Math 122 Sections 4.1-4.2 Worksheet

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Applying the First Derivative Test:

- **Given:** A differentiable function $f(x)$.
- **Goal:** Find the local maxima and local minima of $f(x)$.
- **Approach:**
  - Find the critical points of $f(x)$. Recall that the critical points are the x-values where $f'(x) = 0$.
  - If the derivative $f'(x)$ changes from positive to negative at a critical point c, then c is a local maximum.
  - If the derivative $f'(x)$ changes from negative to positive at a critical point c, then c is a local minimum.

Practice: Find all local maxima and minima for the following functions.

- $f(x) = 3x^2 + 2x + 5$.
- $f(x) = -3x^2 + 2x + 5$.
- $f(x) = x^3$.
- $f(x) = 10x\exp(3 - x^2)$.

Second Derivaitve Test:

- **Given:** A function $f(x)$ that is twice differentiable (that is, both its first and second derivatives exist).
- **Goal:** Find the local maxima and minima of $f(x)$.
- **Approach:**
  - Find the critical points of $f(x)$. Recall that the critical points are the x-values where $f'(x) = 0$.
  - If c is a critical point of $f(x)$ and $f''(c) > 0$, then c is a local minimum of $f(x)$.
  - If c is a critical point of $f(x)$ and $f''(c) < 0$, then c is a local maximum of $f(x)$.

Practice: Find all local maxima and minima for the following functions.

- $f(x) = x^4 - 4x^3$
- $f(x) = -x^3 + 3x^2 + 5$
- $f(x) = x + \dfrac{4}{x}$.
- $f(x) = x^3$.

Point of Inflection: A point of inflection for a twice differentiable function $g(x)$ is a place where the concavity changes from positive to negative, or from negative to positive. So to find points of inflection, find the points that maximize or minimize $g'(x)$. [That is, apply either the first or second derivative test, starting with $f(x) = g'(x)$ as your *given* function.]

Practice: Find the points of inflection for the following functions.

- $f(x) = x^4 - 4x^3$
- $f(x) = -x^3 + 3x^2 + 5$
- $f(x) = x + \dfrac{4}{x}$.
- $f(x) = x^3$.