Instructions: This quiz is closed book, closed note, and an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places. There are 12 possible points; you will be graded out of 10 points.

1. **(2 pts)** Suppose $Z \sim \mathcal{N}(0,1)$; that is, suppose Z is drawn from a standard normal distribution (where the mean $\mu = 0$ and standard deviation $\sigma = 1$). Determine: $\Pr[-2.5 \leq Z \leq 1.76]$.

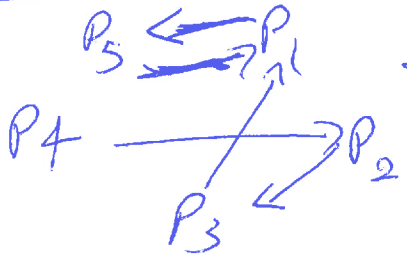
Using the normalcdf function, we have that:

$\Pr[-2.5 \leq Z \leq 1.76] = \text{normalcdf}(-2.5, 1.76, 0, 1)$
$= 0.9546$

2. **(5 pts)** Consider the following instance of the House Allocation Problem. Run the Top-Trading Cycle Procedure to allocate the houses. **Clearly indicate the allocations made at each iteration.**

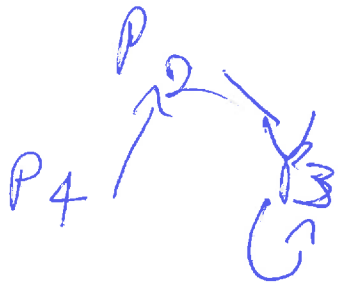
- $p_1 : h_5, h_1, h_4, h_2, h_3$
- $p_2 : h_3, h_5, h_1, h_2, h_4$
- $p_3 : h_1, h_3, h_5, h_4, h_2$
- $p_4 : h_2, h_4, h_3, h_1, h_5$
- $p_5 : h_1, h_3, h_4, h_5, h_2$

Iteration 1



Allocate
$p_1 : h_5$
$p_5 : h_1$

Iteration 2



Allocate
$p_3 : h_3$

Iteration 3
$p_2 \leftrightarrows p_4$

Allocate
$p_2 : h_2$

Iteration 4
p_4

Allocate
$p_4 : h_4$

Final Allocation

$p_1 : h_5$
$p_2 : h_2$
$p_3 : h_3$
$p_4 : h_4$
$p_5 : h_1$

3. (5 pts) Consider the following instance of the Stable Marriage Problem, where we have the sets of players $X = \{x_1, x_2, x_3, x_4\}$ and $Y = \{y_1, y_2, y_3, y_4\}$. The preferences are given below. Run the Gale-Shapley algorithm, using the X players as the proposers. **Clearly indicate the final stable matching.** A correct answer with no work will receive full credit. An incorrect answer with some correct work may receive partial credit. An incorrect answer with no work will receive no credit.

- $x_1 : y_1, y_2, y_3, y_4$
- $x_2 : y_4, y_3$
- $x_3 : y_3, y_1, y_2$
- $x_4 : y_4, y_3, y_2, y_1$
- $y_1 : x_1, x_3, x_4, x_2$
- $y_2 : x_2, x_3, x_4, x_1$
- $y_3 : x_4, x_3, x_2, x_1$
- $y_4 : x_4$

Matches

$x_1 - y_1$

$x_2 - y_3$

$x_3 - y_3$ (x_2, y_3 break up)

$x_4 - y_4$

Final Matching

$x_1 - y_1$

$x_3 - y_3$

$x_4 - y_4$

x_2

y_2