Study Guide 4.4-4.5

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Instructions: Complete the following problems. Justify all your answers in complete sentences, where appropriate.

1 Section 4.4

Problem 1) Find the points of inflection of the following functions.

(a) $f(x) = x^4 - 4x^3$

(b) $f(x) = -x^3 + 3x^2 + 5$

(c) $f(x) = 3x^5 - 5x^3$

(d) $f(x) = x(6 - 2x)^2$

(e) $f(x) = x + \sin(x)$, on the domain $[0, 2\pi]$.

(f) $f(x) = \sin(x)\cos(x)$, on the domain $[0, \pi]$. [**Hint:** It may be helpful, though unnecessary, to re-write $\sin(x)\cos(x)$ using a trig identity. Is there a trig identity involving $2\sin(x)\cos(x)$?]

Problem 2) Suppose $f(x) = x^3 + bx^2 + cx + d$, where b, c, d are fixed, unknown constants. Suppose that there is a critical point at $x = 2$ and an inflection point at $(1, 4)$. Determine b, c, d.

2 Section 4.5

Problem 3) Use L'Hopital's Rule to evaluate the following limits. Clearly identify the indeterminate form each time you apply L'Hopital's Rule. Note that you may need to apply L'Hopital's Rule multiple times for the same problem.

(a) $\lim\limits_{x \to -2} \dfrac{x+2}{x^2 - 4}$

(b) $\lim\limits_{x \to 0} \dfrac{\sin(5x)}{x}$

(c) $\lim\limits_{x \to 0} \dfrac{1 - \cos(x)}{x^2}$

(d) $\lim\limits_{x \to 0} \dfrac{x^4}{2^x}$

(e) Let $n > 1$. Evaluate $\lim\limits_{x \to \infty} \dfrac{\ln(x)}{x^{1/n}}$.

(f) $\lim\limits_{x \to 1^+} \left(\dfrac{1}{x-1} - \dfrac{1}{\ln(x)} \right)$

(g) $\lim\limits_{x \to \infty} (\ln(2x) - \ln(x+1))$. [**Hint:** Use the fact that ln is continuous.]

(h) $\lim_{x \to 0^+} x \ln(x)$

(i) $\lim_{x \to 0^+} \sin(x) \ln(x)$

(j) $\lim_{x \to \infty} (\ln(x))^{1/x}$

(k) $\lim_{x \to 0^+} x^x$

(l) $\lim_{x \to \infty} \left(\frac{x+2}{x-1} \right)^x$ [**Hint:** Use the rules of logs. You should now have two separate limits to evaluate.]

(m) $\lim_{x \to 0} \frac{1 - \cos(x)}{x}$.

(n) $\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n$. [**Note:** This will almost certainly be on Exam 3. You are expected to be able to work out this limit in Math 142.]