Study Guide 4.8

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Instructions: Complete the following problems. Justify all your answers in complete sentences, where appropriate.

1 Sections 4.8 and 5.3

Problem 1) Evaluate the following.

(a) Suppose $n \neq -1$. $\displaystyle\int x^n \, dx = \underline{\hspace{5cm}}$

(b) $\displaystyle\int \frac{1}{x} \, dx = \underline{\hspace{5cm}}$

(c) Let $a > 0$, with $a \neq 1$. $\displaystyle\int a^x \, dx = \underline{\hspace{5cm}}$

(d) $\displaystyle\int e^x \, dx = \underline{\hspace{5cm}}$

(e) $\displaystyle\int \sin(x) \, dx = \underline{\hspace{5cm}}$

(f) $\displaystyle\int \cos(x) \, dx = \underline{\hspace{5cm}}$

(g) $\displaystyle\int \sec^2(x) \, dx = \underline{\hspace{5cm}}$

(h) $\displaystyle\int \csc^2(x) \, dx = \underline{\hspace{5cm}}$

(i) $\displaystyle\int \sec(x)\tan(x) \, dx = \underline{\hspace{5cm}}$

(j) $\displaystyle\int \csc(x)\cot(x) \, dx = \underline{\hspace{5cm}}$

(k) $\displaystyle\int \frac{1}{1+x^2} \, dx = \underline{\hspace{5cm}}$

Problem 2) Compute the following definite integrals.

(a) $\displaystyle\int_{-5}^{-3} \frac{1}{x}\,dx.$

(b) $\displaystyle\int_{-2}^{2} (x^5 + x^3)\,dx.$

(c) $\displaystyle\int_{0}^{1} (3x^2 + 2x + 5)\,dx.$

(d) $\displaystyle\int_{-5}^{5} \sqrt{25 - x^2}\,dx.$

[**Note:** Computing this integral analytically is not possible without techniques from Math 142. Instead, use the area under the curve interpretation of the integral. Start by graphing $\sqrt{25 - x^2}$.]

(e) $\displaystyle\int_{-5}^{5} -\sqrt{25 - x^2}\,dx.$

Problem 3) Suppose that $f(x)$ is an odd function. Let $a > 0$. Prove that

$$\int_{-a}^{a} f(x)\,dx = 0.$$

Problem 4) Suppose that $f(x)$ is an even function. Let $a > 0$. Prove that:

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx.$$

Problem 5) After a foreign substance is introduced into the blood, the rate at which antibodies are made is given by $r(t) = \dfrac{t}{t^2 + 1}$ thousands of antibodies per minute, where t is in minutes. Assuming there are no antibodies at time $t = 0$, find the total quantity of antibodies in the blood at the end of 4 minutes.

Problem 6) A forest fire covers 2000 acres at time $t = 0$. The fire is growing at a rate of $8\sqrt{t}$ acres per hour. How many acres are covered 24 hours later?

Problem 7) Water is pumped out of a holding tank at a rate of $5 - 5e^{-0.12t}$ litres per minute, where t is the number of minutes since the pump started. If the holding tank contains 1000 litres of water when the pump is started, how many litres does it contain an hour later?

Problem 8) Solve the following initial value problems.

(a) $\dfrac{ds}{dt} = \cos(t) + \sin(t)$, with $s(\pi) = 1$.

(b) $\dfrac{dv}{dt} = \dfrac{1}{2}\sec(t)\tan(t)$, with $v(0) = 1$.

Problem 9) Suppose that $\int_0^2 f(x)\,dx = 3$, $\int_0^5 f(x)\,dx = -1$, and $\int_3^5 g(x)\,dx = 4$. Compute the following.

(a) $\displaystyle\int_0^2 3f(x)\,dx.$

(b) $\displaystyle\int_3^5 (f(x) + g(x))\,dx.$

(c) $\displaystyle\int_5^3 g(x)\,dx.$

2   Section 5.4

Problem 10) Use the Fundamental Theorem of Calculus to evaluate the following.

(a) $\displaystyle\frac{d}{dx}\left(\int_0^{\sqrt{x}} \cos(t)\,dt\right).$

(b) $\displaystyle\frac{d}{dx}\left(\int_2^x \sqrt{1 + t^2}\,dt\right)$

(c) $\displaystyle\frac{d}{dt}\left(\int_0^{t^4} \sqrt{u}\,du\right)$