AY250_F16 PS1

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Problem 1

(a) Suppose we consider the two draws sequentially. The prior likelihood for the yellow M&M coming from the 1996 bag vs. the 1994 bag is $P(1994) = P(1996) = 0.5$. That is, before the draw, it is equally likely to come from either bag. The *likelihood* of the yellow M&M coming from the 1994 bag is simply the fractional percentage of yellow M&M's in the bag, $P(Y|1994) = 0.20$. Likewise, $P(Y|1996) = 0.14$. For the green $M\&M$ we adopt identical priors (i.e. equal probability of coming from either bag), and here use the fractional percentage of green M&M's, $P(G|1994) = 0.10$ and $P(G|1996) = 0.20$. Thus, the posterior probabilities are the prior times the likelihood, $P(1994|Y) = P(1994)P(Y|1994) = 0.5 \times 0.2 = 0.01$ and $P(1996|Y) = P(1996)P(Y|1996) = 0.5 \times 0.14 = 0.07$. For the green M&M, $P(1994|G) = 0.05$ and $P(1996|G) = 0.1$.

Suppose we carry out the experiment and get a yellow $M\&M$ and a green $M\&M$. Since we draw one M&M from each bag, we know that only *one* can come from the 1994 bag, and the other *must* come from the 1996 bag. The relative probability of the yellow M&M coming from the 1994 bag rather than the 1996 bag is

$$\frac{P(1994|Y,G)}{P(1996|Y,G)} = \frac{P(1994|Y)P(1996|G)}{P(1996|Y)P(1994|G)} \tag{1}$$

$$= \frac{0.10 \times 0.10}{0.07 \times 0.05} \tag{2}$$

$$= 2.86 \tag{3}$$

or roughly three times more likely to have come from the 1994 bag.

(b) The *normalized* probability is simply the posterior divided by the evidence, where the evidence is given by

$$P(Y,G) = P(1994|Y,G) + P(1996|Y,G) \tag{4}$$

$$= 0.0135 \tag{5}$$

Thus, the normalized probability is

$$P(1994|Y,G) = \frac{P(1994|Y)P(1996|G)}{P(Y,G)} \tag{6}$$

$$= \frac{0.10 \times 0.10}{0.0135} \tag{7}$$

$$= 0.74 \tag{8}$$

or a 74% chance.