4.1/4.2 Basic Matrix Algebra

↳ Matrix addition
↳ Scalar Multiplication
↳ Multiplying Matrices

Matrix Addition Given two matrices with same # rows and same # ~~cols~~ cols, we add componentwise

Ex $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} = \begin{bmatrix} 9 & 11 \\ 13 & 15 \end{bmatrix}$

Ex $\begin{bmatrix} 0 & 1 & 2 \\ 3 & 5 & 7 \\ 11 & 13 & 17 \end{bmatrix} + \begin{bmatrix} 19 & 23 & 31 \\ 37 & 43 & 47 \\ 51 & 53 & 59 \end{bmatrix} =$

$\begin{bmatrix} 19 & 24 & 33 \\ 40 & 48 & 54 \\ 62 & 66 & 76 \end{bmatrix}$

Ex $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 5 & 6 \\ 7 & 9 & 11 \end{bmatrix}$

Cannot Add. The first matrix is a 2×2 matrix, while the second matrix is a 2×3 matrix.

Scalar Multiplication

Ex $4\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 12 & 20 \end{bmatrix}$

Ex $7\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 7 & 0 & 0 \\ 1 & 7 & 0 \\ 1 & 1 & 7 \end{bmatrix}$

Ex

$$\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 0 \\ -1 & 7 & 0 \\ -1 & -1 & 7 \end{bmatrix}$$

Matrix Multiplication

↳ Given M_1 which is an $n \times m$ matrix

M_2 which is an $m \times k$ matrix,

the product $M_1 \cdot M_2$ is an $n \times k$ matrix.

Note The # of cols in M_1 is the same as the # rows in M_2.

Ex

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

2×3 $\quad$ 3×2 $\quad$ 2×2

$1\cdot 7 + 2\cdot 9 + 3\cdot 11 = 58$

$1\cdot 8 + 2\cdot 10 + 3\cdot 12 = 64$

$4\cdot 7 + 5\cdot 9 + 6\cdot 11 = 139$

Ex
$$\begin{bmatrix} 2 & 0 & -1 & 3 \\ 1 & -1 & 2 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 & -8 \\ 1 & 0 & 0 \\ 0 & 5 & 2 \\ -2 & 8 & -1 \end{bmatrix}$$

2×4 $\quad$ 4×3

$$= \begin{bmatrix} -4 & 21 & -21 \\ 4 & -5 & -2 \end{bmatrix}$$

2×3

$2(1) + 0(1) + -1(0) + 3(-2)$

Ex Not all matrices can be multiplied.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

2×2 $\quad$ 4×1

So the first matrix has 2 cols while the 2nd matrix has 4 rows. So cannot multiply matrices

$1\cdot 1 + 0\cdot 1$

Cannot pair these 1's So Cannot multiply.

Ex Matrix Multiplication DOES NOT COMMUTE!

In general, $AB \neq BA$ when A, B are matrices.

$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 0 \\ 5 & -1 \end{bmatrix}$

$AB = \begin{bmatrix} -2 & 1 \\ 10 & -2 \end{bmatrix}$ $BA = \begin{bmatrix} 3 & -3 \\ 5 & -7 \end{bmatrix}$

$3-5$

$0+1$

$0+10$

$0-2$

$3 \cdot 1 + 0$

$3(-1) + 0$

$5(1) + 0$

$5(-1) + (-1)2$

The order in which you multiply matrices matters!