PS1_P2

September 11, 2016

```
In [152]: %matplotlib inline
          import numpy as np
          import matplotlib.pyplot as plt
          from IPython.display import set_matplotlib_formats
          set_matplotlib_formats('png','pdf')
```

0.1 Problem 2: MCMC 1-d Gaussian

We test the Metropolis-Hastings MCMC algorithm here to reproduce a simple 1-d Gaussian with mean μ and variance σ^2.

First, we set those parameters below:

```
In [165]: mu = 5.0
          sigma = 1.0
          params = np.array((mu,sigma*sigma)) #stores the variance rather than std. dev.
```

We then define a likelihood function $P(\theta)$. Specifically, the likelihood function in this case is a normalized Gaussian $P \sim G(\mu, \sigma)$:

$$P(\theta) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(\theta-\mu)^2/2\sigma^2} \tag{1}$$

Since flat priors are assumed, and since the likelihood is normalized, the likelihood is also therefore the properly normalized posterior.

```
In [166]: def likelihood_func(theta):
              p = 1./np.sqrt(2*np.pi*params[1])*np.exp(-.5*(theta-params[0])*(theta-params[0])/params[1
              return p
```

We next define a proposal density function for the next chain element:

$$p(\theta_i) \rightarrow p(\theta_{i+1}) \tag{2}$$

We here choose a Gaussian as well, with mean 0 and variance σ_p^2. In addition, we add another parameter A which increases the step size :

$$Q(\theta_i, \theta_{i+1}) \sim \theta_i + A \cdot G(0, \sigma_p) \tag{3}$$

```
In [155]: def prop_func(t_old,mu_prop,sigma_prop,step_size):
              t_prop = t_old + step_size*np.random.normal(mu_prop,sigma_prop,1)
              return t_prop
```

We are given $(\mu, \sigma) = (5.0, 1.0)$ and want to choose appropriate step sizes to achieve an acceptance fraction $0.25 \lesssim f \lesssim 0.5$.

```
In [331]: t_start = 0.0
          mu_p = 0.0
          sig_p = 1.0
          step_size = 2.5
          def prop(t_old):
              return prop_func(t_old,mu_p,sig_p,step_size)
```

We then define the Metropolis-Hastings component of M-H MCMC. Given a proposal state θ_{i+1}, we define the acceptance fraction α as

$$\alpha(\theta_i, \theta_{i+1}) \equiv \frac{P(\theta_{i+1})}{P(\theta_i)} \tag{4}$$

We accept under two conditions. First, if $\alpha \geq 1$, we "accept" the proposed state as the new state. If $\alpha < 1$, we accept the proposed state θ_{i+1} with probability α, as sampled from a uniform distribution in $[0, 1]$. Otherwise, we add the previous state θ_i to the chain.

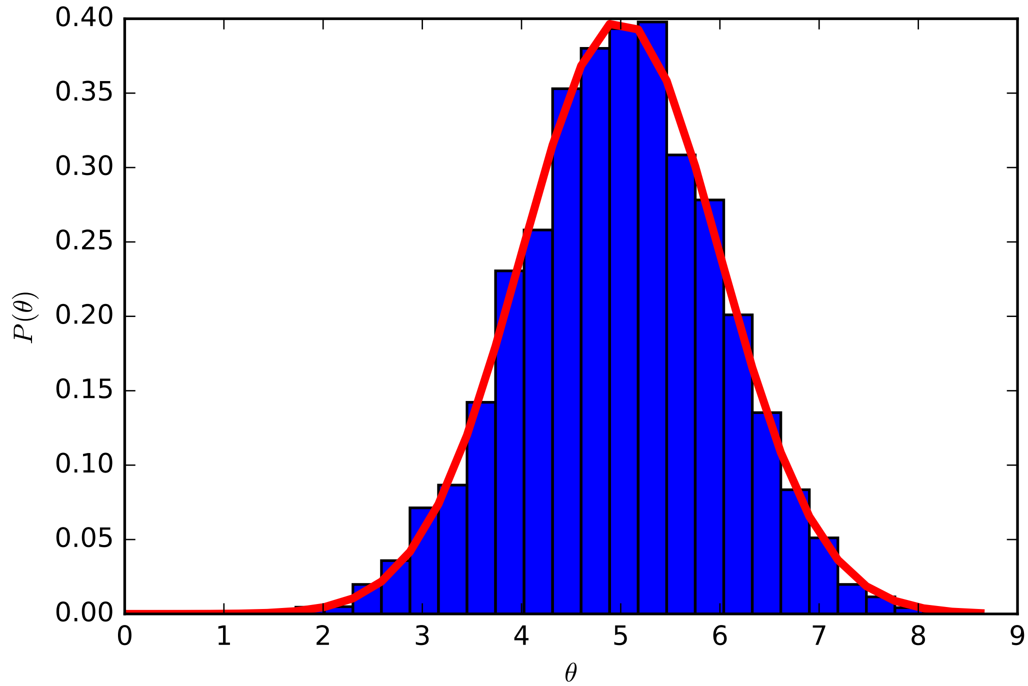
```
In [308]: def new_state(t_old,t_prop):
              alpha = likelihood_func(t_prop)/likelihood_func(t_old)
              if alpha >= 1.0:
                  return t_prop
              else:
                  if alpha >= np.random.rand(1):
                      return t_prop
              return t_old
```

Here we actually do the MCMC iteration to generate the chain. We plot the (normalized) histogram (blue) of the chain, as well as the likelihood function (solid red line) for each value. For 10,000 iterations, the algorithm nicely approximates the 1-d Gaussian, as expected. For a choice of $\sigma_p = 1.0$ and $A = 2.5$ for the step sizes, we get an acceptance fraction $f \approx 0.43$.

```
In [332]: num_theta = 10000

          theta = np.array(t_start)

          t_old = t_start
          t_prop = t_start
          t_new = t_start

          while np.size(theta) < num_theta:
              t_old = t_new
              t_prop = prop(t_old)
              t_new = new_state(t_old,t_prop)
              theta = np.append(theta,t_new)


          accept = np.size(np.unique(theta))/num_theta
          print(accept)
          count, bins, ignored = plt.hist(theta,bins=30,normed=True)
          plt.plot(bins,likelihood_func(bins),linewidth=3,color='red')
          plt.xlabel(r'$\theta$')
          plt.ylabel(r'$P\,(\theta)$')
```

```
0.4311
```

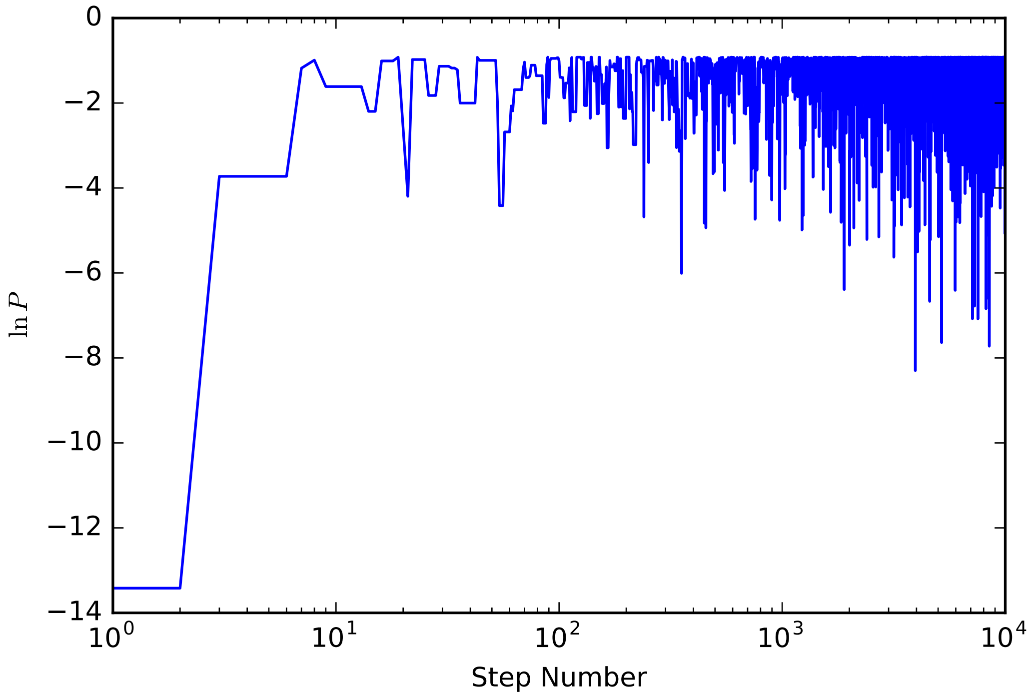
Out[332]: <matplotlib.text.Text at 0x11b761128>



We also plot the step number vs. $\ln P$ and θ vs $\ln P$, which shows proper convergence.

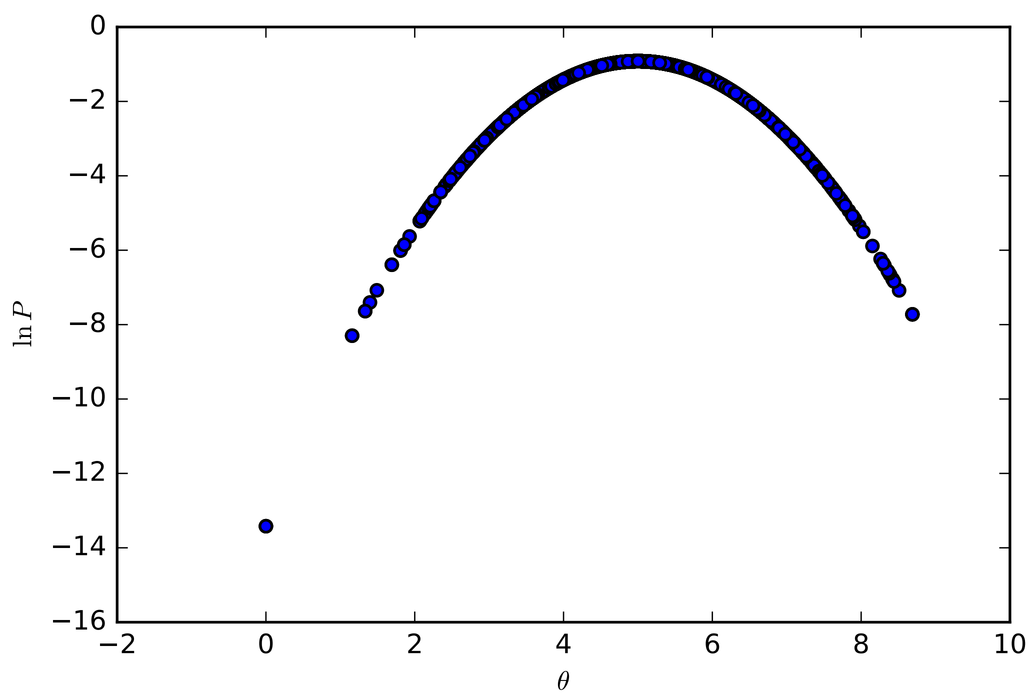
In [322]:
```
plt.plot(range(np.size(theta)),np.log(likelihood_func(theta)))
plt.xscale('log')
plt.xlabel('Step Number')
plt.ylabel(r'$\ln\,P$')
```

Out[322]: <matplotlib.text.Text at 0x11d7fed68>




```
In [329]: plt.scatter(theta,np.log(likelihood_func(theta)))
          plt.xlabel(r'$\theta$')
          plt.ylabel(r'$\ln\,P$')
```

```
Out[329]: <matplotlib.text.Text at 0x11cf05240>
```



```
In [ ]:
```