

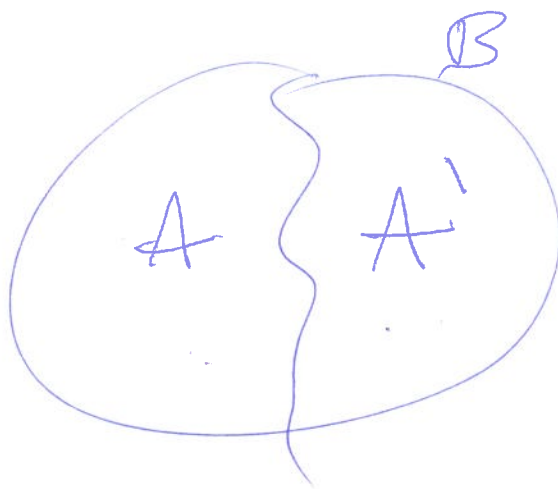
7.6 Bayes' Law

Recall $\Pr[A|B] = \frac{\Pr[A \cap B]}{\Pr[B]}$

Observe $\Pr[A \cap B] = \Pr[B|A] \Pr[A]$
 $= \Pr[A|B] \Pr[B]$



Observe $\Pr[B] = \Pr[B|A] \Pr[A] + \Pr[B|A'] \Pr[A']$



Bayes' Law $\Pr[A|B] = \frac{\Pr[B|A]\Pr[A]}{\Pr[B|A]\Pr[A] + \Pr[B|A']\Pr[A']}$

$\Pr[A \cap B]$ $\Pr[B]$

Ex Test for disease (Outcomes Pos, Neg)

↳ Has 0.95 prob of giving pos result when person has disease ($\Pr[\text{Pos}|\text{Dis}] = 0.95$)

↳ Has 0.1 prob of giving ~~negative~~ pos. result when person does not have disease
 $\Pr[\text{Pos}|\sim\text{Dis}] = 0.1$

↳ $\text{Prob}[\text{Disease}] = 0.005$, so $\text{Prob}[\sim\text{Dis}] = 0.995$

Want $\Pr[\text{Dis}|\text{Pos}] = \frac{\Pr[\text{Pos}|\text{Dis}] \cdot \Pr[\text{Dis}]}{\Pr[\text{Pos}|\text{Dis}]\Pr[\text{Dis}] + \Pr[\text{Pos}|\sim\text{Dis}] \cdot \Pr[\sim\text{Dis}]}$

$= \frac{0.95(0.005)}{0.95(0.005) + 0.1(0.995)}$

Ex Test for Dis

$$\hookrightarrow \Pr[\text{Dis}] = \frac{1}{10,000}$$

$$\hookrightarrow \Pr[\text{Pos}|\text{Dis}] = 0.99$$

$$\hookrightarrow \Pr[\text{Pos}|\sim\text{Dis}] = 0.01 \quad \Pr[\text{Pos}|\text{Dis}] \cdot \Pr[\text{Dis}]$$

Determine $\Pr[\text{Dis}|\text{Pos}] = \frac{0.99 \cdot (\frac{1}{10,000})}{\Pr[\text{Pos}|\text{Dis}] \cdot \Pr[\text{Dis}] + \Pr[\text{Pos}|\sim\text{Dis}] \Pr[\sim\text{Dis}]}$

$$= \frac{0.99 \cdot (\frac{1}{10,000})}{0.99 \cdot (\frac{1}{10,000}) + 0.01 \cdot (\frac{9999}{10,000})}$$

$$\frac{\Pr[\text{Dis} \cap \text{Pos}]}{\Pr[\text{Pos}]}$$