Negative Nominal Interest Rates and Monetary Policy

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    - **Frictions** preventing arbitrage: **Storage/security costs** of holding paper currency
    - Rate of return on currency is effectively negative → **Effective lower bound < 0**

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  - **Frictions** preventing arbitrage: **Storage/security costs** of holding paper currency
  - Rate of return on currency is effectively negative → **Effective lower bound < 0**
- Reducing the effective lower bound on interest rates introduces **trade-offs**
  - **Benefit**: Central banks can expand the set of available interest rates
  - **Cost**: They may have to raise the cost of holding currency to reduce its effective rate of return

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    - Central bank determine the rate of return from withdrawing, holding, and redepositing currency
- I **evaluate the effectiveness** of this policy by developing a micro-founded framework
    - In the model, the lower bound is determined by **costs of holding currency** and a nonpar exchange rate policy

Three research questions and key results

Question #1: Is it possible to reduce the effective lower bound?

→ **Yes**. Central bank can **reduce the lower bound** with nonpar exchange rate policy if the cost of holding currency is sufficiently high

Three research questions and key results

Question #1: Is it possible to reduce the effective lower bound? **Yes**

Question #2: What is the key cost of reducing the lower bound?

→ Reducing the lower bound can **increase the aggregate cost of holding currency**

Nonpar exchange rate → Banks' demand for currency↑ → Currency holding↑

Three research questions and key results

Question #1: Is it possible to reduce the effective lower bound? **Yes**

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Question #3: To what extent should the central bank reduce the effective lower bound?

→ **Never optimal** to reduce the lower bound as it is only costly without any benefits

Three research questions and key results

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Question #3: To what extent should the central bank reduce the effective lower bound?

→ **Never optimal** to reduce the lower bound as it is only costly without any benefits

- Key idea: Only relative interest rates matter (interest rate on reserves relative to currency)
- Suppose the central bank lowers the interest rate by reducing the lower bound

Effective lower bound↓ → Interest rate on reserves relative to currency↑
*→ **Offsets the effects** of lowering the interest rate*

Related literature

- How to reduce the lower bound on interest rates:

  Eisler (1932), Buiter (2010), **Goodfriend (2016)**, **Rogoff (2017a, b)**, Agarwal and Kimball (2015, 2019)

- Implications of a negative interest rate:

  He, Huang, and Wright (2008), **Brunnermeier and Koby (2019)**,

  Eggertsson, Juelsrud, Summers, and Wold (2019), Jung (2019)

- Search-theoretic models with currency and bank deposits:

  Cavalcanti, Erosa, and Temzelides (1999), Williamson (1999, 2012), He, Huang, and Wright (2005, 2008),

  Li (2006, 2011), He, Huang, and Wright (2005, 2008), Sanches and Williamson (2010)

Contribution

1. First work that **formally evaluates** whether the central bank can reduce the lower bound
2. Provides a **novel reason** why a negative interest rate policy can go wrong

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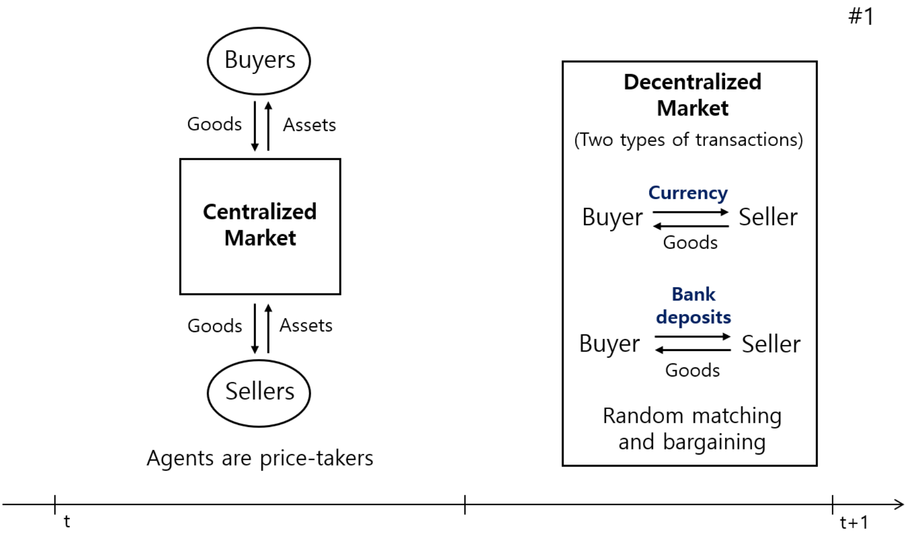
Environment

- Three types of agents – buyers, sellers, and private banks
- Time is discrete, $t = 0, 1, 2, ...$
- **Two sequential markets**
  - *Centralized* Market : Walrasian market
  - *Decentralized* Market : Random matching and bargaining
- **Two types** of transactions in the decentralized market
  - *Currency* transactions & *(private) bank deposit* transactions
  - Transaction type is unknown before matching
- **Limited commitment** and **no memory** (no record-keeping)

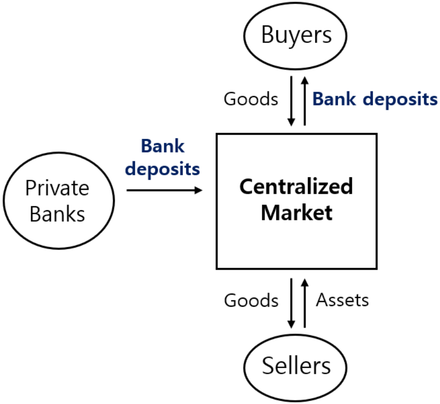
Buyers and sellers

◂ Details



Private banks

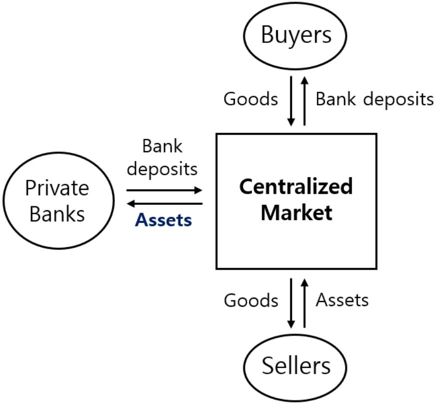
#2



Buyers can withdraw **currency** from their bank accounts

Private banks ◂ Details

#3



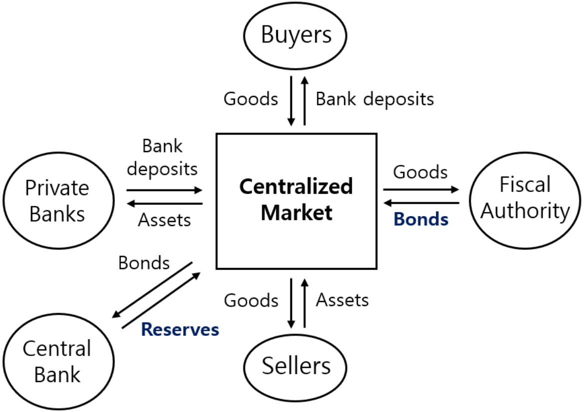
Limited commitment

→ a bank's liabilities must be backed by its assets (collateral)

→ **Incentive constraint**

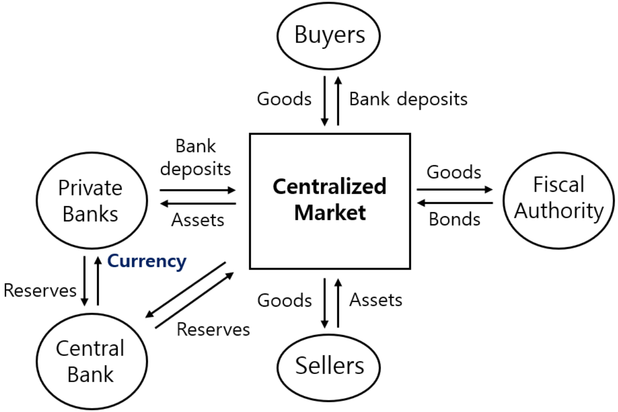
Government

#4



(1) Nominal interest rate on reserves

#5

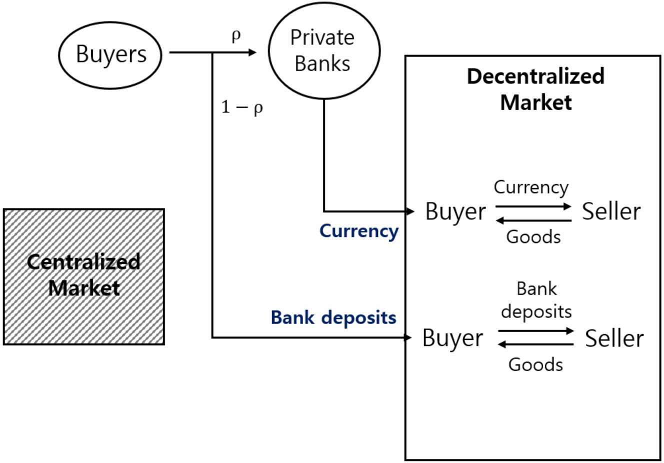


(1) Nominal interest rate on reserves

(2) Nonpar exchange rate of reserves for currency withdrawals

After the centralized market closes

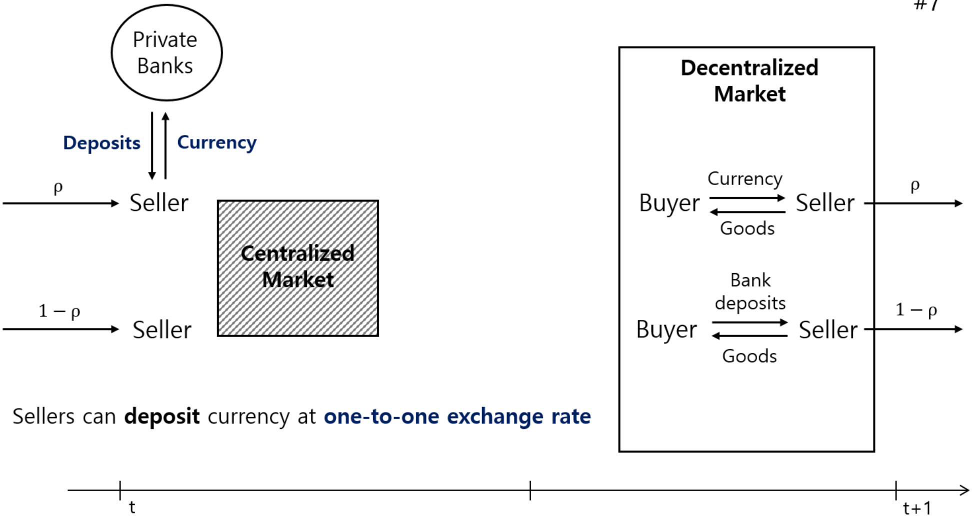
#6



Fraction ρ of buyers **withdraw** currency

Before the next centralized market opens

#7



Sellers can **deposit** currency at **one-to-one exchange rate**

Storage costs of currency

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    - Higher cost → Rate of return on currency ↓ → **Effective lower bound** ↓

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3. **Later**: Add an **endogenous cost** of holding currency

Stationary equilibrium

◂ Definition of stationary equilibrium

- **Price of currency** (in units of reserves) is determined at the centralized market
    - Sellers want to sell currency at a price *higher than* one-to-one exchange rate
    - Private banks want to purchase currency at a price *lower than* nonpar exchange rate

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- When inflation > 0, the market supply of currency cannot meet its demand Case of deflation
  - Private banks must be **indifferent** between purchasing and withdrawing currency

    → Price of currency = **nonpar exchange rate** $\eta > 1$
  - Sellers **strictly prefer** to sell currency in the market rather than depositing with banks

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    → Price of currency = **nonpar exchange rate** $\eta > 1$
  - Sellers **strictly prefer** to sell currency in the market rather than depositing with banks
- Lower bound is determined by **no arbitrage** from holding currency across periods
  - **Two conditions**: No arbitrage for private banks & no arbitrage for sellers

Private bank's problem in equilibrium

Each bank maximizes the representative buyer's expected utility by solving

$$\max_{(k,c',d),(c,m,b)} \mathbb{E}\mathrm{U}_{\text{buyer}}(k,c',d) = -k + \rho u\left(x^c\right) + (1-\rho)u(x^d)$$

subject to

$$\begin{aligned} \mathbb{E}[\text{Payoff}_{\text{bank}}|(k,c',d,c,m,b)] &= 0, \quad \text{(free entry)} \\ \mathbb{E}[\text{Payoff}_{\text{bank}}^{\text{repay}}|(k,c',d,c,m,b)] - \mathbb{E}[\text{Payoff}_{\text{bank}}^{\text{default}}|(k,c',d,c,m,b)] &\geq 0, \quad \text{(bank's IC)} \end{aligned}$$

- (k,c',d) - deposit contract; k - price of contract; c' - **currency offered to buyer**; d - deposit claims
- (c,m,b) - asset portfolio; c - **currency**; m - reserves; b - government bonds
- $x^c = x(c')$ - consumption in currency transaction
- $x^d = x(d)$ - consumption in bank deposit transaction

Private bank's payoffs

$$\mathbb{E}[\text{Payoff}_{\text{bank}}] = k - b - m - \eta \boldsymbol{c} + \beta \left[-(1-\rho)d + \frac{(1+R^m)m + (1+R^b)b + \boldsymbol{c} - \boldsymbol{\rho c'}}{\pi} - \gamma(\boldsymbol{c} - \boldsymbol{\rho c'}) \right]$$

$$\mathbb{E}[\text{Payoff}^{\text{repay}}_{\text{bank}}] = -(1-\rho)d + \frac{(1+R^m)m + (1+R^b)b + \boldsymbol{c} - \boldsymbol{\rho c'}}{\pi}$$

$$\mathbb{E}[\text{Payoff}^{\text{default}}_{\text{bank}}] = \frac{\delta \left[(1+R^m)m + (1+R^b)b + \boldsymbol{c} \right]}{\pi}$$

- π - inflation rate; β - discount factor; γ - proportional storage cost
- δ - fraction of assets not pledged as collateral
- R^m and R^b - nominal interest rates on reserves and bonds, respectively

→ In equilibrium, holding currency across periods is not profitable, i.e., $c = \rho c'$

Private bank's arbitrage: payoff from holding currency $c > \rho c'$

$$-\eta \boldsymbol{c} + \beta \left[\frac{\boldsymbol{c} - \rho c'}{\pi} - \gamma(\boldsymbol{c} - \rho c') \right] + \ldots = 0, \quad \text{(free entry)}$$

$$\frac{(1-\delta)\boldsymbol{c} - \rho c'}{\pi} + \ldots \geq 0, \quad \text{(bank's IC with multiplier } \lambda\text{)}$$

- **No arbitrage** from holding currency across periods implies

$$\text{(FOC w.r.t. } \boldsymbol{c}\text{)} \qquad -\underbrace{\eta}_{\substack{\text{price of} \\ \text{currency}}} \qquad \underbrace{+\beta\left(\frac{1}{\pi} - \gamma\right)}_{\substack{\text{net payoff in} \\ \text{next period}}} \qquad + \underbrace{\frac{\lambda\,(1-\delta)}{\pi}}_{\substack{\text{value as} \\ \text{collateral}}} \qquad \leq 0$$

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- Use other FOCs and define R^c_{banks} as the *net* rate of return on currency for banks FOCs

$$\Rightarrow \boldsymbol{1 + R^m \geq \frac{1}{\eta + \beta\gamma}} \equiv 1 + R^c_{banks}$$

Seller's arbitrage: payoff from holding currency c^s

$$\mathbb{E}[\text{Payoff}_{\text{seller}}] = -b^s - m^s - \eta \boldsymbol{c^s} + \beta \left[\frac{(1+R^m)m^s + (1+R^b)b^s + \eta \boldsymbol{c^s}}{\pi} - \gamma \boldsymbol{c^s} \right]$$

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- Use other FOCs and define $R^c_{sellers}$ as the *net* rate of return on currency for sellers ◂ FOCs

$$\Rightarrow \ \mathbf{1} + \boldsymbol{R^m} \geq \frac{\boldsymbol{\eta}}{(\boldsymbol{\eta} + \boldsymbol{\beta\gamma})[\mathbf{1} + \boldsymbol{\lambda}(\mathbf{1} - \boldsymbol{\delta})/\boldsymbol{\beta}]} \equiv 1 + R^c_{sellers}$$

  - $\lambda(1-\delta)/\beta$ represents the cost of holding currency that is not used as collateral

Effective Lower Bound (ELB) on nominal interest rates

Combining the two no-arbitrage conditions:

$$1 + R^m \geq \quad \max \left[\underbrace{\frac{1}{\eta + \beta\gamma}}_{1+R^c_{banks}}, \quad \underbrace{\frac{\eta}{(\eta + \beta\gamma)[1 + \lambda(1-\delta)/\beta]}}_{1+R^c_{sellers}} \right] \equiv \mathrm{ELB}$$

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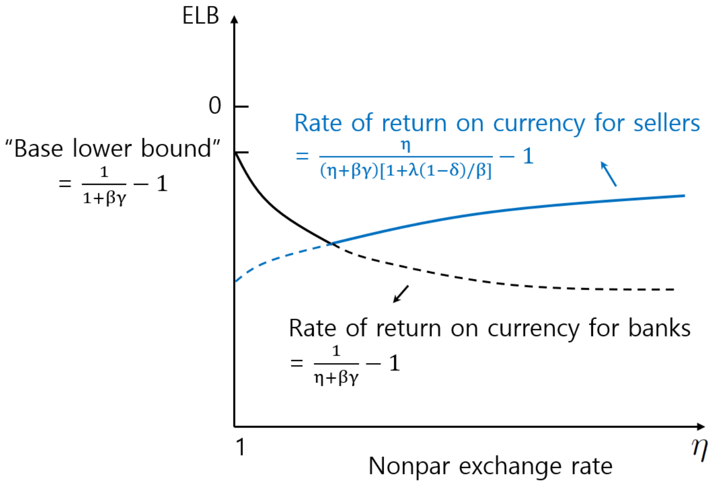
→ Introducing a nonpar exchange rate **reduces** the ELB iff $R^c_{banks} \geq R^c_{sellers}$

because it reduces the rate of return on currency *only for* private banks

Nonpar exchange rate η and the Effective Lower Bound (ELB)

Comparative statics



Why can a nonpar exchange rate policy fail to reduce the ELB?

Nonpar exchange rate $\eta\uparrow$ → Price of currency↑ → Sellers' rate of return on currency $R^c_{sellers}\uparrow$

- For high η or low λ (tightness of the bank's IC), the ELB is determined by $R^c_{sellers}$

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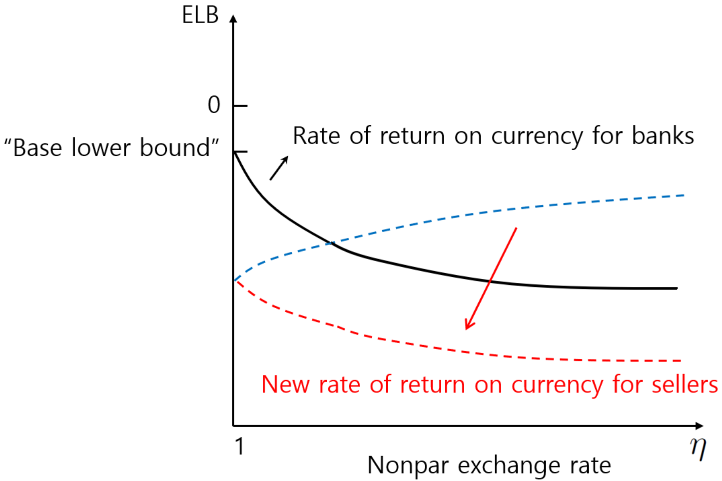
Key assumption: it is **costless** to transport currency in the centralized market

- In practice, transporting currency can be costly
  - E.g., Private banks transport currency using armored vehicles due to the risk of theft
- If theft or other costs arise **endogenously**, this could limit the scope of arbitrage

ELB when there is an endogenous cost of holding currency



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Adding theft as a cost of holding currency ◂ Details

- Between the decentralized market and the next centralized market, add **theft market**

Adding theft as a cost of holding currency

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- At the beginning of the theft market,
  - Buyers can invest in a **theft technology** at a fixed cost κ
  - Sellers can deposit currency to avoid theft

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- At the beginning of the theft market,
    - Buyers can invest in a **theft technology** at a fixed cost κ
    - Sellers can deposit currency to avoid theft
- In the theft market,
    - Random matches between buyers and sellers
    - If the buyer has a theft technology and the seller carries currency in a match, the buyer can steal the seller's currency

Two types of equilibria

◂ Definition of stationary equilibrium

- If the cost of theft κ is sufficiently **high**, then
  - no buyers invest in the theft technology $(\alpha^b = 0)$ → theft does not exist
  - all sellers carry currency to sell in the CM $(\alpha^s = 1)$

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→ Model with a sufficiently high κ is **equivalent** to the baseline model

- If the cost of theft κ is sufficiently **low**, then
  - some buyers invest in the theft technology $(0 < \alpha^b < 1)$ → theft exists
  - some sellers deposit currency to avoid theft $(0 < \alpha^s < 1)$

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  - some sellers deposit currency to avoid theft $(0 < \alpha^s < 1)$

→ Buyers and sellers **become indifferent** between two choices they have in theft market

Effective lower bound (ELB) in an equilibrium with theft

- Sellers are indifferent between depositing currency and carrying it →

$$
\begin{aligned}
\text{Payoff}\{\text{Depositing currency}\} &= \text{Payoff}\{\text{Selling currency at price } \eta \text{ with prob. } 1-\alpha^b\} \\
1 &= \eta(1-\alpha^b)
\end{aligned}
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- ELB on nominal interest rate :

$$1 + R^m \geq \quad \max\left[\underbrace{\frac{1}{\eta+\beta\gamma}}_{1+R^c_{banks}}, \underbrace{\frac{\color{red}{\eta(1-\alpha^b)}}{(\eta+\beta\gamma)[1+\lambda(1-\delta)/\beta]}}_{1+R^c_{sellers}}\right] = \frac{1}{\eta+\beta\gamma}$$

- Therefore, introducing a nonpar exchange rate η **always** reduces the ELB

Equilibrium with theft - Comparative statics

- Holding interest rate R^m constant, an **increase** in the nonpar exchange rate η ...
  - reduces the nominal rate of return on currency and the **ELB**
  - decreases currency c' and increases bank deposits d offered by private banks
    → consumption in currency transactions x^c decreases
    consumption in bank deposit transactions x^d increases
  - increases the fraction of buyers who acquire costly theft technology α^b

	∂ELB	∂x^c	∂x^d	$\partial \pi$	∂r^m	$\partial \alpha^b$	$\partial \alpha^s$
$\partial \eta$	$-$	$-$	$+$	$-$	$+$	$+$	?

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Optimal monetary policy

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  - Welfare measure $\equiv$ the sum of expected utilities across agents Details
- Given a one-to-one exchange rate $\eta = 1$, a **negative interest rate** $R^m < 0$ can be **optimal**
  - Buyers must carry extra currency to compensate for the sellers' costs → Inefficiency
  - Friedman rule ($R^m = 0$) is not optimal due to inefficiency in transactions
  - Lowering R^m further helps **mitigate inefficiency** (by raising the real rate of return on currency)

What if optimal interest rate is constrained by the base lower bound?

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    → no change in the bank's asset portfolio and the allocation of means of payment

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    → inflation rate π **decreases one-for-one** with the interest rate R^m (a pure *Fisher effect*)

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- Therefore, the optimal policy is to set $\eta = 1$ and $R^m = \text{Base lower bound}$

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Further results

- **Quantitative analysis** Details
  - Calibrate the model to the US economy during 2013-2015
  - Ask **how much consumption** individuals would need to be compensated to endure welfare loss
  - In three scenarios where the cost of theft is {2.5%, 5%, 10%} of consumption, reducing the ELB by 5 percentage points **costs** {0.1%, 0.2%, 0.4%} of consumption
    - Cost of 10% inflation has been estimated to be around 1% of consumption
- **Disintermediation** Details
  - Takes place when more consumers opt out of banking system
  - Allow currency and bank deposits to be **substitutable**
  - Nonpar exchange rate policy helps **prevent disintermediation**

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Conclusion

- A model of currency, reserves, and bank deposits is developed to study:
  - the effectiveness of a nonpar exchange rate policy
  - the implications of negative nominal interest rates
- A nonpar exchange rate **reduces the effective lower bound** if the cost of holding currency is sufficiently high
- Reducing the lower bound **increases the aggregate cost** of holding currency
- It is **not desirable** to reduce the effective lower bound because it is only costly without any benefits

Buyers

- Supply labor in the centralized market (CM) and consume goods in the decentralized market (DM) :

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[-H_t + u(x_t)\right]$$

where $0 < \beta < 1$ is discount factor, H_t is labor supply in CM, and x_t is consumption in DM

Choices :

1. Supply labor to produce goods in the CM
2. Exchange goods for assets in the CM
   - Acquire **bank deposits** offered by private banks
   - At the end of CM, learn the transaction type and can **withdraw** currency
3. *Buy* goods with either currency or bank deposits and consume them in the DM

Sellers ◂ back

- Consume goods in the centralized market (CM) and supply labor in the decentralized market (DM) :

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[X_t^s - h_t^s\right]$$

where X_t^s is consumption in CM and h_t^s is labor supply in DM

Choices :

1. At the beginning of CM, currency-holding sellers can either ...
   - deposit currency with a private bank at **one-to-one exchange rate** or
   - choose to sell currency at a market price in the CM
2. Exchange assets for goods and consume them in the CM
3. *Sell* goods in exchange for assets (currency or bank deposits) in the DM

Private banks

- Private banks supply labor and consume goods in the centralized market (CM):

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[-H_t^b + X_t^b \right]$$

  where H_t^b is labor supply and X_t^b is consumption in the CM

Choices :

1. In the CM, issue bank deposits which allow depositors to withdraw currency at the end of CM
   → Deposit contracts are useful as they play an **insurance role** for buyers
2. In the CM, acquire currency either
   - from the central bank at a **nonpar exchange rate** η or
   - from private agents at a market price
3. In the CM, acquire government bonds and reserves

Private banks ◂ back

- At the end of the centralized market (CM), deliver currency for buyers who liquidate their deposits
- At the beginning of the next CM, ...
  - deposit the remaining currency with the central bank at **one-to-one exchange rate**
  - redeem deposits
- Private banks cannot commit to repaying their liabilities (limited commitment)
  - Assets serve as collateral to back their liabilities

→ **Incentive constraint** arises**,** limiting the quantities of currency and deposits offered to buyers

Government ◂ back

- **Central bank** issues currency $\bar{c}$ and reserves $\bar{m}$ and determines:
    - nominal interest rate on reserves R^m (conventional monetary policy)
    - nonpar exchange rate between currency and reserves η
- **Fiscal authority**
    - issues one-period government bonds $\bar{b}$
    - sets the real quantity of government bonds issued by the fiscal authority $v = \eta\bar{c} + \bar{m} + \bar{b}$
    - gives lump-sum transfer τ to each buyer

Definition of stationary equilibrium in the baseline model

Definition: Given fiscal policy v and monetary policy (R^m, η), a stationary equilibrium can be characterized by

- DM consumption quantities (x^c, x^d), asset quantities (k, c', d, c, m, b),
- fraction of sellers carrying currency α^s, price of real currency ϕ
- transfers (τ_0, τ), gross inflation rate π, and nominal interest rate on government bonds R^b,

satisfying consolidated government budget constraints, fiscal policy v, first-order conditions for private bank's problem, no arbitrage conditions for private banks and sellers, incentive compatibility condition for sellers carrying currency, and market clearing conditions

Equilibrium conditions

- Consolidated government budget constraints :

$$\eta\bar{c} + \bar{m} + \bar{b} = \tau_0 \qquad \text{for } t = 0$$

$$\eta\bar{c} + \bar{m} + \bar{b} = \frac{\bar{c} + R^m\bar{m} + R^b\bar{b}}{\pi} + \tau \qquad \text{for } t = 1, 2, \ldots$$

- Fiscal policy rule :

$$v = \eta\bar{c} + \bar{m} + \bar{b}$$

- Incentive compatibility for sellers carrying currency :

$$\begin{array}{llll} \text{if} & \phi < 1, & \text{then} & \alpha^s = 0 \\ \text{if} & \phi = 1, & \text{then} & 0 \leq \alpha^s \leq 1 \\ \text{if} & \phi > 1, & \text{then} & \alpha^s = 1 \end{array}$$

Equilibrium conditions, continued

- No arbitrage condition for sellers :

$$-\phi + \beta \left[\frac{1 - \alpha^s + \alpha^s \phi}{\pi} - \gamma \right] \leq 0$$

- Market clearing conditions :

$$c = \bar{c}; \quad m = \bar{m}; \quad b = \bar{b}$$

First-order conditions for a bank's problem ◂ back

$$
\begin{aligned}
(c') \quad & \frac{\beta[1-\alpha^s+\alpha^s\phi]}{\pi} u'\left(x^c\right) - \phi - \frac{\lambda\delta}{\pi} = 0 \\
(d) \quad & \beta u'(x^d) - \beta - \lambda = 0 \\
(c) \quad & -\phi + \frac{\beta}{\pi} - \beta\gamma + \frac{\lambda\left(1-\delta\right)}{\pi} \leq 0 \\
(m) \quad & -1 + \frac{\beta R^m}{\pi} + \frac{\lambda R^m \left(1-\delta\right)}{\pi} = 0 \\
(b) \quad & -1 + \frac{\beta R^b}{\pi} + \frac{\lambda R^b \left(1-\delta\right)}{\pi} = 0 \\
& \lambda \left[-(1-\rho)d + \frac{(1-\delta)\left(R^m m + R^b b + c\right)}{\pi} - \frac{\rho c'}{\pi} \right] = 0
\end{aligned}
$$

where $x^c = [1-\alpha^s+\alpha^s\phi]\frac{\beta c'}{\pi} - \beta\mu$ and $x^d = \beta d$ are the solutions to the bargaining problems

First-order conditions for a bank's problem

back to bank's arbitrage

back to seller's arbitrage

$$
\begin{aligned}
&(c') \quad \frac{\beta[1-\alpha^s+\alpha^s\phi]}{\pi}u'\left(x^c\right)-\phi-\frac{\lambda\delta}{\pi}=0 \\
&(d) \quad \beta u'(x^d)-\beta-\lambda=0 \\
&(c) \quad -\phi+\frac{\beta}{\pi}-\beta\gamma+\frac{\lambda\left(1-\delta\right)}{\pi}\leq 0 \\
&(m) \quad \boldsymbol{-1+\frac{\beta R^m}{\pi}+\frac{\lambda R^m\left(1-\delta\right)}{\pi}=0} \\
&(b) \quad -1+\frac{\beta R^b}{\pi}+\frac{\lambda R^b\left(1-\delta\right)}{\pi}=0 \\
&\qquad \lambda\left[-(1-\rho)d+\frac{(1-\delta)\left(R^m m+R^b b+c\right)}{\pi}-\frac{\rho c'}{\pi}\right]=0
\end{aligned}
$$

where $x^c=[1-\alpha^s+\alpha^s\phi]\frac{\beta c'}{\pi}-\beta\mu$ and $x^d=\beta d$ are the solutions to the bargaining problems

Stationary equilibrium with deflation ◂ back

- Real value of currency *increases* over time

- Private banks (demanders) *can* acquire a sufficient quantity of currency in the centralized market

  → **No need to withdraw** currency from the central bank

- Sellers (suppliers) will be **indifferent** between depositing currency and selling currency in the centralized market

  → Price of currency = **exchange rate for currency deposits = 1**

Effective Lower Bound (ELB) in an equilibrium with deflation back

$$1 + R^m \geq \max\left[\underbrace{\frac{1}{1+\beta\gamma}}_{1+R^c_{banks}}, \underbrace{\frac{1}{(1+\beta\gamma)[1+\lambda(1-\delta)/\beta]}}_{1+R^c_{sellers}}\right] = \frac{1}{1+\beta\gamma}$$

- β - discount factor; γ - proportional cost of storing currency
- R^m - nominal interest rate on reserves
- R^c_{banks} and $R^c_{sellers}$ - nominal rates of return on currency perceived by banks and sellers

→ Introducing a nonpar exchange rate **does not affect** the ELB

because private banks do not withdraw currency at the nonpar exchange rate

Comparative statics in the baseline model

- With R^m held constant, an **increase** in the nonpar exchange rate η ...
  - increases the rate of return on currency for sellers → Effect on the ELB depends on η
  - increases currency and decreases bank deposits offered by private banks
    → consumption in currency transactions x^c increases
    consumption in bank deposit transactions x^d decreases
  - increases inflation, mitigating an increase in the real rate of return on currency
  - tightens collateral constraints, leading to higher prices of collateralizable assets
    → real interest rate r^m falls

	∂ELB	∂x^c	∂x^d	$\partial \pi$	∂r^m
$\partial \eta$	?	$+$	$-$	$+$	$-$

Comparative statics in the baseline model

- With R^m held constant, an **increase** in the nonpar exchange rate η ...
  - increases the rate of return on currency for sellers → Effect on the ELB depends on η
  - increases currency and decreases bank deposits offered by private banks
    → consumption in currency transactions x^c increases
    consumption in bank deposit transactions x^d decreases ◂ Equilibrium conditions
  - increases inflation, mitigating an increase in the real rate of return on currency
  - tightens collateral constraints, leading to higher prices of collateralizable assets
    → real interest rate r^m falls

	∂ELB	∂x^c	∂x^d	$\partial \pi$	∂r^m
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	∂ELB	∂x^c	∂x^d	$\partial \pi$	∂r^m
$\partial \eta$	?	$+$	$-$	$+$	$-$

Comparative statics in the baseline model

back

- With R^m held constant, an **increase** in the nonpar exchange rate η ...
  - increases the rate of return on currency for sellers → Effect on the ELB depends on η
  - increases currency and decreases bank deposits offered by private banks
    → consumption in currency transactions x^c increases
    consumption in bank deposit transactions x^d decreases
  - increases inflation, mitigating an increase in the real rate of return on currency
  - tightens collateral constraints, leading to higher prices of collateralizable assets
    → real interest rate r^m falls

	∂ELB	∂x^c	∂x^d	$\partial \pi$	∂r^m
$\partial \eta$	?	$+$	$-$	$+$	$-$

Equilibrium conditions in the baseline model

back

$$\eta R^m = \frac{\eta u'(x^c) - \delta u'(x^d) + \delta}{u'(x^d) - \delta u'(x^d) + \delta}$$

$$\left[u'(x^c) + \frac{\delta}{(1-\delta)\eta}\right] \rho(x^c + \beta\mu) + \left[u'\left(x^d\right) + \frac{\delta}{1-\delta}\right](1-\rho)x^d = v$$

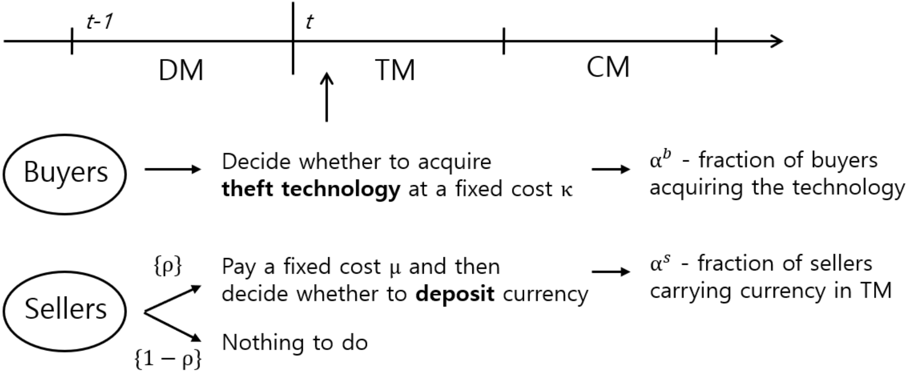
$$\pi = \frac{\beta}{\eta}\left[\eta u'(x^c) - \delta u'(x^d) + \delta\right]$$

$$r^m = \frac{1}{\beta\left[u'(x^d) - \delta u'(x^d) + \delta\right]}$$

$$\kappa \geq \frac{\rho(x^c + \beta\mu)}{\beta}$$

At the beginning of Theft Market (TM)

back



Definition of stationary equilibrium in a model with theft

Definition: Given fiscal policy v and monetary policy (R^m, η), a stationary equilibrium can be characterized by

- DM consumption quantities (x^c, x^d), asset quantities (k, c', d, c, m, b), price of real currency ϕ
- fraction of buyers investing in the theft technology α^b, fraction of sellers carrying currency α^s,
- transfers (τ_0, τ), gross inflation rate π, and nominal interest rate on government bonds R^b,

satisfying consolidated government budget constraints, fiscal policy rule v, first-order conditions for private bank's problem, no arbitrage conditions for private banks and sellers, incentive compatibility conditions for buyers and sellers in TM, and market clearing conditions.

Equilibrium conditions in a model with theft

- Consolidated government budget constraints :

$$\eta\bar{c} + \bar{m} + \bar{b} = \tau_0 \qquad \text{for } t = 0$$

$$\eta\bar{c} + \bar{m} + \bar{b} = \frac{\bar{c} + R^m\bar{m} + R^b\bar{b}}{\pi} + \tau \qquad \text{for } t = 1, 2, \ldots$$

- Fiscal policy rule :

$$v = \eta\bar{c} + \bar{m} + \bar{b}$$

- Incentive compatibility for sellers carrying currency :

$$\begin{aligned}
&\text{if} \quad (1-\alpha^b)\phi < 1, \quad \text{then} \quad \alpha^s = 0 \\
&\text{if} \quad (1-\alpha^b)\phi = 1, \quad \text{then} \quad 0 \leq \alpha^s \leq 1 \\
&\text{if} \quad (1-\alpha^b)\phi > 1, \quad \text{then} \quad \alpha^s = 1
\end{aligned}$$

Equilibrium conditions in a model with theft, continued

- Incentive compatibility for buyers investing in the theft technology :

$$\begin{aligned} &\text{if} \quad \kappa > \frac{\rho\alpha^s\phi c'}{\pi}, \quad \text{then} \quad \alpha^b = 0, \\ &\text{if} \quad \kappa = \frac{\rho\alpha^s\phi c'}{\pi}, \quad \text{then} \quad 0 \leq \alpha^b \leq 1, \\ &\text{if} \quad \kappa < \frac{\rho\alpha^s\phi c'}{\pi}, \quad \text{then} \quad \alpha^b = 1, \end{aligned}$$

- No arbitrage condition for (non-bank) individuals :

$$-\phi + \beta\left[\frac{1-\alpha^s+\alpha^s(1-\alpha^b)\phi}{\pi} - \gamma\right] \leq 0$$

- Market clearing conditions :

$$c = \bar{c}; \quad m = \bar{m}; \quad b = \bar{b}$$

First-order conditions for a bank's problem in a model with theft

back

$$(c') \quad \frac{\beta[1-\alpha^s+\alpha^s(1-\alpha^b)\phi]}{\pi}u'(x^c)-\phi-\frac{\lambda\delta}{\pi}=0$$

$$(d) \quad \beta u'(x^d)-\beta-\lambda=0$$

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$$(b) \quad -1+\frac{\beta R^b}{\pi}+\frac{\lambda R^b(1-\delta)}{\pi}=0$$

$$\lambda\left[-(1-\rho)d+\frac{(1-\delta)\left(R^m m+R^b b+c\right)}{\pi}-\frac{\rho c'}{\pi}\right]=0$$

where $x^c=[1-\alpha^s+\alpha^s(1-\alpha^b)\phi]\frac{\beta c'}{\pi}-\beta\mu$ and $x^d=\beta d$ are the solutions to the bargaining problems

Equilibrium with theft - Comparative statics with respect to R^m back

- Holding the nonpar exchange rate η constant, an **increase** in interest rate R^m ...
  - decreases **currency** c' and increases **bank deposits** d offered by private banks

    → consumption in currency transactions x^c decreases
    consumption in bank deposit transactions x^d increases
  - increases inflation, reducing the real rate of return on currency
  - relaxes collateral constraints, leading to lower prices of collateralizable assets

    → real interest rate r^m rises
  - increases the fraction of sellers carrying currency in the theft market α^s

	∂ELB	∂x^c	∂x^d	$\partial \pi$	∂r^m	$\partial \alpha^b$	$\partial \alpha^s$
∂R^m	$\cdot$	$-$	$+$	$+$	$+$	$\cdot$	$+$

Equilibrium with theft - Comparative statics with respect to R^m

- Holding the nonpar exchange rate η constant, an **increase** in interest rate R^m ...
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	∂ELB	∂x^c	∂x^d	$\partial \pi$	∂r^m	$\partial \alpha^b$	$\partial \alpha^s$
∂R^m	$\cdot$	$-$	$+$	$+$	$+$	$\cdot$	$+$

Equilibrium with theft - Comparative statics with respect to R^m

- Holding the nonpar exchange rate η constant, an **increase** in interest rate R^m ...
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	∂ELB	∂x^c	∂x^d	$\partial \pi$	∂r^m	$\partial \alpha^b$	$\partial \alpha^s$
∂R^m	$\cdot$	$-$	$+$	$+$	$+$	$\cdot$	$+$

Equilibrium with theft - Comparative statics with respect to R^m back

- Holding the nonpar exchange rate η constant, an **increase** in interest rate R^m ...
  - decreases currency c' and increases bank deposits d offered by private banks

    → consumption in currency transactions x^c decreases
    consumption in bank deposit transactions x^d increases
  - increases inflation, reducing the real rate of return on currency
  - relaxes collateral constraints, leading to lower prices of collateralizable assets

    → real interest rate r^m rises
  - increases the fraction of sellers carrying currency in the theft market α^s

	∂ELB	∂x^c	∂x^d	$\partial \pi$	∂r^m	$\partial \alpha^b$	$\partial \alpha^s$
∂R^m	$\cdot$	$-$	$+$	$+$	$+$	$\cdot$	$+$

Equilibrium with theft - Comparative statics, continued

- With R^m held constant, an **increase** in the nonpar exchange rate η ...
  - reduces inflation, mitigating a fall in the real rate of return on currency
  - relaxes collateral constraints, leading to lower prices of collateralizable assets
    → real interest rate r^m rises
  - has an ambiguous effect on the fraction of sellers carrying currency in the theft market α^s

	∂ELB	∂x^c	∂x^d	$\partial \pi$	∂r^m	$\partial \alpha^b$	$\partial \alpha^s$
$\partial \eta$	$-$	$-$	$+$	$-$	$+$	$+$	?

Equilibrium with theft - Comparative statics, continued

- With R^m held constant, an **increase** in the nonpar exchange rate η ...
  - reduces inflation, mitigating a fall in the real rate of return on currency
  - relaxes collateral constraints, leading to lower prices of collateralizable assets
    → real interest rate r^m rises
  - has an ambiguous effect on the fraction of sellers carrying currency in the theft market α^s

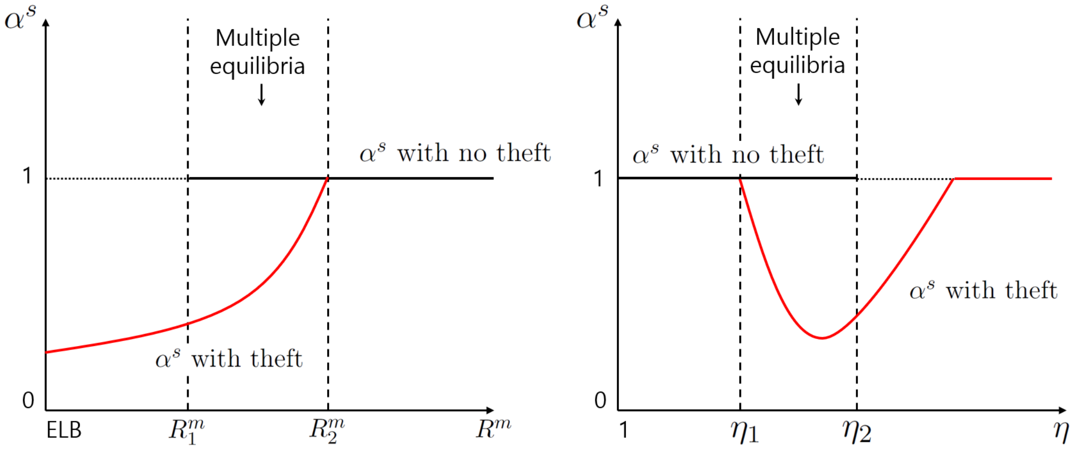
	∂ELB	∂x^c	∂x^d	$\partial \pi$	∂r^m	$\partial \alpha^b$	$\partial \alpha^s$
$\partial \eta$	$-$	$-$	$+$	$-$	$+$	$+$	?

Equilibrium with theft - Comparative statics, continued ◂ back

- With R^m held constant, an **increase** in the nonpar exchange rate η ...
  - reduces inflation, mitigating a fall in the real rate of return on currency
  - relaxes collateral constraints, leading to lower prices of collateralizable assets
    → real interest rate r^m rises
  - has an ambiguous effect on the fraction of sellers carrying currency in the theft market α^s

	∂ELB	∂x^c	∂x^d	$\partial \pi$	∂r^m	$\partial \alpha^b$	$\partial \alpha^s$
$\partial \eta$	$-$	$-$	$+$	$-$	$+$	$+$	?

Equilibrium α^s and Monetary Policy (R^m, η)



Welfare measure ◂ back

- Welfare measure is defined by

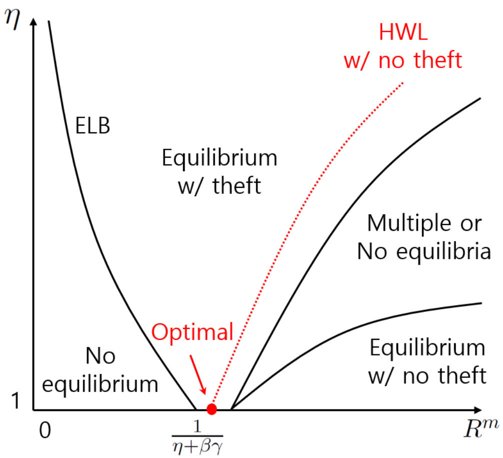
$$\mathcal{W} = \underbrace{0}_{\text{CM surpluses}} \underbrace{+\rho\left[u(x^c) - x^c + \beta\mu\right] + (1-\rho)\left[u(x^d) - x^d\right]}_{\text{DM surpluses}} \underbrace{-(\rho\beta\mu + \alpha^b\kappa)}_{\text{total cost in TM}},$$

  which is the sum of surpluses from trade in the Centralized Market (CM) and the Decentralized Market (DM), net of the total cost incurred in the Theft Market (TM)

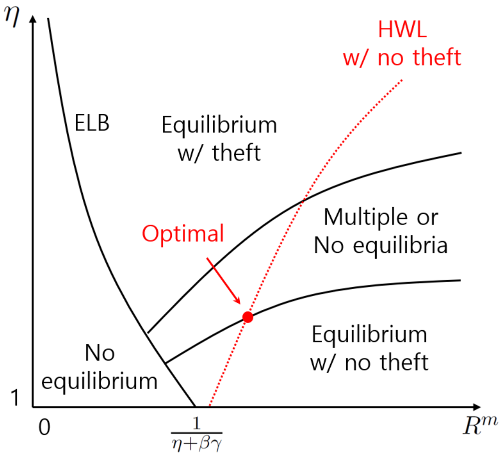
- This is the same as the sum of **expected utilities across agents**

Optimal monetary policy in a model with theft

(For a sufficiently low κ)



(For a sufficiently high κ)



Optimal monetary policy when the cost of theft is sufficiently high

back

- If κ is sufficiently high, the optimal monetary policy is characterized by

$$\eta = \bar{\eta} \qquad \text{and} \qquad 1 + R^m > \frac{1}{\bar{\eta}}$$

where $\bar{\eta}$ is the highest possible level that does not cause theft

Equilibrium conditions in an equilibrium with theft

$$\eta R^m = \frac{u'(x^c) - \delta u'(x^d) + \delta}{u'(x^d) - \delta u'(x^d) + \delta}$$

$$\left[u'(x^c) + \frac{\delta}{1-\delta}\right]\rho(x^c + \beta\mu) + \left[u'\left(x^d\right) + \frac{\delta}{1-\delta}\right](1-\rho)x^d = v$$

$$\pi = \frac{\beta}{\eta}\left[u'(x^c) - \delta u'(x^d) + \delta\right]$$

$$r^m = \frac{1}{\beta\left[u'(x^d) - \delta u'(x^d) + \delta\right]}$$

$$\alpha^b = \frac{\eta - 1}{\eta}$$

$$\alpha^s = \frac{\beta\kappa}{\rho\eta(x^c + \beta\mu)}$$

$$\kappa < \frac{\rho\eta(x^c + \beta\mu)}{\beta}$$

back to comparative statics back to optimal policy

Quantitative analysis - Calibration

- The utility function for consumption in the DM is $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$
- Excluding the cost of theft κ, there are eight parameters to calibrate
- Calibration is based on the U.S. data during 2013-2015
  - the policy rate was close to zero → suitable to discuss the welfare cost of reducing the ELB
  - nominal interest rate on reserves and domestically-held public debt to GDP were stable

Quantitative analysis - Calibration results

- First three parameters are calibrated externally and the last five are calibrated internally
- Sources: Federal Reserve Economic Data (FRED)

Param's	Values	Calibration targets	Sources
β	0.96	Standard in literature	
R^m	1.0025	Avg. interest rate on reserves: 0.25%	FRED
γ	0.00	Lowest target range for fed funds rate: 0-0.25%	FRED
σ	0.17	Money demand elasticity (1959-2007): -4.19	FRED
ρ	0.17	Currency to M1 ratio: 17.22%	FRED; Lucas and Nicolini (2015)
v	1.13	Avg. locally-held public debt to GDP: 66.73%	FRED
δ	0.45	Avg. inflation rate: 1.06%	FRED
μ	0.01	Fixed storage cost: 2% of currency payments	Author's assumption

Quantitative analysis - Counterfactual analysis

back

- Consider three scenarios where
  - Cost of theft κ is (i) 2.5%, (ii) 5%, and (iii) 10% of the current consumption level
- Ask how much consumption (fraction $\Delta_\eta - 1$) would need to be compensated to endure η

η	ELB	R^*_η	$(\Delta_\eta - 1) \times 100$		
			κ = 2.5%	κ = 5%	κ = 10%
1.00	1.000	1.000	-	-	-
1.025	0.976	0.976	0.0561	0.1118	0.2236
1.05	**0.952**	**0.952**	**0.1095**	**0.2183**	**0.4367**
1.10	0.909	0.909	0.2091	0.4168	0.8339

- Cost of 10% inflation has been estimated to be around **1% of consumption**
  - E.g., 0.62% in Chiu-Molico (2010), 0.87% in Lucas (2000), and 1.32% in Lagos-Wright (2005)

Disintermediation

- **Disintermediation** might determine the the effective lower bound
    - ... a situation where more consumers opt out of bank deposit contracts
    - A practical concern : disintermediation might cause **long-run inefficiency** in financial system
- To study the implication of a nonpar exchange rate for potential disintermediation,
    - allow currency and bank deposits to be **substitutable**
    - fraction ρ of sellers accept only currency while $1-\rho$ of sellers accept both means of payment

Disintermediation

- If the cost of theft is sufficiently low, the ELB is given by

$$1 + R^m \geq \frac{u'(x^o)}{\eta\left[(1-\delta)u'(x^d) + \delta\right]}$$

  - x^o - quantity of DM consumption for buyers opting out of bank contracts
  - x^d - quantity of consumption in bank deposit transactions for buyers acquiring bank contracts
  - δ - fraction of assets private banks can abscond with

- Complete disintermediation **cannot** be supported in equilibrium → determines the ELB
  - Due to a shortage of government bonds, central bank cannot meet the demand for currency

Discussion

◂ back

- Which model specification is **more realistic**?
  - There are transactions where both means of payment can be accepted
  - There are also transactions where currency cannot be accepted
  - Reality may be somewhere between the two versions of the model
- **Implication** for the ELB?
  - As there are transactions where currency cannot be used
  - Complete disintermediation **may not** happen → **no arbitrage conditions** will determine ELB

Bibliography

- How to reduce the lower bound on interest rates

  (Eisler, 1932; Buiter, 2010; Goodfriend, 2016; Rogoff, 2017a,b; Agarwal and Kimball, 2015, 2019)

- Implications of a negative interest rate

  (He et al., 2008; Brunnermeier and Koby, 2019; Eggertsson et al., 2019; Jung, 2019)

- Search-theoretic models with currency and bank deposits

  (Cavalcanti et al., 1999; Williamson, 1999; He et al., 2005; Li, 2006; He et al., 2008; Li, 2011; Williamson, 2012)

- Endogenous theft

  (He et al., 2005, 2008; Sanches and Williamson, 2010)

Bibliography i

Agarwal, R. and Kimball, M. S. (2015). Breaking Through the Zero Lower Bound. *IMF Working Paper*, 224:1–39.

Agarwal, R. and Kimball, M. S. (2019). Enabling Deep Negative Rates to Fight Recessions: A Guide. *IMF Working Paper*, 84:1–88.

Brunnermeier, M. K. and Koby, Y. (2019). The Reversal Interest Rate. *Working Paper*, pages 1–42.

Buiter, W. H. (2010). Negative Nominal Interest Rates: Three Ways to Overcome the Zero Lower Bound. *The North American Journal of Economics and Finance*, 20(3):213–238.

Cavalcanti, R. d. O., Erosa, A., and Temzelides, T. (1999). Private Money and Reserve Management in a Random-Matching Model. *Journal of Political Economy*, 107(5):929–945.

Eggertsson, G. B., Juelsrud, R. E., Summers, L. H., and Wold, E. G. (2019). Negative Nominal Interest Rates and the Bank Lending Channel.

Eisler, R. (1932). *Stable Money, the Remedy for the Economic World Crisis: a Programme of Financial Reconstruction for the International Conference, 1933*. Search Publishing Company.

Goodfriend, M. (2016). The Case for Unencumbering Interest Rate Policy at the Zero Bound. In *Designing Resilient Monetary Policy Frameworks for the Future*, pages 127–160, Jackson Hole, Wyoming. Proceedings of the 2016 Economic Policy Symposium.

He, P., Huang, L., and Wright, R. (2005). Money and Banking in Search Equilibrium. *International Economic Review*, 46(2):637–670.

Bibliography ii

He, P., Huang, L., and Wright, R. (2008). Money, Banking, and Monetary Policy. *Journal of Monetary Economics*, 55(6):1013–1024.

Jung, K. M. (2019). Optimal Negative Interest Rate under Uncertainty. *International Journal of Central Banking*, 15(3):1–25.

Li, Y. (2006). Banks, Private Money, and Government Regulation. *Journal of Monetary Economics*, 53(8):2067–2083.

Li, Y. (2011). Currency and Checking Deposits as Means of Payment. *Review of Economic Dynamics*, 14(2):403–417.

Rogoff, K. (2017a). Dealing with Monetary Paralysis at the Zero Bound. *Journal of Economic Perspectives*, 31(3):47–66.

Rogoff, K. (2017b). *The Curse of Cash: How Large-Denomination Bills Aid Tax Evasion and Crime and Constrain Monetary Policy*. Princeton University Press.

Sanches, D. and Williamson, S. D. (2010). Money and Credit with Limited Commitment and Theft. *Journal of Economic Theory*, 145(4):1525–1549.

Williamson, S. D. (1999). Private Money. *Journal of Money, Credit and Banking*, 31(3):469–491.

Williamson, S. D. (2012). Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach. *American Economic Review*, 102(6):2570–2605.