Study Guide- Section 7.7

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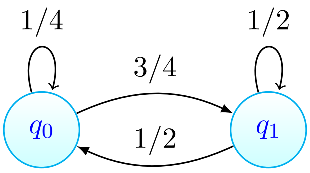
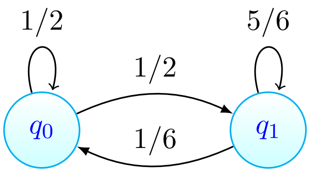
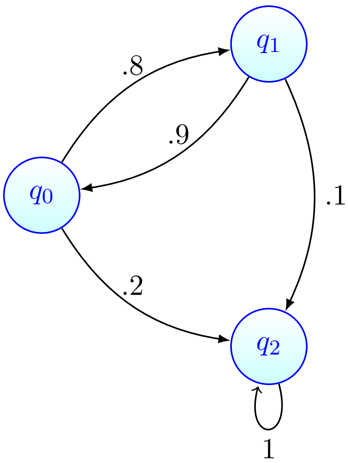
Problem 1) Consider the following state diagram. Determine the transition matrix and the steady-state distribution.



Problem 2) Consider the following state diagram. Determine the transition matrix and the steady-state distribution.



Problem 3) Consider the following state diagram. Determine the transition matrix and the steady-state distribution.



Problem 4) Consider the following transition matrix $P = \begin{pmatrix} 0.5 & 0.5 \\ 0 & 1 \end{pmatrix}$, along with the initial state vector $v_0 = \begin{pmatrix} 1 & 0 \end{pmatrix}$. Find the two-step transition matrix (P^2), as well as the distribution vectors after one, two, and three applications of the Markov Chain.

Problem 5) An auto insurance company classifies each motorist as *high risk* if the motorist has had at least one moving violation during hte past calendar year. Motorists are classified as *low risk* otherwise. According to the company's data, a high risk motorist has a 50% chance of remaining in the high risk category and a 50% chance of moving to the low-risk category. A low-risk motorist has a 10% chance of moving into the high risk category and a 90% chance of remaining in the low-risk category. What is the distribution of the motorists in the long term?

Problem 6) A credit card company classifies its cardholders as falling into one of two credit ratings: *good* and *poor*. Based on its rating criteria, the company finds that a cardholder with a good credit rating has an 80% chance of remaining in that category the following year and a 20% chance of dropping into the poor category. A cardholder with a poor credit rating has a 40% chance of moving into the good rating the following year and a 60% chance of remaining in the poor category. In the long term, what percentage of cardholders fall into each category.

Problem 7) Recall the Gambler's Ruin game from class. At each round, you may choose to play a game. There is a 50% chance of winning; in which case, you have a net gain of $10. Similarly, there is a 50% chance of losing; in which case, you have a net loss of $10. [So if you start a round with $20 and lose, you end with $10.]. You stop playing when you have either $0 or $30. Do the following. [**Note:** There will not be time to discuss this problem during review before the quiz. Please stop by office hours if you wish to discuss this problem.]

(a) Draw the state diagram.

(b) Determine the transition matrix.

(c) Suppose you start with $20. Is it more likely that you will end with $30 or $0? It may help to examine a large number of iterations of the Markov Chain. [**Note:** Your calculator can quickly exponentiate a matrix.]

(d) Suppose you start with $10. Is it more likely that you will end with $30 or $0? It may help to examine a large number of iterations of the Markov Chain.