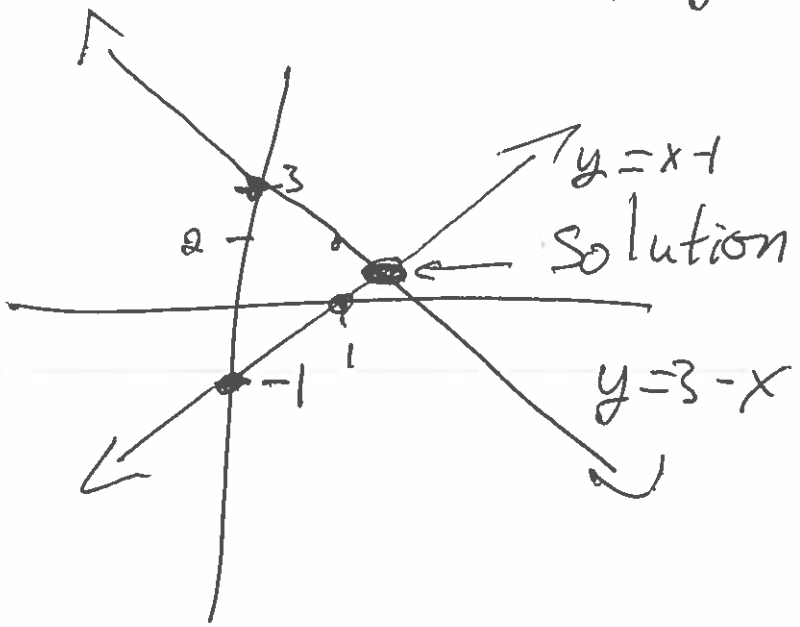
3.1 Solve Systems of Equations

Ex $x+y=3$ $\quad x-y=1$

As lines (slope-intercept)

$y=3-x$

$y=x-1$



Approach 1 Solve for intersection

Set our lines equal: $3-x=x-1$

Solving for x: $4=2x$

$x=2$

To get corresp. y-coord, plug $x=2$ into one of lines

Observe: $3-2=1$ (First line $y=3-x$)

$2-1=1$ (Second line $y=x-1$)

Solution $(2,1)$ (Note Need both x-coord and y-coord)

$(2,1)$: 2 ↑ x-coord, 1 ↑ y-coord

Approach 2 Elimination

$$\begin{array}{r} x+y=3 \\ +\ x-y=1 \\ \hline 2x=4 \end{array}$$

So $x=2$

Plugging $x=2$ into either equation, we obtain $y=1$.

Solution $(2,1)$

Ex $\begin{array}{l} 3x+5y=0 \\ 2x+7y=1 \end{array}$

Want Values for x, y that satisfy both equations.

$$\begin{array}{l} 2(3x+5y=0) \Rightarrow 6x+10y=0 \\ -3(2x+7y=1) \Rightarrow +\ -6x-21y=-3 \\ \hline -11y=-3 \end{array}$$

So $y=\frac{3}{11}$.

We have $3x+5y=0$

Plug in $y=\frac{3}{11}$: $3x+5\left(\frac{3}{11}\right)=0$

So $3x=-5\left(\frac{3}{11}\right)$

$x=\frac{-5}{11}$

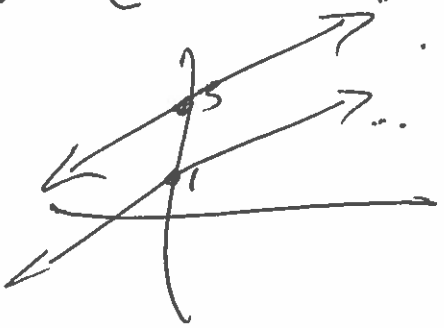
Solutions for Systems of Equations

↳ Case 1 Exactly one solution

↳ Case 2 No solutions (Parallel lines)

$y = x + 1$

$y = x + 3$



↳ Case 3 Infinitely many solutions

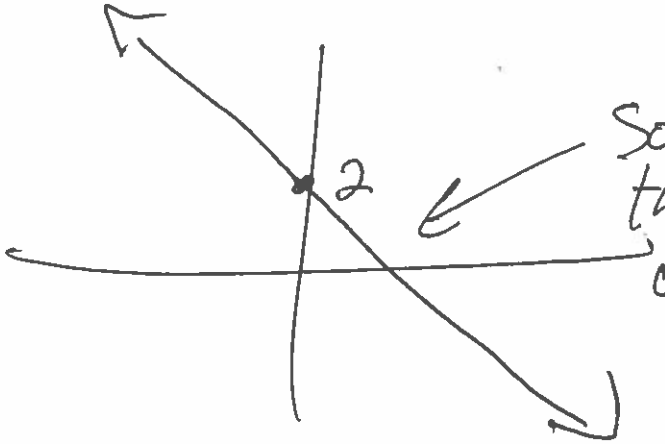
↳ Given multiple equations that represent same line

↳ Ex $x + y = 2$ ($y = 2 - x$)

$2x + 2y = 4$ ($2y = 4 - 2x$, so $y = 2 - x$)

Observe Second equation is multiple of first equation

Solution set: $\{(x, y) \mid y = 2 - x\}$



Solution set is the set of points on line $y = 2 - x$

$$x+y=2$$
$$2x+2y=4$$

Mult $-2(x+y=2) \Rightarrow -2x-2y=-4$

Add
$$\begin{array}{r} -2x-2y=-4 \\ 2x+2y=4 \\ \hline 0+0=0 \end{array}$$

So if you solve a system and obtain $0=0$, then there are inf. many solutions.

Solution set $\{(x,y) \mid y=2-x\}$

Also acceptable Set of points on line $y=2-x$.

Ex No Solutions (Inconsistent System)

$$\begin{array}{l} x-3y=5 \\ -2x+6y=8 \end{array}$$

Mult $2((x-3y)=5) \Rightarrow 2x-6y=10$

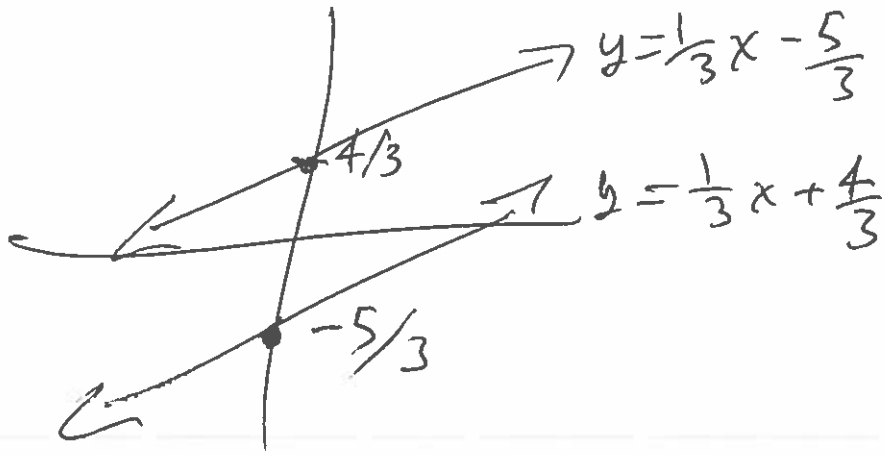
$$\begin{array}{r} + \quad -2x+6y=8 \\ \hline 0+0=18 \end{array}$$

Not true $0=18$.

Inconsistent Solve system and obtain false statement (e.g., $0=18$).

$x - 3y = 5$ $\left(y = \frac{1}{3}x - \frac{5}{3}\right)$

$-2x + 6y = 8$ $\left(y = \frac{1}{3}x + \frac{4}{3}\right)$] Parallel lines



If lines are parallel (same slo
<u>diff</u> y-intercep
the system is inconsistent.

What if lines have <u>same</u> slope and <u>same</u> y-int?
↳ Then both equations represent same line,
So we have inf. many sols.