12/4 - Graph Theory

EC HW (Added to your best exam)

5.2

Ex Loan budget of $25 Million

↳ Condos at 12% interest ↳ $10 Mill for condos or more

↳ Low-Income Housing at 10% interest

↳ $\frac{1}{3}$ of loan amount to low income housing

condos ↙ ↘ Low Income Housing

max $0.12x + 0.1y$ s.t.

$x \geq 10$ (Condos)

$x + y \leq 25$ (Budget)

$\frac{2}{3}y \geq \frac{1}{3}x$ ($\frac{1}{3}$ or more of loans are for Low income Housing)

$x \geq 0$

$y \geq 0$

→ $\frac{2}{3}y - \frac{1}{3}x \geq 0$

$2y - x \geq 0$

max $0.12x + 0.1y$ s.t.

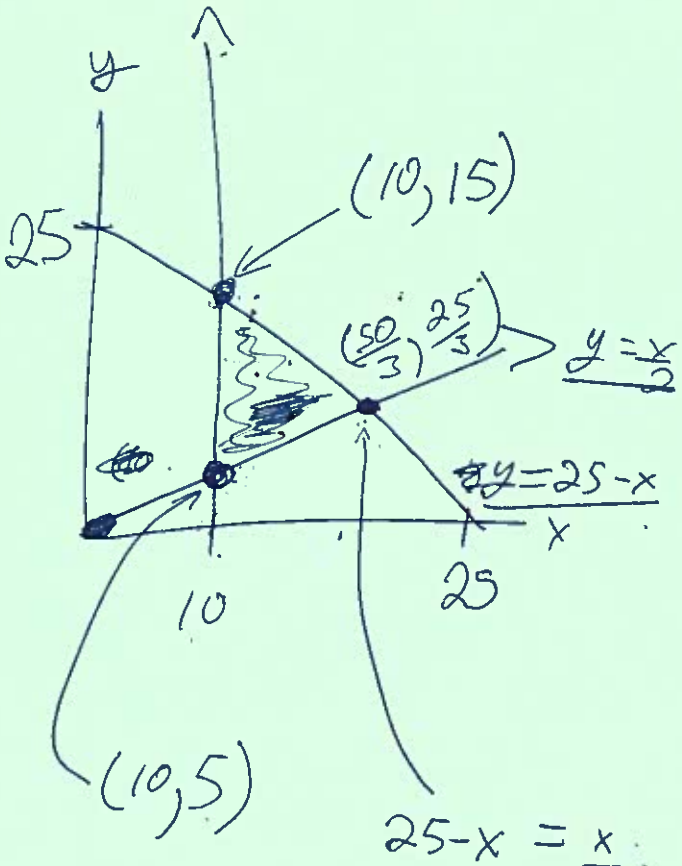
$x \geq 10$

$x + y \leq 25$ $(y \leq 25 - x)$

$2y - x \geq 0$

$x, y \geq 0$

$\rightarrow y \geq \frac{x}{2}$



Want max $0.12x + 0.1y$ in feasible region

Point	$0.12x + 0.1y$
(10,5)	$0.12(10) + 0.1(5) = 1.7$
(10,15)	$0.12(10) + 0.1(15) = 2.7$
$(\frac{50}{3}, \frac{25}{3})$	$0.12(\frac{50}{3}) + 0.1(\frac{25}{3}) = 2.833$

$25 - x = \frac{x}{2}$

$25 = \frac{3}{2}x$

$x = \frac{2}{3}(25) = \frac{50}{3}$

$y = \frac{50}{6} = \frac{25}{3}$

So $0.12x + 0.1y$ achieves max of \$2.833M, by allocating $\$\frac{50}{3}$M to Condos and $\$\frac{25}{3}$M to Low-Income.

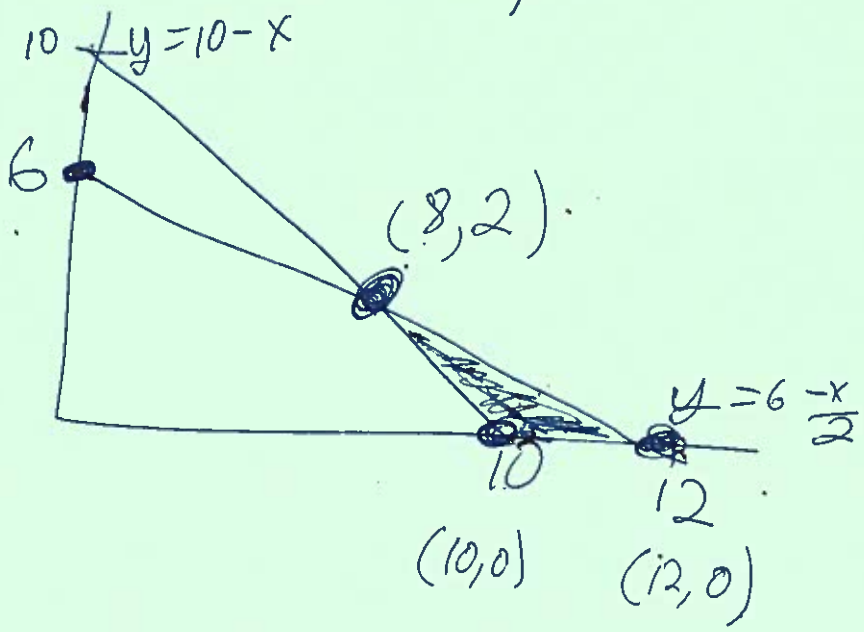
Ex min $2x+3y$ s.t.

$x + y \geq 10$ $\quad (y \geq 10 - x)$.

$x + 2y \leq 12$ $\quad (y \leq 6 - \frac{x}{2})$.

$x \geq 0$

$y \geq 0$



Third Corner Pt

$10 - x = 6 - \frac{x}{2}$

$4 = \frac{x}{2}$

$x = 8$.

$6 - \frac{x}{2} = 0$

$6 = \frac{x}{2}$

Want min $2x+3y$ in feasible region

Point	$2x+3y$
$(10,0)$	$2(10)+3(0)=20$
$(8,2)$	$2(8)+3(2)=22$
$(12,0)$	$2(12)+3(0)=24$

So $2x+3y$ achieves min at $(10,0)$, with min value 20.