Instructions: This quiz is closed book, closed note, and an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places. There are 20 possible points, and you will be graded out of 10 points.

1) A bacteria colony initially has a population of 14 million bacteria. Suppose that t hours later the population is growing at a rate of $f(t) = 2^t$ million bacteria per hour.

- **(3 pts)** What is the total change in population from $t = 0$ to $t = 2$?

  Solution: The total change in population is:
  $$\int_0^2 f(t)\,dt = \frac{1}{\ln(2)} 2^t \Big|_0^2 = \frac{1}{\ln(2)}(2^2 - 2^0) = \frac{3}{\ln(2)}$$

- **(3 pts)** What is the total population at time $t = 2$? [Note: The total population is the initial population plus the net change.]

  Solution: Recall that Total Population = Initial Population + Total Change. So:
  $$\text{Total Population} = 14 + \frac{3}{\ln(2)} \text{ Million bacteria.}$$

(4 pts) 2) Let $f(x) = x^3$. Determine the average value $f(x)$ takes on over the interval $[-3, 2]$.

Solution:
$$\frac{1}{2-(-3)} \int_2^3 x^3\,dx = \frac{1}{5} \cdot \frac{x^4}{4}\Big|_{-3}^{2} = \frac{1}{20}(2^4 - (-3)^4) = -3.25$$

3) Suppose that the consumer demand function is given by $Q(p) = 200 - 3p$ and the producer supply function is given by $S(p) = 4p - 10$.

- **(4 pts)** Find the equilibrium price and quantity.

  Solution: To find equilibrium price and quantity, set: $S(p) = Q(p)$. So:

$$4p - 10 = 200 - 3p$$
$$7p = 210p^* = 30$$

  Now $q^* = Q(p^*) = S(p^*) = 200 - 3(30) = 110$.

- **(3 pts)** Determine the producer surplus. **Show all your work!**
  Solution: We begin by inverting the supply function $S(p)$:

$$q = 4p - 10$$
$$4p = q + 10$$
$$p = \frac{q}{4} + \frac{5}{2}$$

  Now the producer surplus is given by:

$$p^* q^* - \int_0^{q^*} \left(\frac{q}{4} + \frac{5}{2} \right) dq$$
$$= 30 \cdot 110 - \int_0^{110} \left(\frac{q}{4} + \frac{5}{2} \right) dq$$
$$= 1512.5$$

- **(3 pts)** Determine the consumer surplus. **Show all your work!**
  Solution: We begin by inverting the demand function:

$$q = 200 - 3p$$
$$3p = 200 - q$$
$$p = \frac{200}{3} - \frac{q}{3}$$

  The consumer surplus is:

$$\int_0^{q^*} \frac{200 - q}{3} \, dq - p^* q^*$$
$$= \int_0^{110} \frac{200 - q}{3} - 30 \cdot 110$$
$$= \frac{6050}{3} = 2016.67$$