# Solving Portfolio Optimization Problems using AMPL



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**Abstract**

This work presents a new optimization software library which contains a number of financial optimization models. Roughly speaking, the majority of these portfolio allocation models aim to compute the optimal allocation investment weights, and thus they have important applications in stock exchange. Algebraic modeling languages are very well suited for prototyping and developing optimization models. Thus, our proposed financial optimization software package utilizes the flexibility and convenience of the AMPL mathematical programming modeling language. All the financial optimization models have been implemented in AMPL and solved using either Gurobi Optimizer v6.02 or MINOS v5.51 (for those models having general nonlinear objectives). This proposed software library includes several well known portfolio allocation models such as the Markowitz mean variance model, the Konno-Yamazaki absolute deviation model, the Black-Litterman model that aim either to minimize the variance of the portfolios, or maximize the expected returns subject to a number of restrictions, portfolios with a risk-free asset, transaction costs, and others. Furthermore, we also present a literature review of financial optimization software packages and discuss about the benefits and drawbacks of our proposed portfolio allocation model library. Since this is a work in progress, new models are still being added to the proposed library.



**KEYWORDS**

Financial Optimization, Mathematical programming, AMPL.


## 1. INTRODUCTION

In today's complex environment of globalization, constantly increased competition, liberalization of markets and rapid developments in the international economic environment, the use of technology in the fields of information and finance is of vital importance for the quick and effective confrontation of the risks raised by the participation in economic affairs. Therefore the research field of financial optimization utilizes the capabilities of the information technology in order to ensure the investor's portfolio assets.

Due to the reason of the risk in participating in economic-financial affairs various researchers (scientists, economists) tackle these problems by developing mathematical models for financial optimization. The pioneer work by Harry Markowitz in the financial optimization has lead to the most well-known mathematical model of mean-variance optimization (1959) [Markowitz H., 1952] [Markowitz H., 1959] in this research field. A number of mathematical models have been proposed later that were based on the classic Markowitz model. More specifically the variance of the returns of the portfolio's stocks is a measure of risk so the majority of these mathematical models aim to minimize the variance (risk).

Section 2 presents a literature review concerning financial optimization software packages as also the motivation of this research work. Following, in section 3 there is a brief list of all the financial optimization models that are currently implemented in our optimization software library. Implementation details and an example of a financial optimization model in AMPL is given in subsection 3.1. Finally, the last Section concludes our work and some future research directions are discussed.

## 2. LITERATURE REVIEW AND MOTIVATION

There are several optimization software packages for the solution of financial optimization problems. Apart from their price all these financial optimization software packages differ on the number of the mathematical models for financial optimization, in the variety of produced reports, in the speed of the procedure, in its reliability, etc. A list of some of the most well know financial optimization software and packages is briefly presented in this section:

**Microsoft Excel**[1]: To start with Microsoft Excel, it is one of the simplest software for financial optimization. Using the Solver tool we can define the cells which contain the optimal stock allocations by minimizing or maximizing a specific cell which contains our objective function, and also add the necessary constraints. Consequently we have to load data tables of the, e.g., stock returns or covariance matrixes to make the above computations. Although, Microsoft Excel does not have any specific financial optimization package pre-installed, the commercial Hoadley Portfolio Optimizer add-in for Microsoft Excel already exists in the market.
**R**[2]: A number of packages implemented in the R programming language abbreviate the writing of the code for financial optimization models. Some of these packages are the StockPortfolio which is used to download stock data and the Quadprog package (for the solution of quadratic programming problems) which can be used for finding the optimal portfolio.
**LINGO**[3]: The LINGO software by LINDO, is a very useful tool to solve linear and non linear optimization problems. Since Lingo is also a general-purpose mathematical optimization language it can be used for financial problems as well. Interested readers may find online at the web site of Lindo company, a collection of several implementations of financial optimization packages, such as some variations of the classical Markowitz mean-variance model plus other models like factor model, maximizing Sharpe ratio and VaR model.
**Matlab**[4]: Matlab is one of the most well-known commercial optimization languages, which contains the financial toolbox providing us with tools for portfolio optimization, mean-variance portfolio optimization, conditional Value at Risk, portfolio optimization, mean absolute deviation portfolio optimization and portfolio analysis.
**SmartFolio 3**[5]: SmartFolio is a user-friendly analytical tool for assisting inventors regarding their portfolio. It gives the user the ability to manage data and historical returns of the stocks by using four optimization criteria of maximization of an expected utility, minimization of target shortfall probability, maximization of Sharpe ratio and benchmarking. Another interesting feature is that it supports the robust portfolio optimization where in scenario-based optimization, under the worst case scenario the resultant portfolios demonstrate optimal behavior.

Despite the fact that there is a large number of efficient financial optimization software packages, more research efforts are still required in order to develop a more user-friendly non-commercial software package. This is the motivation of this research work; to develop a software library with a large number of the most well-known financial optimization models. The benefits of the proposed financial optimization software library are: first, it utilizes the rich features of the AMPL modeling language (Fourer et al., 2002), second the code of the financial optimization models can be easily extended, and third that the majority of the operational research (OR) scientists are more familiar to general-purpose optimization modeling languages, rather than other programming languages. However, this is a work in progress so new models may be implemented to enrich the existing library.

## 3. Financial optimization mathematical models

This software optimization library [table 1] consists of ten models. More specifically it consists of the most well-known Markowitz's mean-variance model [Markowitz H., 1991] [Steinbach M. C., 2001] which aims to

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[1] http://www.solver.com
[2] http://www.r-project.org
[3] http://www.lingo.com
[4] http://www.mathworks.com
[5] http://www.smartfolio.com

minimize the risk of the portfolio subject to a number of restrictions. In this case, the covariance between the stocks is the measure of risk that has to be minimized. Furthermore there are some variations of the classic Markowitz model such as the Markowitz's mean-variance model with upper bound, the Markowitz's mean-variance model with a risky-free asset and the Markowitz's mean-variance model with transaction costs. Additionally, the Sharpe model [Sharpe W.F., 1989], [Sharpe W. F., 1992], and [Sharpe W. F., 1994] aims to maximize the portfolio's Sharpe ratio subject to budget constraint, and the factor model which minimizes the risk. Furthermore, the formulation of the Black Litermans' model [Black F. and Litterman R., 1992] is exactly the same as that of Markowitz classic mean-variance model, however it also considers the market equilibrium in combination with investor's view. Moreover, the Konno-Yamazaki mean absolute deviation (MAD) model [H. Konno and H. Yamazaki, 1991] aims to minimize the mean absolute deviation (hence the name) subject to a number of restrictions. Finally, the Value at Risk model (VaR) [Rockafellar R. T. and Uryasev S., 2000] aims to maximize the value that is not at risk.

Table 1 Library

| **Name** | **Aim** | **Objective** |
|---|---|---|
| Markowitz(MVO) | Minimize | Risk |
| Markowitz upper bound | Minimize | Risk |
| Markowitz with Risk free Asset | Minimize | Risk |
| Markowitz with Transaction costs | Minimize | Risk |
| Sharpe | Maximize | Sharpe Ratio |
| Factor | Minimize | Risk |
| Black-Litterman | Minimize | Risk |
| Konno-Yamazaki(MAD) | Minimize | Mean absolute deviation |
| Value at Risk (VaR) | Maximize | Value not at risk |
| Young | Maximize | Minimum Returns |

## 3.1 **Young's Minimax model**

In 1998 Martin Young introduced the Minimax model in portfolio optimization (Young M.R., 1998). The central idea of the model is based in the minimax formulation of game theory, so the objective function is to maximize the minimum returns of the portfolio subject to some restrictions.

$$\max_{Mp} M_p$$

subject to

$$\sum_{j=1}^{N} w_j \bar{y}_j \geq G$$

$$\sum_{j=1}^{N} w_j \leq W$$

$$w_j \geq 0,\ j = 1, \ldots, N$$

where $y_{jt}$ = Return on one dollar invested in stock $j$ in time period $t$

$\bar{y}_j$ = Average Return on secuirity $j$ $= \dfrac{1}{T}\sum_{t=1}^{T} y_{jt}$

$x_j$ = Portfolio allocation to stock $j$

$y_{pt}$ = Return on portfolio in time period $t$ $= \sum_{j=1}^{N} x_j y_{jt}$

$M_p$ = Minimum Return on portfolio $= \min_t y_{pt}$

$W$ = Budget

$H$ = Target Return

**Example**: Given the Historical returns for each one of the 12 months for three stocks (A, B, C), Young's minimax model aims to maximize the minimum returns of the portfolio. More specifically the expected returns of every one of the three stocks is the arithmetic average of the historic returns for all 12 months. The minimum returns is the result of subtraction of standard deviation of stock[j] from the expected return of stock[j]. Therefore according to Young, this quantity has to be maximized provided that:

- The sum of the expected returns is equal or exceeds our target return
- The sum of the whole allocation in every stock does not exceed our budget
- The maximum allocation in a single stock does not exceed our threshold
- The minimum allocation in a single stock is 0

The code of the model, the script file and a part of the optimal solution computed by Gurobi solver, is demonstrated below:

Table 2 Example of the Young's minimax AMPL model file

| **AMPL model file** | **AMPL data file** |
|---|---|
| param NStocks > 0; # Number of Stocks<br>param T > 0; # Number of Months<br>param Budget; # Our budget<br>param RetMat{1..T, 1..NStocks}; # Historic monthly returns for five selected shares over one year<br>param upbound; # Upper limit for investing in a single share<br>param TargetRet; # Target return of the portfolio<br>param Mp{1..NStocks}; # Minimum portfolio<br>param ExpRet{1..NStocks}; # Expected returns of stocks<br>param stdv{1..NStocks}; # Standard deviation of stocks<br><br>var x{1..NStocks} >=0;<br><br>maximize MinimumReturn:<br>sum {j in 1..NStocks} x[j]*Mp[j];<br><br>subject to TargetReturn:<br>sum {j in 1..NStocks} ExpRet[j]*x[j] >= TargetRet;<br><br>subject to FullBudget:<br>sum {j in 1..NStocks} x[j] = Budget;<br><br>subject to bounds {j in 1..NStocks}:<br>0 <= x[j] <= upbound; | param T := 12;<br>param NStocks := 3;<br>param upbound := 70000;<br>param Budget := 85000;<br>param TargetRet := 2500;<br><br>param RetMat:<br>1 2 3 :=<br>1 0.053 -0.043 0.064<br>2 0.062 -0.018 0.049<br>3 0.043 0.047 0.045<br>4 -0.039 0.036 0.037<br>5 0.025 0.025 0.04<br>6 0.065 0.034 -0.017<br>7 0.04 0.032 0.034<br>8 0.02 0.04 0.048<br>9 0.052 0.025 -0.043<br>10 0.032 0.052 0.036<br>11 0.032 0.04 0.052<br>12 0.064 0.017 0.038; |

| AMPL script file |
| --- |
| model youngmodel.mod;<br>data youngmodel.dat;<br><br>let {j in 1..NStocks} ExpRet[j] := sum{i in 1..T} RetMat[i,j]/T;<br>let {j in 1..NStocks} stdv[j] := sqrt((sum{i in 1..T} (RetMat[i,j]-ExpRet[j])^2)/T);<br>let {j in 1..NStocks} Mp[j] := ExpRet[j]-stdv[j];<br><br>option solver gurobi_ampl;<br>solve;<br>display x; |
| **Results** |
| >ampl youngmodel.run<br>Gurobi 6.0.2: optimal solution; objective 749.6787276<br>1 simplex iterations<br>x [*] :=<br>1 70000<br>2 0<br>3 15000<br>;<br><br>>Exit code: 0 |

The above Young's Minimax model as well and all the models included in our library have been implemented in AMPL. The majority of the models have linear objectives thus the Gurobi or CPLEX solver was used. However some more complex models due their non-linear objectives, force us to use other solvers such as KNITRO or Minos. Each one mathematical model consists of the model file, the data file and the script file. More specifically the model file consists of the definition of the parameters, variables and the mathematical formulation of the model, the data file contains the parameters that are going to be used and the script file reads those two files and solves the problem using one of the previously mentioned solvers.

## 4. CONCLUSIONS

Finance and decision making theory are among the most useful research fields in today's market. Thus, the focus of this work was to show that the combination of these two research fields, i.e., financial optimization has a plethora of interesting real-world applications. The proposed software library that contains these financial optimization models assists the investor to make the best decision for asset allocations in finance.

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