5.2 Ex

$25 M Loan Budget

↳ Condos 12% rate

↳ Must loan at least $10 M for condos

↳ Low-Income Housing: 10% rate

↳ $\frac{1}{3}$ of total loans (or more) to low-income

↳ $\frac{2}{3}y \geq \frac{1}{3}x$ (For every $2 loaned for Condos, at least $1 for Low income)

x: condos
y: low-income

Want Max profit

max $.12x + .1y$ s.t.

$x + y \leq 25$

$x \geq 10$

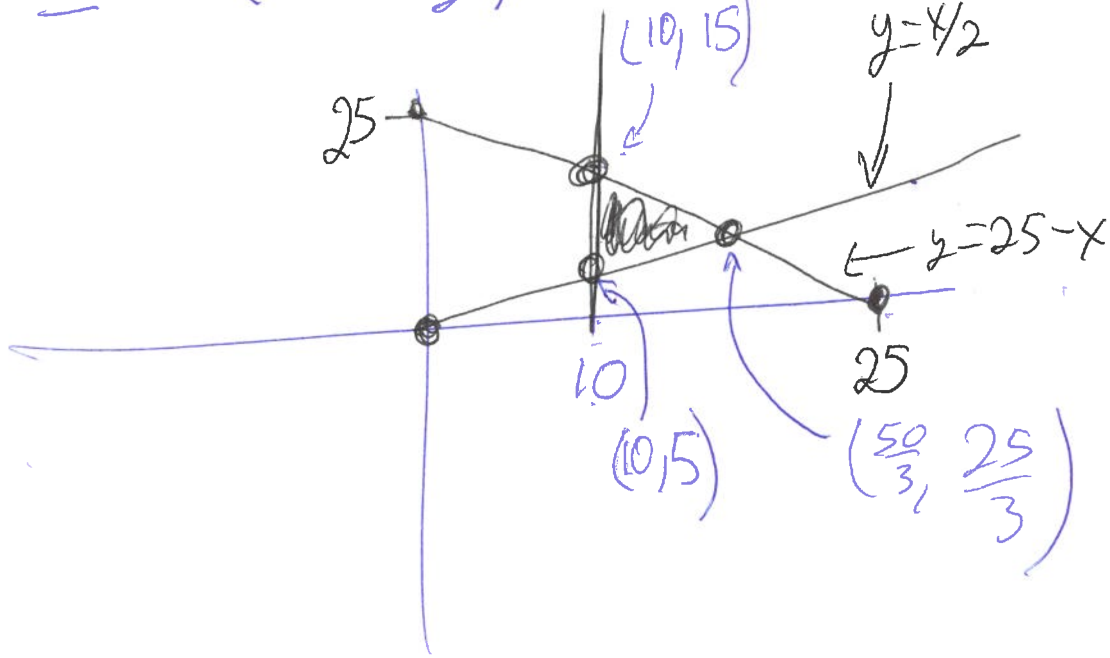
$-\frac{1}{3}x + \frac{2}{3}y \geq 0$ (Third or more to Low-Income Hous.)

$y \geq 0$

$x+y \le 25 \quad (y \le 25-x)$

$-\frac{1}{3}x + \frac{2}{3}y \ge 0 \quad \left(y \ge \frac{x}{2}\right)$

$x \ge 10$



$25 - x = \frac{x}{2}$

$25 = \frac{3x}{2}$

$x = \frac{50}{3}$, so $y = \frac{25}{3}$

Pt	$0.12x + 0.1y$
(10, 5)	1.7
(10, 15)	2.7
$\left(\frac{50}{3}, \frac{25}{3}\right)$	2.833 ← Max profit: \$2.833 M

Maximizer

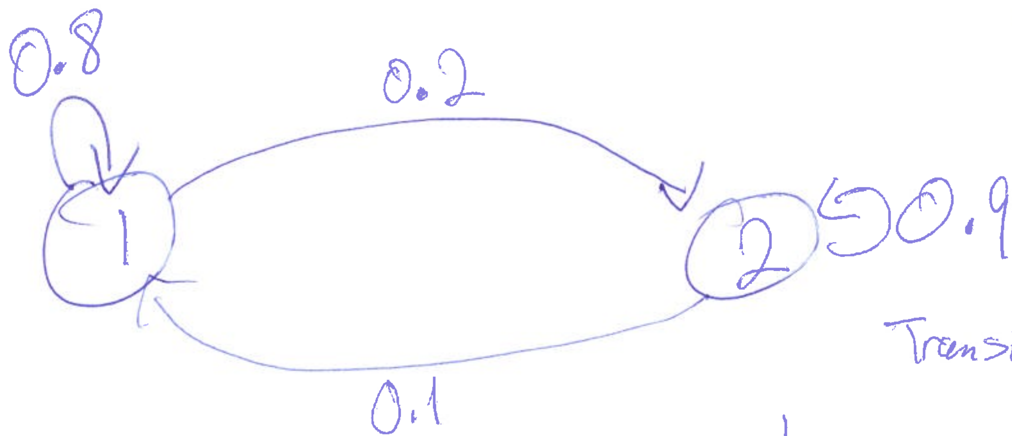
7.7 Markov Chains

Def A Markov Chain has a set of states $[n] := \{1, 2, \dots, n\}$. For each pair of

Notation

states $i, j \in [n]$, we have a prob of transitioning from i to j, denoted P_{ij}

Ex



Transition Matrix

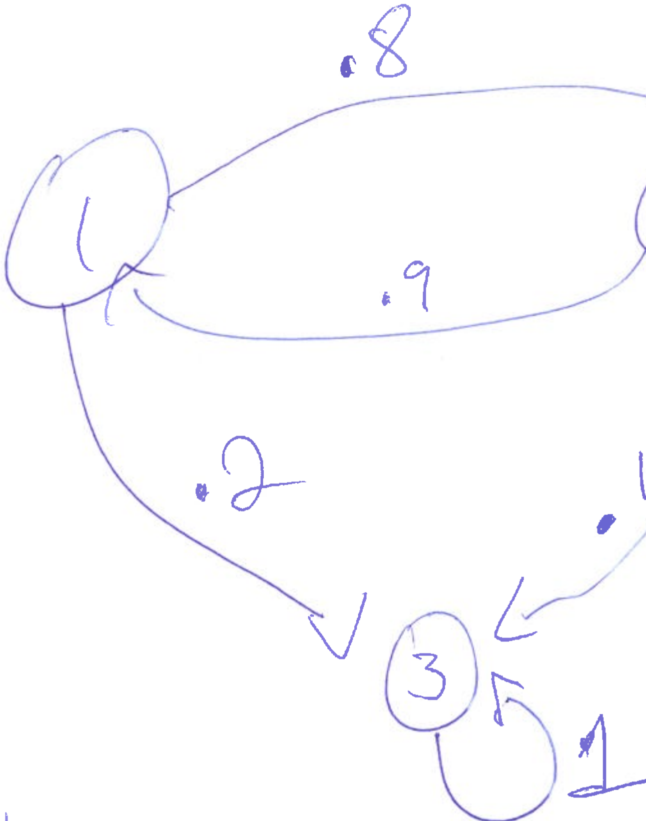
Transition to

$$\begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \text{Start} \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix} \end{array}$$

Q What is sum of entries in given row?

A 1

Ex



$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & .8 & .2 \\ 2 & .9 & 0 & .1 \\ 3 & 0 & 0 & 1 \end{array}$$

Ex Given $P = \begin{bmatrix} 0 & 1 \\ .5 & .5 \end{bmatrix}$, $V_0 = [.2, .8]$

One Iteration $V_1 = V_0 P = [.2 \quad .8] \begin{bmatrix} 0 & 1 \\ .5 & .5 \end{bmatrix}$

$= [.4 \quad .6]$

Two Iterations $V_1 P = (V_0 P) P = V_0 (PP) = V_0 P^2$

$= [.3 \quad .7]$

Three Iterations $V_3 = V_0 P^3 = [.35 \quad .65]$

K Iterations : $V_k = V_0 P^k$

Def Let P be the trans matrix of a given Markov Chain. The steady-state of our Markov Chain $\vec{v} = [v_1, v_2, \cdots, v_n]$ satisfies
$\vec{v}P = \vec{v}$ ($\vec{v}$ is a left-eigenvector)

Ex $P = \begin{bmatrix} .8 & .2 \\ .1 & .9 \end{bmatrix}$, want steady-state

$$[x \quad y] \begin{bmatrix} .8 & .2 \\ .1 & .9 \end{bmatrix} = [x \quad y]$$

$$\left.\begin{array}{l} .8x + .1y = x \rightarrow -.2x + .1y = 0 \\ .2x + .9y = y \rightarrow .2x - 0.1y = 0 \end{array}\right] \text{Same line}$$

$x + y = 1 \qquad x + y = 1$

$$\left[\begin{array}{cc|c} .2 & -0.1 & 0 \\ 1 & 1 & 1 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} \end{array}\right]$$

Steady-State $[\frac{1}{3}, \frac{2}{3}]$

Ex $P = \begin{bmatrix} .2 & .8 \\ .4 & .6 \end{bmatrix}$ Find Steady-State

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} .2 & .8 \\ .4 & .6 \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix}$$

same line

$$.2x + .4y = x \rightarrow -.8x + .4y = 0$$
$$.8x + .6y = y \rightarrow .8x - .4y = 0$$
$$x + y = 1 \qquad x + y = 1$$

$$\left[\begin{array}{cc|c} .8 & -.4 & 0 \\ 1 & 1 & 1 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} \end{array}\right]$$

Steady-State $\left[\frac{1}{3}, \frac{2}{3}\right]$