8.2 Binomial Distribution

↳ n trials
↳ Each trial, we have success (S) or failure (F)
(ie, binary outcomes)
↳ S w/prob p
↳ F w/prob $1-p$

Want $\Pr[X = k \text{ Successes}] = \binom{n}{k} p^k (1-p)^{n-k}$

$\binom{n}{k}$: pick k trials for succ
p^k: prob that those k slots have S
$(1-p)^{n-k}$: prob that rest have F

Ex Toss weighted coin 5 times
↳ 60% chance of H on given toss
↳ 40% chance of T on given toss

Want $\Pr[X = 3\ H] = \binom{5}{3} (0.6)(0.6)(0.6)(0.4)(0.4)$

$\binom{5}{3}$: pick H pos
$(0.6)(0.6)(0.6)$: Prob that 3 selected pos have H
$(0.4)(0.4)$: Prob that rest have T

1	2	3	4	5
0.6		0.6	0.6	

$$\Pr[X = 3\ H] = \binom{5}{3} (0.6)^3 (0.4)^2$$

$\binom{n}{k}$: pick k trials for succ
p^k: prob that those k slots have S
$(1-p)^{n-k}$: prob that rest have F

Q $\Pr[X \leq 3\ H] =$

$$\binom{5}{0}(0.6)^0(0.4)^5$$
$$+ \binom{5}{1}(.6)^1(.4)^4$$
$$+ \binom{5}{2}(.6)^2(.4)^3$$
$$+ \binom{5}{3}(.6)^3(.4)^2$$

$\Pr[X=3\ H] = 0.3456$

↳ binomialpdf(n trials, prob p success, k successes)

$$= \binom{n}{k} p^k (1-p)^{n-k}$$

2nd + Vars

$\Pr[X \leq 3\ H] = 0.66304$

↳ binomialcdf(n trials, prob p, ≤ k succ)

$$= \sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n-i}$$

2nd + vars

Ex $n=100$ times
$p=0.6$

succ btwn 10 and 70

$$\Pr[X \le 70] = \text{binomcdf}(100, 0.6, 70)$$
$$-\Pr[X \le 9] = -\text{binomcdf}(100, 0.6, 9)$$
$$= 0.98522$$

8.3 Measures of Central Tendancy

Sample Mean Given sample $x_1, x_2, \dots, x_n$, then the sample mean

$$\bar{x} = \frac{1}{n}(x_1 + \dots + x_n)$$

Ex 1, 2, 3, 4, 5, 6, then $\bar{x} = \frac{1}{6}(1+2+\dots+6)$
$= 3.5$

Median Given sample $x_1, \dots, x_n$ (in ascending order), the median is
↳ Middle element (n odd)
↳ Avg of middle 2 (n even)

Ex $-3, -1, 2, 2, 4$ — 2 is the median

Ex $-1, 0, 2, 5, 6, 6$ — Avg: 3.5 median

Expected Value Let X be a finite random variable.

The expected value $E[X] = \sum_{i=1}^{n} Pr[X=i] * i$

Ex Fair 6-sided die

$E[X] = \frac{1}{6}(1+2+3+4+5+6) = 3.5$

Ex

X	-1	0	4	5
$Pr[X]$	0.3	0.5	.1	.1

$E[X] = -1(0.3) + 0(0.5) + 4(0.1) + 5(0.1) = 0.6$

For Binomial Random Var (X has binomial distr),

$E[X] = np$ (n is # trials, p is prob success)

Ex $n=20$, $p=.7$

$E[X] = 20(.7) = 14$