

12b) Straight Flush:

↳ 5 cards in row (consec. ranks)

↳ Ace low or high (but not both)

↳ All same suit

Pick Low Card 10 ways

Pick Suit 4 ways

So $10 \cdot 4 = 40$ hands

Straight

↳ 5 consec ranks

↳ Not all same suit

↳ Ace can be low or high (but not both)

Pick Low Rank 10

Pick Suits $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^5$

$10 \cdot 4^5$ hands w/ 5 Consec ranks

bad hands > 40 hands w/ 5 consec ranks and same suit

$$10 \cdot 4^5 - 10 \cdot 4$$

7.3 Probability works a lot like set cardinality.

Rule of Sum Let A, B be mutually exclusive events ($\Pr[A \cap B] = 0$). Then:

$$\Pr[A \cup B] = \Pr[A] + \Pr[B]$$

↑ Prob at least one of A, B occur.

Ex $\Pr[A] = 0.4$ A, B are mutually
 $\Pr[B] = 0.3$ exclusive

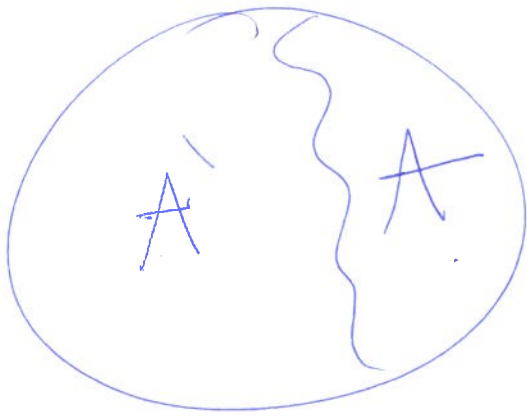
$$\begin{aligned}\Pr[A \cup B] &= \Pr[A] + \Pr[B] \\ &= 0.7\end{aligned}$$

Ex $\Pr[A] = 0.4$ $\Pr[A \cap B] = 0.1$
 $\Pr[B] = 0.3$

$$\begin{aligned}\Pr[A \cup B] &= \Pr[A] + \Pr[B] - \Pr[A \cap B] \\ &= 0.4 + 0.3 - 0.1 \\ &= 0.6\end{aligned}$$

Complements $Pr[A]$

What is $Pr[A']$? $Pr[A'] = 1 - Pr[A]$



Ex $Pr[A] = 0.3$

So $Pr[A'] = 1 - 0.3$
 $= 0.7$

7.4

Relative Frequency = $\frac{\# \text{ objects of interest}}{\# \text{ total objects}}$

Recall There are $\binom{52}{5}$ possible poker hands

Full House

↳ One rank occurs 3 times: $\binom{13}{1} \binom{4}{3}$

↳ Second rank occurs twice: $\binom{12}{1} \binom{4}{2}$

Multi: $\binom{13}{1} \binom{4}{3} \binom{12}{1} \binom{4}{2}$ Full Houses

$$Pr[\text{Full House}] = \frac{\binom{13}{1} \binom{4}{3} \binom{12}{1} \binom{4}{2}}{\binom{52}{5}}$$

Lotto Problem

- ↳ n numbered balls: $\{1, 2, \dots, n\}$
- ↳ Each ball/number appears exactly once
- ↳ "Pick- k lotto", where k of the n -balls are selected (order does not matter)
- ↳ Want prob of matching exactly i of k balls.

Ex Pick-5, with 50 balls.

- ↳ $\binom{50}{5}$ possible tickets ^{exactly}
- ↳ Want # of tickets that match [^]3 winning

numbers:

$$\binom{5}{3} \binom{45}{2}$$

- ↳ Prob of Match Exactly 3 winners:

$$\frac{\binom{5}{3} \binom{45}{2}}{\binom{50}{5}}$$

Ex 407 balls, Pick 24 winners.

↳ Prob of match exactly 5 #s?

$$\frac{\binom{24}{5} \binom{383}{19}}{\binom{407}{24}}$$

$$407 - 24 = \underline{383}$$

407 balls, Pick 24 winners

↳ Prob of matching either 3, 4, or 5 (exactly)
winning numbers

$$\binom{24}{3} \binom{383}{21} + \binom{24}{4} \binom{383}{20} + \binom{24}{5} \binom{383}{19}$$

$$\binom{407}{24}$$