Ch. 7 Discrete Probability

7.1 Sample Spaces and Events

Def A sample space is the set of all poss. be outcomes (ie, our universal set).

Ex Toss a coin: Sample space $= \{H, T\}$
Roll a 6-sided die: Sample space $= \{1, \ldots, 6\}$

Def Let S be a sample space.
An event $E \subseteq S$.

Ex Roll 6-sided die, event rolling ≤ 4.
Event $E = \{1, 2, 3, 4\}$.

Def Let S be a sample space, and let $E, F \subseteq S$ be events. We say that E and F are mutually exclusive if $E \cap F = \emptyset$.

Ex Roll 6-sided die. $E = \{2, 4, 6\}$, $F = \{1, 3, 5\}$
E and F are mutually exclusive.

7.2 Relative Frequency

Idea Let S be a finite sample space, and let $E \subseteq S$ be an event. The probability of E occurring:

$$\Pr[E] = \frac{n(E)}{n(S)}$$

Ex Suppose we roll a single die and get outcomes:

- ↳ 1 (x3)
- ↳ 2 (x3)
- ↳ 3 (x0)
- ↳ 4 (x2)
- ↳ 5 (x1)
- ↳ 6 (x1)

So 10 rolls in total.

X	1	2	3	4	5	6
$\Pr[X]$	$\frac{3}{10}$	$\frac{3}{10}$	0	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Check Do the probabilities add to 1?

$$\frac{3}{10} + \frac{3}{10} + 0 + \frac{2}{10} + \frac{1}{10} + \frac{1}{10} = \frac{10}{10} = 1 \checkmark$$

Note Assume all outcomes equally likely for relative frequency. I.e., Fair die, fair coins, etc.

Ex Suppose we toss fair coin 3 times.
What is prob. of throwing exactly one H?

↳ There are $2 \times 2 \times 2$ 8 possible tosses.

↳ Select pos. for H in $\binom{3}{1} = 3$ ways.

$$\Pr[\#H = 1] = \frac{3}{8}$$

Ex Suppose we toss two distinguishable dice.
What is prob of rolling 2 and 3?

↳ Red and Green Dice

(2, 3) or (3, 2)

$$\Pr[2 \text{ and } 3] = \frac{2}{36} = \frac{1}{18}$$

Ex Roll 2 indistinguishable dice.

↳ Need to determine size of sample space.

↳ Case Distinct rolls. $\left\{\frac{6 \cdot 5}{2}\right\} = 15$ rolls

↳ Case Same rolls: 6 ways.

So sample size $n(S) = 15 + 6 = 21$.

$$Pr[2 \text{ and } 3] = \frac{1}{21}$$

7.3 Probability Models

Ex Weighted 6-sided die: Rolling 1-5 equally likely. Rolling 6 is 3x as likely as any of other #'s.

Q Determine the probability of rolling each #.

x denote Pr[1]

$x + x + x + x + x + 3x = 1$

$8x = 1$

$x = \frac{1}{8} = Pr[1]$

$Pr[6] = \frac{3}{8}$