Kalman Filter Implementation for Hovering Helicopter

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1 Problem Statement

Given a nonlinear motion model of an helicopter which is augmented with Gaussian noise, a provided LQR controller attempts to hover the helicopter in a stationary position. The motion model was linearized around that desired stationary point. However, the linearization and the controller fail to accomplish their mission when the helicopter is slightly perturbed from the stationary position. The main problem is that the controller uses an estimate of the state that is only based on observation without any consideration for previous states or uncertainty. The goal of this lab is to implement a Kalman filter that will render a better state estimate by taking into account previous states and uncertainty.

2 Solution

The solution to this problem would be to implement a Kalman filter, but some investigation need to be done in order to have some insight into the system and what is going on.

2.1 Dynamics and Measurements Free of Uncertainty

When uncertainty is removed by setting the variance of state transition and measurements white noise to zero, the system behaves according to the numbers in the table below.

Table 1. Hover Trim Values

aileron	0.006515
elevator	0.001629
rudder	0.009367
collective	0.4001

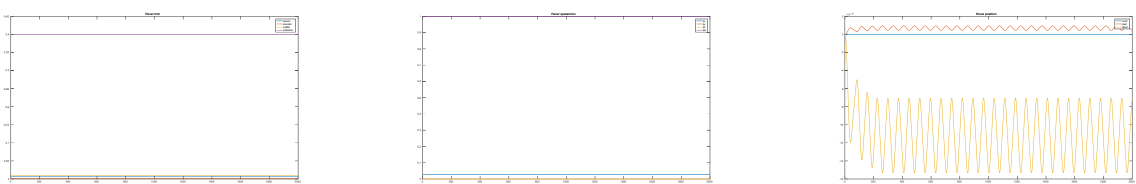
Table 2. Hover Quaternion Values

q_x	0.02906
q_y	0
q_z	0
q_w	0.9996

Table 3. Hover Position Values

Heading	High	Low
North	0	0
East	4.419×10^{-17}	9.57×10^{-17}
Down	-1.531×10^{-15}	-7.07×10^{-16}

The combination of all the values above represent the desired stationary position of the helicopter. The east and down heading are oscillation between 4.419×10^{-17} and 9.57×10^{-17} and -1.531×10^{-15} and -7.07×10^{-16} respectively. Even though there are oscillations, these oscillations are very small, and they are probably due to vibration as the helicopter cannot perfectly stand still in the air without moving.



(a) Hover Trim **(b) Hover Quaternion** **(c) Hover Position**

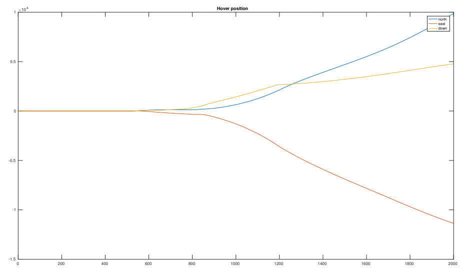
Figure 1. Generated Plots when Uncertainty is Zero

2.2 Uncertainty Added to Dynamics and Measurement

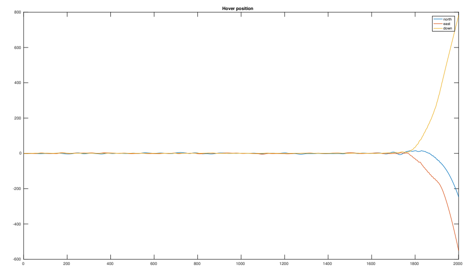
In order to determine at which point the helicopter stops hovering if noise is added to its dynamics, σ_x was increased in increment of 0.05 up until it reached 2.6. From there on, the step size was reduced to 0.01 and another searched was performed in the interval between 2.55 and 2.6. After this second search, it was determined that the helicopter stops hovering when σ_x is around 2.58. The same process was used to determine at which point the helicopter stops hovering if noise is added to its measurement model. It stopped hovering when σ_y reached 0.32 The plots of the helicopter position are shown below

2.3 Kalman Filter Implementation

In order to improve the performance of the system in presence of dynamics and measurement noises, a Kalman filter was implemented, and the controller gained some robustness. The



(a) Due to noise added to State Transition



(b) Due to noise added to Measurement

Figure 2. Helicopter Position: Hovering breakpoint due to noise added to state transition and measurement model respectively

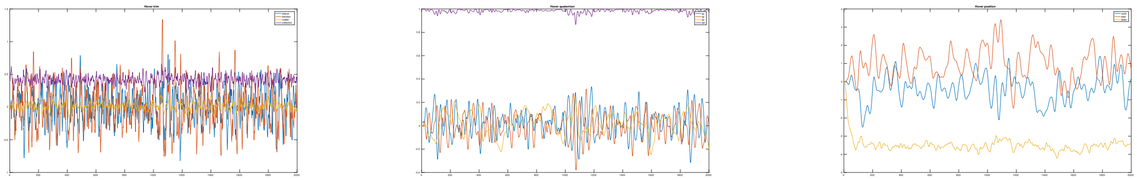
updates equations used are as follow:

$$\begin{aligned}
\bar{\mu}_x^{(t+1)} &= A\mu_x^{(t)} + Bu^t && (1)\\
\bar{\Sigma}^{(t+1)} &= A\Sigma^{(t)}A^T + Q && (2)\\
\mu_x^{(t+1)} &= \bar{\mu}_x^{(t+1)} + \bar{\Sigma}^{(t+1)}C^T\left(R + C\bar{\Sigma}^{(t+1)}C^T\right)^{-1}\left(y^{(t+1)} - C\bar{\mu}_x^{(t+1)}\right) && (3)\\
\Sigma^{(t+1)} &= \bar{\Sigma}^{(t+1)} - \bar{\Sigma}^{(t+1)}C^T\left(R + C\bar{\Sigma}^{(t+1)}C^T\right)^{-1}C\bar{\Sigma}^{(t+1)} && (4)
\end{aligned}$$

The initial parameters μ_x^0 and Σ^0 were chosen as:
$\mu_x^0 = (0.0065, 0.0016, 0.0094, 0.4001, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.0291, 0, 0, 0.9996)^T$ which is just the desired stationary state, which happens to be the first actual state.
Σ^0 was chosen to be the identity matrix of size 21, which is the size of the state. That identity matrix was multiplied by a scalar that was tunned between values from 0.001 to 20. In the end, it was set to 0.2.
Q and R were also manually tunned like Σ^0. They were first set to identity of size 21, and then those identity matrices were multiplied by constant that were tunned by sweeping values between 0.001 and 100 until a somehow robust solution was found. Note the word "somehow", and this is because the implemented filter doesn't work all the time; it works well 95% of the time. It was very hard to find parameters that would work 100% of the time as there is not a systematic way of finding Q and R. In the end, Q was set to the identity multiplied by 0.02 and R was set to the identity multiplied by 1.2. The resulting plot after Kalman filter implementation is shown below.

2.4 Kalman Filter Robustness

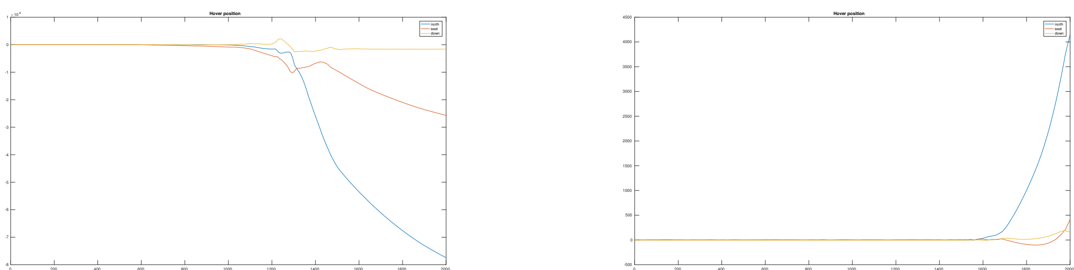
In order to test robustness of the implemented Kalman filter, σ_x and σ_y were increased from their default values in increment of 0.1. Only one of them was varied at a time (i.e. σ_x was kept at 0.1 when σ_y was varied, and σ_y was kept at 0.5 when σ_x was varied).σ_x was increased from 0.1 to 0.5, and the filter and controller were robust. However, when σ_x reached 0.6, the



(a) Hover Trim **(b) Hover Quaternion** **(c) Hover Position**

Figure 3. Generated plots when Kamlam Filter is implemented in presence of Dynamics and Measurement noises

helicopter hovered in 80% of the trials; when it reached 1.2, the helicopter hovered in 35% of the trials. When σ_x was greater than or equal to 1.4, the helicopter failed to hover. On the other hand, σ_y was incremented from 0.5 to 0.6, and the Kalman filter and controller were still robust, but as as soon as it reached 0.7, the helicopter hovered only in 35% of the trials. When σ_y was greater than 0.7, the helicopter failed to hover.



(a) Due to noise added to State Transition **(b) Due to noise added to Measurement**

Figure 4. Helicopter Position: Hovering breakpoint

3 Discussion

The main observations here are that the system is more sensitive to measurement noise than it is to state transition noise as the state transition noise, σ_x, could be increased from 0.1 to 1.2 without having any impact on performance of the Kalman filter. Measurement noise, σ_y, however could only be increased from 0.5 to 0.6 without impacting the Kalman filter's performance. any value greater than or equal to 0.7 severely.
Moreover, the Kalman filter fails at some point because the dynamics of the helicopter are not linear. Even though they were linearized around the point of interest, if noise pushes the linearized dynamics too far from the point of interest such that it is out of the region of attraction of the desired point, the system will become unstable, and it won't be able to come back close to the desired position.