Study Guide 3.10

Michael Levet

Instructions: Complete the following problems. Justify all your answers in complete sentences, where appropriate.

1 Section 3.10

Problem 1) Solve the following related rates problems.

1. If $x = y^3 - y$ and $dy/dt = 5$, what is dx/dt when $y = 2$?
2. If $x^2 + y^2 = 25$ and $dx/dt = -2$, then what is dy/dt when $x = 3$ and $y = -4$?
3. If $x^2y^3 = 4/27$ and $dy/dt = 1/2$, then what is dx/dt when $x = 2$?
4. If $r + s^2 + v^3 = 12$, $dr/dt = 4$, and $ds/dt = -3$, find dv/dt when $r = 3$ and $s = -1$.

Problem 2) A cube's surface area is increasing at a rate of $72\text{in}^2/\text{sec}$. At what rate is the cube's volume changing when the edge length is $x = 3$. Note that the surface area of a cube is $SA = 6x^2$, and the volume is $V = x^3$.

Problem 3) The voltage V (volts), current I (amperes), and resistance R (ohms) of an electric circuit are related by the equation $V = IR$. Suppose that V is increasing at a rate of 1 volt/sec, while I is decreasing at a rate of 1/3 amp/sec. Let t denote time in seconds.

(a) Determine the values of dV/dt and dI/dt.

(b) Find the rate at which R is changing when $V = 12$ volts and $I = 2$ amperes. Is R increasing or decreasing?

Problem 4) The length ℓ of a rectangle is decreasing at the rate of 2 cm/sec, while the width w is increasing at the rate of 2 cm/sec. When $\ell = 12$ cm and $w = 5$ cm, find the rates of change of the following.

(a) The area.

(b) The perimeter.

Problem 5) A 13-ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at a rate of 5 ft/sec. [**Hint:** The hypotenuse of our triangle is the length of the ladder. Is this changing?]

(a) How fast is the ladder sliding down the wall at this point?

(b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing at this point?

Problem 6) Water is flowing out of a conical resevoir (pointed downwards) at a rate of $50\text{m}^3/\text{min}$. The resevoir has base 45 meters and height 6 meters. The volume of a cone is $V = \dfrac{1}{3}\pi r^2 h$.

(a) How fast is the water level falling when the water is 5 meters deep?

(b) How fast is the radius of the water's surface changing when the water is 5 meters deep?

Problem 7) When a circular plate of metal is heated in an oven, its radius increases at the rate of 1/100 cm/min. At what rate is the plate's area increasing? Note that the area of a circle is $A = \pi r^2$.