Instructions: This quiz is closed book, closed note, and an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places.

1. A landscape architect wants to enclose a 2000 square foot rectangular region. She will use shrubs costing \$30/ft along three sides and fencing costing \$20/ft along the fourth side.

(a) **(2 pts)** Determine the function to minimize, as well as the constraints.

Answer: We seek to solve:

$$\begin{aligned} \min\ & 30(2w+\ell) + 20\ell \text{ s.t.} \\ & \ell w = 2000 \\ & \ell, w > 0 \end{aligned}$$

So the function we seek to minimize is: $30(2w+\ell)+20\ell$. Our constraints are:

- Area: $\ell w = 2000$.
- $\ell, w > 0$.

(b) **(3 pts)** Construct the corresponding optimization problem in one variable.

Answer: From part (a), we have the constraint that $\ell w = 2000$. Solving for ℓ, we have that $\ell = 2000/w$. So we have the following optimization problem in one variable:

$$\begin{aligned} \min\ & 30\left(2w + \frac{2000}{w}\right) + 30 \cdot \frac{2000}{w} \text{ s.t.} \\ & w > 0 \end{aligned}$$

(c) **(4 pts)** Determine the global minimizer (this will be either the length or width of the rectangle, based on your answer in part (b)). Justify your answer.

Answer: Denote $f(w) := 30\left(2w + \frac{2000}{w}\right) + 20 \cdot \frac{2000}{w}$. So $f'(w) = 60 - \frac{100000}{w^2}$. Solving for our critical points, we have that:

$$\begin{aligned} 60 - \frac{100000}{w^2} &= 0 \\ 60 &= \frac{100000}{w^2} \\ w^2 &= \frac{100000}{60} \\ w &= \pm\sqrt{\frac{100000}{60}} \end{aligned}$$

We note as well that $w = 0$ is a critical point, as $f'(0)$ is undefined at $w = 0$.

As we have the constraint that $w > 0$, we only consider the critical point $w = \sqrt{\frac{100000}{60}}$. We now verify that $f(w)$ has a local minimum at $w = \sqrt{\frac{100000}{60}}$. We have that $f''(x) = \frac{200000}{w^3}$. So $f''(\sqrt{\frac{100000}{60}}) > 0$. Thus, by the Second Derivative Test, $f(w)$ has a local min at $w = \sqrt{\frac{100000}{60}}$. As we have the constraing $w > 0$, there are no endpoints to check. Furthermore, $f(w)$ has no other critical points greater than 0. Therefore, $f(w)$ has a global minimum at $w = \sqrt{\frac{100000}{60}}$.

(d) **(1 pt)** Determine the minimum cost.

Answer: The minimum cost is $f\left(\sqrt{\frac{100000}{60}}\right) = 4898.98$.