# Ch. 7

## 7.1 Sample Spaces and Events.

**Def** A *sample space* $S$ is the set of all possible outcomes (ie, a universal set).

**Ex** Toss coin. Sample Space $S = \{H, T\}$

Roll 6-sided die. Sample Space is $S = \{1, 2, 3, 4, 5, 6\}$

**Def** Let $S$ be a sample space. An event $E \subseteq S$.

**Ex** Roll 6-sided die, event roll even #
$E = \{2, 4, 6\}$.

**Def** Let $S$ be a sample space, and let $E, F \subseteq S$ be events. We say that $E$ and $F$ are mutually exclusive if $E \cap F = \emptyset$.

Ex Roll 6-sided die.

$E = \{2, 4, 6\}$ $F = \{1, 3, 5\}$

E and F are mutually exclusive events.

## 7.2 Relative Frequency

Idea Let S be a Finite Sample space, and let $E \subseteq S$ be an event.

Then: $Pr[E] = \frac{n(E)}{n(S)}$.

Ex Roll 6-sided die 10 times

↳ 1 (x3) ↳ 4 (x2)

↳ 2 (x3) ↳ 5 (x1)

↳ 3 (~~0~~ x0) ↳ 6 (x1)

| X | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Pr[X] | $\frac{3}{10}$ | $\frac{3}{10}$ | 0 | $\frac{2}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ |

Check $\frac{3}{10} + \frac{3}{10} + 0 + \frac{2}{10} + \frac{1}{10} + \frac{1}{10} = \frac{10}{10} = 1$ ✓

**Note** Assuming each outcome equally likely. (Fair die, fair coins, etc.)

**Ex** Suppose fair coin is tossed 3 times. What is Prob[1 H]?

$\underline{\quad} \times \underline{\quad} \times \underline{\quad} = 2 \times 2 \times 2 = 2^3 = 8$ ← Size of Simple space

8 possible tosses

Pick pos for H in $\binom{3}{1} = 3$ ways

$$Pr[1\ H] = \frac{3}{8}$$

**Ex** Roll 2 distinguishable dice:

one red, one green.

↳ 36 possible outcomes

↳ What is prob of rolling 2 and 3?

(2,3) or (3,2)

$$Prob[\text{Roll 2 and 3}] = \frac{2}{36}$$

Ex Roll 2 indistinguishable dice.

↳ Determine size of Sample space

↳ Case Outcomes on dice are distinct

$\{\overline{6 \cdot 5}\} = 15$ rolls

$(1,2) = (2,1)$ → 2

↳ Case Outcomes are same

6 rolls

Size of Sample Space $15 + 6 = 21$

What is Prob of rolling 2 and 3?

$\frac{1}{21}$

7.3 Ex Weighted 6-sided die, Prob of rolling 1, 2, 3, 4, or 5 is same.
Prob of rolling 6 is 3x that of rolling 1.

Det. Prob. for each outcome.

$X := Pr[1]$ $\quad Pr[6] = 3x$

$Pr[2] = x$ $\quad x + x + x + x + x + 3x = 1$

$8x = 1 \Rightarrow x = \frac{1}{8}$.