• Recall Combinations are subsets of k-element set from n.
Order does not matter.
Elems are distinct.

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The binomial coefficient $\binom{n}{k}$ (or $C(n,k)$) counts the number of k elem subsets of n-element set.

Ex 5 different stocks, want 3 stocks.
How many ways to select 3 stocks?
A $\binom{5}{3}$ (or $C(5,3)$)

Ex 5 different stocks, want 2.
3 different metals, want 2.
8 different bond options, want 5.

Want to select portfolio
↳ Select Stocks: $\binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{5!}{2!\cdot 3!} = \frac{5\cdot 4}{2} = 10$
↳ Select metals: $\binom{3}{2}$
↳ Select Bonds: $\binom{8}{5}$

Rule of Product $\binom{5}{2}\binom{3}{2}\binom{8}{5}$

Poker Hands

↳ Standard deck of playing cards has 52 cards.
  ↳ 4 suits (~~Ace~~ Heart, Diamond, Clubs, Spades)
  ↳ 13 ranks/values (Ace, 2, 3, ..., 10, J, Q, K)

Ex How many 5-card hands exist? $\binom{52}{5}$

Ex Full House has:
  ↳ 3 cards of one rank
  ↳ 2 cards of another rank

(a) 3 Q's, 2 A's.
  $\binom{4}{3}$ ways to pick Q's
  $\binom{4}{2}$ ways to pick A's
  Rule of Product $\binom{4}{3}\binom{4}{2}$ such hands

(b) 3 Q's, need to select second rank:
  ↳ $\binom{4}{3}$ ways to pick Q's
  ↳ $\binom{12}{1}\binom{4}{2}$ ways to pick two cards of 2nd rank
  Pick 2nd rank
  Rule of Product $\binom{4}{3}\binom{12}{1}\binom{4}{2}$ such hands

(c) Want How many Full Houses?

↳ $\binom{13}{1}\binom{4}{3}$

Sel First rank

↳ $\binom{12}{1}\binom{4}{2}$

2 cards of 2nd rank

Rule of Product $\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$

3-cards of one rank

Ex A one-pair has two cards of the same rank, and three cards of three different ranks (all different than the First rank).

(a) Pair of 5's:

↳ $\binom{4}{2}$ ways to select 5's

↳ $\binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1}$

Select remaining 3 ranks

Rule of Product $\binom{4}{2}\binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1}$ such hands

b) How many one-pairs?

↳ Select pair: $\binom{13}{1}\binom{4}{2}$

↳ Select Remaining 3 cards: $\binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1}$

So $\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3$ one-hands