International Effects of Quantitative Easing and Foreign Exchange Intervention

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Abstract

A two-country general equilibrium model is developed to study the global consequences of quantitative easing and foreign exchange intervention. The model incorporates financial frictions such as limited commitment, differential pledgeability of assets as collateral, and a low supply of collateralizable assets. Due to differential asset pledgeability, financial intermediaries acquire different asset portfolios particular to their home country. Quantitative easing can reduce long-term nominal interest rates, mitigate financial frictions globally, and depreciate the currency of the country that supplies more pledgeable assets. The international effects of foreign exchange intervention depend on the implementing country. If implemented by the country that supplies more pledgeable assets, such intervention can ease financial frictions and enhance welfare globally.

Keywords: Quantitative easing; Foreign exchange intervention; Collateral constraint; Banking

JEL Codes: E4, E5, F3, F4

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1 Introduction

After the global financial crisis of 2007-08, major central banks such as the US Federal Reserve, the European Central Bank, and the Bank of England turned to unconventional policy measures when short-term nominal interest rates were at their lower bounds. One element in unconventional monetary policy is quantitative easing (QE), which involves large-scale purchases of assets, with the intention of stimulating economic activity by directly reducing long-term nominal interest rates. For example, between 2009 and 2014, the Federal Reserve purchased approximately $2 trillion worth of US Treasury securities, equivalent to 13% of GDP, and the Fed implemented a large QE program during the global pandemic, beginning in early 2020.

Large QE programs in rich countries, particularly the United States, have sparked intense debate among policymakers regarding potentially adverse effects in other countries. While some have argued that other countries benefit from QE in the United States simply because this lowers global long-term interest rates (e.g., Bernanke, 2017), others have expressed concerns about substantial capital inflows and currency appreciations in emerging market economies. Empirical evidence appears to support some of these concerns, but the alleged positive impact on output from the international transmission of QE needs further study.[1]

My contribution to this debate involves the construction of a two-country general equilibrium model that enhances our understanding of the international transmission of QE.[2] In this model, financial intermediaries acquire portfolios of assets comprising currency, short-term government bonds, and long-term government bonds issued by each country. These intermediaries write deposit contracts that provide insurance, in a manner similar to what banks do in Diamond and Dybvig (1983).[3] Central banks can implement QE by purchasing long-term local government bonds, and QE matters as it alters the composition of assets held by intermediaries in the global economy.

The model incorporates three key financial frictions. First, intermediaries operate under limited commitment, implying that their assets must be pledged as collateral to back their liabilities. Second, different assets have varying degrees of pledgeability as collateral, following the work of Kiyotaki and Moore (2005), Venkateswaran and Wright (2014), and Williamson (2016). In practice, short-term, local currency-denominated assets are considered more pledgeable than long-term, foreign currency-denominated assets.[4] Accounting for this feature, the model generates a term

[1]See, for example, Bauer and Neely (2014), Neely (2015), and Rogers et al. (2014), who find that QE implemented by the US Federal Reserve decreased long-term bond yields internationally. See also Bhattarai et al. (2021), who find significant spillover effects of QE by the Federal Reserve on financial markets in emerging market economies with no positive effects on their outputs.

[2]Understanding how QE can spill over to other countries has become even more important because many central banks in developed and emerging market economies implemented QE or QE-like interventions during the COVID-19 pandemic in 2020 (Hartley and Rebucci, 2020; Arslan et al., 2020).

[3]The basic structure of the model comes from Lagos and Wright (2005). Details of the structure of financial intermediaries and fiscal policy are related to Williamson (2012, 2016) and Andolfatto and Williamson (2015), while the structure of international trade is related to Gomis-Porqueras et al. (2013) and Gomis-Porqueras et al. (2017).

[4]For example, when the US Federal Reserve extends discount window lendings to depository institutions, 99% of the market value can be pledged as collateral for US Treasury securities with a duration of fewer than three years, and 95% can be pledged for US Treasury securities with duration more than ten years. For foreign government bonds

premium (an upward-sloping nominal yield curve) and a home bias in asset portfolios. Finally, collateralizable assets are in short supply, as in Andolfatto and Williamson (2015) and Williamson (2016), in that the total value of safe assets falls below what is required to support the first-best allocation, the result being inefficiency. The low supply of safe assets leads to binding collateral constraints, a liquidity premium on assets, and low real interest rates.

The nominal exchange rate between the two currencies in the model is determined by relative inflation rates and relative intertemporal marginal rates of substitution, as in the standard international asset pricing model (e.g., Lucas, 1982). However, uncovered interest parity does not hold due to varying liquidity premia on assets, as in Lee and Jung (2020) and Bianchi et al. (2021).[5] In the model, the two countries possess different economic fundamentals, causing inefficiency resulting from low safe asset supply—measured by the tightness of collateral constraints—to differ internationally. As the difference in low asset supplies leads to differential liquidity premia and real interest rates, the nominal exchange rate between the two currencies may deviate from uncovered interest parity.

Different degrees of inefficiency caused by low asset supply, along with the differential pledgeability of assets, also determine which assets migrate to which country. Each asset is allocated to the country where it is most valuable as collateral. When the degree of inefficiency due to short asset supply is similar across countries, assets issued in each country (denominated in the local currency) are held solely by local intermediaries, resulting in no cross-border capital flows. However, when there is a significant difference in the degree of inefficiency, intermediaries in the country with higher asset-supply-induced inefficiency (*higher-asset-market-inefficiency country*, hereafter) acquire foreign assets in addition to local assets, while those in the country with lower asset-supply-induced inefficiency (*lower-asset-market-inefficiency country*) acquire only local assets. This leads to capital flows from the higher-asset-market-inefficiency country to the other.

Financial intermediaries in the higher-asset-market-inefficiency country can purchase collateralizable assets issued in the lower-asset-market-inefficiency country to mitigate their collateral constraints. Conversely, financial intermediaries in the lower-asset-market-inefficiency country face tighter collateral constraints due to international capital flows. In other words, the higher-asset-market-inefficiency country can effectively transmit its low asset supply problem to the lower-asset-market-inefficiency country, similar to the findings in Caballero et al. (2020).[6]

I find that a central bank's QE in one country has spillover effects on the other country when asset markets are connected through international capital flows. As intermediaries in the higher-asset-market-inefficiency country hold both local and foreign assets, a change in local asset prices impacts foreign asset prices through portfolio rebalancing by these intermediaries. Hence, QE

maturing within three years, the percentage of the market value pledgeable as collateral is 97-98% if denominated in the US dollar and 94% if denominated in a foreign currency.

[5] Lee and Jung (2020) and Bianchi et al. (2021) provide a liquidity-based explanation of why a relatively higher interest-yielding currency can appreciate in practice (or the uncovered interest parity puzzle).

[6] Caballero et al. (2020) develop a two-country model with a shortage of safe assets. However, they do not address how a central bank's policy involves asset swaps and affects the effective stock of safe assets in the financial market.

aimed at lowering the long-term nominal interest rate and the term premium in the local bond market spills over to the foreign bond market, mitigating its initial effects on the local market, as in Alpanda and Kabaca (2020) and Kolasa and Wesolowski (2020).[7] Real interest rates, however, rise as opposed to what occurs in their models. In my model, QE tends to reduce both short-term and long-term nominal interest rates, so the central bank must sell its holdings of short-term bonds to maintain the short-term rate at the target level. Consequently, QE involves the central bank's swap of better collateral (short-term bonds) for worse collateral (long-term bonds), leading to an increase in the effective stock of collateral held by the public. This relaxes the collateral constraints of intermediaries, as in Dedola et al. (2013), resulting in lower liquidity premia and higher real interest rates in the global economy.[8]

In the model, QE expands the central bank's balance sheet, increasing the real stock of currency in circulation.[9] Therefore, to incentivize private agents to hold more currency in equilibrium, inflation must decrease (or the rate of return on currency must increase). The balance sheet of the *foreign* central bank also expands in response to QE. This occurs because higher real interest rates put upward pressure on the *foreign* short-term nominal interest rate, making the foreign central bank conduct open market purchases to maintain its policy rate at the target level. Therefore, the *foreign* inflation rate must also decrease due to an increase in the real stock of foreign currency outstanding.

Although the inflation rate falls in both countries, the magnitude of this inflation reduction is consistently greater in the higher-asset-market-inefficiency country, regardless of where QE is implemented. This discrepancy arises from the larger size of the central bank's open market purchases in the higher-asset-market-inefficiency country. Consequently, only the higher-asset-market-inefficiency country can depreciate its local currency through QE. This result differs from the findings of Alpanda and Kabaca (2020) and Kolasa and Wesolowski (2020), where QE always leads to local currency depreciation.

The model developed in this paper can also be utilized to examine the international effects of foreign exchange (FX) interventions. FX interventions, which aim to prevent local currency appreciation, can involve substantial asset purchases similar to QE. For instance, between 2002 and 2007, total foreign exchange reserves held by central banks rose by \$4.7 trillion and then again by \$4.9 trillion between 2008 and 2014, driven in part by demand from central banks in emerging market economies. While FX interventions and currency wars have been a longstanding issue, recent research has highlighted concerns about their global consequences for safe asset shortages and low

[7] Alpanda and Kabaca (2020) and Kolasa and Wesolowski (2020) develop two-country general equilibrium models with asset market segmentation. Unlike their models that impose imperfect substitutability across different assets, I assume different pledgeability of assets as collateral and show that asset substitutability is determined endogenously.

[8] Dedola et al. (2013) adopt limited commitment of financial intermediaries with limited pledgeability of assets, as I do, to capture the international transmission of QE. Unlike their model, which focuses on real economic activity, a monetary model developed in my paper allows further analysis of the behaviors of inflation rates and the nominal exchange rate.

[9] Note that in practice, QE involves a swap of reserves for assets, expanding the central bank's balance sheet.

real interest rates.[10]

My findings reveal that the international effects of FX intervention, specifically a local currency depreciation, can vary depending on the implementing country. Intermediaries in the higher-asset-market-inefficiency country play a crucial role in interconnected asset markets, so assets issued in that country are considered more pledgeable collateral than those issued in the other. If the central bank in the lower-asset-market-inefficiency country intervenes in the FX market, it swaps less pledgeable local assets for more pledgeable foreign ones, thereby downgrading the quality of collateral in international asset markets. Therefore, this intervention tightens collateral constraints, decreasing global real interest rates and welfare, as in Fanelli and Straub (2021).

This effect of tight collateral constraints on real interest rates and welfare has implications for FX interventions by emerging market economies because these countries typically accumulate more safe assets (such as US Treasury securities) by issuing less safe and sometimes risky local liabilities. In such instances, the Fed's overnight reverse repurchase agreement facility and liquidity swap lines can alleviate safe asset shortages and increase welfare globally. These Fed interventions effectively enable other central banks to hold reserve accounts at the Fed, thereby disincentivizing their accumulation of US Treasury securities.

However, if the central bank in the higher-asset-market-inefficiency country implements the FX intervention, the result is reversed. In this case, FX intervention serves to take worse collateral (less pledgeable foreign assets) out of markets and replace it with better collateral (more pledgeable local assets). With relaxed collateral constraints, global real interest rates and welfare increase.

This novel finding can be applied to the Swiss National Bank's (SNB) unconventional FX intervention launched after the global financial crisis. To prevent excessive appreciation of the Swiss franc, the SNB increased its foreign reserve holdings from roughly 10% of GDP in 2010 to over 100% in 2016. The SNB's intervention involved exchanging Swiss government liabilities, including sight deposits at the SNB, for foreign government bonds, corporate bonds, and equity.[11] My findings suggest that the SNB's FX intervention can benefit the global economy by improving the quality of collateral in international asset markets.

The remainder of the paper is organized as follows: Section 2 describes the model, and Section 3 defines and characterizes a stationary equilibrium with nonbinding collateral constraints. In Section 4, two types of equilibria with binding collateral constraints are analyzed. Section 5 concludes, and the Appendix contains proofs, additional details, and discussions.

[10] See Del Negro et al. (2019) and Jorda et al. (2017), who empirically find that the secular decline in global real interest rates since the 1990s is driven primarily by the shortage of safe assets. Obstfeld (2013) discusses the adverse effect of FX interventions on global interest rates. Also, see Fanelli and Straub (2021), who develop a theoretical framework to show how FX interventions can lead to inefficiently low interest rates on safe assets.

[11] At the end of 2021Q1, the SNB holds 66% of its foreign exchange reserves as government bonds, 11% as other bonds, and 23% as equities.

2 Model

Suppose there are two countries, *Home* and *Foreign*. Each country has three types of agents: *buyers*, *sellers*, and *banks*. For each type, there is a continuum of agents with unit mass. Home parameters and variables are denoted with a subscript h and without an asterisk, while Foreign parameters and variables are denoted with a subscript f and an asterisk. Time is discrete and indexed by t = 0, 1, 2, ..., and all agents discount the future at rate $\beta \in (0, 1)$. For convenience, I describe the model environment from the perspective of the Home country while keeping in mind that there is a symmetric Foreign counterpart.

The model is based on Lagos and Wright (2005) and Rocheteau and Wright (2005). Each period consists of two subperiods—the centralized market (CM) followed by the decentralized market (DM). In the CM, all agents in both countries interact, and debts are settled at the beginning. Then, Home and Foreign agents produce and consume homogeneous perishable goods and trade assets and goods internationally in a perfectly competitive market. Buyers and banks can produce goods, but sellers cannot in the CM. Specifically, buyers incur disutility H from producing H units of *CM goods,* while sellers receive utility X from consuming X units of CM goods. Banks' per-period preferences are $X - H$, where X and H are units of CM goods consumed and produced, respectively. As goods and assets are traded internationally, the CM can be referred to as a *tradeable sector.*[12]

In the DM, random matches occur between buyers and sellers within each country. That is, trades take place exclusively between Home buyers and Home sellers, as well as between Foreign buyers and Foreign sellers. Therefore, the DM can be considered a *non-tradeable sector.*[13] In this subperiod, sellers can produce goods, but buyers cannot. Buyers receive utility $u(x)$ from consuming x units of *DM goods,* while sellers incur disutility h from producing h units of DM goods. The function $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u'(0) = \infty$, $u'(\infty) = 0$, and $-\frac{xu''(x)}{u'(x)} < 1$. Denote the first-best quantity by $\hat{x} \in (0, \infty)$ that solves $u'(\hat{x}) = 1$.

In all DM matches, there is no memory or record keeping, and a matched buyer and seller are subject to limited commitment. As unsecured credit cannot be supported in equilibrium, some assets must be exchanged on the spot or posted as collateral for trade in the DM. However, there are two types of limitations on the available means of payment. First, in the DM, no technology permits sellers to recognize liabilities issued by foreign institutions, including foreign currency and bank deposits.[14] Second, some sellers have information technology that enables them to accept

[12]The structure of the tradeable and non-tradeable sectors is derived from Gomis-Porqueras et al. (2013) and Gomis-Porqueras et al. (2017). The tradeable sector in the model is consistent with the one in existing international monetary models, as mentioned in Gomis-Porqueras et al. (2013). Examples of such models include Schlagenhauf and Wrase (1995) and Chari et al. (2002).

[13]In this model, international trade takes place only in the CM. However, in practice, the existence of an over-the-counter market for international trade can explain why foreign reserve assets are in high demand. Although the introduction of decentralized international trade is beyond the scope of this paper, interested readers can refer to Geromichalos and Simonovska (2014) and Geromichalos and Jung (2018), among others, for more information.

[14]Without this assumption, the two currencies become perfect substitutes, leading to indeterminacy in the nominal exchange rate, as discussed in Kareken and Wallace (1981). Alternatively, a special trading mechanism such as the

local bank deposits as a means of payment, while others do not, as in Williamson (2012, 2016). Specifically, in a fraction ρ of each country's DM meetings, the seller cannot verify the buyer's asset holdings other than local currency. So, only local currency can be accepted as a means of payment in those meetings, which I will refer to as *currency transactions*. In a fraction $1-\rho$ of DM meetings, the seller can verify any liabilities issued by local institutions. This implies that local currency, local bank deposits, or both can be used in those meetings, which I will refer to as *non-currency transactions*. In any match, the buyer makes a take-it-or-leave-it offer to the seller.

The type of transaction buyers will make in the following DM is unknown when they write contracts with banks at the beginning of the CM. Buyers learn the type at the end of the CM after trades in goods and assets have taken place. A buyer's type of transaction is private information, and each buyer can meet with their respective bank after learning the type. Eventually, the interaction between a buyer and a bank at the end of the CM will only occur for the execution of the contract written earlier in the CM.[15]

There are three types of assets issued by each country's government—currency, short-term bonds, and long-term bonds.[16] First, each country's central bank produces a perfectly divisible and storable currency. Home (Foreign) currency is issued by the Home (Foreign) central bank with a price ϕ (ϕ^*) in units of CM goods. The nominal exchange rate is denoted by e and measures the price of Foreign currency in units of Home currency. As agents can trade goods and assets internationally without friction, the law of one price holds in the tradeable sector, i.e., $\phi^* = e\phi$.[17] Also, each country's fiscal authority issues government bonds denominated in the local currency with two different maturities. I will refer to the Home currency-denominated bonds as *Home bonds* and their Foreign counterparts as *Foreign bonds*. A short-term Home (Foreign) bond sells at a price z_s (z_s^*) in units of the local currency in the CM and pays off one unit of the local currency in the following CM. A long-term Home (Foreign) bond sells at a price z_l (z_l^*) in units of the local currency in the CM and pays off one unit of the local currency in every future CM.[18]

In addition to government liabilities, there are bank liabilities that emerge endogenously in the private sector. Due to the lack of commitment and memory, bank liabilities must be backed by collateral. However, the pledgeability of assets as collateral is limited because, at the beginning of the CM, a bank can abscond with some portions of the collateralized assets. A key assumption

one introduced by Zhu and Wallace (2007) could be utilized to generate the same result, as shown in Rocheteau and Nosal (2017, chapter 12.1.2) and Lee and Jung (2020). However, incorporating this mechanism would only complicate the model without adding any useful implications in this context. In contrast to the DM, I assume that there is a publicly available technology in the CM that helps verify liabilities issued by foreign institutions.

[15]This assumption essentially imposes spatial separation between agents at the end of the CM. For banks to play a Diamond and Dybvig (1983) insurance role, restrictions on side-trading are necessary because otherwise, side trades would undo the banking arrangements (See Jacklin, 1987; Wallace, 1988).

[16]Private assets could also be introduced in the model. However, including them would only make the model more complicated without changing the main results of the paper.

[17]It is worth noting that the law of one price holds in the tradeable sector but not in the non-tradeable sector. Introducing some frictions in the international market could generate deviations from the law of one price. However, the main results of the paper would remain unchanged.

[18]These long-term government bonds can be considered as Consols, which the British government initially issued in 1751. All the remaining British Consols were fully redeemed in 2015.

is that the degree of pledgeability varies across different assets, as in Kiyotaki and Moore (2005), Venkateswaran and Wright (2014), and Williamson (2016). Specifically, a bank in country $i = h, f$ can abscond with fraction θ_{is} (θ^*_{is}) of Home (Foreign) currency and short-term Home (Foreign) bonds, and fraction θ_{il} (θ^*_{il}) of long-term Home (Foreign) bonds. Banks can acquire any assets in their portfolios, but the fraction of an asset that a Home bank can abscond with differs from that of a Foreign bank, i.e., $\theta_{hj} \neq \theta_{fj}$ and $\theta^*_{hj} \neq \theta^*_{fj}$ for $j = s, l$.[19]

In practice, assets with higher volatility in their market value tend to receive larger haircuts when pledged as collateral. Therefore, assets with longer maturities or denominated in foreign currency typically receive larger haircuts than those with shorter maturities or denominated in the local currency. For example, when the US Federal Reserve extends discount window lending to depository institutions, 99% of the market value can be pledged as collateral for US Treasury securities with a duration of fewer than three years, whereas 95% can be pledged for US Treasury securities with a duration exceeding ten years. Regarding foreign government bonds maturing within three years, the percentage of the market value pledgeable as collateral is 97-98% if denominated in the US dollar and 94% if denominated in a foreign currency.[20]

Based on empirical observations, I assume that the degrees of pledgeability satisfy the following regularity conditions: (i) long-term bonds are less pledgeable as collateral than short-term bonds with the same denomination ($\theta_{il} > \theta_{is}$ and $\theta^*_{il} > \theta^*_{is}$, $i = h, f$), (ii) foreign currency-denominated bonds are less pledgeable than local currency-denominated bonds with the same maturity ($\theta^*_{hj} > \theta_{hj}$ and $\theta_{fj} > \theta^*_{fj}$, $j = s, l$), and (iii) the pledgeability of short-term bonds relative to long-term bonds is higher for foreign currency-denominated bonds ($\frac{1-\theta_{fs}}{1-\theta_{fl}} > \frac{1-\theta_{hs}}{1-\theta_{hl}}$ and $\frac{1-\theta^*_{hs}}{1-\theta^*_{hl}} > \frac{1-\theta^*_{fs}}{1-\theta^*_{fl}}$).[21]

2.1 Private Banks

Banks endogenously create deposit contracts that effectively allocate currency to currency transactions and other higher-yielding assets to non-currency transactions, as in Williamson (2012, 2016). Therefore, deposit contracts play a role in providing insurance to depositors, similar to Diamond and Dybvig (1983).[22] However, there are key differences in this model: agents in the two countries

[19] In the model, the different degrees of pledgeability are given exogenously. Although it would be interesting to endogenize the degrees of pledgeability by introducing country-specific aggregate shocks into the model, such an analysis is beyond the scope of this paper.

[20] For more details, refer to the Discount Window & Payment System Risk Collateral Margins Table, available at https://www.frbdiscountwindow.org/Home/Pages/Collateral/. Readers can also find the Collateral Margin Requirements for the Bank of Canada's Standing Liquidity Facility at https://www.bankofcanada.ca/2020/04/assets-eligible-collateral-standing-liquidity-facility-090420/.

[21] The first two assumptions have empirical relevance. The last assumption suggests that if agents need to utilize foreign currency-denominated assets as collateral, they would prefer short-term bonds over long-term ones.

[22] To understand how a deposit contract can improve social welfare in this model, consider a scenario where banking activity is prohibited. In such a case, each buyer would acquire currency and government bonds in the CM. Then, the buyer would hold idle government bonds in a DM currency transaction since only currency would be accepted as a means of payment. In a DM non-currency transaction, the buyer would hold some currency, which allows for less consumption compared to government bonds. Although buyers can opt out of deposit contracts in the model, they choose to hold a deposit contract as it effectively allocates currency exclusively to currency transactions and government bonds solely to non-currency transactions.

interact internationally in the CM, and the pledgeability of each asset varies across countries. As a result, assets are allocated to the country where they are most valuable as collateral.

Consider a bank in the Home country (*Home bank*, hereafter) that issues deposit contracts to buyers in the Home country (*Home buyers*, hereafter).[23] At the beginning of the CM, the Home bank writes a deposit contract before buyers realize the type of transaction (currency or non-currency) in the following DM. The deposit contract provides buyers with two options. Once they learn the type of their transaction, a buyer can either contact the bank to withdraw the specified amount of currency as stated in the contract, thereby relinquishing any other claims on the bank, or they can receive a deposit claim that can be redeemed in the subsequent CM for a predetermined quantity of CM goods. Since a deposit contract essentially functions as a debt contract that enables the bank to borrow from buyers, the bank's liabilities must be backed by its asset holdings.

Then, the Home bank's problem in equilibrium can be expressed as

$$\max_{k,c,d,b_{hs},b_{hl},b^*_{hs},b^*_{hl}} \left[-k + \rho u\left(\frac{\beta\phi_{+1}c}{\phi}\right) + (1-\rho)u\left(\beta d\right)\right] \tag{1}$$

subject to

$$\begin{aligned}
&k - \rho c - z_s b_{hs} - z_s^* b_{hs}^* - z_l b_{hl} - z_l^* b_{hl}^* - \beta(1-\rho)d \\
&+ \beta\frac{\phi_{+1}}{\phi}\left\{b_{hs} + (1+z_{l,+1})b_{hl}\right\} + \beta\frac{\phi^*_{+1}}{\phi^*}\left\{b^*_{hs} + (1+z^*_{l,+1})b^*_{hl}\right\} \geq 0, &(2)\\
&-(1-\rho)d + \frac{\phi_{+1}}{\phi}\left\{b_{hs} + (1+z_{l,+1})b_{hl}\right\} + \frac{\phi^*_{+1}}{\phi^*}\left\{b^*_{hs} + (1+z^*_{l,+1})b^*_{hl}\right\} \\
&\geq \frac{\phi_{+1}}{\phi}\left\{\theta_{hs}(\rho c + b_{hs}) + (1+z_{l,+1})\theta_{hl}b_{hl}\right\} + \frac{\phi^*_{+1}}{\phi^*}\left\{\theta^*_{hs}b^*_{hs} + (1+z^*_{l,+1})\theta^*_{hl}b^*_{hl}\right\}, &(3)\\
&k, c, d, b_{hs}, b_{hl}, b^*_{hs}, b^*_{hl} \geq 0. &(4)
\end{aligned}$$

In the above problem, (k, c, d) represents a deposit contract, where k is the quantity of goods deposited by the buyer in the CM, c is the real quantity of Home currency the buyer can withdraw at the end of the CM, and d is the quantity of claims to goods in the following CM that the buyer receives if currency has not been withdrawn. Additionally, b_{hs} and b_{hl} (b^*_{hs} and b^*_{hl}) are short-term and long-term Home (Foreign) bonds acquired by the Home bank.[24] All quantities in the problem are denoted in terms of current CM goods, except for d, which is denoted in terms of following CM goods.

The buyer's take-it-or-leave-it offer in each DM meeting implies that in a currency transaction, the buyer exchanges c units of Home currency for $\frac{\beta\phi_{+1}c}{\phi}$ units of goods, whereas in a non-currency transaction, the buyer exchanges d units of claims for βd units of goods, as expressed in the objective function (1). The Home bank's discounted net payoff from the deposit contract is represented by

[23] Recall that the Home bank liabilities are useless in the DM meetings between Foreign agents. So, Home banks issue deposit contracts only for Home buyers in equilibrium.

[24] The Home bank does not hold Foreign currency across periods because holding Foreign bonds is always weakly preferred to holding Foreign currency.

the value on the left-hand side of inequality (2). In the current CM, the bank receives k units of deposits from the buyer and obtains an asset portfolio consisting of Home currency ρc, Home bonds, and Foreign bonds. At the end of the CM, a fraction ρ of buyers realize they will engage in currency transactions and withdraw c units of Home currency, while the remaining fraction $1-\rho$ of buyers participate in non-currency transactions and receive deposit claims. In the subsequent CM, the bank pays off d units of goods to each holder of the deposit claims and receives the payoff from its asset holdings. The collateral constraint, inequality (3), ensures that the Home bank's net payoff from repaying its liabilities must not be smaller than the payoff from defaulting.[25]

The above maximization problem shows that, in equilibrium, the Home bank must choose a contract that maximizes the representative Home buyer's expected utility, considering the bank's nonnegative payoff constraint (2), collateral constraint (3), and nonnegativity constraints (4). If the optimal contract did not solve this problem, then an alternative contract would emerge to attract all Home buyers and yield a higher expected payoff for the bank. Therefore, the solution to problem (1) subject to constraints (2)-(4) consists of a Nash equilibrium. A Foreign bank's problem is analogous to the Home bank's and is relegated to Appendix.

2.2 Fiscal Authority and Central Bank

I will confine attention to stationary equilibria where all real variables are constant across periods. This implies that $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ and $\frac{\phi^*_{t+1}}{\phi^*_t} = \frac{1}{\mu^*}$ for all t, where μ and μ^* denote the gross inflation rates in the Home and Foreign countries, respectively. Each fiscal authority has the ability to impose lump-sum taxes on buyers in their respective countries and issues short-term and long-term bonds denominated in the local currency. Each central bank issues the local currency through open market purchases of local and foreign government bonds and transfers any profits to the local fiscal authority. The consolidated government budget constraints for the two countries in period 0 are given by

$$\bar{c} + \sum_{i=s,l} \left[z_i \bar{b}_i - z_i^* a_i^* \right] = \tau_0,$$
$$\bar{c}^* + \sum_{i=s,l} \left[z_i^* \bar{b}_i^* - z_i a_i \right] = \tau_0^*,$$

where $\bar{c}$, $\bar{b}_s$, and $\bar{b}_l$ ($\bar{c}^*$, $\bar{b}_s^*$, and $\bar{b}_l^*$) denote the real quantities of Home (Foreign) currency and short-term and long-term Home (Foreign) bonds held by the public. Additionally, a_s^* and a_l^* (a_s and a_l) denote the real quantities of short-term and long-term Foreign (Home) bonds held by the Home (Foreign) central bank, and τ_0 (τ_0^*) denotes the lump-sum transfer to each buyer in the Home (Foreign) country. Then, the consolidated government budget constraints for each succeeding

[25] If the bank defaults, it will not permit currency withdrawals at the end of the CM, as in Williamson (2022).

period, $t = 1, 2, \ldots$, are given by

$$\bar{c} + \sum_{i=s,l} \left[z_i \bar{b}_i - z_i^* a_i^* \right] = \frac{1}{\mu} \left[\bar{c} + \bar{b}_s + (1 + z_l) \bar{b}_l \right] - \frac{1}{\mu^*} \left[a_s^* + (1 + z_l^*) a_l^* \right] + \tau,$$

$$\bar{c}^* + \sum_{i=s,l} \left[z_i^* \bar{b}_i^* - z_i a_i \right] = \frac{1}{\mu^*} \left[\bar{c}^* + \bar{b}_s^* + (1 + z_l^*) \bar{b}_l^* \right] - \frac{1}{\mu} \left[a_s + (1 + z_l) a_l \right] + \tau^*,$$

where τ (τ^*) denotes the lump-sum transfer to each Home (Foreign) buyer for $t = 1, 2, \ldots$ In both equations, the left-hand side represents the total value of consolidated government liabilities issued in each period, while the right-hand side represents the sum of the redemption value of the liabilities and the transfers to buyers.

The behavior of fiscal authorities plays a crucial role in determining equilibrium, as this influences the aggregate supply of collateral. One key empirical observation is the persistently low real interest rates on government liabilities, which can be explained in this model by binding collateral constraints due to an inefficiently low aggregate supply of collateralizable assets. Specifically, I assume that each fiscal authority determines the real value of the consolidated government liabilities in each country, following the approach of Andolfatto and Williamson (2015) and Williamson (2016). That is, the Home and Foreign fiscal authorities set V and V^*, respectively, where

$$V = \bar{c} + \sum_{i=s,l} \left[z_i \bar{b}_i - z_i^* a_i^* \right], \tag{5}$$

$$V^* = \bar{c}^* + \sum_{i=s,l} \left[z_i^* \bar{b}_i^* - z_i a_i \right]. \tag{6}$$

Then, (5) and (6) imply that $\tau_0 = V$ and $\tau_0^* = V^*$ while τ and τ^* are determined endogenously in equilibrium. In other words, each fiscal authority passively determines the lump-sum transfer in $t = 1, 2, \ldots$ in order to achieve the target level of real value for the consolidated government liabilities. Since my focus is on cases where the aggregate supply of government liabilities is inefficiently low, I will eventually analyze the effects of monetary policy interventions under suboptimal fiscal policies (specifically, sufficiently small V and V^*).[26]

Fiscal authorities also determine the outstanding value of local government bonds of each maturity. Let V_s and V_l (V_s^* and V_l^*) denote the values of short and long-term Home (Foreign) bonds issued by the Home (Foreign) fiscal authority, where $V = V_s + V_l$ ($V^* = V_s^* + V_l^*$). Then, the following conditions must hold in equilibrium:

$$0 \leq z_i \bar{b}_i \leq V_i; \qquad 0 \leq z_i^* \bar{b}_i^* \leq V_i^*,$$

for $i = s, l$. Given fiscal policies (V, V_s, V_l) and (V^*, V_s^*, V_l^*), central banks' balance sheets are well-defined, as illustrated in Table 1. The fiscal policies help define conventional and unconventional

[26]This fiscal policy rule can be interpreted as a debt ceiling or debt limit policy, similar to the concept observed in the United States. The equivalence between this fiscal policy rule and a debt ceiling policy is discussed in the Appendix, which also includes further discussions on alternative fiscal and monetary policy rules.

<Home Central Bank>

Assets	Liabilities
ST Home bonds $V_s - z_s\bar{b}_s$	Home
LT Home bonds $V_l - \omega_l$	currency
ST Foreign bonds κ_s^*	$\bar{c}$
LT Foreign bonds κ_l^*	

<Foreign Central Bank>

Assets	Liabilities
ST Foreign bonds $V_s^* - z_s^*\bar{b}_s^*$	Foreign
LT Foreign bonds $V_l^* - \omega_l^*$	currency
ST Home bonds κ_s	$\bar{c}^*$
LT Home bonds κ_l	

Table 1: Balance sheets of central banks

Notes: ST - short-term; LT - long-term

monetary policies in a plausible way and allow for tractability when analyzing the effects of monetary policies.

Each central bank's monetary policy has three dimensions. Consider the Home central bank's policy, bearing in mind that the Foreign central bank's policy is defined symmetrically. First, the Home central bank determines the price of short-term Home bonds z_s (or equivalently, pegs the short-term nominal interest rate R_s). Also, the Home central bank sets $\omega_l = z_l\bar{b}_l$, the value of long-term Home bonds held by the public. By choosing ω_l, the Home central bank determines the value of its holdings of long-term Home bonds $V_l - \omega_l$ (quantitative easing or tightening). Lastly, the Home central bank determines $\kappa_i^* = z_i^* a_i^*$ for $i = s, l$, the values of its holdings of short-term and long-term Foreign bonds (foreign exchange reserves).

Monetary policy variables must be consistent with feasibility conditions. For instance, from the Home central bank's balance sheet in Table 1, the real value of short-term Home bonds acquired by the Home central bank, which can also be expressed as $\bar{c} - V_l + \omega_l - \sum_{i=s,l}\kappa_i^*$, must be nonnegative and cannot exceed the value of short-term Home bonds issued by the fiscal authority V_s. Therefore, variables $(\omega_l, \omega_l^*, \{\kappa_i^*, \kappa_i\}_{i=s,l})$ must satisfy the following conditions:

$$\max\left[0, V_l - \bar{c} + \kappa_s^* + \kappa_l^*\right] \le \omega_l \le \min\left[V_l, V - \bar{c} + \kappa_s^* + \kappa_l^*\right], \tag{7}$$

$$\max\left[0, V_l^* - \bar{c}^* + \kappa_s + \kappa_l\right] \le \omega_l^* \le \min\left[V_l^*, V^* - \bar{c}^* + \kappa_s + \kappa_l\right]. \tag{8}$$

3 Equilibrium Characterization and Plentiful Collateral

In this section, I will define a stationary equilibrium and characterize an equilibrium with nonbinding collateral constraints. The collateral constraints of private banks do not bind in equilibrium if there is a sufficiently high supply of collateralizable assets in financial markets. As fiscal policies V and V^* determine the value of collateralizable assets available in financial markets, a sufficiently large value of V and V^* leads to nonbinding collateral constraints in equilibrium.

3.1 Characterization of Equilibrium

Let λ and λ^* denote the multipliers related to the collateral constraints of Home and Foreign banks, respectively. Then, noting that $\frac{\phi_{+1}}{\phi} = \frac{1}{\mu}$ and $\frac{\phi_{+1}^*}{\phi^*} = \frac{1}{\mu^*}$, the first-order conditions for a Home bank's problem (problem (1) subject to constraints (2)-(4)) can be written as

$$\frac{\beta}{\mu}u'\left(\frac{\beta c}{\mu}\right)-1-\frac{\lambda\theta_{hs}}{\mu}=0, \tag{9}$$

$$\beta u'(\beta d)-\beta-\lambda=0, \tag{10}$$

$$-z_s+\frac{\beta}{\mu}+\frac{\lambda\left(1-\theta_{hs}\right)}{\mu}\leq 0, \tag{11}$$

$$-z_s^*+\frac{\beta}{\mu^*}+\frac{\lambda\left(1-\theta_{hs}^*\right)}{\mu^*}\leq 0, \tag{12}$$

$$-z_l+\frac{\beta\left(1+z_l\right)}{\mu}+\frac{\lambda\left(1-\theta_{hl}\right)\left(1+z_l\right)}{\mu}\leq 0, \tag{13}$$

$$-z_l^*+\frac{\beta\left(1+z_l^*\right)}{\mu^*}+\frac{\lambda\left(1-\theta_{hl}^*\right)\left(1+z_l^*\right)}{\mu^*}\leq 0, \tag{14}$$

and

$$\lambda\left\{-(1-\rho)d+\frac{1}{\mu}\left[-\theta_{hs}\rho c+\left(1-\theta_{hs}\right)b_{hs}+\left(1+z_l\right)\left(1-\theta_{hl}\right)b_{hl}\right]\right.$$
$$\left.+\frac{1}{\mu^*}\left[\left(1-\theta_{hs}^*\right)b_{hs}^*+\left(1+z_l^*\right)\left(1-\theta_{hl}^*\right)b_{hl}^*\right]\right\}=0. \tag{15}$$

Similarly, the first-order conditions for a Foreign bank's problem can be written as

$$\frac{\beta}{\mu^*}u'\left(\frac{\beta c^*}{\mu^*}\right)-1-\frac{\lambda^*\theta_{fs}^*}{\mu^*}=0, \tag{16}$$

$$\beta u'(\beta d^*)-\beta-\lambda^*=0, \tag{17}$$

$$-z_s+\frac{\beta}{\mu}+\frac{\lambda^*\left(1-\theta_{fs}\right)}{\mu}\leq 0, \tag{18}$$

$$-z_s^*+\frac{\beta}{\mu^*}+\frac{\lambda^*(1-\theta_{fs}^*)}{\mu^*}\leq 0, \tag{19}$$

$$-z_l+\frac{\beta\left(1+z_l\right)}{\mu}+\frac{\lambda^*\left(1-\theta_{fl}\right)\left(1+z_l\right)}{\mu}\leq 0, \tag{20}$$

$$-z_l^*+\frac{\beta\left(1+z_l^*\right)}{\mu^*}+\frac{\lambda^*\left(1-\theta_{fl}^*\right)\left(1+z_l^*\right)}{\mu^*}\leq 0, \tag{21}$$

and

$$\lambda^*\left\{-(1-\rho)d^*+\frac{1}{\mu^*}\left[-\theta_{fs}^*\rho c^*+(1-\theta_{fs}^*)b_{fs}^*+(1+z_l^*)(1-\theta_{fl}^*)b_{fl}^*\right]\right.$$
$$\left.+\frac{1}{\mu}\left[\left(1-\theta_{fs}\right)b_{fs}+\left(1+z_l\right)\left(1-\theta_{fl}\right)b_{fl}\right]\right\}=0. \tag{22}$$

Also, it is convenient to characterize an equilibrium using the quantities of consumption in DM transactions. In particular, let $x_1=\frac{\beta c}{\mu}$ and $x_1^*=\frac{\beta c^*}{\mu^*}$ denote the consumption quantities in DM currency transactions for Home and Foreign buyers, and let $x_2=\beta d$ and $x_2^*=\beta d^*$ denote the corresponding consumption quantities in DM non-currency transactions.

In equilibrium, asset markets clear so that the sum of the demands for each asset is equal to the corresponding supply. That is,

$$\rho c = \bar{c}; \quad b_{hi} + b_{fi} + a_i = \bar{b}_i; \quad \rho c^* = \bar{c}^*; \quad b^*_{hi} + b^*_{fi} + a^*_i = \bar{b}^*_i, \tag{23}$$

for $i = s, l$. Then, an equilibrium can be defined as follows.

Definition *Given fiscal policies* $(V, \{V_i\}_{i=s,l})$ *and* $(V^*, \{V^*_i\}_{i=s,l})$ *and monetary policies* $(z_s, \omega_l, \{\kappa^*_i\}_{i=s,l})$ *and* $(z^*_s, \omega^*_l, \{\kappa_i\}_{i=s,l})$*, a stationary equilibrium consists of DM consumption quantities* $(\{x_k, x^*_k\}_{k=1,2})$*, asset quantities* $(c, c^*, d, d^*, \{\bar{b}_i, \bar{b}^*_i\}_{i=s,l}, \{b_{ji}, b^*_{ji}\}_{i=s,l;\, j=h,f})$*, prices of long-term government bonds* (z_l, z^*_l)*, gross inflation rates* (μ, μ^*)*, and the nominal depreciation rate of Home currency* $\frac{e_{+1}}{e}$*, satisfying (5)-(23).*

3.2 Equilibrium with Plentiful Collateral

What happens if fiscal authorities choose a sufficiently large value of V and V^* so that collateralizable assets are collectively plentiful in equilibrium? The following proposition characterizes such an equilibrium.

Proposition 1 (Existence) *If the sum of* V *and* V^* *is sufficiently large, then there exists a stationary equilibrium where collateral constraints do not bind, and no international trades take place. In this equilibrium, the quantities of DM consumption are given by* $x_1 = (u')^{-1}[1/z_s]$*,* $x^*_1 = (u')^{-1}[1/z^*_s]$ *and* $x_2 = x^*_2 = \hat{x}$*. Gross inflation rates are* $\mu = \frac{\beta}{z_s}$ *and* $\mu^* = \frac{\beta}{z^*_s}$*, prices of long-term government bonds are* $z_l = \frac{\beta}{\mu - \beta}$ *and* $z^*_l = \frac{\beta}{\mu^* - \beta}$*, and the nominal depreciation rate of Home currency is given by* $\frac{e_{+1}}{e} = \frac{z^*_s}{z_s}$*.*

Proposition 1 demonstrates that private banks' collateral constraints do not bind in equilibrium when there is a sufficiently high supply of collateralizable assets. It also characterizes equilibrium prices and allocations. Notably, there is no inefficiency in DM non-currency transactions since buyers consume the first-best quantity of consumption goods $\hat{x}$. Additionally, a change in the total value of consolidated government debt outstanding (a change in V or V^*) has no effect on equilibrium prices and allocations.

Conventional monetary policies matter as they influence the quantities of consumption in DM currency transactions, as is standard in monetary models. For example, consider a scenario where the Home central bank decreases the price of short-term Home bonds z_s, which is equivalent to an increase in the short-term nominal interest rate on Home bonds R_s since $1 + R_s = \frac{1}{z_s}$. As the Home inflation rate can be expressed as $\mu = \beta(1 + R_s)$, an increase in R_s leads to a one-for-one increase in μ (a pure *Fisher effect*). Then, the quantity of consumption in DM currency transactions x_1

declines as a higher μ induces a decrease in the real quantity of Home currency held by the public ρc.

Other types of monetary policies have no impact on equilibrium prices and allocations if conventional policy variables z_s and z_s^* remain constant. This is because, in equilibrium, central banks can only adjust the composition of collateralizable assets—local and foreign government bonds with short and long maturities—held by the public. However, the composition is irrelevant when collateralizable assets are plentiful in financial markets.[27]

In what follows, I will examine the welfare implications of these monetary policies. To do so, I measure welfare using the sum of lifetime utility across local agents in each country, as is standard in the monetary economics literature. For instance, the welfare measure for the Home country is defined by

$$W = \underbrace{\frac{1}{1-\beta}\left\{\rho[u(x_1)-x_1]+(1-\rho)[u(x_2)-x_2]\right\}}_{\text{DM surplus}} + \underbrace{\bar{X}_0 - \bar{X}_0^* + \frac{\beta}{1-\beta}(\bar{X}-\bar{X}^*)}_{\text{CM surplus}},$$

where $\bar{X}_0$ and $\bar{X}$ denote the quantities of CM goods imported from the Foreign country and consumed by Home agents, respectively, in period 0 and in each subsequent period. Also, $\bar{X}_0^*$ and $\bar{X}^*$ denote the corresponding quantities of CM goods produced by Home agents and exported to the Foreign country. The first term in the welfare function W is the sum of Home agents' lifetime utilities in the DM, equivalent to DM surplus, while the second term represents the sum of utilities in the CM or CM surplus. In the CM, the sum of local agents' utilities from consumption can differ from the sum of their disutilities from production. For example, the former exceeds the latter if there are net imports in the Home country, indicating that Home agents consume more than they produce. The welfare measure for the Foreign country is similarly defined.[28]

4 Equilibrium with a Low Effective Supply of Collateral

In this section, I examine cases in which the total value of consolidated government liabilities in both countries (represented by the sum of V and V^*) is inefficiently small, resulting in binding collateral constraints in equilibrium. Additionally, for the sake of simplicity, I assume that central

[27]A central bank's purchases of long-term local government bonds only result in swaps between short-term government bonds and long-term ones in equilibrium. This change in portfolio held by the public is irrelevant, similar to Wallace (1981). Similarly, a central bank's foreign asset purchases through sterilized intervention merely alter the relative supplies of local and foreign currency-denominated bonds in equilibrium. Thus, sterilized interventions are irrelevant, in line with Backus and Kehoe (1989).

[28]It is worth noting that this model does not address transitional dynamics, similar to standard models based on Lagos and Wright (2005) and Rocheteau and Wright (2005). Quasi-linear preferences and unconstrained labor supplies allow buyers and banks to choose their asset portfolios in the CM (control variables) independently of the asset holdings from the previous period (state variables). Relaxing either of these assumptions would enable the study of out-of-steady-state dynamics, but that falls beyond the scope of this paper.

banks are unable to hold long-term bonds as foreign exchange reserves, i.e., $\kappa_l = \kappa_l^* = 0$.[29] In some cases, local currency-denominated assets are exclusively held by local agents, leading to no international capital flows. In other cases, private banks in one country acquire both local and foreign government bonds in equilibrium. This occurs when the supply of collateralizable assets in one country is significantly lower compared to the collateral supply in the other country. Private banks in the country with higher asset market inefficiency are willing to pay a premium for foreign currency-denominated assets, causing assets to migrate to the country where they are valued most as collateral.

As I will demonstrate later, binding collateral constraints make an asset price higher than its fundamental price determined by the payoff structure and agents' preferences. In this model, an asset is considered in low supply when supply falls short of the demand at its fundamental price due to binding collateral constraints, creating an asset market inefficiency. Since the degree of asset market inefficiency may vary across countries, several scenarios arise regarding private banks' asset portfolios, depending on the relative degree of asset market inefficiency. For the sake of simplicity, I will focus on cases where asset market inefficiency is more severe in the Home country. The degree of asset market inefficiency can be measured by the multipliers associated with collateral constraints, λ and λ^*, which can be expressed as, from (10) and (17),

$$\lambda = \beta \left[u'(x_2) - 1 \right],$$
$$\lambda^* = \beta \left[u'(x_2^*) - 1 \right].$$

Then, a higher degree of asset market inefficiency in the Home country implies that $\lambda \geq \lambda^*$. This restriction also implies that Foreign banks do not purchase Home bonds, while Home banks may be willing to purchase Foreign bonds.

With $\lambda \geq \lambda^*$, there are four remaining cases to study. When λ is not significantly higher than λ^*, Home banks do not acquire Foreign bonds at prices that Foreign banks are willing to pay. However, as λ increases, Home banks become willing to pay a higher price to acquire Foreign bonds. Specifically, when λ becomes sufficiently high but not too high, Home banks purchase short-term Foreign bonds in addition to Home bonds of all maturities. Moreover, if Home and Foreign banks are willing to acquire a short-term Foreign bond at the same price, then short-term Foreign bonds are held in both countries. However, if Home banks are willing to pay a higher price for a short-term Foreign bond than Foreign banks, then short-term Foreign bonds are exclusively held by Home banks. Lastly, when λ becomes very high, Home banks acquire all types of government bonds issued in both countries, while Foreign banks only hold long-term Foreign bonds. Figure 1 illustrates the relationship between the degrees of asset market inefficiency and private banks' bond portfolios. I will analyze the first two cases in the following sections and present the other two cases in the Appendix.

[29] The implications of central banks' purchases of long-term foreign government bonds are discussed in the Appendix.

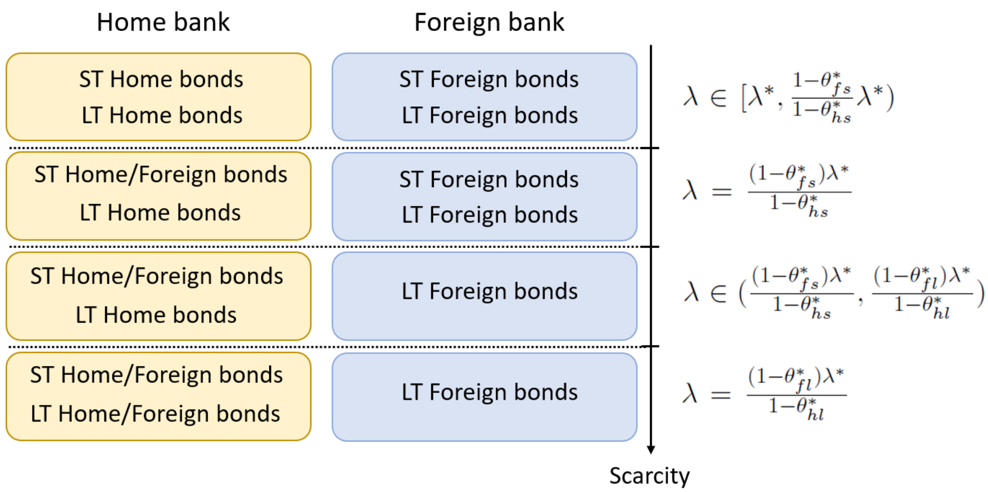

Figure 1: Asset market inefficiency and private banks' bond portfolios

Notes: ST - short-term; LT - long-term

4.1 Bond Yields and Term Premia

For convenience, I will focus solely on Home bonds when discussing nominal/real yields and the corresponding liquidity premia. Foreign counterparts are described in the Appendix. Noting that (11) and (13) hold with equality, the nominal bond yield for each maturity can be expressed as

$$R_j = \frac{\mu}{\beta\left[\left(1-\theta_{hj}\right)u'\left(x_2\right)+\theta_{hj}\right]} - 1, \tag{24}$$

for $j = s, l$. A term premium is defined by the difference between long-term and short-term bond yields. So, the nominal term premium can be expressed as

$$R_l - R_s = \frac{\mu\left(\theta_{hl}-\theta_{hs}\right)\left[u'\left(x_2\right)-1\right]}{\beta\left[\left(1-\theta_{hl}\right)u'\left(x_2\right)+\theta_{hl}\right]\left[\left(1-\theta_{hs}\right)u'\left(x_2\right)+\theta_{hs}\right]}. \tag{25}$$

The nominal term premium is strictly positive due to larger haircuts applied to long-term bonds compared to short-term bonds, coupled with inefficiently low consumption in DM non-currency transactions. The larger haircuts on long-term bonds, indicating their lower pledgeability relative to short-term bonds, are captured by $\theta_{hl} > \theta_{hs}$. Additionally, a collectively low supply of collateral combined with binding collateral constraints leads to inefficiencies in DM non-currency transactions, as measured by $\lambda > 0$ or $u'(x_2) > 1$. In other words, the low supply of collateralizable assets results in a lower quantity of goods exchanged in DM non-currency transactions compared to the first-best quantity $\hat{x}$.

In this model, the liquidity premium of a specific asset can be defined as the disparity between its fundamental yield and its actual yield. The fundamental yield of each government bond is determined by the bond's payoff structure and the preferences of agents acquiring the bond in equilibrium. Given quasi-linear preferences that effectively render private agents risk-neutral toward asset payoffs, the fundamental yield for Home bonds is given by $\frac{\mu}{\beta}-1$. Consequently, the liquidity premium for Home bonds of each maturity can be expressed as

$$L_j = \frac{\mu}{\beta} - 1 - R_j = \frac{\mu\left(1-\theta_{hj}\right)\left[u'\left(x_2\right)-1\right]}{\beta\left[\left(1-\theta_{hj}\right)u'\left(x_2\right)+\theta_{hj}\right]},$$

for $j = s, l$. Note that the liquidity premium increases with the associated pledgeability. For instance, the liquidity premium for short-term bonds exceeds that of long-term bonds, as $1-\theta_{hs} > 1-\theta_{hl}$. Hence, in this model, the term premium arises from the disparity in liquidity premia among bonds with different maturities.

Using (24), I can derive the real bond yields and the real term premium as

$$r_j = \frac{1}{\beta\left[\left(1-\theta_{hj}\right)u'\left(x_2\right)+\theta_{hj}\right]} - 1, \tag{26}$$

$$r_l - r_s = \frac{\left(\theta_{hl}-\theta_{hs}\right)\left[u'\left(x_2\right)-1\right]}{\beta\left[\left(1-\theta_{hl}\right)u'\left(x_2\right)+\theta_{hl}\right]\left[\left(1-\theta_{hs}\right)u'\left(x_2\right)+\theta_{hs}\right]}, \tag{27}$$

for $j = s, l$. Since the fundamental real bond yield is $\frac{1}{\beta}-1$ for all types of government bonds, the real liquidity premium for Home bonds can be computed as

$$l_j = \frac{1}{\beta} - 1 - r_j = \frac{\left(1-\theta_{hj}\right)\left[u'\left(x_2\right)-1\right]}{\beta\left[\left(1-\theta_{hj}\right)u'\left(x_2\right)+\theta_{hj}\right]},$$

for $j = s, l$.

4.2 Nominal Exchange Rate

Given that the law of one price holds in the CM, the nominal depreciation rate of the Home currency between current and future periods can be expressed as

$$\frac{e_{+1}}{e} = \frac{\mu}{\mu^*}. \tag{28}$$

Therefore, the depreciation rate of the Home currency is determined solely by the inflation rate ratio between the two countries. However, it is important to note that the model functions in a manner similar to the standard international asset pricing model by Lucas (1982). In the standard model, the nominal exchange rate is determined by

$$\frac{e_{+1}}{e} = \frac{m^*}{m}\frac{\mu}{\mu^*},$$

where m and m^* denote the intertemporal marginal rates of substitution (IMRS) of the representative Home and Foreign buyers. However, in my model, the IMRS is a constant β due to the quasi-linear preferences, which render buyers risk-neutral towards payoffs on assets in the CM.

From (24), (26), and (28), the ratio of the gross nominal yield on Home bonds relative to the corresponding yield on Foreign bonds can be expressed as

$$\frac{1+R_j}{1+R_j^*} = \frac{e_{+1}}{e} \cdot \frac{1+r_j}{1+r_j^*} = \frac{e_{+1}}{e} \cdot \frac{(1-\theta_{fj}^*)u'(x_2^*)+\theta_{fj}^*}{(1-\theta_{hj})\,u'(x_2)+\theta_{hj}} \tag{29}$$

for each maturity $j = s, l$. Equation (29) demonstrates that the uncovered interest parity (UIP) condition does not generally hold due to differing real interest rates on government bonds. A real interest rate differential arises in this model because frictions, characterized by varying degrees of pledgeability and binding collateral constraints, create distinct liquidity premia across government bonds.[30]

4.3 Equilibrium with Segmented Asset Markets

If the degree of asset market inefficiency in the Home country is not significantly higher than in the Foreign country, private banks will acquire assets denominated only in the local currency, i.e., $b_{hi}^* = b_{fi} = 0$ for $i = s, l$. This implies that there is no international asset trade between private agents and that international asset markets are effectively segmented. In this case, first-order conditions (11), (13), (19), and (21) hold with equality, while (12), (14), (18), and (20) do not. From (12) and (19), a necessary condition for the existence of this equilibrium is given by

$$\lambda^* \leq \lambda < \frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*}. \tag{30}$$

Substituting in the Home bank's collateral constraint (15) using (5), (9)-(11), (13), and (23) and noting that $b_{fj} = b_{hj}^* = 0$ for $j = s, l$, I can rewrite the collateral constraint as

$$\begin{aligned} 0 = &\left[u'(x_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho x_1 + \left[u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)x_2 \\ &- \left\{V + \kappa_s^* - \kappa_s - \frac{(\theta_{hl}-\theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(x_2)+\theta_{hl}\right]}\right\}. \end{aligned} \tag{31}$$

The first two terms on the right-hand side represent the demands for collateral, derived from the quantities of DM consumption in currency and non-currency transactions, while the last negative term represents the total supply of collateral. Therefore, equation (31) effectively states that there is no excess demand for collateral in equilibrium. From (9)-(11), and (13), the first-order conditions

[30] This finding is closely related to the work of Lee and Jung (2020), who identify the role of differential liquidity premia as a source of deviation from the UIP. They develop a two-country model based on Lagos and Wright (2005), similar to the one presented here. However, their focus is on the different functions of government bonds in transactions (as direct means of payment or collateralizable assets) while abstracting from international capital flows and banking activities. Bianchi et al. (2021) also construct a two-country model to provide a liquidity-based explanation for exchange rate dynamics.

for the Home bank's problem can be rewritten as

$$z_s = \frac{u'(x_2) - \theta_{hs} u'(x_2) + \theta_{hs}}{u'(x_1) - \theta_{hs} u'(x_2) + \theta_{hs}}, \tag{32}$$

$$z_l = \frac{(1-\theta_{hl}) u'(x_2) + \theta_{hl}}{u'(x_1) - \theta_{hs} u'(x_2) + \theta_{hs} - [(1-\theta_{hl}) u'(x_2) + \theta_{hl}]}, \tag{33}$$

$$\mu = \beta \left[u'(x_1) - \theta_{hs} u'(x_2) + \theta_{hs}\right]. \tag{34}$$

Similarly, using (6), (16), (17), (19), (21), and (23), the Foreign bank's collateral constraint (22) can be rewritten as

$$0 = \left[u'(x_1^*) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right] \rho x_1^* + \left[u'(x_2^*) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right] (1-\rho) x_2^* - \left\{V^* + \kappa_s - \kappa_s^* - \frac{(\theta_{fl}^* - \theta_{fs}^*)\omega_l^*}{(1-\theta_{fs}^*)[(1-\theta_{fl}^*)u'(x_2^*) + \theta_{fl}^*]}\right\}. \tag{35}$$

From (16), (17), (19), and (21), the first-order conditions for the Foreign bank's problem can be rewritten as

$$z_s^* = \frac{u'(x_2^*) - \theta_{fs}^* u'(x_2^*) + \theta_{fs}^*}{u'(x_1^*) - \theta_{fs}^* u'(x_2^*) + \theta_{fs}^*}, \tag{36}$$

$$z_l^* = \frac{(1-\theta_{fl}^*) u'(x_2^*) + \theta_{fl}^*}{u'(x_1^*) - \theta_{fs}^* u'(x_2^*) + \theta_{fs}^* - [(1-\theta_{fl}^*) u'(x_2^*) + \theta_{fl}^*]}, \tag{37}$$

$$\mu^* = \beta \left[u'(x_1^*) - \theta_{fs}^* u'(x_2^*) + \theta_{fs}^*\right]. \tag{38}$$

In addition to condition (30), monetary policy variables $(\omega_l, \kappa_s^*, \omega_l^*, \kappa_s)$ must satisfy conditions (7) and (8) for this equilibrium to exist. Another necessary condition is that the nominal interest rates on government bonds must be nonnegative in equilibrium. Since currencies always yield zero nominal interests and there are no frictions preventing arbitrage, negative nominal interest rates on government bonds cannot be sustained in equilibrium. This implies that central banks cannot choose short-term nominal interest rates lower than zero, i.e., $z_s \leq 1$ and $z_s^* \leq 1$.

Proposition 2 (Existence) *There exists a nonempty set of parameter values that support a stationary equilibrium with binding collateral constraints, characterized by equations (31)-(38).*

Solving the model follows a straightforward process. First, equations (31) and (32) solve for the quantities of Home DM consumption x_1 and x_2 given fiscal/monetary policies $(V, z_s, \omega_l, \kappa_s^*, \kappa_s)$. Then, (33) solves for the price of long-term Home bonds z_l, and (34) determines the Home inflation rate μ. Similarly, equations (35) and (36) solve for x_1^* and x_2^* given fiscal/monetary policies $(V^*, z_s^*, \omega_l^*, \kappa_s, \kappa_s^*)$. Subsequently, (37) solves for the price of long-term Foreign bonds z_l^*, (38) solves for the Foreign inflation rate μ^*, and (28) determines the nominal depreciation rate of the Home currency $\frac{e_{+1}}{e}$. Finally, conditions (7) and (8) establish upper and lower bounds on x_1, x_2, x_1^*, and

x_2^* while noting that $\bar{c} = \rho x_1 \left[u'(x_1) - \theta_{hs} u'(x_2) + \theta_{hs}\right]$ and $\bar{c}^* = \rho x_1^* [u'(x_1^*) - \theta_{fs}^* u'(x_2^*) + \theta_{fs}^*]$ from (9) and (16).

4.3.1 Conventional Monetary Policy

This section examines the impact of conventional monetary policy in an equilibrium with segmented international asset markets. The Home central bank conducts conventional monetary policy by adjusting z_s, the price of short-term Home bonds. What happens if the Home central bank decreases z_s (or increases the short-term nominal interest rate R_s) while keeping the value of long-term Home bonds outstanding ω_l and the value of short-term Foreign bonds acquired by the central bank κ_s^* constant?[31]

Proposition 3 (Conventional tightening) *Suppose there is a decrease in z_s in an equilibrium characterized by (31)-(38). Then, x_1 decreases and x_2 increases. Also, μ, R_l, r_s, r_l, and $\frac{e_{+1}}{e}$ increase, while $r_l - r_s$ and W fall. However, the effect on $R_l - R_s$ is ambiguous. The other variables remain unchanged.*

The effects of conventional monetary policy on DM consumption quantities and asset prices in the Home country align with the findings of the closed economy model in Williamson (2016), but only when the degree of asset market inefficiency is not significantly different across countries in equilibrium. Specifically, as z_s decreases, the quantity of consumption in DM currency transactions x_1 decreases, while the quantity of consumption in DM non-currency transactions x_2 increases in equilibrium. This happens because the decrease in z_s is achieved through the Home central bank's open market sale of short-term Home bonds. As a result, there is less currency outstanding in real terms, leading to lower consumption in currency transactions x_1. However, the quantity of Home bonds available for use as collateral increases in the private sector, effectively relaxing the collateral constraints of Home banks and increasing consumption in non-currency transactions x_2.

The relaxation of collateral constraints reduces the liquidity premium on Home bonds and increases the real interest rates r_s and r_l. However, the increase in the real interest rate on Home bonds of each maturity does not outweigh the increase in the corresponding nominal interest rate, leading to a rise in the inflation rate μ. The nominal interest rate on long-term Home bonds R_l also increases. The effect on the nominal term premium is ambiguous, as the rise in the inflation rate tends to increase the nominal term premium, while the decrease in the real term premium works in the opposite direction. These findings are summarized in Table 2.

It is important to note that the markets for Foreign currency-denominated assets are completely insulated from what happens in the markets for Home currency-denominated assets. However, the rise in the short-term nominal interest rate R_s increases the expected depreciation of the Home currency $\frac{e_{+1}}{e}$ in equilibrium. This aligns with the UIP relation, as an increase in the nominal interest

[31] It is important to note that monetary and fiscal policy variables are exogenously determined in this model. The following comparative statics analyses focus on the effects of a marginal change in a policy variable while keeping all other policy variables constant.

x_1	x_2	μ	R_s	R_l	$R_l - R_s$	r_s	r_l	$r_l - r_s$	W	
$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	?	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	
x_1^*	x_2^*	μ^*	R_s^*	R_l^*	$R_l^* - R_s^*$	r_s^*	r_l^*	$r_l^* - r_s^*$	W^*	$\frac{e_{+1}}{e}$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\uparrow$

Table 2: Effects of an increase in R_s

rate appreciates the local currency in the current period, accompanied by an expected depreciation in the future. The UIP's prediction holds true since, from (29), real interest rates rise by less than nominal interest rates, leading to an expected future depreciation of the local currency.

4.3.2 Unconventional Monetary Policy

The unconventional monetary policy I study in this model is also called quantitative easing (QE). Suppose that the Home central bank decreases ω_l, the value of long-term Home bonds held by the public, with other policy variables, z_s and κ_s^*, held constant. This policy is effectively QE as it involves an increase in the value of long-term Home bonds acquired by the central bank $V_l - \omega_l$.

Proposition 4 (Quantitative easing) *Suppose that there is a decrease in ω_l in an equilibrium characterized by equations (31)-(38). Then, x_1, x_2, r_s, r_l, and W increase, while μ, R_l, $R_l - R_s$, $r_l - r_s$, and $\frac{e_{+1}}{e}$ decrease. The other variables remain unchanged.*

Similar to the previous policy experiment, the effects of unconventional monetary policy in the Home country align with the findings of Williamson (2016). Specifically, the quantities of consumption in Home DM transactions x_1 and x_2 increase in equilibrium. Initially, the Home central bank increases its holdings of long-term Home bonds $V_l - \omega_l$ through swaps of Home currency for long-term Home bonds. However, open market purchases typically decrease nominal interest rates. To maintain the short-term nominal interest rate R_s at the target level, the Home central bank must conduct open market sales of short-term Home bonds. As a result, the Home central bank effectively swaps short-term Home bonds and Home currency for long-term Home bonds in equilibrium. With a higher real quantity of Home currency outstanding, the quantity of consumption in Home DM currency transactions x_1 increases. Also, this unconventional intervention increases the effective stock of collateral in the private sector, as short-term bonds are better collateral than long-term bonds ($\theta_{hs} < \theta_{hl}$). Consequently, the quantity of consumption in Home DM non-currency transactions x_2 increases.

A larger stock of collateral held by Home banks relaxes their collateral constraints, leading to a reduction in liquidity premia and an increase in real interest rates r_s and r_l. With no change in the short-term nominal interest rate R_s, an increase in the short-term real interest rate r_s implies lower inflation μ in equilibrium. While there are no effects on the prices of Foreign currency-denominated assets, a decrease in ω_l leads to a lower depreciation rate of the Home currency $\frac{e_{+1}}{e}$. This finding is consistent with the UIP, as a decline in the long-term nominal interest rate, caused by the central

x_1	x_2	μ	R_s	R_l	$R_l - R_s$	r_s	r_l	$r_l - r_s$	W	
$\uparrow$	$\uparrow$	$\downarrow$	$\cdot$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	
x_1^*	x_2^*	μ^*	R_s^*	R_l^*	$R_l^* - R_s^*$	r_s^*	r_l^*	$r_l^* - r_s^*$	W^*	$\frac{e_{+1}}{e}$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\downarrow$

Table 3: Effects of a decrease in ω_l

bank's purchase of long-term bonds, results in a depreciation of the local currency in the current period. These results are summarized in Table 3.

It is important to note that the Home central bank's conventional and unconventional monetary interventions do not spill over and affect the Foreign inflation rate or the nominal/real interest rates on Foreign bonds. This lack of spillover effects is due to the segmentation of international asset markets, where private banks participate only in local currency-denominated asset markets based on their own choices. However, these policy interventions do affect the nominal depreciation rate (exchange rate effect) since there are no frictions in goods markets that prevent the law of one price.

4.3.3 Foreign Exchange Intervention

Another policy experiment that can be conducted in this model involves a central bank's foreign exchange intervention coupled with foreign asset purchases. These purchases entail the central bank swapping local currency-denominated assets for foreign ones, which can have global implications, particularly when collateralizable assets are scarce. Consider the scenario where the Home central bank increases its holdings of short-term Foreign bonds κ_s^* while holding other policy variables z_s and ω_l constant. The following proposition outlines the effects of this intervention.

Proposition 5 (Foreign exchange intervention) *Suppose that there is an increase in κ_s^* in an equilibrium characterized by equations (31)-(38). Then, x_1 and x_2 increase, while x_1^* and x_2^* decrease. Also, r_s, r_l, μ^*, R_l^*, $R_l^* - R_s^*$, and $r_l^* - r_s^*$ increase, while μ, R_l, $R_l - R_s$, $r_l - r_s$, r_s^*, r_l^*, and $\frac{e_{+1}}{e}$ decrease. Finally, if β is sufficiently high, then W increases, and W^* decreases.*

The process begins with the central bank raising κ_s^* through the exchange of Home currency for short-term Foreign bonds. This intervention leads to an increase in the real quantity of currency outstanding, which in turn results in a proportional increase in x_1. However, according to equations (24) and (34), the increase in x_1 coincides with a decline in the inflation rate μ and the short-term nominal interest rate R_s. To maintain the policy rate R_s at its targeted level, the central bank must sell its holdings of short-term Home bonds. As a consequence, in equilibrium, the real quantities of Home currency and short-term Home bonds held by Home banks increase, ultimately causing x_1 and x_2 to rise.

Furthermore, the increased stock of collateral held by Home banks relaxes their collateral constraints, leading to higher real interest rates on Home bonds r_s and r_l (a liquidity effect). However, the rise in the long-term real interest rate is insufficient to offset the decline in the inflation rate.

x_1	x_2	μ	R_s	R_l	$R_l - R_s$	r_s	r_l	$r_l - r_s$	W	
$\uparrow$	$\uparrow$	$\downarrow$	$\cdot$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	
x_1^*	x_2^*	μ^*	R_s^*	R_l^*	$R_l^* - R_s^*$	r_s^*	r_l^*	$r_l^* - r_s^*$	W^*	$\frac{e_{+1}}{e}$
$\downarrow$	$\downarrow$	$\uparrow$	$\cdot$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$

Table 4: Effects of an increase in κ_s^*

As a result, both the long-term nominal interest rate R_l and the nominal term premium $R_l - R_s$ decrease.

It is important to note that although the Home central bank conducts open market operations to "sterilize" the potential impact of Foreign asset purchases on the short-term nominal interest rate R_s, these operations cannot fully neutralize the effects on other asset prices. This is because the central bank's intervention alters the relationship between the short-term nominal interest rate and the inflation rate, eventually causing changes in nominal and real interest rates beyond the short-term nominal rate. Unlike in the case of plentiful collateral where sterilized foreign exchange market interventions are inconsequential, in this scenario, they matter due to the low supply of collateralizable assets and the binding collateral constraints.

In contrast to the Home country, the Foreign country experiences a decrease in consumption in currency transactions x_1^* and consumption in non-currency transactions x_2^*. This implies opposite effects on the Foreign asset markets compared to those observed in the Home country. It is worth noting that the Home central bank's purchases of short-term Foreign bonds tend to reduce the short-term nominal interest rate on Foreign bonds R_s^*. Consequently, the Foreign central bank must conduct an open market sale of short-term Foreign bonds to maintain its policy rate R_s^*. This leads to a decrease in the quantity of currency outstanding and, subsequently, a reduction in x_1^*. Additionally, as the quantity of short-term Foreign bonds supplied by the Foreign central bank is smaller than the quantity purchased by the Home central bank, the supply of collateral in the Foreign country decreases, resulting in lower consumption in non-currency transactions x_2^*. Moreover, real bond yields r_s^* and r_l^* decline as the reduced supply of collateral tightens Foreign banks' collateral constraints.

The Home central bank's foreign exchange intervention leads to the appreciation of the Foreign currency in the current period, followed by its future depreciation. Furthermore, the Foreign country experiences a net import in the current CM as the Home central bank exchanges Home currency for Foreign bonds, and its counterpart (a Foreign bank) trades Home currency for CM goods produced by Home agents. Therefore, the current CM surplus in the Foreign country increases, as Foreign agents consume more in the current period without a corresponding increase in production. However, this effect is temporary, as the Foreign country experiences a net export in each subsequent CM.[32] For a sufficiently high β, the total decrease in DM surplus (due to a decline in x_1^* and x_2^*) and future CM surplus (a decrease in $\bar{X}^* - \bar{X}$) outweigh the increase in current CM surplus (an increase in

[32]The Home country experiences a net import in each subsequent period, as long as the redemption value of a Foreign bond exceeds the cost of its new acquisition, i.e., $z_s^* < 1$.

$\bar{X_0}^* - \bar{X_0}$). Thus, a *beggar-thy-neighbor* effect occurs, whereby higher welfare in the Home country is accompanied by lower welfare in the Foreign country. These results are summarized in Table 4.

4.4 Equilibrium with Interconnected Asset Markets

In this section, I analyze an equilibrium with interconnected asset markets, specifically focusing on a case where short-term Foreign bonds are held by private banks in both countries. Despite Home bonds being exclusively held by Home banks and long-term Foreign bonds exclusively held by Foreign banks, the prices of Home bonds are responsive to those of Foreign bonds and vice versa. In other words, asset markets are effectively interconnected across the two countries.

In this equilibrium, first-order conditions (11)-(13), (19), and (21) hold with equality. From (10), (12), (17), and (19), a necessary condition for the existence of this equilibrium can be written as

$$\lambda = \frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*}$$

or

$$(1-\theta_{hs}^*)\, u'(x_2) + \theta_{hs}^* = \left(1-\theta_{fs}^*\right) u'(x_2^*) + \theta_{fs}^*. \tag{39}$$

Equation (39) indicates that the degrees of inefficiency in DM non-currency transactions in the two countries, represented by $u'(x_2)$ and $u'(x_2^*)$, are positively correlated. Also, the collateral constraints faced by private banks in the two countries are interconnected, with all banks subject to the same international collateral constraint. From (5), (6), and (9)-(23), noting that $b_{fj} = b_{hl}^* = 0$ for $j = s, l$, the international collateral constraint can be expressed as

$$\mathcal{F}(\{x_k, x_k^*\}_{k=1,2}, \omega_l, \omega_l^*, \kappa_s, \kappa_s^*, V, V^*) = \mathcal{D}(\{x_k, x_k^*\}_{k=1,2}) - \mathcal{S}(x_2, x_2^*, V, V^*, \omega_l, \omega_l^*, \kappa_s^*, \kappa_s) = 0. \tag{40}$$

Here, $\mathcal{D}$ represents the aggregate demand for collateral, and $\mathcal{S}$ represents the aggregate supply. Thus, equation (40) implies that the excess demand in aggregate must be zero in equilibrium. Specifically, the aggregate demand for collateral is given by

$$\begin{aligned}\mathcal{D} = \left[u'(x_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho x_1 + \left[u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)x_2 \\ + \left[u'(x_1^*) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right]\Omega\rho x_1^* + \left[u'(x_2^*) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right]\Omega(1-\rho)x_2^*,\end{aligned}$$

while the aggregate supply of collateral is given by

$$\begin{aligned}\mathcal{S} = V + \Omega V^* + (1-\Omega)(\kappa_s^* - \kappa_s) \\ - \frac{(\theta_{hl}-\theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(x_2) + \theta_{hl}\right]} - \frac{\Omega(\theta_{fl}^*-\theta_{fs}^*)\omega_l^*}{(1-\theta_{fs}^*)[(1-\theta_{fl}^*)u'(x_2^*) + \theta_{fl}^*]},\end{aligned} \tag{41}$$

where

$$\Omega = \frac{u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}}{u'(x_2) + \frac{\theta^*_{hs}}{1-\theta^*_{hs}}}.$$

The factor Ω, augmented by the demand and supply from the Foreign country, is less than one because Home banks consider short-term Home bonds more pledgeable than short-term Foreign bonds, i.e., $\theta_{hs} < \theta^*_{hs}$.

The other first-order conditions for private banks' problems in this equilibrium lead to the same equilibrium conditions as those in an equilibrium with segmented asset markets, as given by (32)-(34) and (36)-(38). Additionally, monetary policy variables $(\omega_l, \kappa^*_s, \omega^*_l, \kappa_s)$ must satisfy conditions (7) and (8).

Proposition 6 (Existence) *There exists a nonempty set of parameter values that support a stationary equilibrium with binding collateral constraints that can be characterized by equations (32)-(34) and (36)-(40). Furthermore, the function $\mathcal{F}$ in (40) is strictly decreasing in the last three arguments (κ^*_s, V, V^*) and strictly increasing in the other arguments.*

The model can be solved using the following approach. First, equations (32), (36), (39), and (40) solve for the DM consumption quantities x_k and x^*_k for $k = 1, 2$, given monetary/fiscal policies $(V, V^*, z_s, z^*_s, \omega_l, \omega^*_l, \kappa^*_s, \kappa_s)$. Subsequently, (33) and (37) solve for z_l and z^*_l, (34) and (38) solve for μ and μ^*, and (28) determines $\frac{e_{+1}}{e}$.

For graphical illustration, consider cases where $\theta^*_{hs} = \theta^*_{fs}$, implying that from (39), $x_2 = x^*_2$ in equilibrium. Figure 2 provides a graphical depiction of the international collateral constraint (40) in equilibrium, represented by the curves IC in the (x_1, x_2) space and IC^* in the (x^*_1, x^*_2) space. The curves $z_s = z$ and $z^*_s = z^*$ represent equations (32) and (36), respectively, if $z_s = z < 1$ and $z^*_s = z^* < 1$. Thus, the solution for $\{x_k, x^*_k\}_{k=1,2}$ is uniquely determined by the intersection of the curves IC and $z^s = z$, as well as the intersection of the curves IC^* and $z^*_s = z^*$. Conditions (7) and (8) impose upper and lower bounds on the DM consumption quantities.

In this interconnected equilibrium, where Home banks hold short-term Foreign bonds, there are international capital flows from the Home country to the Foreign country. In particular, Home banks receive deposits from Home buyers and exchange CM goods for b^*_{hs} units of short-term Foreign bonds, in real terms, at price $z^*_s \leq 1$. This implies that, from the Home country's perspective, there are net exports (a positive entry in the current account) and net capital outflows (a negative entry in the capital account) in the CM of period 0. In each subsequent CM, Home banks receive b^*_{hs} units of Foreign currency from the Foreign fiscal authority and purchase the same quantity of short-term Foreign bonds at price $z^*_s \leq 1$. Consequently, there are net capital inflows with a value of $(1-z^*_s)b^*_{hs}$, accompanied by net imports of the same value.

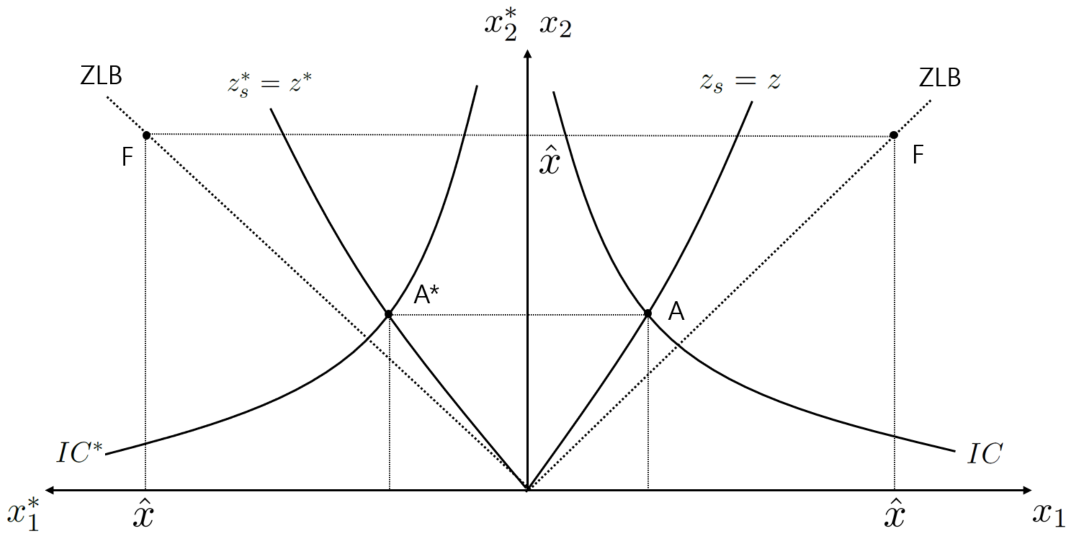

Figure 2: Equilibrium with short-term Foreign bonds held in both countries

Proposition 7 (Capital control) *Suppose that an equilibrium is characterized by (32)-(34) and (36)-(40), and β is sufficiently high. Then, restricting international capital mobility decreases W and increases W^*.*

To assess whether the free movement of international capital is mutually beneficial, consider a hypothetical equilibrium where international capital flows are strictly prohibited. Compared to this autarky equilibrium, the expected utility of Home buyers from consuming DM goods would be higher in an equilibrium with freely mobile international capital, as Home banks can acquire a larger quantity of collateral to provide a larger quantity of deposit claims to buyers. Although Home agents would experience some disutility from producing CM goods for the Foreign country in the current period, it would not exceed the increase in the discounted expected utility in the DM for every period, provided that agents are sufficiently patient. Therefore, welfare in the Home country W would be higher in an equilibrium with international capital flows.

However, for Foreign agents, the higher utility in the current CM does not sufficiently compensate for the lower discounted expected utility in every DM. Hence, welfare in the Foreign country W^* would be lower in an equilibrium with international capital flows. In essence, introducing a policy that restricts international capital mobility would increase W^* at the expense of a decrease in W. These effects arise because international asset markets effectively allow the Home country to alleviate its asset market inefficiency by importing assets from the Foreign country, similar to the findings in Caballero et al. (2020).

4.4.1 Conventional Monetary Policy

Consider the conventional monetary policy of the Home central bank.[33] Specifically, suppose that the price of short-term Home bonds z_s decreases from z^0 to z^1 (or the short-term nominal interest rate R_s increases) while other policy variables remain constant.

Proposition 8 (Conventional tightening) *Suppose that there is a decrease in z_s in an equilibrium characterized by equations (32)-(34) and (36)-(40). Then, x_1 decreases and x_2 increases, while x_1^* and x_2^* both increase. Additionally, μ, R_l, r_s, r_l, r_s^*, r_l^*, and $\frac{e+1}{e}$ increase, while $r_l - r_s$, μ^*, R_l^*, $R_l^* - R_s^*$, and $r_l^* - r_s^*$ decrease. However, the effect on $R_l - R_s$ is ambiguous. Finally, if β is sufficiently high, then W decreases and W^* increases.*

The impact of a decrease in z_s on the consumption quantities in DM transactions is illustrated in Figure 3. From (36) and (40), the curves IC_0, IC_0^*, and $z_s^* = z^*$ remain unchanged with a decrease in z_s, while from (32), the curve $z_s = z^0$ shifts up to $z_s = z^1$. Thus, consumption in DM currency transactions x_1 falls and consumption in DM non-currency transactions x_2 rises (from point A to B), similar to what happens in an equilibrium with segmented asset markets.

However, in an equilibrium with interconnected asset markets, the increase in x_2 is accompanied by an increase in x_2^* and the shift of the curve IC^* from IC_0^* to IC_1^*. This occurs because the Home central bank can only increase R_s through its open market sale of short-term Home bonds, which tends to increase the supply of collateral in the global economy. However, points B and B^* do not constitute an equilibrium because, from (36), the increase in x_2^* tends to increase the short-term nominal interest rate on Foreign bonds R_s^*. To maintain R_s^* at the target level, the Foreign central bank needs to purchase short-term Foreign bonds (represented by a transition from point B^* to C^*), shifting the curve IC_0 down to IC_1. This effect arises as the Foreign central bank's open market purchase reduces the quantity of collateral held in the global economy.

Nevertheless, points C and C^* do not represent an equilibrium either, as R_s tends to decrease. Therefore, the Home central bank must sell more of its holdings of short-term Home bonds to achieve the target level of R_s, causing the curve IC_1^* to shift up again. This process continues until a new equilibrium is reached at points E and E^*. Consequently, in the Home country, x_1 falls more and x_2 rises less compared to an equilibrium with segmented asset markets (due to the effect from point B to E), while in the Foreign country, the consumption quantities in DM transactions, x_1^* and x_2^*, increase (from point A^* to E^*) due to the international spillover effects.

In this equilibrium, the effects of conventional monetary intervention on the prices of Home currency-denominated assets are qualitatively identical to those in an equilibrium with segmented asset markets. Specifically, the Home inflation rate μ rises, the real interest rates on Home bonds r_s and r_l increase, and the real term premium $r_l - r_s$ falls. However, international spillovers lead to a further increase in μ, as an increase in R_s requires a larger quantity of open market sales by the

[33] The effects of the Foreign central bank's conventional monetary intervention are qualitatively symmetric to those of the Home central bank's intervention.

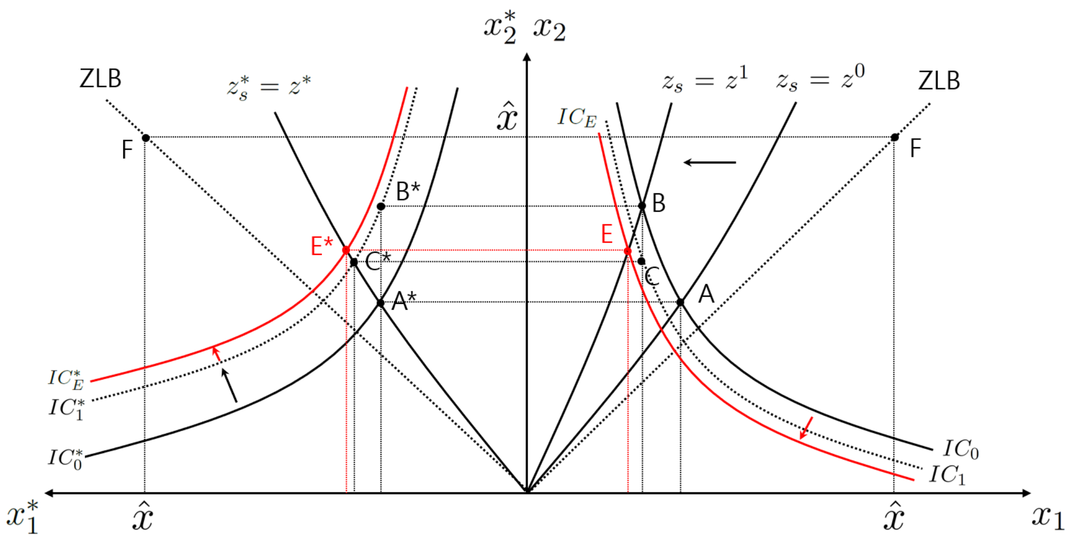

Figure 3: Conventional monetary policy: an increase in R_s

x_1	x_2	μ	R_s	R_l	$R_l - R_s$	r_s	r_l	$r_l - r_s$	W	
$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	?	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	

x_1^*	x_2^*	μ^*	R_s^*	R_l^*	$R_l^* - R_s^*$	r_s^*	r_l^*	$r_l^* - r_s^*$	W^*	$\frac{e_{+1}}{e}$
$\uparrow$	$\uparrow$	$\downarrow$	$\cdot$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$

Table 5: Effects of an increase in R_s

Home central bank. Since there is a smaller quantity of Home currency outstanding compared to an equilibrium with segmented asset markets, the rate of return on currency must be lower, resulting in a higher μ.

Also, international spillovers tend to decrease real interest rates r_s and r_l, because the Foreign central bank's conventional open market operation reduces the supply of short-term Foreign bonds, thereby tightening the international collateral constraint. However, the decrease in r_s and r_l resulting from the Foreign central bank's intervention does not outweigh their increase caused by the Home central bank's intervention. Consequently, r_s and r_l increase in equilibrium, but to a lesser extent than in an equilibrium with segmented asset markets.

The conventional monetary intervention by the Home central bank also affects asset prices in the Foreign country and the nominal exchange rate. Specifically, the Foreign inflation rate μ^* decreases, the real interest rates on Foreign bonds r_s^* and r_l^* increase, and the real term premium $r_l^* - r_s^*$ falls. Moreover, the nominal interest rate on long-term Foreign bonds R_l^* and the nominal term premium $R_l^* - R_s^*$ decrease, while the depreciation rate of the Home currency $\frac{e_{+1}}{e}$ rises.

The effect on the nominal exchange rate aligns with the prediction of the UIP in that an increase in R_s leads to a current appreciation of the Home currency, followed by an expected future depreciation. However, the effect on R_l^* appears non-standard, as capital outflows from the Foreign country typically increase the nominal interest rate on long-term Foreign bonds. In this model, there is indeed upward pressure on the long-term nominal interest rate due to the increase in the short-term interest rate on Home bonds and resulting capital flows. However, there is also downward pressure from the Foreign central bank's conventional open market operation, which increases the real quantity of Foreign currency outstanding and reduces the Foreign inflation rate. As the decrease in the Foreign inflation rate exceeds the increase in the long-term real interest rate, the long-term nominal interest rate decreases in equilibrium. These results are summarized in Table 5.

4.4.2 Unconventional Monetary Policy

Here, I analyze the effects of unconventional monetary policies implemented by the Home and Foreign central banks. Consider first the Home central bank's quantitative easing (QE), which involves an increase in the value of its holdings of long-term Home bonds $V_l - \omega_l$. This can be achieved by decreasing ω_l while holding z_s and κ_s^* constant.

Proposition 9 (Quantitative easing) *Suppose there is a decrease in ω_l in an equilibrium characterized by equations (32)-(34) and (36)-(40). Then, x_1, x_2, x_1^*, and x_2^* all increase. Also, μ, μ^*, R_l, R_l^*, $R_l - R_s$, $R_l^* - R_s^*$, $r_l - r_s$, $r_l^* - r_s^*$, and $\frac{e+1}{e}$ decrease, while r_s, r_l, r_s^*, and r_l^* increase. Moreover, if β is sufficiently high, then W and W^* increase.*

The effects of a decrease in ω_l on the quantities of DM consumption are depicted in Figure 4. From (32) and (36), the curves $z_s = z$ and $z_s^* = z^*$ remain fixed. However, the curves representing equation (40) shift up from IC_0 to IC_1 in the (x_1, x_2) space and from IC_0^* to IC_1^* in the (x_1^*, x_2^*) space. Similar to an equilibrium with segmented asset markets, a decrease in ω_l relaxes the collateral constraints of private banks. This occurs because the Home central bank effectively swaps better collateral (short-term Home bonds) for worse collateral (long-term Home bonds) in equilibrium, thereby increasing the effective stock of collateral held by the public.

If the Foreign central bank does not intervene through its open market operation, x_1 and x_2 would increase from point A to B, as observed in an equilibrium with segmented asset markets. However, there is upward pressure on the nominal interest rate on short-term Foreign bonds R_s^*, causing the Foreign central bank to swap Foreign currency for short-term Foreign bonds to maintain its policy rate constant. The Foreign central bank's open market purchase tightens the collateral constraint, mitigating the original effects of a decrease in ω_l. Therefore, in equilibrium at points E and E^*, the quantities of DM consumption in the Home country x_1 and x_2 rise but to a lesser extent than in an equilibrium with segmented asset markets. However, the corresponding Foreign consumption quantities x_1^* and x_2^* also increase due to international spillover effects.

In this equilibrium, the effects of a decrease in ω_l on the prices of Home currency-denominated assets are qualitatively identical to those in an equilibrium with segmented asset markets. Specifically, the inflation rate μ falls, the real interest rates r_s and r_l rise, and the real term premium $r_l - r_s$ decreases. Also, the long-term nominal interest rate R_l and the nominal term premium $R_l - R_s$ both

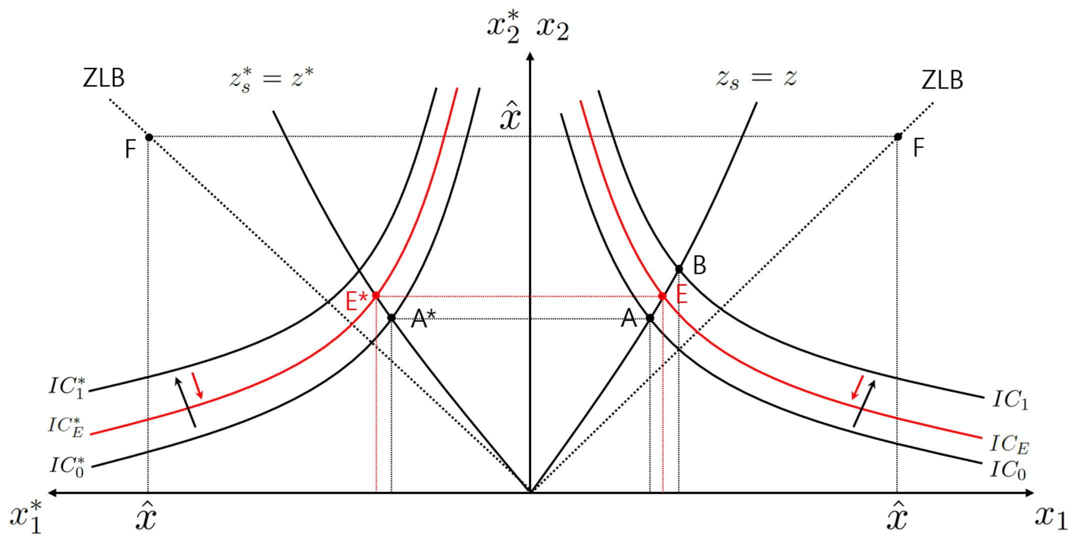

Figure 4: Quantitative easing or foreign asset purchases: an increase in $V_l - \omega_l$ or κ_s^*

x_1	x_2	μ	R_s	R_l	$R_l - R_s$	r_s	r_l	$r_l - r_s$	W	
$\uparrow$	$\uparrow$	$\downarrow$	$\cdot$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	
x_1^*	x_2^*	μ^*	R_s^*	R_l^*	$R_l^* - R_s^*$	r_s^*	r_l^*	$r_l^* - r_s^*$	W^*	$\frac{e_{+1}}{e}$
$\uparrow$	$\uparrow$	$\downarrow$	$\cdot$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$

Table 6: Effects of an increase in $V_l - \omega_l$ or κ_s^*

decrease. However, these effects are quantitatively smaller than those in an equilibrium with segmented asset markets due to the Foreign central bank's conventional intervention, which mitigates the original effects of a decrease in ω_l on the asset prices.

The Home central bank's unconventional monetary intervention also influences the prices of Foreign currency-denominated assets due to spillover effects. In particular, the prices of Foreign assets change in the same direction as those of Home assets. This means that the real interest rates r_s^* and r_l^* rise, the long-term nominal interest rate R_l^* and the nominal term premium $R_l^* - R_s^*$ decrease, and the inflation rate μ^* falls. Additionally, the expected depreciation rate of Home currency $\frac{e_{+1}}{e}$ decreases, indicating a current depreciation of the Home currency accompanied by an expected future appreciation. Although both central banks' open market purchases result in larger real quantities of currencies outstanding, the size of the Home central bank's open market purchases of long-term Home bonds (net of its sales of short-term Home bonds) is larger than that of the Foreign central bank's purchases. As a result, the Home inflation rate μ falls more than the Foreign inflation rate μ^*, leading to a decline in the depreciation rate of the Home currency $\frac{e_{+1}}{e}$. These findings are summarized in Table 6.

Notably, QE implemented by the Home central bank reduces the long-term nominal interest rates, flattens the yield curves internationally, and causes an immediate depreciation of the Home currency. These results are consistent with those of Alpanda and Kabaca (2020) and Kolasa and

Wesolowski (2020), who develop two-country DSGE models incorporating asset market segmentation with "preferred habitat" or "portfolio balance" theories to examine the international effects of QE. However, contrary to their models, the real interest rates rise, and the inflation rates fall in my model. The increase in real interest rates and enhanced global welfare arise from a relaxation of the international collateral constraint faced by Home and Foreign banks. This result aligns with the findings of Dedola et al. (2013) in that QE mitigates the financial constraints of both domestic and foreign private banks, thereby generating positive spillover effects.

Interestingly, the effects of the Foreign central bank's unconventional monetary intervention are qualitatively symmetric to those of the Home central bank's intervention, except for the impact on the nominal exchange rate. Specifically, a decrease in ω_l^* (reflecting the Foreign central bank's QE) leads to a current appreciation of the Foreign currency. Since the Home country exhibits a higher degree of asset market inefficiency, the Home central bank's open market purchases have a more substantial effect on the Home inflation rate than the Foreign central bank's purchases have on the Foreign inflation rate. Consequently, QE implemented by the Foreign central bank, or the central bank in the country with lower asset market inefficiency, results in a current appreciation of the local currency followed by an expected future depreciation.

4.4.3 Foreign Exchange Intervention

What are the global effects of central banks' foreign asset purchases? Similar to the QE experiment, it is crucial to understand whether a central bank's intervention increases or decreases the supply of collateralizable assets in financial markets. The following corollary, derived from (41) and Proposition 6, sheds light on the matter.

Corollary 1 *The function $\mathcal{S}$ is strictly decreasing in $(\omega_l, \omega_l^*, \kappa_s)$ and strictly increasing in (κ_s^*, V, V^*).*

According to Corollary 1, decreasing ω_l, ω_l^*, or κ_s, and increasing κ_s^*, V, or V^* effectively increase the supply of collateral in the interconnected asset market. Therefore, an increase in κ_s^* (the Home central bank's purchases of short-term Foreign bonds), a decrease in κ_s (the Foreign central bank's sales of short-term Home bonds), and a decrease in ω_l (the Home central bank's purchases of long-term Home bonds) have the same qualitative effects on equilibrium prices and allocations, given that conventional policy rates R_s and R_s^* remain constant.

Consider the Home central bank's foreign exchange intervention. If there is an increase in κ_s^* (the value of short-term Foreign bonds held by the Home central bank), the curves representing (40) shift up from IC_0 to IC_1 in the (x_1, x_2) space and from IC_0^* to IC_1^* in the (x_1^*, x_2^*) space, while the curves $z_s = z$ and $z_s^* = z^*$ remain constant (as depicted in Figure 4). Unlike in an equilibrium with segmented asset markets where an increase in κ_s^* has a beggar-thy-neighbor effect, this intervention increases the effective supply of collateral in the global financial market.

Notice that the Foreign central bank must conduct open market operations to eliminate upward pressure on the short-term nominal interest rate on Foreign bonds R_s^*, shifting down IC_1 and IC_1^*.

Similar to the effects of a decrease in ω_l, all consumption quantities in DM transactions increase, while inflation rates fall. The long-term nominal interest rates fall, and short-term and long-term real interest rates rise, leading to a decrease in both nominal and real term premia. Furthermore, the Home currency experiences a current depreciation followed by an expected future appreciation, as the depreciation rate of the Home currency falls.

In contrast, an increase in κ_s (the value of short-term Home bonds held by the Foreign central bank) decreases the effective supply of collateral in the global economy, resulting in opposite effects compared to an increase in κ_s^*. The key factor determining the international effects of foreign exchange (FX) intervention lies in the relative market value of assets issued or sold by a central bank as collateral in exchange for foreign assets. When the Home central bank purchases Foreign bonds, it swaps short-term Home bonds for short-term Foreign bonds in equilibrium. Since short-term Home bonds are considered better collateral than short-term Foreign bonds in the interconnected asset market ($\theta_{hs} < \theta_{hs}^*$), Home banks reduce their holdings of short-term Foreign bonds, effectively increasing the stock of collateral held by Foreign banks and relaxing the international collateral constraint. However, when the Foreign central bank purchases Home bonds, it swaps short-term Foreign bonds for short-term Home bonds. As a result, the private sector holds a smaller quantity of Home bonds and a larger quantity of Foreign bonds, causing Home banks to acquire a larger quantity of short-term Foreign bonds, thereby tightening the international collateral constraint.

To properly evaluate the international effect of FX intervention, it is important to consider the relative market value of assets, as collateral, that a central bank issues or sells in exchange for foreign assets. For example, the Swiss National Bank (SNB)'s foreign exchange reserves at the end of the first quarter of 2021 consist of government bonds (66%), other bonds (11%), and equities (23%). Among the fixed-income assets, 61% are AAA-rated, 20% are AA-rated, and 19% are below AA.[34] Considering that the credit rating for Swiss government bonds has been AAA, the SNB's swaps of local government liabilities for foreign assets are likely to alleviate the shortage of safe assets and global financial frictions.[35]

However, FX intervention in some emerging market and developing economies can exacerbate the global shortage of safe assets. This can happen if their government liabilities are considered less valuable as collateral in global financial markets compared to the assets they purchase, which are mostly liquid and safe. In such cases, policies that allow foreign central banks to hold reserve accounts at central banks in advanced economies (the issuers of safe assets) can mitigate the global shortage of safe assets. For instance, the Federal Reserve's overnight reverse repurchase agreement facility and liquidity swap lines enable foreign central banks to hold reserve accounts at the Fed or have options to withdraw US dollars. These policies reduce foreign central banks' incentives to purchase US Treasury securities in financial markets, thereby relaxing the collateral constraints of

[34] See Table for Investment Structure of Foreign Exchange Reserves and Swiss Franc Bond Investments at https://www.snb.ch/en/iabout/assets/id/assets_reserves.

[35] See Moody's Investors Service's 2020 rating outlook for Government of Switzerland bonds, which is available at https://www.moodys.com/research/Moodys-affirms-Switzerlands-Aaa-ratings-maintains-stable-outlook--PR_436243.

private banks.

5 Conclusion

I have developed a two-country general equilibrium model that incorporates limited commitment, differential pledgeability of collateral, and a low supply of collateralizable assets. In this model, short-term government debt is considered more pledgeable as collateral compared to long-term government debt, resulting in term premia. Additionally, financial intermediaries exhibit a home bias in their asset portfolios, favoring local assets over foreign assets due to their higher pledgeability.

Given the low supply of collateralizable assets in interconnected financial markets, quantitative easing can reduce long-term bond yields and term premia and improve welfare on an international scale. By enhancing the quality of collateral available in the global financial market, quantitative easing relaxes the collateral constraints faced by financial intermediaries, leading to an increase in real bond yields. Foreign exchange intervention can have different implications depending on the nature of financial market segmentation. In cases where financial markets are endogenously segmented in equilibrium, foreign exchange intervention can result in beggar-thy-neighbor outcomes. However, when financial markets are globally interconnected in equilibrium, foreign exchange intervention can sometimes enhance global welfare. Generally, when a central bank engages in foreign asset purchases, it involves swapping local currency-denominated assets for foreign currency-denominated assets. If local assets are considered more valuable as collateral than foreign assets, foreign exchange intervention can effectively increase the supply of collateral in financial markets, alleviating the collateral constraints faced by financial intermediaries.

While this paper sheds light on the impact of domestic monetary policy on global asset prices and welfare, there are avenues for further research. Specifically, exploring the fundamental factors that contribute to the differential pledgeability of assets would be a valuable area for future research. A more in-depth modeling of these factors could provide additional insights and contribute to the advancement of research in this field.

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A Appendix

A.1 A Foreign Bank's Problem

Similarly to problem (1) subject to (2)-(4), a Foreign bank's problem in equilibrium can be expressed as

$$\max_{k^*,c^*,d^*,b_{fs},b_{fl},b^*_{fs},b^*_{fl}} -k^* + \rho u\left(\frac{\beta\phi^*_{+1}c^*}{\phi^*}\right) + (1-\rho)u\left(\beta d^*\right)$$

subject to

$$\begin{aligned}
& k^* - \rho c^* - z_s b_{fs} - z^*_s b^*_{fs} - z_l b_{fl} - z^*_l b^*_{fl} - \beta(1-\rho)d^* \\
& + \beta\frac{\phi_{+1}}{\phi}\left\{b_{fs} + (1+z_{l,+1})b_{fl}\right\} + \beta\frac{\phi^*_{+1}}{\phi^*}\left\{b^*_{fs} + (1+z^*_{l,+1})b^*_{fl}\right\} \geq 0, \\
& -(1-\rho)d^* + \frac{\phi_{+1}}{\phi}\left\{b_{fs} + (1+z_{l,+1})b_{fl}\right\} + \frac{\phi^*_{+1}}{\phi^*}\left\{b^*_{fs} + (1+z^*_{l,+1})b^*_{fl}\right\} \\
& \geq \frac{\phi_{+1}}{\phi}\left\{\theta_{fs}b_{fs} + (1+z_{l,+1})\theta_{fl}b_{fl}\right\} + \frac{\phi^*_{+1}}{\phi^*}\left\{\theta^*_{fs}(\rho c^* + b^*_{fs}) + (1+z^*_{l,+1})\theta^*_{fl}b^*_{fl}\right\}, \\
& k^*, c^*, d^*, b_{fs}, b_{fl}, b^*_{fs}, b^*_{fl} \geq 0,
\end{aligned}$$

where (k^*, c^*, d^*) is the deposit contract of the Foreign bank, which is analogous to (k, c, d) of the Home bank, and b^*_{fs} and b^*_{fl} (b_{fs} and b_{fl}) are, respectively, short-term and long-term Foreign (Home) bonds acquired by the Foreign bank.

A.2 Discussions on Alternative Fiscal and Monetary Policies

The fiscal policy rule presented in this paper can be interpreted as the debt ceiling or debt limit in the United States. To understand this, let $\hat{b}_s$ and $\hat{b}_l$ denote the Home central bank's holdings of short-term and long-term Home bonds, respectively, in period 0. The Home central bank purchases Home and Foreign bonds by issuing Home currency in period 0 and then transfers its profits to the Home fiscal authority in every following period. As the central bank capital is zero, the real value of the central bank's assets must be equal to that of liabilities, i.e.,

$$\sum_{i=s,l}\left[z_i\hat{b}_i + z^*_i a^*_i\right] = \bar{c},$$

where a^*_i is the real quantity of Foreign bonds held by the central bank for $i = s, l$ and $\bar{c}$ is the real value of Home currency outstanding. From equation (5), the fiscal policy rule in the Home country can be rewritten as

$$V = \sum_{i=s,l} z_i\left[\bar{b}_i + \hat{b}_i\right].$$

Therefore, the Home fiscal authority effectively sets the total value of Home bonds issued in period 0. If $\bar{V}$ represents the level of debt ceiling, I can demonstrate that a welfare-maximizing fiscal authority would set the value of Home bonds at $\bar{V}$ in equilibrium, provided that $\bar{V}$ is sufficiently small.

In practice, central banks set a short-term nominal interest rate (typically, an overnight rate or an interest rate on reserves) to achieve a desired rate of inflation. Hence, conventional monetary policy in this paper aligns with reality as central banks exogenously adjust short-term nominal interest rates R_s and R_s^* so that inflation rates μ and μ^* are determined endogenously. This setup also allows studying the effects of other types of monetary policies, such as quantitative easing (QE) and foreign exchange (FX) intervention. For example, it enables the analysis of the international effect of QE when short-term nominal interest rates are constrained by zero lower bounds, as in Alpanda and Kabaca (2020) and Kolasa and Wesolowski (2020). Additionally, the current setup makes it possible to differentiate between sterilized FX intervention and non-sterilized intervention.[36]

In this model, fiscal authorities exogenously determine V and V^*, the real values of consolidated government liabilities held by the public. Hence, levels of taxes τ and τ^* are endogenously determined. However, fiscal authorities might want to choose the quantity of government debt outstanding given the tax levels. Therefore, it may be useful to consider τ and τ^* as exogenous variables, and V and V^* as endogenous variables following the fiscal-theory-of-the-price-level literature (See Leeper, 1991). A key advantage of the current setup is that, given V and V^*, the balance sheets of central banks and the real values of government bonds held by private banks are well-defined. Consequently, this allows for analyzing how monetary policies change the composition of assets held by private banks given V and V^* in a tractable manner.

A.3 Foreign Bond Yields and Term Premia

Note that (19) and (21) hold with equality. So, the nominal yield on Foreign bonds of each maturity can be expressed as

$$R_j^* = \frac{\mu^*}{\beta[(1-\theta_{fj}^*)u'(x_2^*) + \theta_{fj}^*]} - 1, \tag{42}$$

for $j = s, l$. As a term premium is the difference between long-term and short-term bond yields, the nominal term premium for Foreign bonds can be expressed as

$$R_l^* - R_s^* = \frac{\mu^*(\theta_{fl}^* - \theta_{fs}^*)\left[u'(x_2^*) - 1\right]}{\beta[(1-\theta_{fl}^*)u'(x_2^*) + \theta_{fl}^*][(1-\theta_{fs}^*)u'(x_2^*) + \theta_{fs}^*]}. \tag{43}$$

A liquidity premium is the difference between the fundamental yield and the actual yield on a particular asset. Since the fundamental yield on Foreign bonds is given by $\frac{\mu^*}{\beta} - 1$, the liquidity

[36]Some readers may be interested in studying the effects of QE and FX intervention in an economy where inflation rates are exogenously set by central banks (and thus, R_s and R_s^* are endogenous). In this model, incorporating such a policy is straightforward, as constant inflation rates imply constant consumption quantities in DM currency transactions x_1 and x_1^*. Consequently, the effects on asset prices discussed in the paper will be amplified given constant inflation rates.

premia for Foreign bonds can be expressed as

$$L_j^* = \frac{\mu^*}{\beta} - 1 - R_j^* = \frac{\mu^*(1-\theta_{fj}^*)\left[u'\left(x_2^*\right)-1\right]}{\beta[(1-\theta_{fj}^*)u'\left(x_2^*\right)+\theta_{fj}^*]},$$

for $j = s, l$. The associated real bond yields, real term premium, and real liquidity premia can be expressed as

$$r_j^* = \frac{1}{\beta[(1-\theta_{fj}^*)u'\left(x_2^*\right)+\theta_{fj}^*]} - 1, \tag{44}$$

$$r_l^* - r_s^* = \frac{(\theta_{fl}^* - \theta_{fs}^*)\left[u'\left(x_2^*\right)-1\right]}{\beta[(1-\theta_{fl}^*)u'\left(x_2^*\right)+\theta_{fl}^*][(1-\theta_{fs}^*)u'\left(x_2^*\right)+\theta_{fs}^*]}, \tag{45}$$

$$l_j^* = \frac{1}{\beta} - 1 - r_j^* = \frac{(1-\theta_{fj}^*)\left[u'\left(x_2^*\right)-1\right]}{\beta[(1-\theta_{fj}^*)u'\left(x_2^*\right)+\theta_{fj}^*]},$$

for $j = s, l$.

A.4 Accumulating Long-Term Bonds as Foreign Exchange Reserves

Suppose that central banks can purchase long-term foreign government bonds as foreign exchange reserves, i.e., $\kappa_l^* > 0$ and $\kappa_l > 0$. Also, confine attention to cases discussed in Section 4: Equilibrium with Scarce Collateral.

First, consider an equilibrium with segmented asset markets. I can obtain the following collateral constraints:

$$\begin{aligned} 0 = &\left[u'(x_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho x_1 + \left[u'\left(x_2\right) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)x_2 \\ &- \left\{V + \kappa_s^* - \kappa_s + \kappa_l^* - \mathit{\Gamma}\kappa_l - \frac{(\theta_{hl}-\theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'\left(x_2\right)+\theta_{hl}\right]}\right\}, \end{aligned} \tag{46}$$

for Home banks, where

$$\mathit{\Gamma} = \frac{u'\left(x_2\right) + \frac{\theta_{hs}}{1-\theta_{hs}}}{u'\left(x_2\right) + \frac{\theta_{hl}}{1-\theta_{hl}}} < 1,$$

and

$$\begin{aligned} 0 = &\left[u'(x_1^*) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right]\rho x_1^* + \left[u'\left(x_2^*\right) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right](1-\rho)x_2^* \\ &- \left\{V^* + \kappa_s - \kappa_s^* + \kappa_l - \mathit{\Gamma}^*\kappa_l^* - \frac{(\theta_{fl}^*-\theta_{fs}^*)\omega_l^*}{(1-\theta_{fs}^*)[(1-\theta_{fl}^*)u'\left(x_2^*\right)+\theta_{fl}^*]}\right\}, \end{aligned} \tag{47}$$

for Foreign banks, where

$$\Gamma^{*}=\frac{u'\left(x_{2}\right)+\frac{\theta_{fs}^{*}}{1-\theta_{fs}^{*}}}{u'\left(x_{2}\right)+\frac{\theta_{fl}^{*}}{1-\theta_{fl}^{*}}}<1.$$

Then, equations (32) and (46) determine x_1 and x_2 in equilibrium, while (36) and (47) determine x_1^* and x_2^*. In this case, an increase in κ_l^*, the value of long-term Foreign bonds held by the Home central bank, has the same qualitative effects as does an increase in κ_s^*, the value of short-term Foreign bonds held by the Home central bank. That is, x_1, x_2, W, r_s, r_l, μ^*, R_l^*,$R_l^* - R_s^*$, and $r_l^* - r_s^*$ increase while x_1^*, x_2^*, W^*, μ, R_l, $R_l - R_s$, $r_l - r_s$, r_s^*, r_l^*, and $\frac{e+1}{e}$ decrease. Notice that the effects of an increase in κ_l^* on the Home country are quantitatively identical to those of an increase in κ_s^*, but the effects on the Foreign country are quantitatively smaller than those of an increase in κ_s^* because $\Gamma^* < 1$ in (47).

Next, consider an equilibrium where short-term Foreign bonds are held in both countries. In this equilibrium, the international collateral constraint can be expressed as

$$\mathcal{F}(\{x_k,x_k^*\}_{k=1,2},\omega_l,\omega_l^*,\{\kappa_i,\kappa_i^*\}_{i=s,l},V,V^*)=\mathcal{D}(\{x_k,x_k^*\}_{k=1,2})-\mathcal{S}(x_2,x_2^*,V,V^*,\omega_l,\omega_l^*,\{\kappa_i^*,\kappa_i\}_{i=s,l})=0, \tag{48}$$

where $\mathcal{D}$ is the aggregate demand for collateral, identical to the one in Section 4.4, and $\mathcal{S}$ is the aggregate supply of collateral such that

$$\begin{aligned}\mathcal{S}=\ &V+\Omega V^*+(1-\Omega)(\kappa_s^*-\kappa_s)-(\Gamma-\Omega)\kappa_l+(1-\Omega\Gamma^*)\kappa_l^*\\&-\frac{\left(\theta_{hl}-\theta_{hs}\right)\omega_l}{\left(1-\theta_{hs}\right)\left[(1-\theta_{hl})u'\left(x_2\right)+\theta_{hl}\right]}-\frac{\Omega(\theta_{fl}^*-\theta_{fs}^*)\omega_l^*}{(1-\theta_{fs}^*)[(1-\theta_{fl}^*)u'\left(x_2^*\right)+\theta_{fl}^*]}.\end{aligned} \tag{49}$$

Then, equations (32), (36), (39), and (49) determine the DM consumption quantities in equilibrium, i.e., x_k and x_k^* for $k = 1, 2$. In this case, an increase in κ_l^* leads to an increase in x_k, x_k^*, r_i, r_i^*, W, and W^* and a decrease in μ, μ^*, R_l, R_l^*, $R_l - R_s$, $R_l^* - R_l^*$, $r_l - r_s$, $r_l^* - r_s^*$, and $\frac{e+1}{e}$ for $k = 1, 2$ and $i = s, l$. So, the effects of an increase in κ_l^* are qualitatively the same as, but quantitatively larger than, those of an increase in κ_s^* since $1 - \Omega\Gamma^* > 1 - \Omega$. However, the effects of an increase in κ_l (the Foreign central bank's holdings of long-term Home bonds) depend on θ_{hs}^* and θ_{hl}. If long-term Home bonds are more pledgeable than short-term Foreign bonds for Home banks ($\theta_{hs}^* > \theta_{hl}$), then $\Gamma - \Omega > 0$. In this case, an increase in κ_l decreases the supply of collateral in the global economy as does an increase in κ_s. So, x_k, x_k^*, r_i, r_i^*, W, and W^* decrease while μ, μ^*, R_l, R_l^*, $R_l - R_s$, $R_l^* - R_l^*$, $r_l - r_s$, $r_l^* - r_s^*$, and $\frac{e+1}{e}$ increase for $k = 1, 2$ and $i = s, l$. In contrast, if short-term Foreign bonds are more pledgeable than long-term Home bonds for Home banks ($\theta_{hs}^* < \theta_{hl}$), then $\Gamma - \Omega < 0$ and an increase in κ_l effectively increases the supply of collateral in the global economy. Therefore, the effects of an increase in κ_l are opposite to those of an increase in κ_s.

A.5 Other Types of Equilibrium with Interconnected Asset Markets

A.5.1 Equilibrium with $\lambda \in (\frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*}, \frac{(1-\theta_{fl}^*)\lambda^*}{1-\theta_{hl}^*})$

In this equilibrium, Home bonds and short-term Foreign bonds are held only by Home banks, while long-term Foreign bonds are held only by Foreign banks. Then, first-order conditions (11)-(13), and (21) must hold with equality in equilibrium. These equations can be rewritten as (32)-(34), (37), and (38). From (10), (12), (16), and (17), I obtain the following equation:

$$z_s^* = \frac{u'(x_2) - \theta_{hs}^* u'(x_2) + \theta_{hs}^*}{u'(x_1^*) - \theta_{fs}^* u'(x_2^*) + \theta_{fs}^*}. \tag{50}$$

Also, from (5), (6), (15), (23), (32)-(34), (37)-(38), and (50), noting that $b_{fj} = b_{fs}^* = b_{hl}^* = 0$ for $j = s, l$, the Home bank's collateral constraint can be rewritten as

$$\begin{aligned} 0 = &\left[u'(x_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho x_1 + \left[u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)x_2 - \{V + \Omega V^* + (1-\Omega)(\kappa_s^* - \kappa_s) \\ &-\Omega\omega_l^* - \left[u'(x_1^*) - \theta_{fs}^* u'(x_2^*) + \theta_{fs}^*\right]\Omega\rho x_1^* - \frac{(\theta_{hl} - \theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(x_2) + \theta_{hl}\right]}\right\}, \end{aligned} \tag{51}$$

where

$$\Omega = \frac{u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}}{u'(x_2) + \frac{\theta_{hs}^*}{1-\theta_{hs}^*}} < 1.$$

Finally, from (22), (37)-(38), the Foreign bank's collateral constraint can be rewritten as

$$\theta_{fs}^*\rho x_1^* + (1-\rho)x_2^* = \frac{\left(1-\theta_{fl}^*\right)\omega_l^*}{(1-\theta_{fl}^*)u'(x_2^*) + \theta_{fl}^*}. \tag{52}$$

As first-order conditions (14) and (18)-(20) do not hold with equality, a necessary condition for this equilibrium to exist is given by

$$\frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*} < \lambda < \frac{(1-\theta_{fl}^*)\lambda^*}{1-\theta_{hl}^*}. \tag{53}$$

Therefore, if (53) holds, an equilibrium can be characterized by equations (32)-(34), (37)-(38), and (50)-(52).

Proposition A.1 *There exists a nonempty set of parameter values that support a stationary equilibrium with binding collateral constraints that can be characterized by equations (32)-(34), (37)-(38), and (50)-(52).*

A.5.2 Equilibrium with $\lambda = \frac{(1-\theta^*_{fl})\lambda^*}{1-\theta^*_{hl}}$

In this equilibrium, Home banks acquire all types of government bonds issued in two countries while Foreign banks acquire only long-term Foreign bonds. Then, first-order conditions (11)-(14), and (21) must hold with equality in equilibrium. This leads to equations (32)-(34), (37)-(38), and (50). From (14) and (21), a necessary condition for this equilibrium to exist is given by

$$(1-\theta^*_{hl})\, u'(x_2) + \theta^*_{hl} = \left(1-\theta^*_{fl}\right) u'(x^*_2) + \theta^*_{fl}. \tag{54}$$

Also, from (5), (6), (23), (32)-(34), (37), (38), (50), and (54), noting that $b_{fj} = b^*_{fs} = 0$ for $j = s, l$, I can rewrite the Home and Foreign banks' collateral constraints as the form

$$\mathcal{D}(x_1, x_2, x^*_1, x^*_2) - \mathcal{S}(x_2, x^*_2, V, V^*, \omega_l, \omega^*_l, \kappa^*_s, \kappa_s) = 0, \tag{55}$$

where $\mathcal{D}$ denotes the aggregate demand for collateral and $\mathcal{S}$ denotes the aggregate supply, implying that the excess demand in aggregate is zero in equilibrium. The aggregate demand for collateral is given by

$$\begin{aligned}\mathcal{D} = &\left[u'(x_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho x_1 + \left[u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)x_2 \\ &+ \left[u'(x^*_1) - \theta^*_{fs}u'(x^*_2) + \theta^*_{fs}\right]\Omega\rho x^*_1 + \left[u'(x_2) + \frac{\theta^*_{hs}}{1-\theta^*_{hs}}\right]\left(\frac{1-\theta^*_{hl}}{1-\theta^*_{fl}}\right)\Omega\left[(1-\rho)x^*_2 + \theta^*_{fs}\rho x^*_1\right],\end{aligned} \tag{56}$$

and the aggregate supply of collateral is given by

$$\begin{aligned}\mathcal{S} = &V + \Omega V^* + (1-\Omega)(\kappa^*_s - \kappa_s) \\ &- \frac{(\theta_{hl} - \theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(x_2) + \theta_{hl}\right]} - \frac{\Omega(\theta^*_{hl} - \theta^*_{hs})\omega^*_l}{(1-\theta^*_{hs})[(1-\theta^*_{hl})u'(x_2) + \theta^*_{hl}]},\end{aligned} \tag{57}$$

where

$$\Omega = \frac{u'(x_2) + \frac{\theta_{hs}}{1-\theta_{hs}}}{u'(x_2) + \frac{\theta^*_{hs}}{1-\theta^*_{hs}}}.$$

Therefore, an equilibrium can be characterized by equations (32)-(34), (37)-(38), (50), and (54)-(55).

Proposition A.2 *There exists a nonempty set of parameter values that support a stationary equilibrium with binding collateral constraints that can be characterized by equations (32)-(34), (37)-(38), (50), and (54)-(55).*

A.6 Omitted Proofs

Proof of Proposition 1: Suppose that the sum of V and V^* is sufficiently large so that collateral constraints do not bind in equilibrium. In this case, from (15) and (22), the Lagrange multipliers to the collateral constraints must be zero, that is, $\lambda = \lambda^* = 0$. Then, from (9)-(14) and (16)-(21), I can obtain

$$\begin{aligned} x_1 &= (u')^{-1}\left[\frac{1}{z_s}\right], \\ x_2 &= \hat{x}, \\ \mu &= \frac{\beta}{z_s}, \\ z_l &= \frac{\beta}{\mu - \beta}, \end{aligned}$$

for the Home country and

$$\begin{aligned} x_1^* &= (u')^{-1}\left[\frac{1}{z_s^*}\right], \\ x_2^* &= \hat{x}, \\ \mu^* &= \frac{\beta}{z_s^*}, \\ z_l^* &= \frac{\beta}{\mu^* - \beta}, \end{aligned}$$

for the Foreign country. Also, the law of one prices must hold, implying that

$$\frac{e_{+1}}{e} = \frac{\mu}{\mu^*}.$$

Finally, a necessary condition for collateral constraints to not bind is given by

$$\begin{aligned} &-(1-\rho)d - (1-\rho)d^* + \frac{1}{\mu}\left[-\theta_{hs}\rho c + (1-\theta_{hs})\, b_{hs} + (1+z_l)\,(1-\theta_{hl})\, b_{hl}\right] \\ &\qquad + \frac{1}{\mu^*}\left[-\theta_{fs}^*\rho c^* + (1-\theta_{fs}^*)b_{fs}^* + (1+z_l^*)(1-\theta_{fl}^*)b_{fl}^*\right] \geq 0. \end{aligned}$$

Using the equilibrium consumption quantities and asset prices, together with the fiscal policies given by (5)-(6), the above inequality can be rewritten as

$$\begin{aligned} V + V^* \geq{}& \rho\left(\frac{1}{z_s} + \frac{\theta_{hs}}{1-\theta_{hs}}\right)(u')^{-1}\left[\frac{1}{z_s}\right] + \rho\left(\frac{1}{z_s^*} + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right)(u')^{-1}\left[\frac{1}{z_s^*}\right] \\ &+ \frac{(1-\rho)\hat{x}}{1-\theta_{hs}} + \frac{(1-\rho)\hat{x}}{1-\theta_{fs}^*} + \frac{(\theta_{hl}-\theta_{hs})\omega_l}{1-\theta_{hs}} + \frac{(\theta_{fl}^*-\theta_{fs}^*)\omega_l^*}{1-\theta_{fs}^*}. \end{aligned} \tag{58}$$

Therefore, for an equilibrium with nonbinding collateral constraints to exist, the sum of V and V^* must be sufficiently large to satisfy the above inequality. $\square$

Proof of Proposition 2: In order for equations (31)-(38) to characterize an equilibrium, V and V^* must be sufficiently small so that (58) does not hold. That is,

$$V + V^* < \rho\left(\frac{1}{z_s} + \frac{\theta_{hs}}{1-\theta_{hs}}\right)(u')^{-1}\left[\frac{1}{z_s}\right] + \rho\left(\frac{1}{z_s^*} + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right)(u')^{-1}\left[\frac{1}{z_s^*}\right] + \frac{(1-\rho)\hat{x}}{1-\theta_{hs}} + \frac{(1-\rho)\hat{x}}{1-\theta_{fs}^*} + \frac{(\theta_{hl}-\theta_{hs})\omega_l}{1-\theta_{hs}} + \frac{(\theta_{fl}^*-\theta_{fs}^*)\omega_l^*}{1-\theta_{fs}^*}. \tag{59}$$

Also, a necessary condition for this equilibrium to exist is given by

$$\lambda^* \leq \lambda < \frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*}.$$

Let $(\bar{x}_1, \bar{x}_2)$ denote the solution to (31)-(32) and $(\bar{x}_1^*, \bar{x}_2^*)$ denote the solution to (35)-(36). Then, from (10) and (17), the above condition can be rewritten as

$$\bar{x}_2 \leq \bar{x}_2^*,$$

and

$$u'(\bar{x}_2) < \frac{[(1-\theta_{fs}^*)u'(\bar{x}_2^*) + \theta_{fs}^*] - \theta_{hs}^*}{1-\theta_{hs}^*}.$$

The first inequality implies that

$$V \leq \left[u'(\dot{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\dot{x}_1 + \left[u'(\dot{x}_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\dot{x}_2 - \kappa_s^* + \kappa_s + \frac{(\theta_{hl}-\theta_{hs})\,\omega_l}{(1-\theta_{hs})\,[(1-\theta_{hl})u'(\dot{x}_2) + \theta_{hl}]}, \tag{60}$$

where $(\dot{x}_1, \dot{x}_2)$ is the solution to

$$\dot{x}_2 = \bar{x}_2^*,$$
$$z_s = \frac{u'(\dot{x}_2) - \theta_{hs}u'(\dot{x}_2) + \theta_{hs}}{u'(\dot{x}_1) - \theta_{hs}u'(\dot{x}_2) + \theta_{hs}}.$$

The second inequality implies that

$$V > \left[u'(\tilde{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\tilde{x}_1 + \left[u'(\tilde{x}_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\tilde{x}_2 - \kappa_s^* + \kappa_s + \frac{(\theta_{hl}-\theta_{hs})\,\omega_l}{(1-\theta_{hs})\,[(1-\theta_{hl})u'(\tilde{x}_2) + \theta_{hl}]}, \tag{61}$$

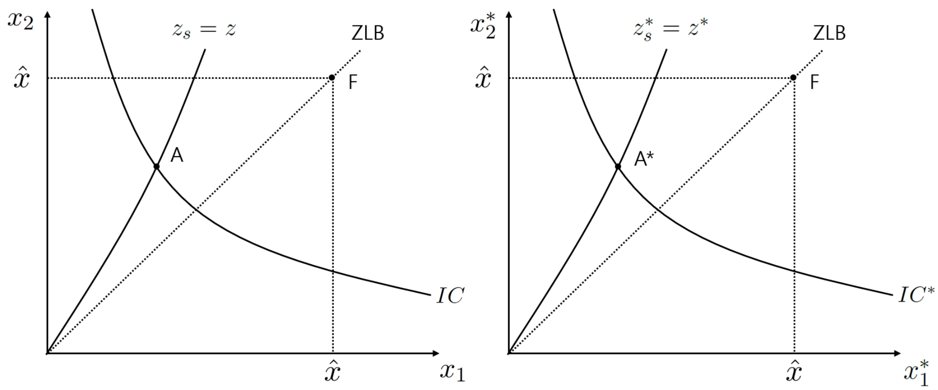

Figure 5: Equilibrium with no international capital flows

where $(\tilde{x}_1, \tilde{x}_2)$ is the solution to

$$u'(\tilde{x}_2) = \frac{[(1-\theta^*_{fs})u'(\bar{x}^*_2) + \theta^*_{fs}] - \theta^*_{hs}}{1-\theta^*_{hs}},$$
$$z_s = \frac{u'(\tilde{x}_2) - \theta_{hs}u'(\tilde{x}_2) + \theta_{hs}}{u'(\tilde{x}_1) - \theta_{hs}u'(\tilde{x}_2) + \theta_{hs}},$$

Therefore, given V and V^* that satisfy (59), (60), and (61), there exists an equilibrium that can be characterized by equations (31)-(38). $\square$

Proof of Proposition 3: The collateral constraints (31) and (35) can be expressed, respectively, as

$$\mathcal{C}_{\mathcal{H}}(x_1, x_2, a_l, \kappa_s, \kappa^*_s, V) = 0, \tag{62}$$
$$\mathcal{C}_{\mathcal{F}}(x^*_1, x^*_2, a^*_l, \kappa^*_s, \kappa_s, V^*) = 0, \tag{63}$$

where both functions $\mathcal{C}_{\mathcal{H}}$ and $\mathcal{C}_{\mathcal{F}}$ are strictly increasing in the first four arguments and strictly decreasing in the last two arguments. In equation (32), x_1 increases with an increase in x_2 and similarly, x^*_1 increases with x^*_2 in (36). Notice that, given fiscal/monetary policies, these four equations characterize equilibrium consumption quantities x_1, x_2, x^*_1, and x^*_2, and are illustrated in Figure 5.

In the left panel of the figure, the locus IC is generated by (62), and the locus $z_s = z$ is generated by (32). Analogously, the locus IC^* in the right panel is generated by (63), and the locus $z^*_s = z^*$ is generated by (36). Therefore, the solution for (x_1, x_2) is determined by the intersection, at point A, of the curve IC and the curve $z^s = z$, and the solution for (x^*_1, x^*_2) is determined at point A^*, the intersection of the curve IC^* and the curve $z^*_s = z^*$.

Suppose that there is a decrease in z_s from z^0 to z^1, with (ω_l, κ^*_s) held constant. Then, the curves $z^*_s = z^*$, IC, and IC^* do not shift, but the curve $z_s = z^0$ shifts up to $z_s = z^1$, as illustrated in Figure 6. So, x_1 falls and x_2 rises in equilibrium. Then, from (34) μ rises, and from (24) and (34) R_l rises. From (26), r_s and r_l rise, and from (27) the real term premium $r_l - r_s$ falls. From (25), the effect on the nominal term premium $R_l - R_s$ is ambiguous, and from (28) $\frac{e_{+1}}{e}$ rises.

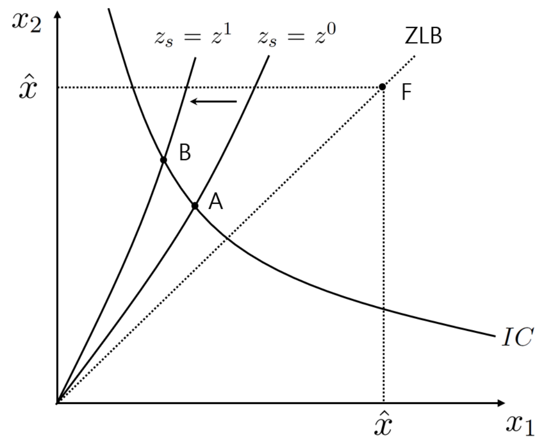

Figure 6: Conventional monetary policy: a decrease in z_s

To find the welfare implication of a decrease in z_s, differentiate the welfare measure W with respect to z_s. As $\frac{d\bar{X}}{dz_s} = \frac{d\bar{X}^*}{dz_s} = 0$ in an equilibrium with completely segmented asset markets, the derivative of W is given by

$$\frac{dW}{dz_s} = \rho[u'(x_1) - 1]\frac{dx_1}{dz_s} + (1-\rho)[u'(x_2) - 1]\frac{dx_2}{dz_s}.$$

For convenience, let $\sigma = -\frac{xu''(x)}{u'(x)}$ where $0 < \sigma < 1$. Totally differentiating (31) and (32) with respect to z_s gives

$$\frac{dx_1}{dz_s} = \frac{-(1-\rho)\left[(1-\sigma)(1-\theta_{hs})u'(x_2) + \theta_{hs}\right]\left[u'(x_1) - \theta_{hs}u'(x_2) + \theta_{hs}\right]^2}{\left\{\begin{array}{c} \rho u''(x_2)\left[(1-\sigma)(1-\theta_{hs})u'(x_1) + \theta_{hs}\right]\left[(1-\theta_{hs})u'(x_1) + \theta_{hs}\right] \\ +(1-\rho)u''(x_1)\left[(1-\sigma)(1-\theta_{hs})u'(x_2) + \theta_{hs}\right]\left[(1-\theta_{hs})u'(x_2) + \theta_{hs}\right] \end{array}\right\}},$$

$$\frac{dx_2}{dz_s} = \frac{\rho\left[(1-\sigma)(1-\theta_{hs})u'(x_1) + \theta_{hs}\right]\left[u'(x_1) - \theta_{hs}u'(x_2) + \theta_{hs}\right]^2}{\left\{\begin{array}{c} \rho u''(x_2)\left[(1-\sigma)(1-\theta_{hs})u'(x_1) + \theta_{hs}\right]\left[(1-\theta_{hs})u'(x_1) + \theta_{hs}\right] \\ +(1-\rho)u''(x_1)\left[(1-\sigma)(1-\theta_{hs})u'(x_2) + \theta_{hs}\right]\left[(1-\theta_{hs})u'(x_2) + \theta_{hs}\right] \end{array}\right\}},$$

so the derivative of W can be written as

$$\frac{dW}{dz_s} = \frac{\rho(1-\rho)\left[(1-\sigma)(1-\theta_{hs}) + \theta_{hs}\right]\left[u'(x_2) - u'(x_1)\right]\left[u'(x_1) - \theta_{hs}u'(x_2) + \theta_{hs}\right]^2}{\left\{\begin{array}{c} \rho u''(x_2)\left[(1-\sigma)(1-\theta_{hs})u'(x_1) + \theta_{hs}\right]\left[(1-\theta_{hs})u'(x_1) + \theta_{hs}\right] \\ +(1-\rho)u''(x_1)\left[(1-\sigma)(1-\theta_{hs})u'(x_2) + \theta_{hs}\right]\left[(1-\theta_{hs})u'(x_2) + \theta_{hs}\right] \end{array}\right\}}.$$

From (32), the zero lower bound constraint, or $z_s \leq 1$, implies that $u'(x_2) \leq u'(x_1)$ in equilibrium. This in turn implies that $\frac{dW}{dz_s} \geq 0$. Therefore, a decrease in z_s leads to a decrease in W. $\square$

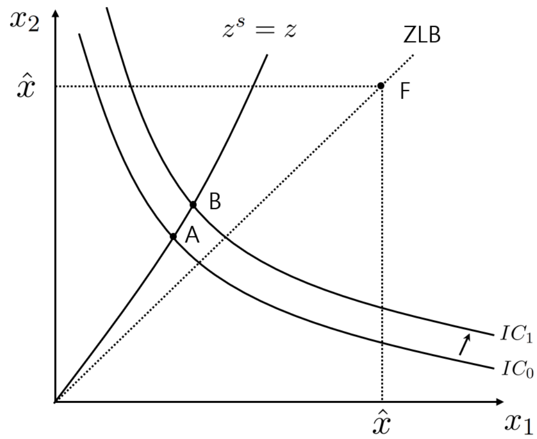

Figure 7: Quantitative easing: a decrease in ω_l

Proof of Proposition 4: The effects of a decrease in ω_l on the DM consumption quantities x_1 and x_2 in the Home country are illustrated in Figure 7. Note that the curves IC^* and $z_s^* = z^*$ in Figure 5 do not shift in response to a change in ω_l. From (32), the curve $z_s = z$ remains fixed, but the curve that depicts (62) shifts upward from IC_0 to IC_1. As a result, both x_1 and x_2 rise in equilibrium. Then, from (26) r_s and r_l rise, and from (27) the real term premium $r_l - r_s$ falls. From (24), (25), (32), and (34), the nominal interest rate on long-term Home bonds and the nominal term premium can be rewritten as, respectively,

$$R_l = \frac{(1-\theta_{hs})\, u'(x_2) + \theta_{hs}}{z_s\left[(1-\theta_{hl})\, u'(x_2) + \theta_{hl}\right]} - 1, \tag{64}$$

$$R_l - R_s = \frac{(\theta_{hl} - \theta_{hs})\left[u'(x_2) - 1\right]}{z_s\left[(1-\theta_{hl})\, u'(x_2) + \theta_{hl}\right]}. \tag{65}$$

Since each differentiation of the right-hand sides of (64) and (65) with respect to x_2 are both negative, R_l and $R_l - R_s$ both decrease. Totally differentiating (32) and (34) gives

$$\frac{d\mu}{dx_1} = \frac{\beta(1-\theta_{hs})u''(x_1)}{z_s\theta_{hs} + (1-\theta_{hs})} < 0, \tag{66}$$

so μ falls with an increase in x_1. From (28), $\frac{e+1}{e}$ falls, and finally, W increases as both x_1 and x_2 increase while $\bar{X}_0, \bar{X}, \bar{X^*}_0$, and $\bar{X}^*$ remain unchanged. $\square$

Proof of Proposition 5: The effects of an increase in κ_s^* on the DM consumption quantities x_1, x_2, x_1^*, and x_2^* are illustrated by Figure 8. In the figure, the curves $z_s = z$ and $z_s^* = z^*$ that depict, respectively, (32) and (36) remain fixed, while the curve that represents (62) shifts up from IC_0 to IC_1, and the curve that represents (63) shifts down from IC_0^* to IC_1^*. Therefore, x_1 and x_2 increase, but x_1^* and x_2^* decrease. Then, from (24) and (25), R_l and $R_l - R_s$ both decrease. From (26), r_s and r_l rise, and from (27) $r_l - r_s$ falls. From (42) and (43), R_l^* and $R_l^* - R_s^*$ both increase.

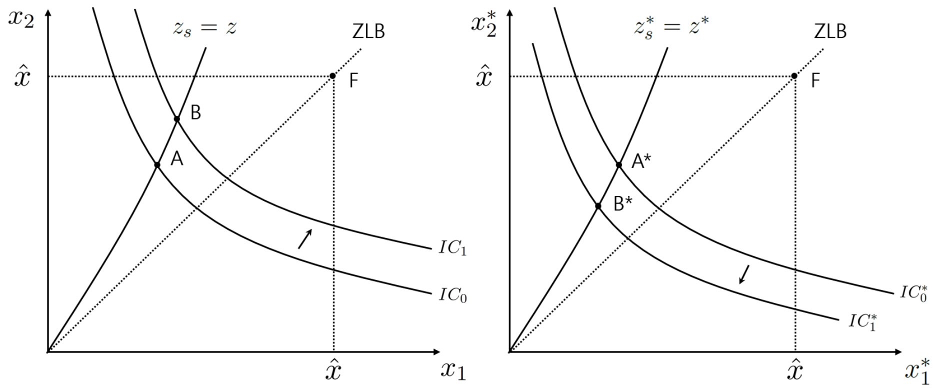

Figure 8: Foreign asset purchases by the Home central bank: an increase in κ_s^*

From (44) and (45), r_s^* and r_l^* fall and $r_l^* - r_s^*$ rises. From (34), μ falls, and from (38) μ^* rises, so from (28), $\frac{e_{+1}}{e}$ falls.

Letting $\sigma = -\frac{xu''(x)}{u'(x)}$ where $0 < \sigma < 1$, the total differentiation of (31) and (32) with respect to κ_s^* is given by

$$\frac{dx_1}{d\kappa_s^*} = \frac{(1-\theta_{hs})(1-\theta_{hs}+z_s\theta_{hs})u''(x_2)}{\left\{\begin{array}{c}\rho u''(x_2)(1-\theta_{hs}+z_s\theta_{hs})\left[(1-\sigma)(1-\theta_{hs})u'(x_1)+\theta_{hs}\right] \\ +(1-\rho)z_s u''(x_1)\left[(1-\sigma)(1-\theta_{hs})u'(x_2)+\theta_{hs}\right] - z_s\Gamma u''(x_1)u''(x_2)\end{array}\right\}} > 0,$$

$$\frac{dx_2}{d\kappa_s^*} = \frac{(1-\theta_{hs})z_s u''(x_1)}{\left\{\begin{array}{c}\rho u''(x_2)(1-\theta_{hs}+z_s\theta_{hs})\left[(1-\sigma)(1-\theta_{hs})u'(x_1)+\theta_{hs}\right] \\ +(1-\rho)z_s u''(x_1)\left[(1-\sigma)(1-\theta_{hs})u'(x_2)+\theta_{hs}\right] - z_s\Gamma u''(x_1)u''(x_2)\end{array}\right\}} > 0,$$

where

$$\Gamma = \frac{(\theta_{hl}-\theta_{hs})(1-\theta_{hl})\omega_l}{[(1-\theta_{hl})u'(x_2)+\theta_{hl}]^2} > 0.$$

Then, the derivative of W with respect to κ_s^* can be written as

$$\frac{dW}{d\kappa_s^*} = \frac{1}{1-\beta}\left\{\rho[u'(x_1)-1]\frac{dx_1}{d\kappa_s^*} + (1-\rho)[u'(x_2)-1]\frac{dx_2}{d\kappa_s^*}\right\} - 1 + \frac{\beta}{1-\beta}\left(\frac{1}{z_s^*}-1\right).$$

Notice that the first term (DM surplus) and third term (CM surplus in future periods) are positive while the second term (CM surplus in the current period) is negative. So, an increase in κ_s^* increases W for sufficiently high β. Similarly, the derivative of W^* with respect to κ_s^* can be written as

$$\frac{dW^*}{d\kappa_s^*} = \frac{1}{1-\beta}\left\{\rho[u'(x_1^*)-1]\frac{dx_1^*}{d\kappa_s^*} + (1-\rho)[u'(x_2^*)-1]\frac{dx_2^*}{d\kappa_s^*}\right\} + 1 - \frac{\beta}{1-\beta}\left(\frac{1}{z_s^*}-1\right),$$

where $\frac{dx_1^*}{d\kappa_s^*} < 0$ and $\frac{dx_2^*}{d\kappa_s^*} < 0$. Therefore, an increase in κ_s^* leads to a decrease in W^* for sufficiently high β. $\square$

Proof of Proposition 6: Suppose that V and V^* are sufficiently small to satisfy (59). From (23), a necessary condition for an equilibrium where short-term Foreign bonds are held in both countries to exist is given by

$$0 < b_{hs}^* < \bar{b}_s^* - \frac{\kappa_s^*}{z_s^*},$$

that is, both Home and Foreign banks must hold positive quantities of short-term Foreign bonds in equilibrium. Note that, if $b_{hs}^* = 0$, then the equilibrium becomes the one with segmented asset markets described in Section 4.3. If $b_{hs}^* = b_s^* - \frac{\kappa_s^*}{z_s^*}$, then $b_{fs}^* = 0$ and $\lambda \in (\frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*}, \frac{(1-\theta_{fl}^*)\lambda^*}{1-\theta_{hl}^*})$ must hold in equilibrium. Then, as shown in Appendix A.5.1, an equilibrium can be characterized by equations (32)-(34), (37)-(38), and (50)-(52). For $b_{hs}^* > 0$ in equilibrium, it must be satisfied that, from (61),

$$\begin{aligned} V \leq & \left[u'(\tilde{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\tilde{x}_1 + \left[u'\left(\tilde{x}_2\right) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\tilde{x}_2 \\ & - \kappa_s^* + \kappa_s + \frac{\left(\theta_{hl} - \theta_{hs}\right)\omega_l}{\left(1-\theta_{hs}\right)\left[(1-\theta_{hl})u'\left(\tilde{x}_2\right) + \theta_{hl}\right]}, \end{aligned} \tag{67}$$

where $(\tilde{x}_1, \tilde{x}_2)$ is the solution to

$$\begin{aligned} u'(x_2) =& \frac{[(1-\theta_{fs}^*)u'(\bar{x}_2^*) + \theta_{fs}^*] - \theta_{hs}^*}{1-\theta_{hs}^*}, \\ z_s =& \frac{u'\left(x_2\right) - \theta_{hs}u'\left(x_2\right) + \theta_{hs}}{u'\left(x_1\right) - \theta_{hs}u'\left(x_2\right) + \theta_{hs}}. \end{aligned}$$

Similarly, for $b_{fs}^* > 0$ or $b_{hs}^* < b_s^* - \frac{\kappa_s^*}{z_s^*}$, it must be satisfied that, from (51),

$$\begin{aligned} V \geq & \left[u'(\ddot{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\ddot{x}_1 + \left[u'(\ddot{x}_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\ddot{x}_2 - \Omega V^* - (1-\Omega)(\kappa_s^* - \kappa_s) \\ & + \Omega\omega_l^* + \left[u'(\ddot{x}_1^*) - \theta_{fs}^*u'(\ddot{x}_2^*) + \theta_{fs}^*\right]\Omega\rho\ddot{x}_1^* + \frac{\left(\theta_{hl} - \theta_{hs}\right)\omega_l}{\left(1-\theta_{hs}\right)\left[(1-\theta_{hl})u'\left(\ddot{x}_2\right) + \theta_{hl}\right]}, \end{aligned} \tag{68}$$

where $(\ddot{x}_1^*, \ddot{x}_2^*)$ is the solution to

$$z_s^* = \frac{u'\left(x_2^*\right) - \theta_{fs}^*u'\left(x_2^*\right) + \theta_{fs}^*}{u'\left(x_1^*\right) - \theta_{fs}^*u'\left(x_2^*\right) + \theta_{fs}^*}, \tag{69}$$

$$\theta_{fs}^*\rho x_1^* + (1-\rho)x_2^* = \frac{(1-\theta_{fl}^*)\omega_l^*}{(1-\theta_{fl}^*)u'\left(x_2^*\right) + \theta_{fl}^*}, \tag{70}$$

and $(\ddot{x}_1, \ddot{x}_2)$ is the solution to (32) and (51) given $(\ddot{x}_1^*, \ddot{x}_2^*)$. Therefore, given V and V^* that satisfy (59), (67), and (68), there exists an equilibrium where short-term Foreign bonds are held in both countries and the equilibrium can be characterized by (32)-(34), (36)-(38), (39), and (40).

It seems obvious that the function is $\mathcal{F}$ is strictly increasing in ω_l, ω_l^*, and κ_s, and strictly decreasing in κ_s^*, V, and V^*. But, with respect to x_1, x_2, x_1^*, and x_2^*, it seems less obvious how $\mathcal{F}$ moves with these arguments. The derivatives of $\mathcal{F}$ with respect to x_1, x_1^*, and x_2^* are given by

$$\frac{\partial \mathcal{F}}{\partial x_1} = \rho \left[u'(x_1) \left\{ 1 + \frac{x_1 u''(x_1)}{u'(x_1)} \right\} + \frac{\theta_{hs}}{1-\theta_{hs}} \right] > 0,$$

$$\frac{\partial \mathcal{F}}{\partial x_1^*} = \rho\Omega \left[u'(x_1^*) \left\{ 1 + \frac{x_1^* u''(x_1^*)}{u'(x_1^*)} \right\} + \frac{\theta_{fs}^*}{1-\theta_{fs}^*} \right] > 0,$$

$$\frac{\partial \mathcal{F}}{\partial x_2^*} = (1-\rho)\,\Omega \left[u'(x_2^*) \left\{ 1 + \frac{x_2^* u''(x_2^*)}{u'(x_2^*)} \right\} + \frac{\theta_{fs}^*}{1-\theta_{fs}^*} \right] - \frac{(1-\theta_{fl}^*)(\theta_{fl}^*-\theta_{fs}^*)\Omega\omega_l^* u''(x_2^*)}{(1-\theta_{fs}^*)\left[(1-\theta_{fl}^*)u'(x_2^*)+\theta_{fl}^*\right]^2} > 0.$$

The derivative of $\mathcal{F}$ with respect to x_2 is given by

$$\begin{aligned}\frac{\partial \mathcal{F}}{\partial x_2} = {} & (1-\rho) \left[u'(x_2) \left\{ 1 + \frac{x_2 u''(x_2)}{u'(x_2)} \right\} + \frac{\theta_{hs}}{1-\theta_{hs}} \right] - \frac{(1-\theta_{hl})(\theta_{hl}-\theta_{hs})\omega_l u''(x_2)}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(x_2)+\theta_{hl}\right]^2} \\ & + \frac{\partial\Omega}{\partial x_2}\left[\left\{ u'(x_1^*) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*} \right\} \rho x_1^* + \left\{ u'\left(x_2^*\right) + \frac{\theta_{fs}^*}{1-\theta_{fs}^*} \right\} (1-\rho) x_2^* \right. \\ & \left. - \left\{ V^* + \kappa_s - \kappa_s^* - \frac{(\theta_{fl}^*-\theta_{fs}^*)\omega_l^*}{(1-\theta_{fs}^*)[(1-\theta_{fl}^*)u'\left(x_2^*\right)+\theta_{fl}^*]} \right\} \right] > 0,\end{aligned}$$

where

$$\frac{\partial\Omega}{\partial x_2} = \frac{(\theta_{hs}^*-\theta_{hs})u''(x_2)}{(1-\theta_{hs}^*)(1-\theta_{hs})} < 0.$$

If asset markets were segmented, Foreign banks' demand for collateralizable assets would be equal to the supply of Foreign currency-denominated assets, so the last term in the above derivative would be zero. However, in this equilibrium, Home banks purchase some Foreign assets implying that Foreign bank's holdings of collateralizable assets must be lower than the supply of Foreign assets. So, the last term must be positive. □

Proof of Proposition 7: Without loss of generality, assume that $z_s = z_s^* = 1$. Then, $\bar{X}_0 - \bar{X}_0{}^* = -b_{hs}^*$, $\bar{X} - \bar{X}^* = 0$, $x_1 = x_2 = x$ and $x_1^* = x_2^* = x^*$ in equilibrium. Let x_a and x_a^* denote the would-be quantities of DM consumption in the Home and Foreign countries, if capital did not flow across countries. Then, from (15), (22) and (40),

$$\begin{aligned} b_{hs}^* = {} & x_a^* u'(x_a^*) + \frac{\theta_{fs}^* x_a^*}{1-\theta_{fs}^*} - x^* u'(x^*) - \frac{\theta_{fs}^* x^*}{1-\theta_{fs}^*}, \\ = {} & \frac{1}{\Omega}\left[x u'(x) + \frac{\theta_{hs} x}{1-\theta_{hs}} - x_a u'(x_a) + \frac{\theta_{hs} x_a}{1-\theta_{hs}} \right], \end{aligned}$$

where $x > x_0$, $x_0^* > x^*$, and

$$\Omega = \frac{u'(x) + \frac{\theta_{hs}}{1-\theta_{hs}}}{u'(x) + \frac{\theta_{hs}^*}{1-\theta_{hs}^*}}.$$

So, the welfare measures for the two countries can be written as

$$W = \frac{1}{1-\beta}\left[u(x) - x\right] - \frac{1}{\Omega}\left[xu'(x) + \frac{\theta_{hs}x}{1-\theta_{hs}} - x_a u'(x_a) + \frac{\theta_{hs}x_a}{1-\theta_{hs}}\right],$$

$$W^* = \frac{1}{1-\beta}\left[u(x^*) - x^*\right] + \left[x_a^* u'(x_a^*) + \frac{\theta_{fs}^* x_a^*}{1-\theta_{fs}^*} - x^* u'(x^*) - \frac{\theta_{fs}^* x^*}{1-\theta_{fs}^*}\right].$$

Then, I can obtain

$$\frac{dW}{dx} = \frac{1}{1-\beta}\left[u'(x) - 1\right] - \frac{1}{\Omega}\left[xu''(x) + u'(x) + \frac{\theta_{hs}}{1-\theta_{hs}}\right], \tag{71}$$

$$\frac{dW^*}{dx^*} = \frac{1}{1-\beta}\left[u'(x^*) - 1\right] - \left[x^*u''(x^*) + u'(x^*) + + \frac{\theta_{fs}^*}{1-\theta_{fs}^*}\right]. \tag{72}$$

For sufficiently high β, the welfare measures W and W^* increase with x and x^*, respectively. Since international capital flows lead to an increase in x and a decrease in x^*, welfare in the Home country W increases while welfare in the Foreign country W^* decreases. $\square$

Proof of Proposition 8: The equilibrium quantities of DM consumption in two countries x_1, x_2, x_1^*, and x_2^* are determined by equations (32), (36), (39), and (40). These four equations can be expressed, respectively, by

$$\mathcal{Z}(x_1, x_2; z_s) = 0, \tag{73}$$

$$\mathcal{Z}^*(x_1^*, x_2^*; z_s^*) = 0, \tag{74}$$

$$\mathcal{G}(x_2, x_2^*) = 0, \tag{75}$$

$$\mathcal{F}(x_1, x_2, x_1^*, x_2^*; \omega_l, \omega_l^*, \kappa_s, \kappa_s^*, V, V^*) = 0, \tag{76}$$

where

$$\mathcal{Z}(x_1, x_2; z_s) = \frac{u'(x_2) - \theta_{hs}u'(x_2) + \theta_{hs}}{u'(x_1) - \theta_{hs}u'(x_2) + \theta_{hs}} - z_s,$$

$$\mathcal{Z}^*(x_1^*, x_2^*; z_s^*) = \frac{u'(x_2^*) - \theta_{fs}^*u'(x_2^*) + \theta_{fs}^*}{u'(x_1^*) - \theta_{fs}^*u'(x_2^*) + \theta_{fs}^*} - z_s^*,$$

$$\mathcal{G}(x_2, x_2^*) = (1 - \theta_{hs}^*)\, u'(x_2) + \theta_{hs}^* - \left(1 - \theta_{fs}^*\right) u'(x_2^*) - \theta_{fs}^*.$$

Then, it is straightforward to obtain the following:

$$\frac{\partial \mathcal{Z}}{\partial x_1} > 0, \qquad \frac{\partial \mathcal{Z}}{\partial x_2} < 0, \qquad \frac{\partial \mathcal{Z}}{\partial z_s} < 0,$$
$$\frac{\partial \mathcal{Z}^*}{\partial x_1^*} > 0, \qquad \frac{\partial \mathcal{Z}^*}{\partial x_2^*} < 0, \qquad \frac{\partial \mathcal{Z}^*}{\partial z_s^*} < 0,$$
$$\frac{\partial \mathcal{G}}{\partial x_2} < 0, \qquad \frac{\partial \mathcal{G}}{\partial x_2^*} > 0.$$

Noting that the function $\mathcal{F}$ is increasing in x_1, x_2, x_1^*, and x_2^*, implicitly differentiating the above four equations with respect to z_s gives

$$\frac{dx_1}{dz_s} = \frac{\frac{\partial \mathcal{Z}}{\partial z_s}\Phi}{\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi} > 0,$$
$$\frac{dx_2}{dz_s} = -\frac{\frac{\partial \mathcal{Z}}{\partial z_s}\frac{\partial \mathcal{F}}{\partial x_1}}{\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi} < 0,$$
$$\frac{dx_1^*}{dz_s} = -\frac{\frac{\partial \mathcal{Z}^*}{\partial x_2^*}\frac{\partial \mathcal{G}}{\partial x_2}\frac{\partial \mathcal{Z}}{\partial z_s}\frac{\partial \mathcal{F}}{\partial x_1}}{\frac{\partial \mathcal{Z}^*}{\partial x_1^*}\frac{\partial \mathcal{G}}{\partial x_2^*}\left[\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi\right]} < 0,$$
$$\frac{dx_2^*}{dz_s} = \frac{\frac{\partial \mathcal{G}}{\partial x_2}\frac{\partial \mathcal{Z}}{\partial z_s}\frac{\partial \mathcal{F}}{\partial x_1}}{\frac{\partial \mathcal{G}}{\partial x_2^*}\left[\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi\right]} < 0,$$

where

$$\Phi = \frac{\partial \mathcal{F}}{\partial x_2} - \frac{\frac{\partial \mathcal{G}}{\partial x_2}}{\frac{\partial \mathcal{G}}{\partial x_2^*}}\left[\frac{\partial \mathcal{F}}{\partial x_2^*} - \frac{\frac{\partial \mathcal{Z}^*}{\partial x_2^*}}{\frac{\partial \mathcal{Z}^*}{\partial x_1^*}}\right] > 0.$$

Therefore, a decrease in z_s decreases x_1 and increases x_2, x_1^*, and x_2^*. Then, from (34) μ rises, from (26) r_s and r_l rise, and from (27) the real term premium $r_l - r_s$ falls. From (24) and (34) R_l rises, but from (25) the effect on the nominal term premium is ambiguous. Further, from (38) μ^* falls, and from (44) and (45) r_s^* and r_l^* rise while $r_l^* - r_s^*$ falls. From (36), (38), (42), and (43), the nominal interest rate on long-term Foreign bonds and the nominal term premium can be rewritten, respectively, as

$$R_l^* = \frac{(1-\theta_{fs}^*)u'(x_2^*) + \theta_{fs}^*}{z_s^*[(1-\theta_{fl}^*)u'(x_2^*) + \theta_{fl}^*]} - 1, \tag{77}$$
$$R_l^* - R_s^* = \frac{(\theta_{fl}^* - \theta_{fs}^*)\left[u'(x_2^*) - 1\right]}{z_s^*[(1-\theta_{fl}^*)u'(x_2^*) + \theta_{fl}^*]}, \tag{78}$$

so R_l^* and $R_l^* - R_s^*$ both decrease with the increase in x_2^*. Then, from (28), $\frac{e+1}{e}$ rises. Proposition 3 shows that W decreases in response to a decrease in z_s in an equilibrium with segmented asset markets. That is, in Figure 3, W decreases if the equilibrium moves from point A to B. However, W is lower at point E than B because x_1 and x_2 are both smaller at E. Therefore, as z_s decreases, W decreases to a larger extent than it does in an equilibrium with segmented asset markets. Finally, from (72) W^* increases as both x_1^* and x_2^* increase. $\square$

Proof of Proposition 9: Implicitly differentiate equations (73)-(76) with respect to ω_l to obtain

$$\begin{aligned}
\frac{dx_1}{d\omega_l} &= -\frac{\frac{\partial \mathcal{F}}{\partial \omega_l}\frac{\partial \mathcal{Z}}{\partial x_2}}{\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi} < 0, \\
\frac{dx_2}{d\omega_l} &= \frac{\frac{\partial \mathcal{F}}{\partial \omega_l}\frac{\partial \mathcal{Z}}{\partial x_1}}{\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi} < 0, \\
\frac{dx_1^*}{d\omega_l} &= \frac{\frac{\partial \mathcal{Z}^*}{\partial x_2^*}\frac{\partial \mathcal{G}}{\partial x_2}\frac{\partial \mathcal{F}}{\partial \omega_l}\frac{\partial \mathcal{Z}}{\partial x_1}}{\frac{\partial \mathcal{Z}^*}{\partial x_1^*}\frac{\partial \mathcal{G}}{\partial x_2^*}\left[\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi\right]} < 0, \\
\frac{dx_2^*}{d\omega_l} &= -\frac{\frac{\partial \mathcal{G}}{\partial x_2}\frac{\partial \mathcal{F}}{\partial \omega_l}\frac{\partial \mathcal{Z}}{\partial x_1}}{\frac{\partial \mathcal{G}}{\partial x_2^*}\left[\frac{\partial \mathcal{F}}{\partial x_1}\frac{\partial \mathcal{Z}}{\partial x_2} - \frac{\partial \mathcal{Z}}{\partial x_1}\Phi\right]} < 0.
\end{aligned}$$

Therefore, a decrease in ω_l increases all x_1, x_2, x_1^*, and x_2^*. Then, from (34) and (38), both μ and μ^* fall, and from (26) and (44) r_s, r_l, r_s^*, and r_l^* all rise. From (27) and (45), both $r_l - r_s$ and $r_l^* - r_s^*$ fall. Also, from (64) and (65), R_l and $R_l - R_s$ decrease, and from (77) and (78) R_l^* and $R_l^* - R_s^*$ decrease. From (29) and (39), I obtain

$$\frac{e_{+1}}{e} = \frac{z_s^*\left[(1-\theta_{hs})u'(x_2) + \theta_{hs}\right]}{z_s\left[(1-\theta_{hs}^*)u'(x_2) + \theta_{hs}^*\right]},$$

and differentiating the above equation gives

$$\frac{d\left(e_{+1}/e\right)}{dx_2} = \frac{z_s^* u''(x_2)(\theta_{hs}^* - \theta_{hs})}{z_s[(1-\theta_{hs}^*)u'(x_2) + \theta_{hs}^*]^2} < 0,$$

which implies that, as x_2 rises in response to a decrease in ω_l, $\frac{e_{+1}}{e}$ falls. Finally, from (71) and (72) both W and W^* increase because x_1, x_2, x_1^*, and x_2^* all increase. $\square$

Proof of Proposition A.1: Suppose that V and V^* are sufficiently small to satisfy (59). A necessary condition for this equilibrium to exist is given by

$$\frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*} < \lambda < \frac{(1-\theta_{fl}^*)\lambda^*}{1-\theta_{hl}^*}.$$

For $\lambda > \frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*}$ to hold in equilibrium, from (51) the following inequality must be satisfied:

$$\begin{aligned}
V < &\left[u'(\ddot{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\ddot{x}_1 + \left[u'(\ddot{x}_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\ddot{x}_2 - \Omega V^* - (1-\Omega)(\kappa_s^* - \kappa_s) \\
&+ \Omega\omega_l^* + \left[u'(\ddot{x}_1^*) - \theta_{fs}^* u'(\ddot{x}_2^*) + \theta_{fs}^*\right]\Omega\rho\ddot{x}_1^* + \frac{(\theta_{hl} - \theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(\ddot{x}_2) + \theta_{hl}\right]}, \qquad (79)
\end{aligned}$$

where $(\ddot{x}_1^*, \ddot{x}_2^*)$ is the solution to (69) and (70), and $(\ddot{x}_1, \ddot{x}_2)$ is the solution to (32) and (51) given $(\ddot{x}_1^*, \ddot{x}_2^*)$. Similarly, for $\lambda < \frac{(1-\theta_{fl}^*)\lambda^*}{1-\theta_{hl}^*}$ to hold in equilibrium, it must be satisfied that, from (51),

$$V > \left[u'(\dddot{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\dddot{x}_1 + \left[u'(\dddot{x}_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\dddot{x}_2 - \Omega V^* - (1-\Omega)(\kappa_s^* - \kappa_s)$$
$$+ \Omega\omega_l^* + \left[u'(\dddot{x}_1^*) - \theta_{fs}^* u'(\dddot{x}_2^*) + \theta_{fs}^*\right]\Omega\rho\dddot{x}_1^* + \frac{(\theta_{hl} - \theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(\dddot{x}_2) + \theta_{hl}\right]}, \quad (80)$$

where $(\dddot{x}_1, \dddot{x}_2, \dddot{x}_1^*, \dddot{x}_2^*)$ is the solution to (32), (50), (52), and (54). Therefore, given V and V^* that satisfy (59), (79), and (80), there exists an equilibrium that can be characterized by equations (32)-(34), (37)-(38), and (50)-(52). □

Proof of Proposition A.2: Suppose V and V^* satisfy (59). From (14), a necessary condition for equations (32)-(34), (37)-(38), (50), and (54)-(55) to characterize an equilibrium is

$$0 < b_{hl}^* < b_l^*,$$

that is, both Home and Foreign banks must hold positive quantities of long-term Foreign bonds in equilibrium. Note that, if $b_{hl}^* = 0$, the economy is in an equilibrium with $\lambda \in (\frac{(1-\theta_{fs}^*)\lambda^*}{1-\theta_{hs}^*}, \frac{(1-\theta_{fl}^*)\lambda^*}{1-\theta_{hl}^*})$. Also, note that $b_{hl}^* = b_l^*$ cannot be supported as an equilibrium since the asset market inefficiency in the Foreign country diverges to infinity as b_{fl}^* gets close to zero. For $b_{hl}^* > 0$, it must be satisfied that, from (51),

$$V \leq \left[u'(\dddot{x}_1) + \frac{\theta_{hs}}{1-\theta_{hs}}\right]\rho\dddot{x}_1 + \left[u'(\dddot{x}_2) + \frac{\theta_{hs}}{1-\theta_{hs}}\right](1-\rho)\dddot{x}_2 - \Omega V^* - (1-\Omega)(\kappa_s^* - \kappa_s)$$
$$+ \Omega\omega_l^* + \left[u'(\dddot{x}_1^*) - \theta_{fs}^* u'(\dddot{x}_2^*) + \theta_{fs}^*\right]\Omega\rho\dddot{x}_1^* + \frac{(\theta_{hl} - \theta_{hs})\,\omega_l}{(1-\theta_{hs})\left[(1-\theta_{hl})u'(\dddot{x}_2) + \theta_{hl}\right]}, \quad (81)$$

where $(\dddot{x}_1, \dddot{x}_2, \dddot{x}_1^*, \dddot{x}_2^*)$ is the solution to (32), (50), (52), and (54). Therefore, given V and V^* that satisfy (59) and (81), there exists an equilibrium that can be characterized by equations (32)-(34), (37)-(38), (50), and (54)-(55). □