6.4 Combinations

Collections of distinct elements, where order does not matter.

Recall: The binomial coefficient $\binom{n}{k}$ (or $C(n,k)$), counts the # of k-element subsets of $\{1, 2, \ldots, n\}$.

Ex: 8 different stocks, want 3.
There $\binom{8}{3}$ such portfolios.

Ex: 8 different stocks, want 3.
5 different metals, want 2.
7 different bonds, want 4.

Q: How many portfolios?
↳ Select stocks: $\binom{8}{3}$
↳ Select Metals: $\binom{5}{2}$
↳ Select Bonds: $\binom{7}{4}$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Rule of Product: $\binom{8}{3}\binom{5}{2}\binom{7}{4}$ portfolios

$$\binom{8}{3} = \frac{8!}{5!\cdot 3!} = \frac{8\cdot 7\cdot 6\,(5\cdot 4\cdot 3\cdot 2\cdot 1)}{5!\cdot 3!} = \frac{8\cdot 7\cdot 6\cdot \cancel{5!}}{\cancel{5!}\cdot 3!}$$

$$= \frac{8\cdot 7\cdot 6}{3!} = \frac{8\cdot 7\cdot 6}{6} = 8\cdot 7 = 56$$

· Poker Hands

↳ Deck of 52 playing cards.
  ↳ 4 suits (Heart, Diamonds, Clubs, Spades)
  ↳ 13 ranks/values (Ace, 2, 3, ..., 10, J, Q, K)

Q How many 5-card hands exist?

A $\binom{52}{5}$

Ex A full-house is a 5-card hand with
  ↳ 3 cards of one rank
  ↳ 2 cards of second rank

(a) How many Full-Houses w/ 3 Q's and 2 A's?
  ↳ Sel Q's: $\binom{4}{3} = \frac{4!}{3! \cdot 1!}$
  ↳ Sel A's: $\binom{4}{2}$

Rule of Product $\binom{4}{3}\binom{4}{2}$

(b) 3 Q's, need to pick 2^{nd} rank
  ↳ Sel Q's: $\binom{4}{3}$
  ↳ Sel 2^{nd} rank: $\binom{12}{1}\binom{4}{2}$

Rule of Product: $\binom{4}{3}\binom{12}{1}\binom{4}{2}$ hands

c) How Many Full-Houses?

↳ Sel First Rank: $\binom{13}{1}\binom{4}{3}$

↳ Sel 2^{nd} Rank: $\binom{12}{1}\binom{4}{2}$

<u>Rule of Product</u> $\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$ such hands

<u>Ex</u> A <u>one-pair</u> has two cards of same rank and 3 cards of 3 different ranks (all different than rank of pair).

↳ (a) Suppose 5 appears twice.

↳ Sel 5's: $\binom{4}{2}$

↳ Sel last 3 cards: $\binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1}$

Sel ranks

<u>Rule of Product</u> $\binom{4}{2}\binom{12}{3}\binom{4}{1}^3$

(b) How many one pairs?

↳ Sel Pair: $\binom{13}{1}\binom{4}{2}$

↳ Sel last 3 cards: $\binom{12}{3}\binom{4}{1}^3$

<u>Rule of Product</u>: $\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3$