of reflection: $x=0 \quad (x, y) \rightarrow (-x, y)$

| | | | | | |
|-----|---------|-----|---------|-----|----------|
| A | (3, -2) | B | (1, -6) | C | (6, -10) |
| A' | (3, 2) | B' | (1, 6) | C' | (6, 10) |
| A'' | (-3, 2) | B'' | (-1, 6) | C'' | (-6, 10) |

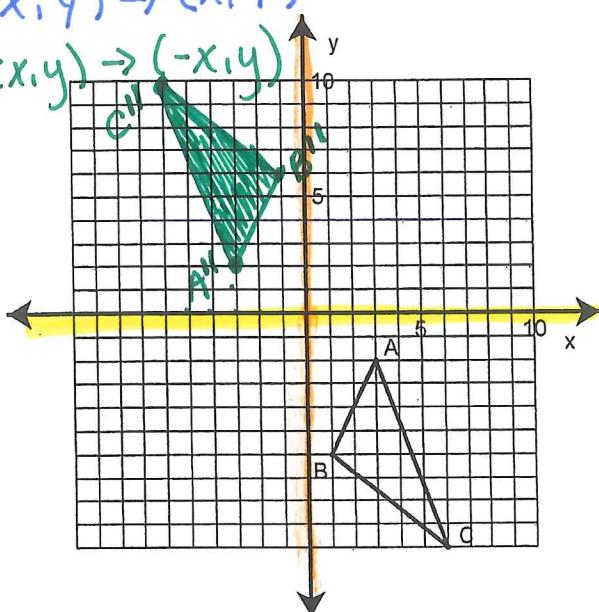
What transformation occurred from this composite? In other words, what transformation would transform ΔABC to $\Delta A''B''C''$ without using any reflections?

Rotation

180° clockwise AND counter clockwise about the origin

Give an image formula for this transformation.

$$\begin{aligned} A(3, -2) &\longrightarrow A''(-3, 2) \\ (x, y) &\longrightarrow (-x, -y) \end{aligned}$$



12. Notice in numbers 10 and 11, the same transformation occurred but different lines were used. Make a conjecture on what you think must be true about the lines used as lines of reflection.

It is a reflection over 2 lines which are \perp . It did not matter what order they needed to be.

10 - 11 Use the image formulas written in numbers 1 – 4 to first find the coordinates of $\Delta A'B'C'$ and then to find the coordinates of $\Delta A''B''C''$ for the given composites. Then graph ONLY $\Delta A''B''C''$.

10. Composite: $(r_{y=x} \circ r_{y=-x})(\Delta ABC)$

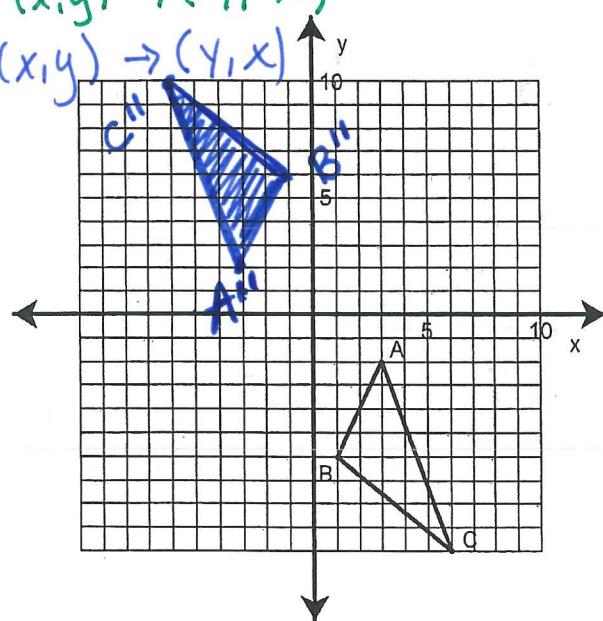
Equation of the first line of reflection: $y = -x$ $(x,y) \rightarrow (-y,-x)$

Equation of the second line of reflection: $y = x$ $(x,y) \rightarrow (y,x)$

| | | | | | |
|-----|---------|-----|----------|-----|-----------|
| A | (3, -2) | B | (1, -6) | C | (6, -10) |
| A' | (-2, 3) | B' | (-1, 6) | C' | (-6, 10) |
| A'' | (-3, 2) | B'' | (-1, -6) | C'' | (-6, -10) |

What transformation occurred from this composite? In other words, what transformation would transform ΔABC to $\Delta A''B''C''$ without using any reflections?

180° Rotation around the origin



Give an image formula for this transformation.

$$A(3, -2) \longrightarrow A''(-3, 2)$$

$$(x, y) \longrightarrow (-x, -y)$$