5:30

Thm (Rule of Product). Let A, B be finite sets. Then $n(A \times B) = n(A) * n(B)$.

Def We say that an alphabet Σ is a finite set.

Ex $\Sigma = \{A, B, \ldots, Z\}$

$\Sigma = \{0, 1\}$

$\Sigma = \{0, 1, \ldots, k-1\}$ (k-letter alphabet).

Def Let Σ be an alphabet. A word or string of length k over Σ is an ordered sequence $w = (w_1, w_2, w_3, \ldots, w_k)$, where $w_1, w_2, w_3, \ldots, w_k \in \Sigma$.

Remark The string $w \in \Sigma^k = \underbrace{\Sigma \times \Sigma \times \cdots \times \Sigma}_{k\text{-times}}$

Ex Let $\Sigma = \{0, 1\}$ be the binary alphabet.

↳ $(0,1)$ is a binary string of length 2.

$(0,1) \in \Sigma \times \Sigma$

↳ (1,0,1) is a binary string of length 3
$(1,0,1) \in \Sigma \times \Sigma \times \Sigma$

<u>Q</u> How many binary strings of length 3?
↳ $\Sigma = \{0,1\}$ is binary alphabet
↳ $\Sigma \times \Sigma \times \Sigma$ is the set of <u>all</u> binary strings of length 3.

<u>Want</u> $n(\Sigma \times \Sigma \times \Sigma) = n(\Sigma) * n(\Sigma) * n(\Sigma)$
$2 * 2 * 2$
$= 2^3$ binary strings of length 3
Leave as is

$(\underset{2}{\_}, \underset{2}{\_}, \underset{2}{\_})$
$2 \cdot 2 \cdot 2 = 8 = 2^3$

The binary strings of length 3:

(0,0,0) (1,0,0)
(0,0,1) (1,0,1)
(0,1,0) (1,1,0)
(0,1,1) (1,1,1)

Ex Suppose Σ is 5-letter alphabet.

How many strings of length 8 exist over Σ?

$$(\_\_, \_\_, \_\_, \_\_, \_\_, \_\_, \_\_, \_\_)$$

$$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^8 \text{ strings}$$

$5^8 = 625 \cdot 625$

preferred

~~#~~ Ex Two 6-sided dice.

↳ one red

↳ one green

Q How many possible rolls?

$$(\underset{\text{red}}{\_\_}, \underset{\text{green}}{\_\_})$$

$$6 \times 6 = 36 \text{ possible rolls}$$

Ex Two indistinguishable 6-sided die.

$(3,2) = (2,3)$

Q How many possible rolls?

↳ Case 1 Both die have same value. 6 rolls

↳ Case 2 Two dice have different values

$(\_ , \_)$

$\frac{6 \cdot 5}{2} = \frac{30}{2} = 15$ rolls

Total Possible Rolls $6 + 15 = 21$

Def Let A, B be sets. We say that A is a subset of B (denoted $A \subseteq B$) if every elem of A is also an elem of B

Ex $A = \{1,2,3\}$ $\quad A \subseteq B$

$B = \{1,2,3,4,5\}$. $\quad C \subseteq B$

$C = \{2,3,5\}$

Q Is $A \subseteq C$?

No. $1 \in A$, but $1 \notin C$.

6.2 Set Cardinality

Thm (Rule of Sum). Let A, B be finite sets such that A and B share no common elems ($A \cap B = \emptyset$). Then $n(A \cup B) = n(A) + n(B)$.

Ex $A = \{a, \underline{b, c, d}\}$ $\quad B = \{\underline{b, c, d}, e, f\}$

$A \cup B = \{a, b, c, d, e, f\}$, $n(A \cup B) = 6$

$$n(A \cup B) = n(A) + n(B) - \underline{n(A \cap B)}$$
$$6 = 4 + 5 - 3$$

Instance of Principle of Inclusion-Exclusion (PIE)

Ex Amazon has 132,000 cookbooks.
- ↳ 5000 vegetarian
- ↳ 20,000 regional
- ↳ $\underline{24,000}$ veget $\underline{\text{or}}$ regional

Q How many are neither vegetarian nor regional?

$132,000 - 24,000 = 108,000$

Q How many cookbooks are not both (veget and regional)?

$24000 = 5000 + 20000 - n(A \cap B)$

So $n(A \cap B) = 1000$ cookbooks that both veget and regional

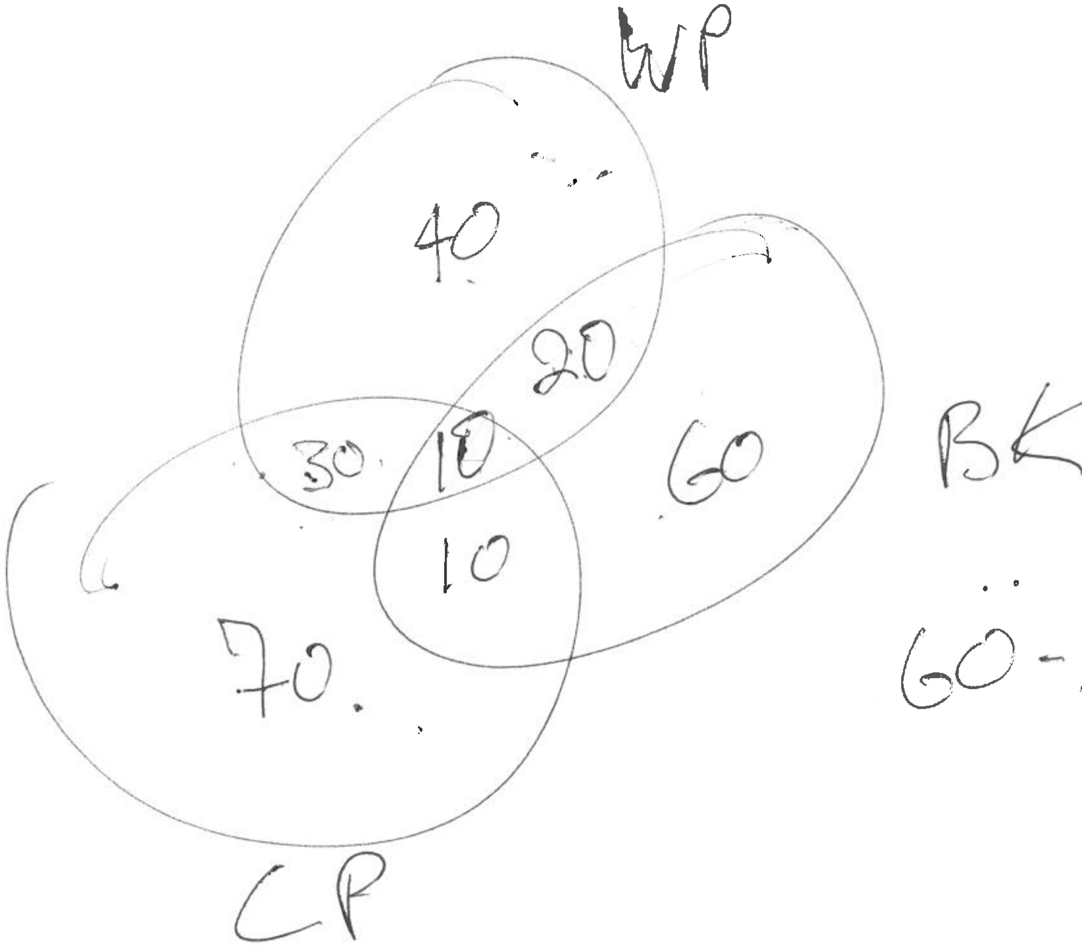
$\Rightarrow$ $132{,}000 - 1000 = 131{,}000$ cookbooks not both (veget and regional).

Ex 300 college students surveyed.

- 100 ~~__~~ read War and Peace (WP).
- 120 Crime and Punishment (CP).
- 100 The Brothers Karamazov (BK)

Sets may overlap

- 40 ~~only~~ read only WP
- 70 read WP but not BK
- 80 read BK but not CP
- Only 10 read all 3
- Some could have read none of the books



60 - None