Advanced Algorithms - Problem set 8

Whenever you give an algorithm, also argue for its running time and correctness. Always feel free to ask on MS Teams if you're stuck, need a clarification, or suspect a typo.

1. Show that an algorithm with running time $n^{o(\sqrt{k})}$ for deciding the existence of a k-clique in an n-vertex graph refutes the exponential-time hypothesis ETH. (Hint: You can follow the proof shown in the lecture, but that proof can be simplified somewhat for this concrete running time.)

2. Path and tree decompositions:

   (a) Describe a path decomposition of width $n-1$ for the perfect binary tree on n levels. (In this tree, every non-leaf vertex has two children.)

   (b) Implement an algorithm that outputs a nice path decomposition of an $m \times n$ square grid. Recall that a nice path decomposition is a nice tree decomposition in which the underlying tree is a path. (We need this for the next exercises.)

3. Implement the DP algorithm for finding a maximum independent set in bounded-treewidth graphs and use it to find the maximum independent set size in a 30×10 square grid. Check whether the output matches your expectations.

4. Determine the number of independent sets in the 12×12 square grid. To this end, modify the DP algorithm for finding a maximum independent set in bounded-treewidth graphs so it doesn't find a maximum-weight independent set, but instead counts them. Here are some hints:

   - Weights are irrelevant in this setting.
   - In our application, you will not need join nodes. As a bonus, you can explain how to handle them.
   - You may need to import support for large integers. (Not in Python.)