Ch. 3 Systems of Equations

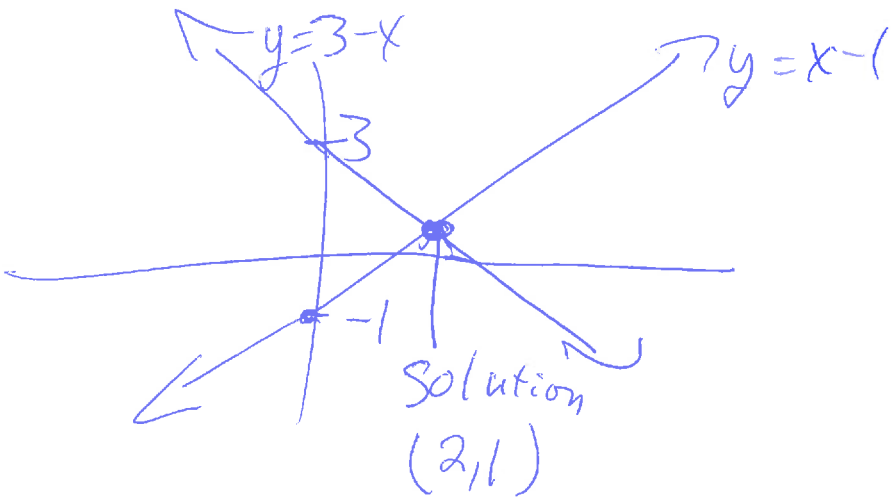
3.1

Ex $x+y=3$ $(y=3-x)$

$+$ $x-y=1$ $(y=x-1)$

$2x=4 \Rightarrow x=2$

So $y=1$



Solutions for Systems of Equations

Case 1 Exactly one solution (ie, unique solution)

↳ Ex $x+y=3$ Sol: (2,1)
$x-y=1$

Case 2 No solutions (ie, Parallel lines)

↳ Ex $y=x+1$
$y=x+3$

$x - y = -1$
$x - y = -3$

So
$$\begin{array}{r} x - y = -1 \\ +\ -x + y = 3 \\ \hline 0x + 0y = 2 \end{array}$$

$0 = 2$ (False statement)

↑ No solution!

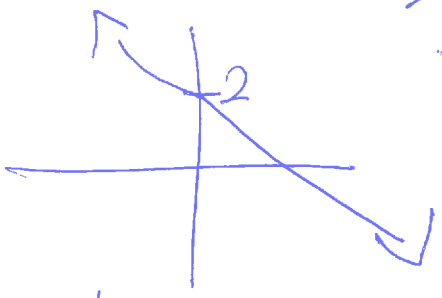
Case 3 Infinitely many solutions

↳ Given same line multiple times

Ex $x + y = 2$ $(y = 2 - x)$

$2x + 2y = 4$ $(4 - 2x = 2y,\ \text{so } y = 2 - x)$

Solution set Geometrically, sols are pts on line



$\text{Sol} = \{(x, y) \mid y = 2 - x\}$

3.2/3.3 Using Matrices to Solve Systems of Equations

Ex $2x - y = 3$
$-x + 2y = -4$

Show

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & -4 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 2/3 \\ 0 & 1 & -5/3 \end{bmatrix}$$

$x = \frac{2}{3}, \; y = \frac{-5}{3}$

Solution

Calc
2nd Matrix
Edit
Go Home
2nd Matrix
↳ Math
↳ rref
2nd Matrix
Select your matrix

Recall $2x - y = 3$ ✓
$-x + 2y = -4$ ✓

Check $2\left(\frac{2}{3}\right) - \left(\frac{-5}{3}\right) = \frac{4}{3} + \frac{5}{3} = \frac{9}{3} = 3$

$-\frac{2}{3} + 2\left(\frac{-5}{3}\right) = -\frac{2}{3} - \frac{10}{3} = -\frac{12}{3}$

$= -4$

Ex $-\frac{2}{3}x + \frac{1}{2}y = -3$

$\frac{1}{4}x - y = \frac{11}{4}$

$$\left[\begin{array}{cc|c} -2/3 & 1/2 & -3 \\ 1/4 & -1 & 11/4 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array}\right]$$

$x = 3,\ y = -2$

Ex $2x + y + 3z = 1$

$4x + 2y + 4z = 4$

$x + 2y + z = 4$

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 4 & 2 & 4 & 4 \\ 1 & 2 & 1 & 4 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$x = 1,\ y = 2,\ z = -1$

Ex $x + y + z = 1$

$\frac{1}{4}x - \frac{1}{2}y + \frac{3}{4}z = 0$

$x + 7y - 3z = 3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1/4 & -1/2 & 3/4 & 0 \\ 1 & 7 & -3 & 3 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 5/3 & 2/3 \\ 0 & 1 & -2/3 & 1/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$0x + 0y + 0z = 0$

Infinitely many sols

Ex $x + y + z = 1$

$2x - y + z = 0$

$4x + y + 3z = 3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -1 & 1 & 0 \\ 4 & 1 & 3 & 3 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 2/3 & 0 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$0x + 0y + 0z = 1$

No sols

Ex Purchase Airplanes to Fulfill 4800 person Capacity

Plane A: 320 pass, costs $200M

Plane B: 250 pass, costs $125M

Plane C: 275 pass, costs $200 M

Cost of Fleet $3.1 Bill ($3,100 M) Given

Given Twice as many Plane C's as Plane B's.

↳ $c = 2b \Rightarrow 2b - c = 0$

System

$0a + 2b - c = 0$ (2x C's as B's)

$320a + 250b + 275c = 4800$ (Capacity)

$200a + 125b + 200c = 3100$

$$\left[\begin{array}{ccc|c} 0 & 2 & -1 & 0 \\ 320 & 250 & 275 & 4800 \\ 200 & 125 & 200 & 3100 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 8 \end{array}\right]$$

$a = 5, b = 4, c = 8$