Between Bandit and Full-Information: Regret bounds for Online Gradient Descent

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1 Introduction

Online convex optimization (OCO) is an area of great interest with wide ranging applications in machine learning and controls. In general, the goal of OCO is to minimize a stream of convex loss functions without prior information about the function. This can also be viewed as a game between the algorithm (player) and the environment (adversary). The algorithm chooses an action $x_t \in \mathcal{K} \subseteq \mathbb{R}^d$ and the environment gives a loss $\ell_t : \mathcal{K} \to \mathbb{R}$.

The algorithm may have several different levels of access to the loss function, corresponding to different problem settings. The most typical setting is that the algorithm has access to gradient information. However, here we focus on the Bandit setting where the algorithm only has access to the functional evaluation(s) of the action(s) that the algorithm plays.

We measure the performance of various algorithms with regret. However, here in contrast to the majority of the work on Bandit OCO, we focus on dynamic or time-varying regret:

$$\sum_{t=1}^{T} \ell_t(x_t) - \sum_{t=1}^{T} \min_{x_t^* \in \mathcal{K}} \ell_t(x_t^*).$$

In the Bandit setting, the algorithm cannot compete against a completely adaptive adversary who chooses the loss function ℓ_t after observing the action x_t Agarwal et al. [2010]. Hence for this setting, we will show an algorithm that is no-regret against an adaptive adversary that can only observe information of the $t-1$ rounds preceding round t.

Overall in the literature we observe a large gap between between the optimal regret bounds between the full information and bandit settings. In particular, in the full information there are algorithms that are no-regret against a completely adaptive adversary. This raises the question of studying a range of problem between bandit (single function evaluation) and full information extremes. We thus begin with studying a generic k-point bandit feedback setting. Then we show that for $k = d + 1$-point feedback, there is a deterministic algorithm that is no-regret against a completely adaptive adversary, therefore matching the full-information bound.

For this k-point feedback setting, we define the expected regret:

Definition 1. k-point Bandit Regret

$$\mathbb{E}\frac{1}{k}\sum_{t=1}^{T}\sum_{i=1}^{k} \ell_t(y_{t,i}) - \mathbb{E}\sum_{t=1}^{T} \min_{x_t^* \in \mathcal{K}} \ell_t(x_t^*)$$

where the expectation is taken over the randomness of the player.

As is typical in Bandit OCO, the algorithm relies on Online Gradient Descent (OGD) which uses a gradient estimator $\tilde{g}_t$. Thus the regret bounds are based on the regret bounds of OGD. Here we draw

extensively from Agarwal et al. [2010] which studies our problem but with static regret and Mokhtari et al. [2016] that gives dynamic regret bounds for OGD. To our knowledge our problem has not been studied before.

2 Preliminaries

2.1 Problem Setting

First we formally introduce the problem. Our objective is:

$$\min_{x_t \in \mathcal{K}} \ell_t(x_t)$$

over rounds $t = 1, \ldots, T$. Where $\mathcal{K} \subset \mathbb{R}^d$ is our action set and ℓ_t are adversarially chosen time varying loss functions. The key in our setting is that we do not have first order gradient information $\nabla \ell_t$ but we are able to get zeroth-order (bandit) feedback with $k = d + 1$ points. First we outline our assumptions.

We assume that $\mathcal{K}$ is compact and has a nonempty interior (otherwise project $\mathcal{K}$ to a lower dimensional space). For this work, when we implicitly or explicitly refer to norms, we will be using the euclidean norm. We also have the following assumptions for each loss function ℓ_t:

Assumption 1. Let $\mathcal{B}$ denote the unit ball centered at the origin. There exists $r, D > 0$ such that

$$r\mathcal{B} \subseteq \mathcal{K} \subseteq D\mathcal{B}$$

Assumption 2. Strong Convexity. For $\mu \geq 0$, ℓ_t is μ-strongly convex over the set $\mathcal{K}$:

$$\ell_t(x) \geq \ell_t(y) + \nabla \ell_t(y)^\intercal (x - y) + \frac{\mu}{2}\|x - y\|^2, \quad \forall x, y \in \mathcal{K}$$

Assumption 3. ℓ_t is L-smooth on $\mathcal{K}$ if it is differentiable on an open set containing $\mathcal{K}$ and its gradient is Lipschitz continuous with constant L:

$$\|\nabla \ell_t(x) - \nabla \ell_t(y)\| \leq L\|x - y\|, \quad \forall x, y \in \mathcal{K}$$

.

Assumption 4. ℓ_t is G-Lipschitz:

$$\|\ell_t(x) - \ell_t(y)\| \leq G\|x - y\|, \quad \forall x, y \in \mathcal{K}$$

.

Hence the gradient of ℓ_t over $\mathcal{K}$ is bounded:

$$\|\nabla \ell_t(x)\| \leq G \quad \forall t, \forall x \in \mathcal{K}$$

We also use the notation $\mathbb{E}_t$ to denote the conditional expectation conditioned on all randomness in the first $t - 1$ rounds.

3 Projected Gradient Descent with k queries per round

Algorithm 1 Generic k-point Bandit Online Gradient Descent

Input: Step size η, exploration parameter δ and shrinkage coefficient ξ.
Set $x_1 = 0$
for $t = 1, ..., T$ **do**
  Observe randomized queries around x_t: $\ell_t(y_{t,1}), \ldots, \ell_t(y_{t,k})$
  Estimate the gradient $\tilde{g}_t = g(\ell_t(y_{t,1}), \ldots, \ell_t(y_{t,k}))$.
  Update $x_{t+1} = proj_{(1-\xi)\mathcal{K}}(x_t - \eta\tilde{g}_t)$.
end for

In this section we present a no-regret result for any algorithm that is an instantiation of Algorithm 1 for some k and where its randomized gradient estimate satisfies boundedness and closeness of its expectation to the true gradient. We assume the reader is familiar with OGD.

We first explain the algorithm. It plays k actions, constructs a gradient estimate $\tilde{g}_t$ from its k actions in round t and performs the OGD step to produce the next action x_{t+1}:

$$x_{t+1} = proj_{(1-\xi)\mathcal{K}}(x_t - \eta\tilde{g}_t),$$

where $\xi \in (0,1)$ and $(1-\xi)\mathcal{K}$ is shorthand for $\{(1-\xi)x : x \in \mathcal{K}\}$. Note that our gradient estimators will randomly query around the action x_t in order to estimate the gradient. Thus the projection is made onto the shrunk set to ensure that the random queries around the point x_{t+1} belong to $\mathcal{K}$. For any $x \in (1-\mathcal{K})$ and any unit vector u it holds that $(x + \delta u) \in \mathcal{K}$ for any $\delta \in [0, \xi r]$ [Flaxman et al., 2004].

We now present the main result. But first, we need two Lemmas the first is the dynamic regret bound of OGD and the second is the difference in regret between playing x_t on $\mathcal{K}$ and $\{y_{t,i}\}_{i=1}^k$ on $(1-\xi)\mathcal{K}$.

Lemma 1. [Mokhtari et al., 2016]. Assume that the functions h_t are strongly convex and L-smooth. Assume further that the gradient norms are bounded and the step size is chosen such that $\eta \leq 1/L$. Then, the dynamic regret $Regret_T^d$ for the sequence of actions x_t generated by OGD is bounded by

$$Regret_T^d(OGD, G) := \sum_{t=1}^{T} h_t(x_t) - \sum_{t=1}^{T} \min_{x_t^* \in \mathcal{K}} h_t(x_t^*) \leq GK_1 \sum_{t=2}^{T} \left\| x_t^* - x_{t-1}^* \right\| + GK_2$$

where the constants K_1 and K_2 are explicitly given by

$$K_1 := \frac{\|x_1 - x_1^*\| - \rho\|x_T - x_T^*\|}{(1-\rho)}, \quad K_2 := \frac{1}{1-\rho}.$$

Where $0 \leq \rho := (1 - \eta\mu)^{1/2} < 0$. Is our linear convergence constant.

Lemma 2. For any point $x \in \mathcal{K}$,

$$\frac{1}{k}\sum_{i=1}^{k} \ell_t(y_{t,i}) - \ell_t(x) \leq \ell_t(x_t) - \ell_t((1-\xi)x) + G\delta + GD\xi.$$

Proof. By assumption of Lipschitz continuity,

$$\ell_t(y_{t,i}) \leq \ell_t(x_t) + G\delta.$$

We also have that by the Lipschitz property and $\|x\| \leq D$, for all $x \in \mathcal{K}$,

$$\ell_t((1-\xi)x) \leq \ell_t(x) + GD\xi.$$

Combining the above two inequalities we get

$$\frac{1}{k}\sum_{i=1}^{k}\ell_t(y_{t,i}) + \ell_t((1-\xi)x) \leq \ell_t(x_t) + \ell_t(x) + G\delta + GD\xi.$$

Rearranging terms gives us the Lemma.

□

Theorem 1. Suppose the assumptions in the preliminaries hold. Suppose on round t the algorithm plays k random queries $y_{t,1}, ..., y_{t,k}$, constructs a gradient estimator $\tilde{g}_t = g(y_{t,1}, ..., y_{t,k})$ and uses the the algorithmic step $x_{t+1} = proj_{(1-\xi)\mathcal{K}}(x_t - \eta\tilde{g}_t)$ with $\eta \leq \frac{1}{2G_1}$, $\delta = \frac{\log(T)}{T}$, and $\xi = \frac{\delta}{r}$. If the gradient estimator satisfies the following conditions for all $t \geq 1$:

1. $\|x_t - y_{t,i}\| \leq \delta$ for $i = 1, ...k$.
2. $\|\tilde{g}_t\| \leq G_1$ for some constant G_1.
3. $\|\mathbb{E}_t\tilde{g}_t - \nabla\ell_t(x_t)\| \leq c\delta$ for some constant c.

Then for any sequence $\{x_t^*\}_{t=1}^T, x_t^* \in \mathcal{K}$ we have

$$\mathbb{E}\frac{1}{k}\sum_{t=1}^{T}\sum_{i=1}^{k}\ell_t(y_{t,i}) - \mathbb{E}\sum_{t=1}^{T}\min_{x_t^*\in\mathcal{K}}\ell_t(x_t^*) \leq G_1K_1\sum_{t=2}^{T}\left\|x_t^* - x_{t-1}^*\right\| + G_1K_2 + G\log(T)\big(1 + 2c + \frac{D}{r}\big).$$

where the constants K_1 and K_2 are explicitly given by

$$K_1 := \frac{\|x_1 - x_1^*\| - \rho\|x_T - x_T^*\|}{(1-\rho)}, \quad K_2 := \frac{1}{1-\rho}.$$

Where $0 \leq \rho := (1 - \eta\mu)^{1/2} < 0$ is our linear convergence constant.

Proof. Start by defining $h_t(x) = \ell_t(x) + (\tilde{g}_t - \nabla\ell_t(x))^\intercal x$. Then h_t has the same convexity properties as ℓ_t and is L_1-smooth. Note also that $\nabla h_t(x_t) = \tilde{g}_t$. So we can pretend that the algorithm is actually performing deterministic gradient descent, as if with full information, on the functions h_t restricted to $(1-\xi)\mathcal{K}$. Using the OGD regret bound from Lemma 1 we have that since $\nabla h_t(x_t) = \tilde{g}_t$ and hence $\|\tilde{g}_t\| \leq G_1$ (So h_t is $2G_1$-smooth since ∇h_t is $2G_1$-Lipschitz),

$$\sum_{t=1}^{T}h_t(x_t) - \sum_{t=1}^{T}h_t(x_t^*) \leq G_1K_1\sum_{t=2}^{T}\left\|x_t^* - x_{t-1}^*\right\| + 2G_1K_2 := Regret_T^d(OGD, G_1).$$

Then taking expectations,

$$\begin{aligned}
\mathbb{E}\sum_{t=1}^{T}[\ell_t(x_t) - \ell_t(x_t^*)] &= \mathbb{E}\sum_{t=1}^{T}[h_t(x_t) - h_t(x_t^*)] + \mathbb{E}\sum_{t=1}^{T}[\ell_t(x_t) - h_t(x_t) - \ell_t(x_t^*) + h_t(x_t^*)] \\
&\leq Regret_T^d(OGD, G_1) + \mathbb{E}\sum_{t=1}^{T}(\mathbb{E}_t\tilde{g}_t - \nabla\ell_t(x_t))^\intercal(x_t - x_t^*) \\
&\leq Regret_T^d(OGD, G_1) + 2c\delta DT.
\end{aligned}$$

Where the first inequality is by the convexity of ℓ_t and h_t. Now we use Lemma 2 and obtain

Algorithm 2 Gradient descent with deterministic estimator based on $d+1$ points

Input: Step size η, exploration parameter δ and shrinkage coefficient ξ.
Set $x_1 = 0$
for $t = 1, ..., T$ **do**
  Observe $\ell_t(x_t)$, $\ell_t(x_t + \delta e_i)$ for $i = 1, ..., d$.
  Set $\tilde{g}_t = \frac{1}{\delta}\sum_{i=1}^{d}(\ell_t(x_t + \delta e_i) - \ell_t(x_t))e_i$.
  Update $x_{t+1} = proj_{(1-\xi)\mathcal{K}}(x_t - \eta\tilde{g}_t)$.
end for

$$\mathbb{E}\frac{1}{k}\sum_{t=1}^{T}\sum_{i=1}^{k}\ell_t(y_{t,i}) - \mathbb{E}\sum_{t=1}^{T}\min_{x_t^*\in\mathcal{K}}\ell_t(x_t^*) \leq Regret_T^d(OGD, G_1) + 2c\delta DT + TG\delta + GDT\xi.$$

To finish the proof, plug in the values for δ and ξ.

□

4 $d+1$ point feedback

In this section, we show that we can construct a deterministic gradient estimator using $d+1$ point feedback. Thus we obtain a deterministic version of Theorem 1. Hence, the algorithm is no-regret even against completely adaptive adversaries meaning that the adversary can choose the loss ℓ_t after the algorithm plays x_t. Hence we match the full-information bound.

The algorithm constructs the deterministic gradient estimator

$$\tilde{g}_t = \frac{1}{\delta}\sum_{i=1}^{d}(\ell_t(x_t + \delta e_i) - \ell_t(x_t))e_i.$$

Where e_i's are the standard unit basis vectors. We further need only the assumptions on strong convexity and L-smoothness, since they imply a bound on the gradient which we denote G. We can thus derive a bound on the norm of the estimator

$$\begin{aligned}\|\tilde{g}_t\| &= \left\|\frac{1}{\delta}\sum_{i=1}^{d}(\ell_t(x_t + \delta e_i) - \ell_t(x_t))e_i\right\| \\ &\leq \frac{d}{\delta}\max_i\|\ell_t(x_t + \delta e_i) - \ell_t(x_t)\| \\ &\leq \frac{d}{\delta}\delta G \\ &= dG.\end{aligned}$$

Where the second inequality is by the Lipschitz property. We can also derive the divergence of the estimator:

$$\begin{aligned}\|\tilde{g}_t - \nabla\ell_t(x_t)\| &= \sqrt{|\frac{1}{\delta}\sum_{i=1}^{d}(\ell_t(x_t + \delta e_i) - \ell_t(x_t))e_i - \langle\nabla\ell_t(x_t), e_i\rangle|^2} \\ &\leq \sqrt{\frac{d}{\delta}\max_i\{|(\ell_t(x_t + \delta e_i) - \ell_t(x_t))e_i - \langle\nabla\ell_t(x_t), e_i\rangle|^2\}}\end{aligned}$$

By the smoothness assumption, we have for all i

$$\ell_t(x_t + \delta e_i) \leq \ell_t(x_t) + \delta\langle \nabla \ell_t(x_t), e_i\rangle + \frac{L\delta^2}{2}.$$

And by convexity we have $\ell_t(x_t + \delta e_i) \geq \ell_t(x_t) + \delta\langle \nabla \ell_t(x_t), e_i\rangle$. Hence

$$|\frac{1}{\delta}(\ell_t(x_t + \delta e_i) - \ell_t(x_t))e_i - \langle \nabla \ell_t(x_t), e_i\rangle|^2 \leq \frac{L^2\delta^2}{4}.$$

So we conclude that

$$\|\tilde{g}_t - \nabla \ell_t(x_t)\| \leq \frac{\sqrt{d}L\delta}{2}$$

Hence we have that the properties of $\tilde{g}_t$ are the deterministic version of the properties of the estimator outline in theorem 1. Hence we have an algorithm that can guarantee no-regret against a completely adaptive adversary.

Theorem 2. Suppose a completely adaptive adversary chooses the sequence of loss functions $\{\ell_t\}_{t=1}^T$ subject to the same assumptions as above. If Algorithm 2 is run with the $\eta \leq \frac{1}{2dG}$, $\delta = \frac{\log(T)}{T}$, and $\xi = \frac{\delta}{r}$, then

$$\sum_{t=1}^{T} \frac{1}{d+1}(\ell_t(x_t) + \sum_{i=1}^{d} \ell_t(x_t + \delta e_i)) - \sum_{t=1}^{T} \ell_t(x_t^*) \leq Regret_T^d(OGD, dG) + G\log(T)\big(1 + \frac{\sqrt{d}L\delta}{2} + \frac{D}{r}\big).$$

Proof. This is a modification of the proof of Theorem 1. Define $h_t(x) = \ell_t(x) + (\tilde{g}_t - \nabla \ell_t(x))^\intercal x$. Then since $\|\nabla h_t(x_t)\| \leq dG$. Hence from Lemma 1 for any sequence $\{x_t^*\}_{t=1}^T$,

$$\sum_{t=1}^{T} h_t(x_t) - h_t(x_t^*) \leq Regret_T^d(OGD, dG).$$

Then we proceed as in the proof of Theorem 1, use $\|\tilde{g}_t - \nabla \ell_t(x_t)\| \leq \frac{\sqrt{d}L\delta}{2}$. Apply Lemma 2 and plug in our parameters and this gives us the result.

□

5 Experiments

In this section we present experimental data on the performance of Algorithm 2. We also compare its performance to an algorithm that uses the full gradient information.

We consider a time-varying least-squares problem of the form:

$$\min_{x \in \mathcal{B}} \frac{1}{2} \|Ax - b_t\|^2$$

where $\mathcal{B}$ is the unit ball, $A \in \mathbb{R}^{n \times d}$ is a given regression matrix, and b_t is a time-varying vector. We now outline how the loss functions $\ell_t = \frac{1}{2}\|Ax - b_t\|^2$ are generated.

We generate A by defining its singular value decomposition: for its left and right-singular vectors, sample two orthogonal matrices $U \in \mathbb{R}^{n \times n}$ and $V \in \mathbb{R}^{d \times d}$. Then we let the singular values be equally spaced from $1/\sqrt{\kappa}$ to 1. We consider $\kappa = 100$.

We generate b_t with

$$b_t = Ax_t^* + w_t$$

where x_t^* is generated using a random walk with step size of 1 ($\left\|x_t^* - x_{t-1}^*\right\| = 1 \ \ \forall t$) and w_t is a zero-mean Gaussian vector with covariance matrix $10^{-3} I_n$. Hence $\sum_{t=1}^{T} \left\|x_t^* - x_{t-1}^*\right\| = T$.

Clearly the ℓ_t's are μ-strongly convex and L-smooth with $\mu = 1/\sqrt{\kappa}$ and $L \leq \|A\| = 1$. We also have that ℓ_t is G-Lipschitz with $G = 6\|A^\intercal A\|^2 = 6$.

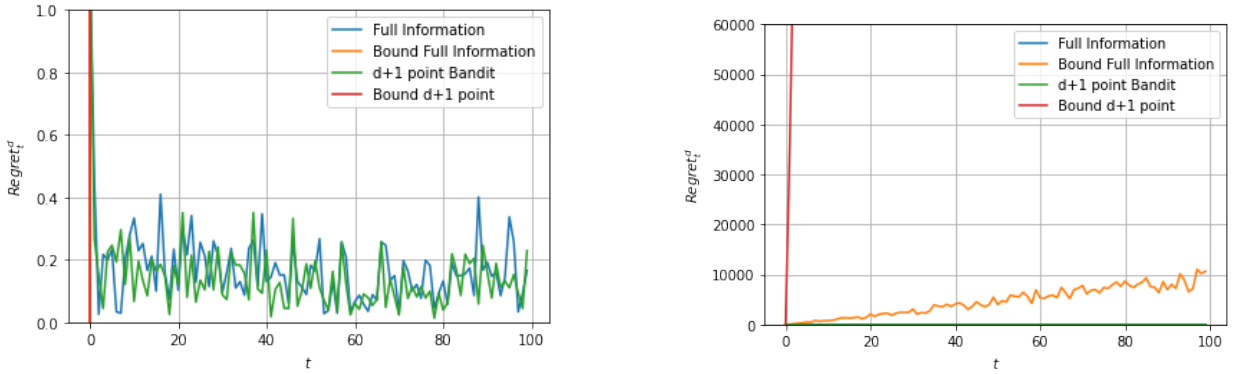

Figure 1: Plot showing the regret of Algorithm 2 and the full-information OGD along with bounds for both. Note that both the bounds diverge almost immediately.

References

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Aryan Mokhtari, Shahin Shahrampour, Ali Jadbabaie, and Alejandro Ribeiro. Online optimization in dynamic environments: Improved regret rates for strongly convex problems. In *2016 IEEE 55th Conference on Decision and Control (CDC)*, pages 7195–7201. IEEE, 2016.

A Code

generator.py

```
import numpy as np
from numpy import linalg

from scipy.stats import ortho_group
# Return a random orthogonal matrix, drawn from the O(N) Haar distribution (the only unifor


class ls_loss:
    def __init__(self, A, b_t, xstar, d):
        self.A = A
        self.b = b_t
        self.xstar = xstar
        self.d = d

    def evaluate(self, x):
        A = self.A
        b_t = self.b
        return 0.5 * linalg.norm(A @ x - b_t) ** 2

    def gradient(self, x):
        A = self.A
        b_t = self.b
        return A.T @ (A @ x - b_t)

    def tracking_error(self, x):
        return linalg.norm(x - self.xstar)

    def regret(self, x):
        return self.evaluate(x) - self.evaluate(self.xstar)


def generate_ls_seq(n, d, xstar_gen, iters = 500):
    #  generate list containing sequence of loss objects
    # ball xstar_gen = xstar2(d)

    losses = []
    A = generate_A(n, d)
    b_gen = generate_bt(A, n, xstar_gen)  #generator for b

    for i in range(iters):
        b_t, xstar_t = next(b_gen)
        losses.append(ls_loss(A, b_t, xstar_t, d))
    return losses


def generate_A(n, d):
    U = ortho_group.rvs(n)
    V = ortho_group.rvs(d)
```

```
    D = np.diagflat(np.flip(np.linspace(1 / np.sqrt(100), 1, min(n, d))))
    D = np.vstack((D, np.zeros([n - d, d])))
    return U @ D @ V


def generate_bt(A, n, x_gen):
    '''generator for b_t'''
    while True:
        xstar = next(x_gen)
        w = np.random.normal(0, 10 ** (-3), n)
        yield A @ xstar + w, xstar


def xstar1(sigma, d):
    '''generator for x_t^* for unconstrained'''
    x = np.zeros(d)
    while True:
        yield x
        x += sigma * sample_n_sphere_surface(d)


def xstar2(d):
    '''generator for x_t^* for unit ball'''
    x = np.zeros(d)
    while True:
        yield x
        x = xstar2_helper(x, d)


def xstar2_helper(x, d):
    step = sample_n_sphere_surface(d)  # step size 1
    while linalg.norm(x + step) >= 1.0:  # resample the step
        step = sample_n_sphere_surface(d)
    return x + step


def sample_n_sphere_surface(ndim, norm_p=2):
    """sample random vector from S^n-1 with norm_p"""

    vec = np.random.randn(ndim)
    vec = vec / linalg.norm(vec, norm_p)  # create random vector with norm 1
    return vec
```

optimizer.py

```
import numpy as np
from numpy import linalg
import generator as gen


#run experiment over sequence of loss function objects
```

```
def gradient_descent(loss_seq, projection, alpha):
    tracking_error = []
    regret = []

    x_t = np.ones(loss_seq[0].d)

    for loss in loss_seq:
        tracking_error.append(loss.tracking_error(x_t))
        regret.append(loss.regret(x_t))
        x_t = x_t - alpha * loss.gradient(x_t)
        x_t = projection(x_t)

    return tracking_error, regret


def bandit_descent(loss_seq, projection, alpha, delta, xi):
    tracking_error = []
    regret = []

    x_t = np.ones(loss_seq[0].d)  # init play

    for loss in loss_seq:
        tracking_error.append(loss.tracking_error(x_t))
        gradient, d1_loss = d1_point_gradient_loss(x_t, delta, loss)
        regret.append(d1_loss - loss.evaluate(loss.xstar))

        x_t = x_t - alpha * gradient
        x_t = projection(x_t, xi)  # project onto set scaled by xi

    return tracking_error, regret


def d1_point_gradient_loss(x_t, delta, loss):
    d = loss.d

    g = np.zeros(d)
    reg_query = 0

    lxt = loss.evaluate(x_t)
    for i in range(d):
        e_i = np.zeros(d)
        e_i[i] = 1.0
        point_loss = loss.evaluate(x_t + delta * e_i)
        reg_query += point_loss
        g += (point_loss - lxt) * e_i
    return 1 / delta * g, 1 / (d+1) * (reg_query + lxt)


def k_point_gradient(x_t, delta, d, loss):
    pass
```

```
def project_ball(x, radius=1):
    norm = linalg.norm(x)
    if norm > radius:
        return x / norm * radius
    else:
        return x
```

Experiment runner

```
#!/usr/bin/env python
# coding: utf-8

# # Experimental data for Bandit OGD with d+1 point Feedback
#
# Dhamma Kimpara

# In[ ]:


get_ipython().run_line_magic('load_ext', 'autoreload')
get_ipython().run_line_magic('autoreload', '2')


# In[2]:


import generator as gen
import optimizer as opt
import numpy as np
import scipy

import matplotlib.pyplot as plt
get_ipython().run_line_magic('matplotlib', 'inline')


# In[3]:


kappa = 100
n = 20
d = 5
iters = 100
loss_seq = gen.generate_ls_seq(n, d, gen.xstar2(d), iters)

G1 = d*6

eta = 1 / (2 * G1)
delta = np.log(iters)/iters
xi = delta/1.0

G = 6
```

```
tracking_errs0, regs0 = opt.gradient_descent(loss_seq, opt.project_ball, 0.9)
tracking_errs1, regs1 = opt.bandit_descent(loss_seq, opt.project_ball, eta, delta, xi)


# In[8]:


def regret_sequence(tracking_errs, G, rho):
    reg_bound = [0]
    for i in range(len(tracking_errs) - 1):
        k1 = (tracking_errs[0] - rho * tracking_errs[i + 1]) / (1-rho)
        k2 = 1/(1-rho)
        reg_bound.append(k1*G*i + G*k2)
    return reg_bound
def regret_sequence_bandit(tracking_errs, G1, G, rho, d): #G1 is gradient bound of estimat
    reg_bound = [0]
    for i in range(len(tracking_errs) - 1):
        k1 = (tracking_errs[0] - rho * tracking_errs[i + 1]) / (1-rho)
        k2 = 1/(1-rho)
        reg_bound.append(k1*G1*i + G1*k2 + G * np.log(i+1)*(1+2*d*G + 1))
    return reg_bound


# In[9]:


rho = max([np.abs(1 - 0.9/np.sqrt(kappa)), np.abs(1 - 0.9)])
rho_bandit = np.sqrt(1 - eta / np.sqrt(kappa))


bounds0 = regret_sequence(tracking_errs0, G,rho)
bounds1 = regret_sequence_bandit(tracking_errs1, G1, G, rho_bandit, d)


### Results for $|sigma=1$

# In[19]:


plt.plot(range(len(regs0)), regs0, label="Full Information")
plt.plot(range(len(regs0)), bounds0, label="Bound Full Information")

plt.plot(range(len(regs1)), regs1, label="d+1 point Bandit")
plt.plot(range(len(regs1)), bounds1, label="Bound d+1 point")


#plt.axhline(1/(1-rho), linestyle='--', label="Asymptotic bound")
plt.grid()
plt.ylim([0, 60000.0])
```

```
plt.legend()

plt.xlabel("$t$")
plt.ylabel("$Regret_t^d$")


# In[ ]:
```