PS1_P3

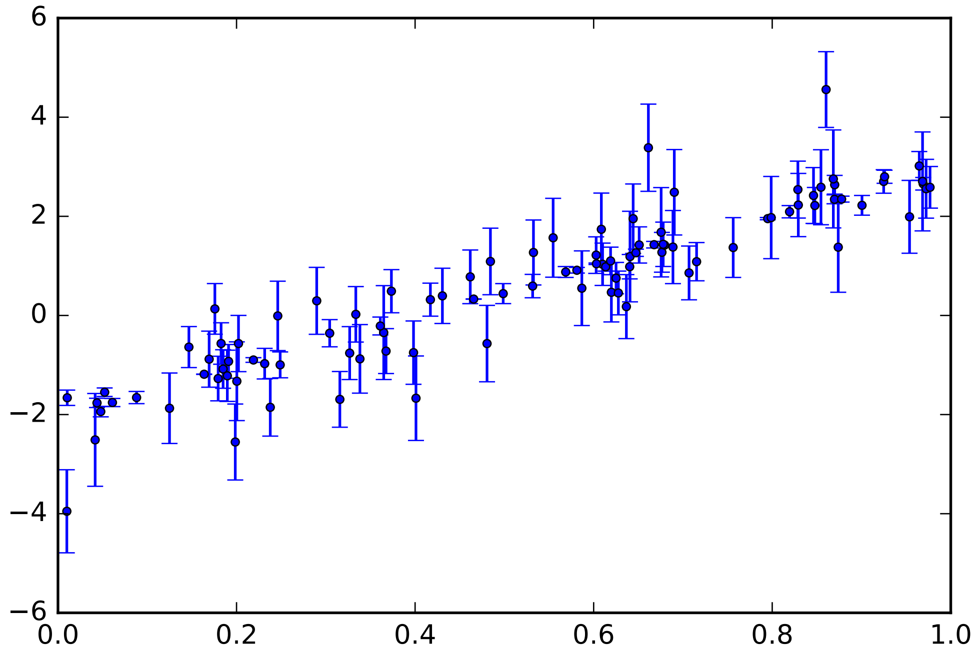
September 13, 2016

```
In [1]: %matplotlib inline
        import numpy as np
        import matplotlib.pyplot as plt
        from IPython.display import set_matplotlib_formats
        set_matplotlib_formats('png','pdf')
```

0.0.1 Problem 3: Linear Best-Fit Parameters

First, we generate some fake data with Gaussian noise, given linear parameters $\theta = (m, b) = (5.0, -2.0)$

```
In [465]: m,b = 5.0, -2.0

          def gen_data(num):
              xdat = 1.*np.random.rand(num)
              ydat = m*xdat+b
              sigma = np.random.rand(num)
              y_err = np.random.normal(0.0,1.*sigma,num)
              return np.stack((xdat,ydat+y_err,sigma),axis=-1)


          data = gen_data(100)
          plt.errorbar(data[:,0],data[:,1],yerr=data[:,2],fmt='o',markersize=3)

Out[465]: <Container object of 3 artists>
```



Next, we define the likelihood, proposition, and Markov chain functions as in Problem 2. The main differences are:

1.) We transform to log-space for the likelihood. The reason for this is, assuming Gaussian errors, the log probability becomes identical to chi-squared minimization and makes the math easier (see Hogg 2010).

2.) The proposition function becomes multivariate. We here opt to assign the same variances for m and b, which may not be correct.

In [412]:
```
#plt.errorbar(data[:,0],data[:,1],yerr=data[:,2],fmt='o',markersize=3)

def log_likelihood(theta,data):
    chi_sq = np.sum((data[:,1]-theta[0]*data[:,0]-theta[1])**2/(data[:,2]*data[:,2]))
    return -.5*chi_sq

def prop(theta_old,mc_params):
    mean = [0,0]
    cov = [[mc_params[0],0],[0,mc_params[0]]]
    theta_prop = theta_old + mc_params[1]*np.random.multivariate_normal(mean,cov,1)
    return theta_prop[0]

def new_state(t_old,t_prop,data):
    alpha = log_likelihood(t_prop,data)-log_likelihood(t_old,data)
    if alpha >= np.log(1.0):
        return t_prop
    else:
        if alpha >= np.log(np.random.rand(1)):
            return t_prop
    return t_old
```

Here we define the actual run function, where we pick an offset starting point $(3.0, -3.0)$ to test the code's capability to properly converge to the expected result.

This function returns a set containing the vector of parameters (dimensions $2xN$), the log likelihoods of the parameters, and the acceptance ratio.

```
In [419]: def run_mcmc(num_dat,num_theta,mc_params):
              dat = gen_data(num_dat)
              t_start = [3.0,-3.0]
              theta = np.array((t_start))
              ln_l = np.array((log_likelihood(t_start,dat)))

              theta_old = t_start
              theta_prop = t_start
              theta_new = t_start

              while np.size(ln_l) < num_theta:
                  theta_old = theta_new
                  theta_prop = prop(theta_old,mc_params)
                  theta_new = new_state(theta_old,theta_prop,dat)
                  theta = np.vstack((theta,theta_new))
                  ln_l = np.append(ln_l,log_likelihood(theta_new,dat))

              accept = np.size(np.unique(ln_l))/num_theta

              return [theta,ln_l,accept]
```

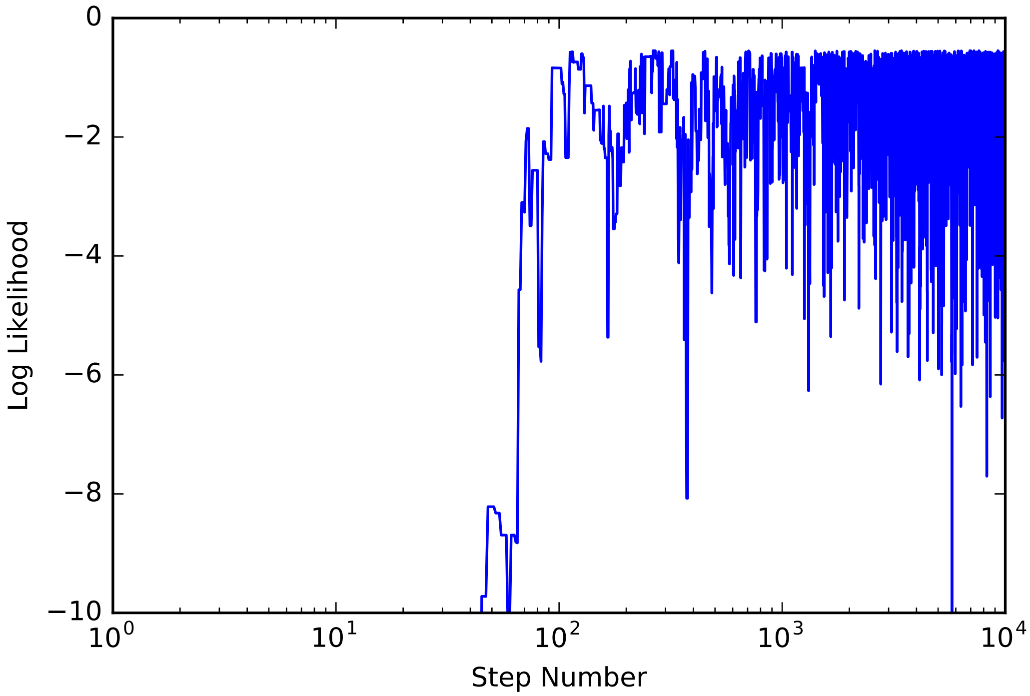
Run for 10 data points:

```
In [421]: run_10 = run_mcmc(10,10000,(2.0,0.1))
          print(run_10[2])
```

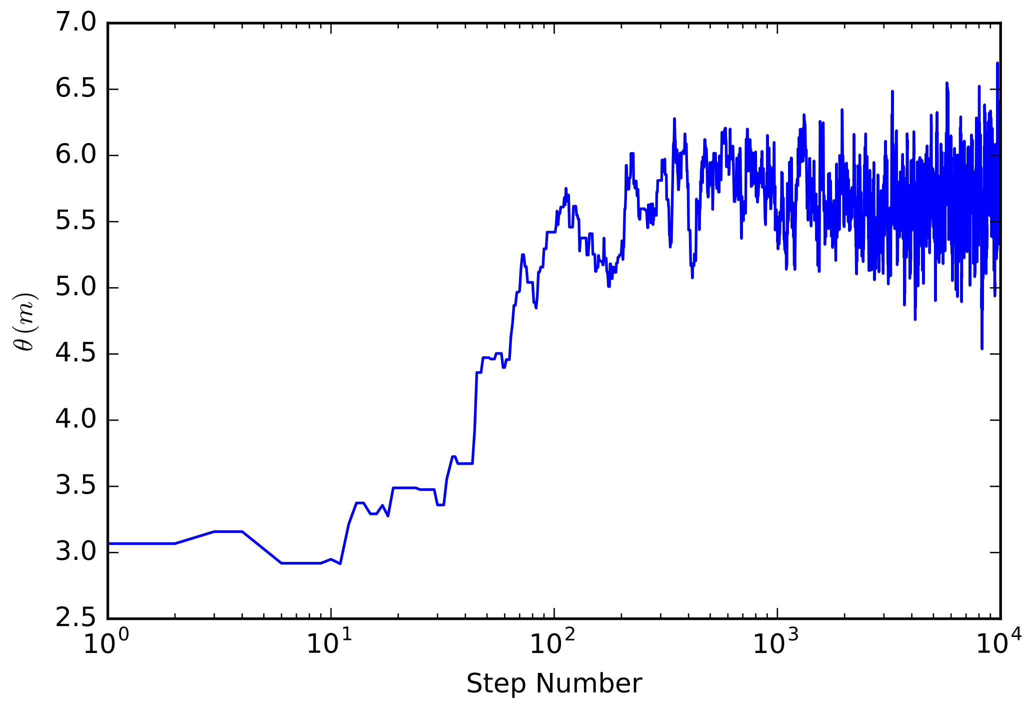
```
0.3681
```

```
In [466]: plt.plot(range(10000),2.5+run_10[1])
          plt.ylim(-10,0)
          plt.xscale('log')
          plt.xlabel("Step Number")
          plt.ylabel("Log Likelihood")
```

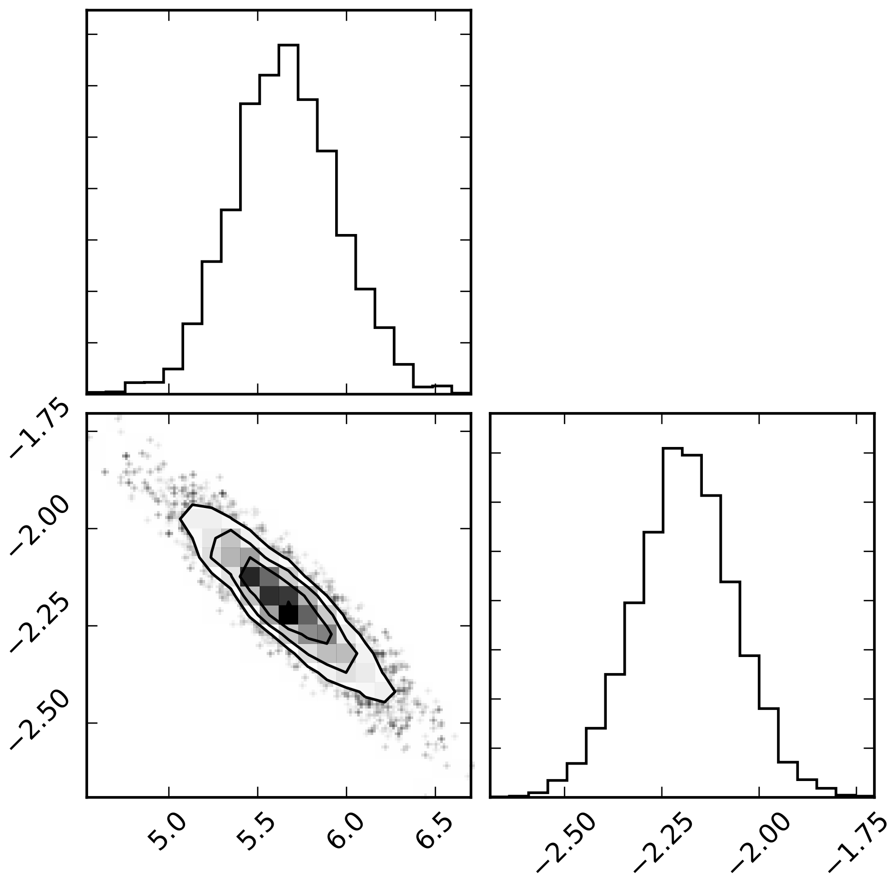
```
Out[466]: <matplotlib.text.Text at 0x115442208>
```



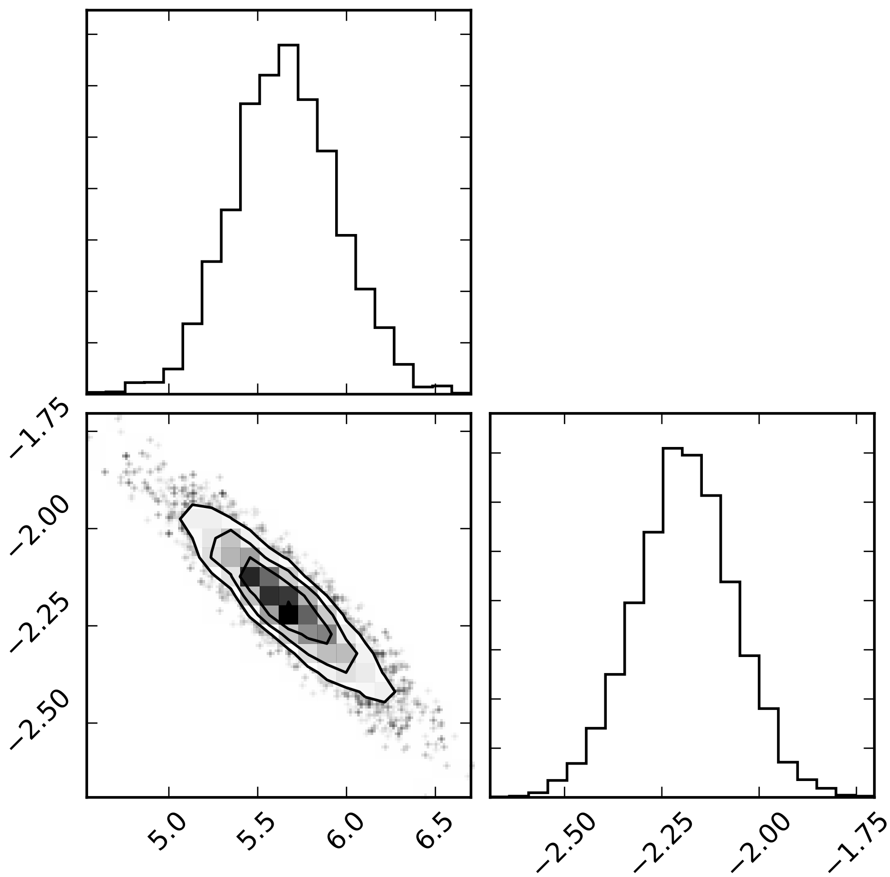

```
In [469]: plt.plot(range(10000),run_10[0][:,0])
          plt.xscale('log')
          plt.xlabel("Step Number")
          plt.ylabel(r'$\theta\,(m)$')

Out[469]: <matplotlib.text.Text at 0x1121d5ef0>
```




```
In [481]: import corner
          corner.corner(run_10[0][1000:9999,])

Out[481]:
```



−1.75
−2.00
−2.25
−2.50
5.0
5.5
6.0
6.5
−2.50
−2.25
−2.00
−1.75



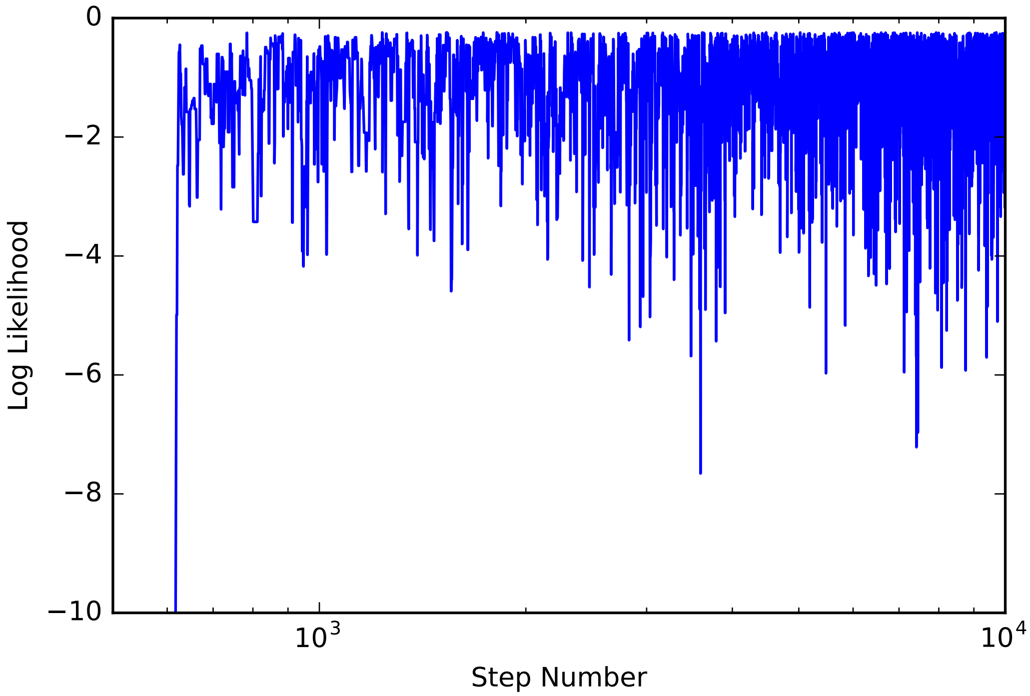
Run for 100 data points:

```
In [423]: run_100 = run_mcmc(100,10000,(1.0,0.0125))
          print(run_100[2])
```

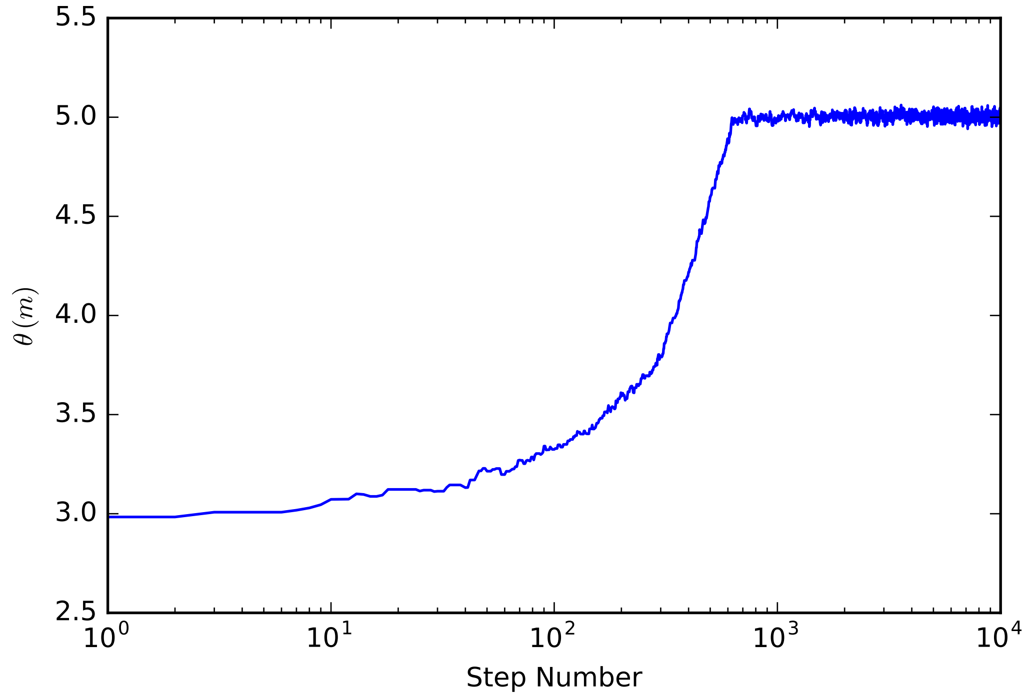
```
0.4185
```

```
In [470]: plt.plot(range(10000),60+run_100[1])
          plt.ylim(-10,0)
          plt.xscale('log')
          plt.xlim(5e2,1e4)
          plt.xlabel("Step Number")
          plt.ylabel("Log Likelihood")
```

```
Out[470]: <matplotlib.text.Text at 0x11343ca20>
```

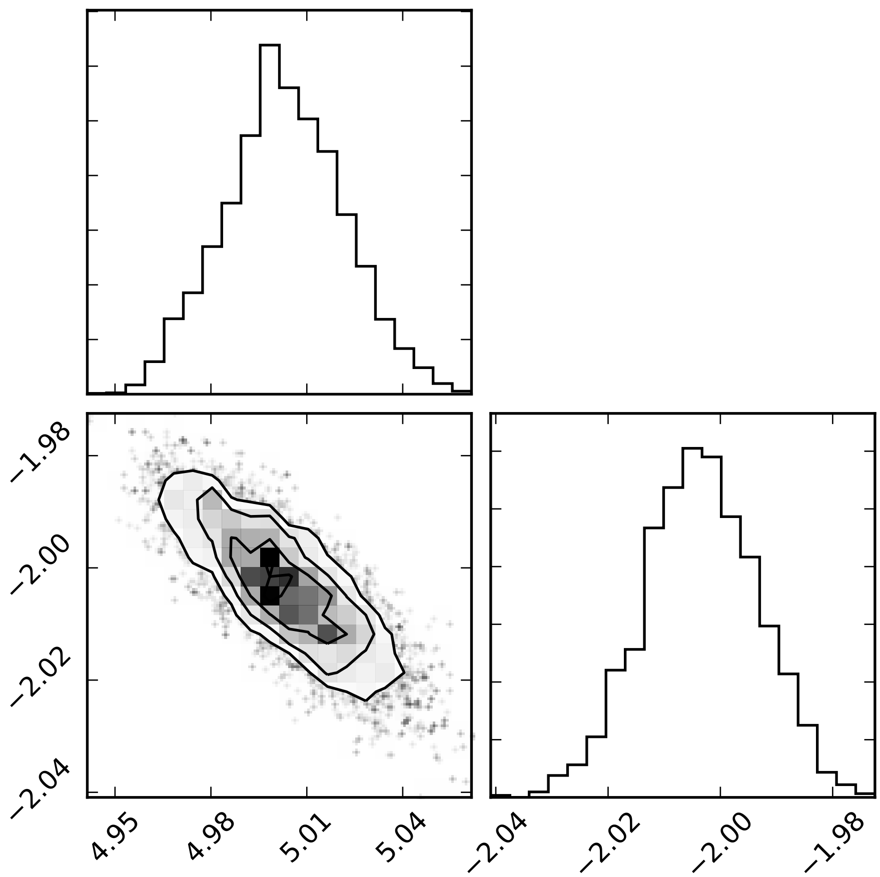


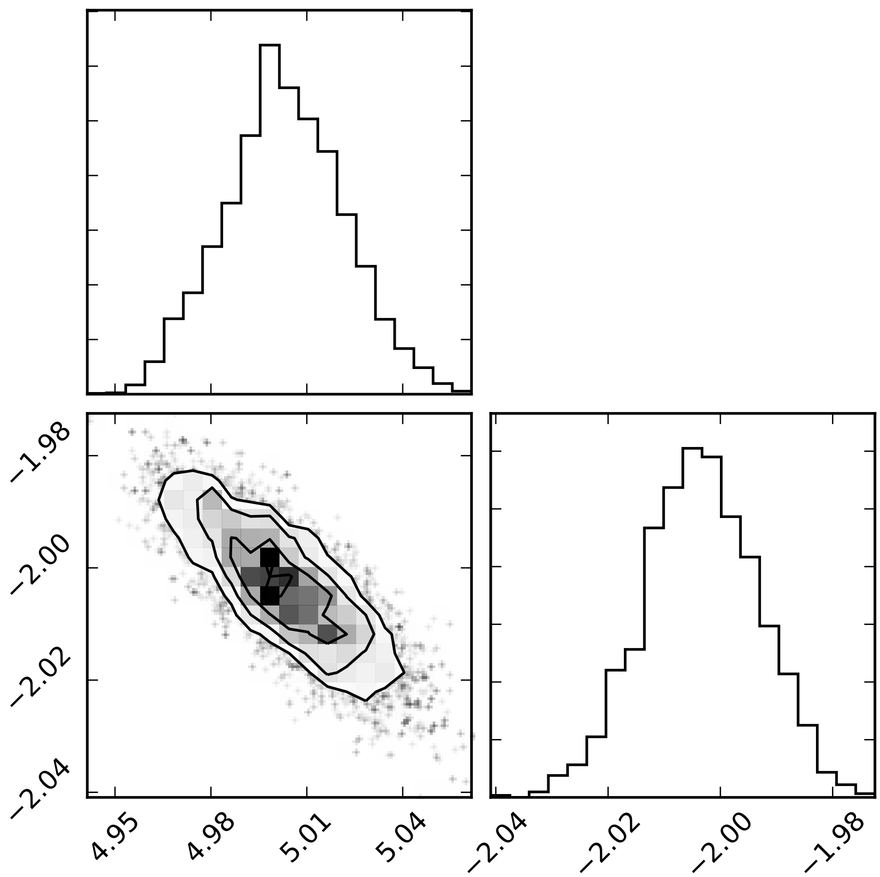

```
In [471]: plt.plot(range(10000),run_100[0][:,0])
          plt.xscale('log')
          plt.xlabel("Step Number")
          plt.ylabel(r'$\theta\,(m)$')

Out[471]: <matplotlib.text.Text at 0x1135e1dd8>
```




```
In [480]: corner.corner(run_100[0][1000:9999,])
```

Out[480]:


−1.98
−2.00
−2.02
−2.04
4.95
4.98
5.01
5.04
−2.04
−2.02
−2.00
−1.98



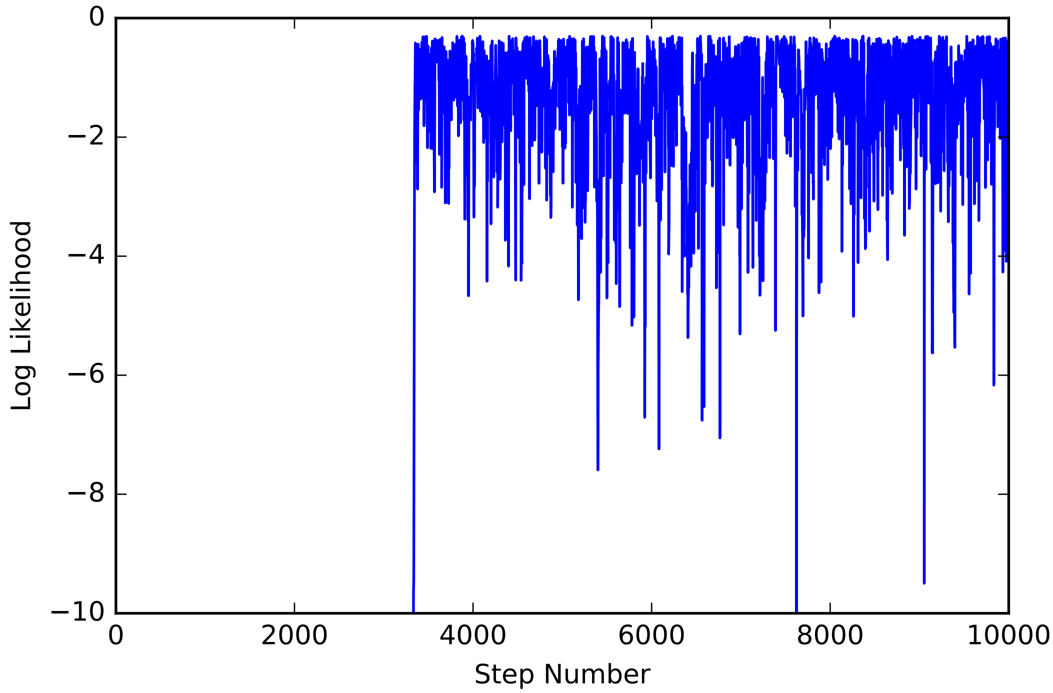
Run for 1000 data points:

```
In [438]: run_1000 = run_mcmc(1000,10000,(5.0,.001))
          print(run_1000[2])
```

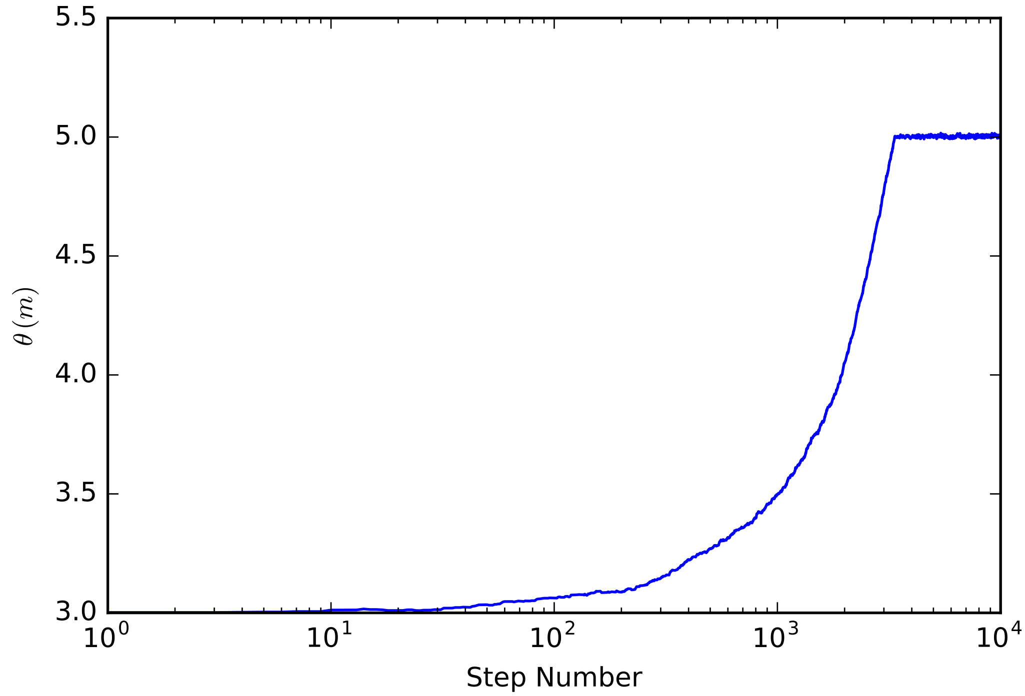
```
0.445
```

```
In [472]: plt.plot(range(10000),475+run_1000[1])
          plt.ylim(-10,0)
          plt.xlabel("Step Number")
          plt.ylabel("Log Likelihood")
```

```
Out[472]: <matplotlib.text.Text at 0x112b24f28>
```

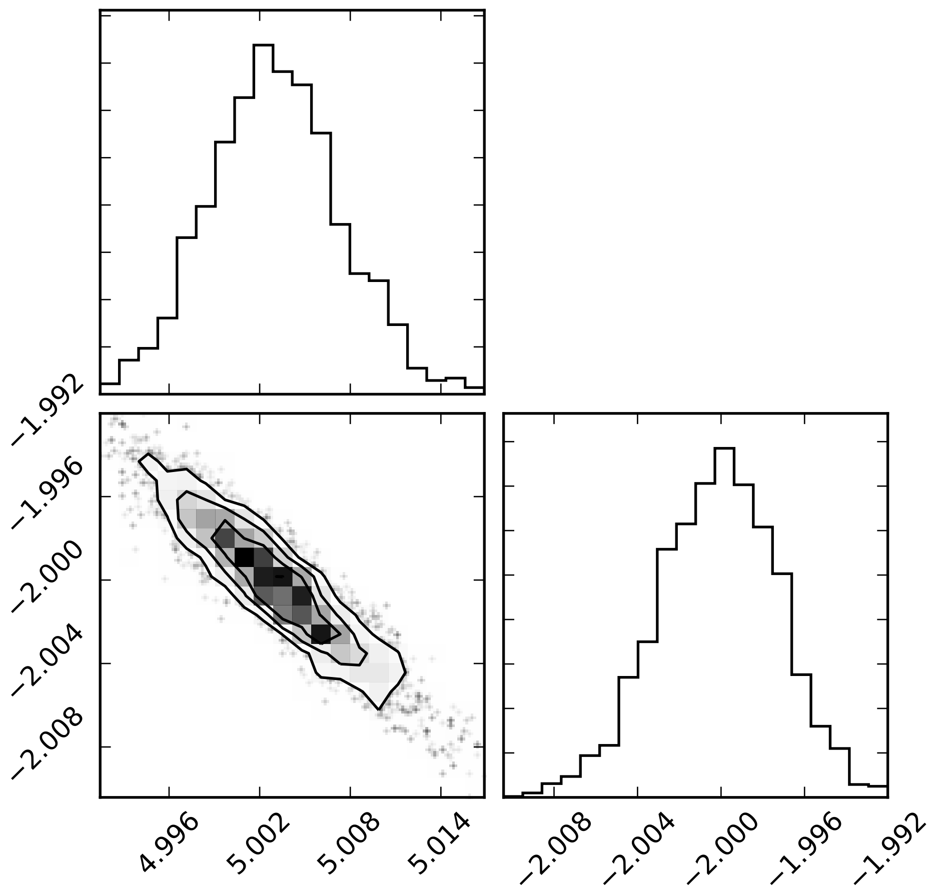



```
In [473]: plt.plot(range(10000),run_1000[0][:,0])
          plt.xscale('log')
          plt.xlabel("Step Number")
          plt.ylabel(r'$\theta\,(m)$')

Out[473]: <matplotlib.text.Text at 0x11537b748>
```

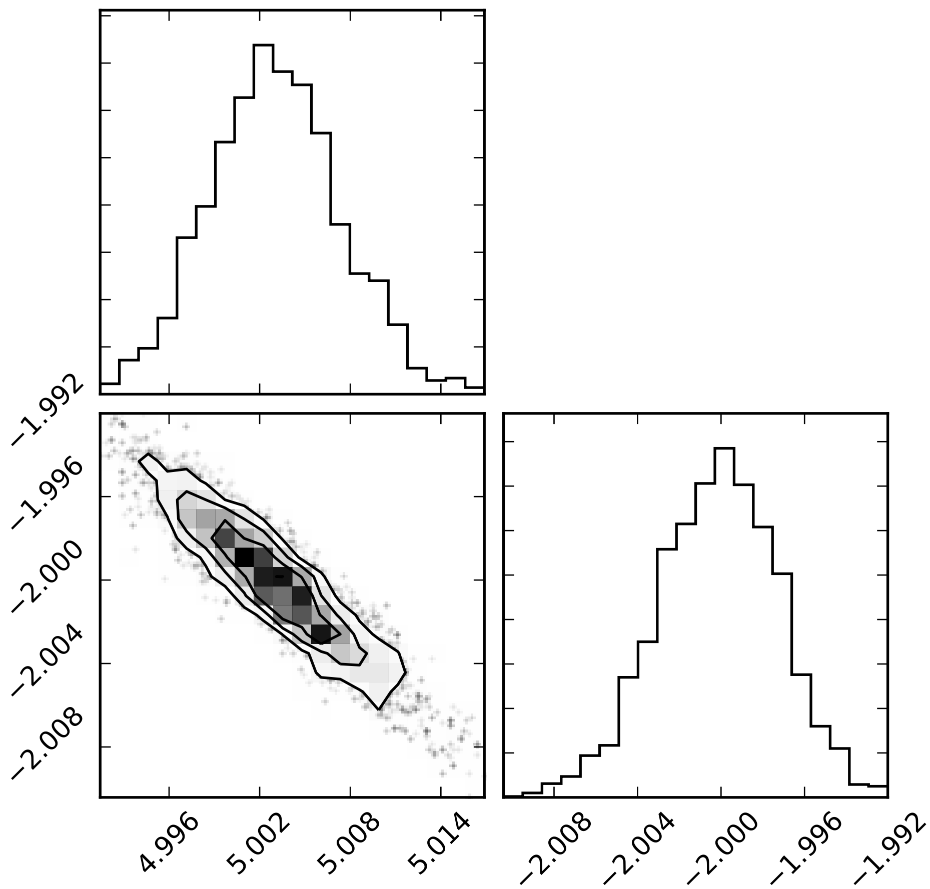



```
In [484]: corner.corner(run_1000[0][4000:9999,])
```

```
Out[484]:
```



−1.992
−1.996
−2.000
−2.004
−2.008
4.996
5.002
5.008
5.014
−2.008
−2.004
−2.000
−1.996
−1.992




```
In [485]: import emcee

In [498]: ndim, nwalkers = 2, 6
          p0 = [np.random.rand(ndim) for i in range(nwalkers)]

In [499]: data = gen_data(100)

          def lnprob(theta):
              return log_likelihood(theta,data)

In [502]: sampler = emcee.EnsembleSampler(nwalkers,ndim,lnprob)
          pos, prob, state = sampler.run_mcmc(p0,1000)
          sampler.reset()
          sampler.run_mcmc(p0,8000)

Out[502]: (array([[ 5.03319632, -2.0158658 ],
                  [ 5.03664352, -2.01525742],
                  [ 5.00926287, -2.00181753],
                  [ 5.07036828, -2.0214412 ],
                  [ 4.97610477, -2.00032446],
                  [ 5.01626723, -1.99940283]]),
           array([-39.40844419, -38.90450013, -38.87164836, -39.09543152,
```

```
      -41.23293594, -39.9099519 ]),
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2320261930, 1583615904, 1331016175, 3934564216], dtype=uint32), 487, 0, 0.0))
```
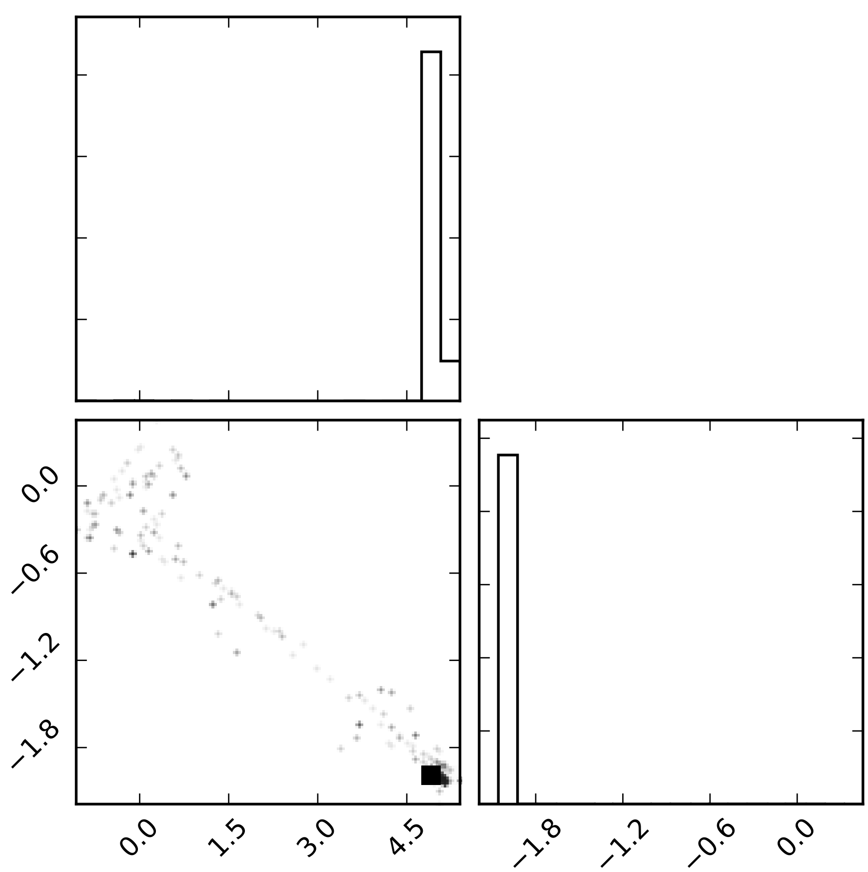
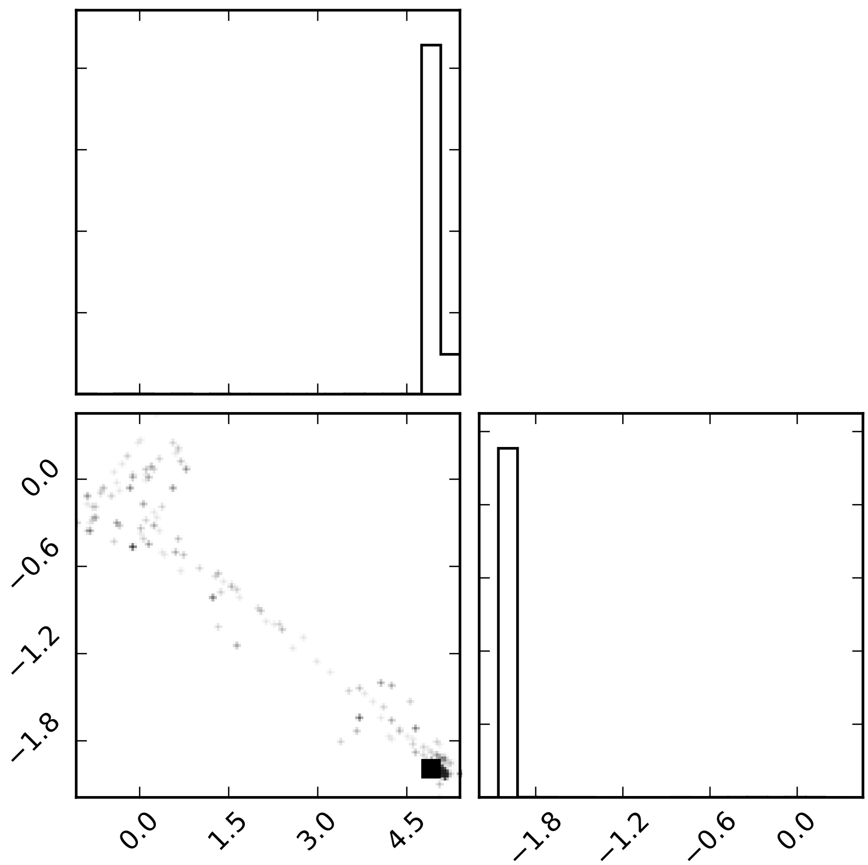

In [503]:
```
import corner
corner.corner(sampler.flatchain)
```

Out[503]:






```
In [ ]:
```