1. Is it possible that an event is independent of itself? If so, when?

Ans:

Researchers and statisticians have found that the the only events that are independent of themselves are those events that occur with probability 0 Or with probability 1. This idea stems from the fact that 0 and 1 are the only probability values, that when squared, equal themselves.

For example, the event of flipping a coin and getting heads is independent of itself. This is because the probability of getting heads on one flip of the coin is 0.5, and the probability of getting heads on two flips of the coin is also 0.5.

Another example is the event of rolling a die and getting a 6. The probability of getting a 6 on one roll of the die is 1/6, and the probability of getting a 6 on two rolls of the die is also 1/6.

In general, any event that has a probability of 0 or 1 is independent of itself.

2. Is it always true that if A and B are independent events, then Ac and Bc are independent

events? Show that it is, or give a counterexample.

Ans:

Yes, it is true that if events A and B are independent, then their complements Ac and Bc are also independent. This property can be demonstrated using the definition of independence and some basic probability rules.

Definition of Independence:

Events A and B are independent if and only if the probability of their intersection is the product of their individual probabilities:

P(A ∩ B) = P(A) \* P(B)

Now let's consider the complements:

Ac: The complement of event A.

Bc: The complement of event B.

We want to show that if A and B are independent, then Ac and Bc are also independent:

P(Ac ∩ Bc) = P(Ac) \* P(Bc)

Using set theory and probability rules, we can rewrite the above expression:

P(Ac ∩ Bc) = 1 - P(A ∪ B) (by De Morgan's law)

= 1 - (P(A) + P(B) - P(A ∩ B)) (by the inclusion-exclusion principle)

= 1 - (P(A) + P(B) - P(A) \* P(B)) (since A and B are independent)

= 1 - P(A) - P(B) + P(A) \* P(B)

Now, let's calculate the individual probabilities on the right-hand side of the equation:

P(Ac) = 1 - P(A)

P(Bc) = 1 - P(B)

Substitute these values back into the expression:

P(Ac) \* P(Bc) = (1 - P(A)) \* (1 - P(B))

= 1 - P(A) - P(B) + P(A) \* P(B)

Comparing this result to our earlier calculation for P(Ac ∩ Bc), we see that they are the same:

P(Ac) \* P(Bc) = P(Ac ∩ Bc)

This demonstrates that if A and B are independent events, then their complements Ac and Bc are also independent events.