Warm-Up Exam broken into two parts (both required)

- Part A: Answer 10 T/F or 4 MC questions, (but not both) where each MC has 5 options.
- Part B: 8 T/F or 5 MC (but not both), each MC has 4 options.

How many ways are there of completing the exam?
(Assume you cannot omit questions)

Part A $2^{10}+5^4$

10 T/F: 2^{10}

4 MC (5 options): 5^4

$\underline{5}\times\underline{5}\times\underline{5}\times\underline{5}=5^4$

$\underline{\quad}\ \underline{\quad}\ \underline{\quad}$

T/F T/F T/F

$2\times 2\times 2=2^3$

TTT
TTF
TFT
TFF
FTT
FTF
FFT
FFF

Part B 2^8+4^5

8 T/F → 2^8

MC → 4^5

Exam #Part A × #Part B

$(2^{10}+5^4)\times(2^8+4^5)$

6.4 <u>Notation</u> Let $n \in \mathbb{N}$ ($n \geq 0$, n is an integer)

The factorial (pronounced "n-factorial")

$$n! = n(n-1)(n-2)(n-3)\cdot \cdots \cdot 2 \cdot 1$$

$$0! = 1.$$

<u>Ex</u>

$$2! = 2\cdot 1 = 2$$

$$3! = 3\cdot 2\cdot 1 = 6$$

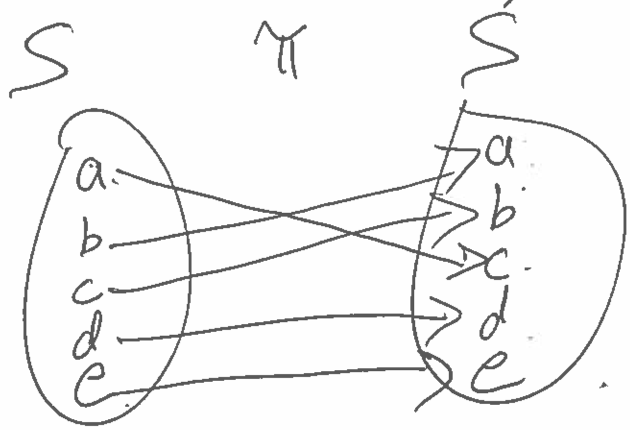
$$4! = 4\cdot 3\cdot 2\cdot 1 = 24$$

$$5! = 5\cdot \underline{4\cdot 3\cdot 2\cdot 1} = 120$$

$$6! = 6\cdot 5! = 720$$

<u>Def</u> Let S be a <u>finite</u> set. A permutation π is a one-to-one function from S to itself.

<u>Recall</u> π is one-to-one if for any distinct $x, y \in S$, then $\pi(x) \neq \pi(y)$



$a \to c$ 5

$b \to a$ 4

$c \to b$ 3

$d \to d$ 2

$e \to e$ 1

$5!$

Fact There are $n!$ permutations of an n-element set.

$0!$

↳ $\emptyset$ has cardinality 0

↳ There is only one permutation of $\emptyset$

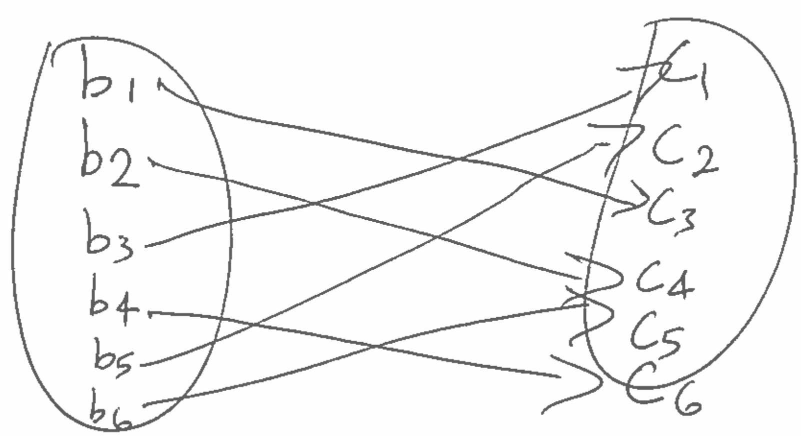
↳ So $0! = 1$

Ex I have 24 songs on playlist. How many orderings are there?

A: $24!$

Ex Assign 6 different books to 6 children, so each child gets one book.

Books Children



b_1 : 6

b_2 : 5

b_3 : 4

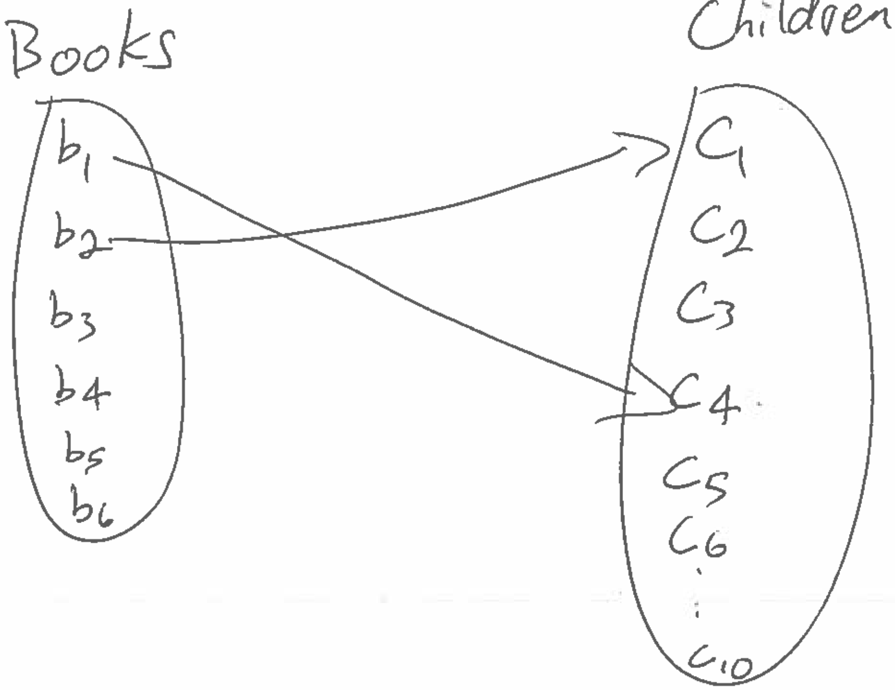
b_4 : 3

b_5 : 2

b_6 : 1

A $6!$

Ex 6 different books, 10 children, each child gets at most 1 book



b_1: 10
b_2: 9
b_3: 8
b_4: 7
b_5: 6
b_6: 5

$$10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5 = \frac{10!}{4!} = \frac{10!}{(10-6)!}$$

$$\frac{10!}{4!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot \cancel{4\cdot 3\cdot 2\cdot 1}}{\cancel{4\cdot 3\cdot 2\cdot 1}}$$

<u>Restricted Permutation</u> Assignment of k distinct objects to n distinguishable people, s.t. each person received at most one object. ~~($0\le k\le n$)~~

($0\le k\le n$)

$$P(n,k) = \frac{n!}{(n-k)!} \qquad P(10,6) = \frac{10!}{4!}$$

Section 6.3: All

Section 6.4: 6, 8, 10

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