5.2 Linear programming (LP)

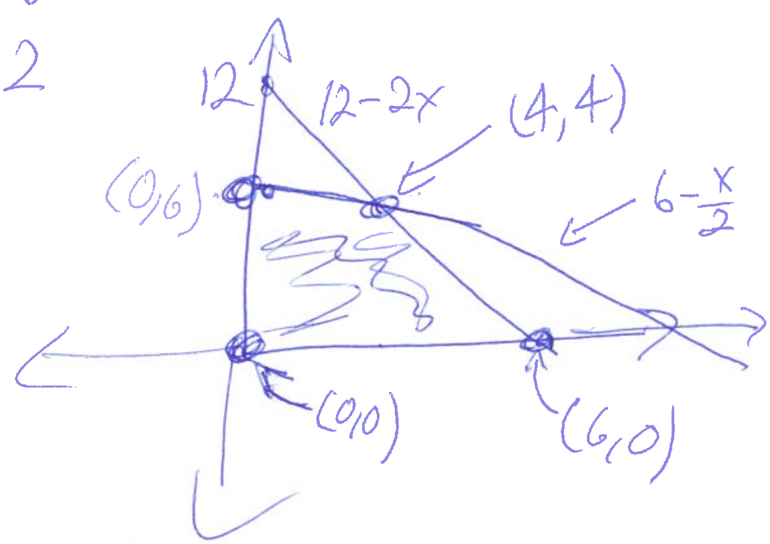
An **LP** is an optimization problem, where the constraints are linear inequalities.

E.g. Constraints look like and **NOT**

$3x - 2y \geq 0$	$x^2 y > 0$
$x + 2y \leq 0$	$xy^3 > 14$
$x \geq 0$	

Ex max $x+y$ Subject to (S.t.)

$$2x + y \leq 12$$
$$x + 2y \leq 12$$
$$x \geq 0$$
$$y \geq 0$$



$$12 - 2x = 6 - \frac{x}{2}$$
$$6 = \frac{3x}{2}$$
$$x = \frac{2}{3}(6) = 4$$
$$12 - 2(4) = 4 = y$$

Pt	$x+y$
(0,0)	0
(6,0)	6
(0,6)	6
(4,4)	8

maximizer (4,4)

8 ← maximum value of x+y given constraints

Ex Two types of Juice

↳ Type X: 30 oz water, 2 oz concentrate
profit: \$0.20/unit

↳ Type Y: 20 oz water, 12 oz concentrate
profit: \$0.30/unit

Objective function $0.2x + 0.3y$ (want to max profit)

Told 30,000 oz water available
3600 oz concentrate available

LP

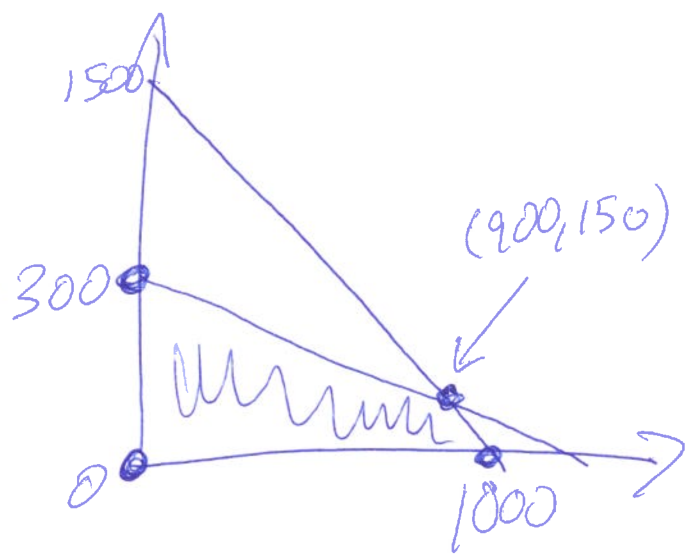
$$\max\ 0.2x + 0.3y \quad \text{s.t.}$$

$$30x + 20y \le 30{,}000 \quad \text{(Water)}$$

$$2x + 12y \le 3600 \quad \text{(Concentrate)}$$

$$x \ge 0$$

$$y \ge 0$$



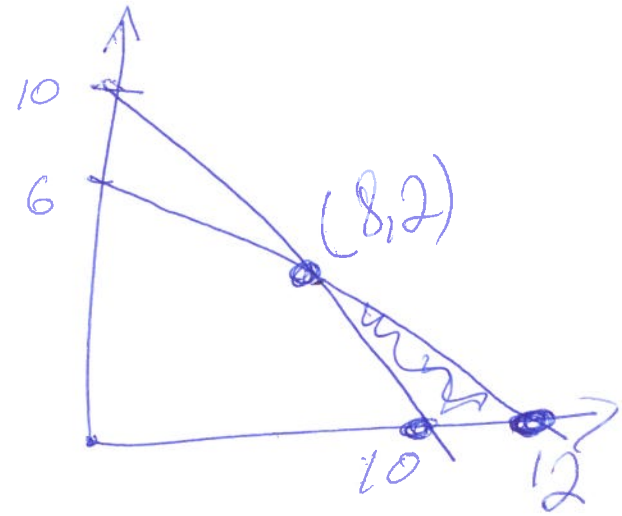
Pt	$0.2x + 0.3y$
(0,0)	0
(0,300)	$0 + 0.3(300) = 90$
(1000,0)	$0.2(1000) + 0 = 200$
(900,150)	$0.2(900) + 0.3(150) = 180 + 45 = 225$

(900,150) ← maximizer

225 ← maximum profit

Ex min $2x+3y$ s.t.

$$x+y \geq 10$$
$$x+2y \leq 12$$
$$x \geq 0$$
$$y \geq 0$$



minimizer

pt	$2x+3y$	
(10,0)	20	← minimum
(12,0)	24	
(8,2)	22	