Stable-Marriage Problem

↳ Two disjoint sets of players

$X = \{x_1, x_2, \ldots, x_n\}$

$Y = \{y_1, y_2, \ldots, y_m\}$

The X players can <u>only</u> be matched w/ Y players (and vice versa).

Everyone can be single, and everyone starts single.

↳ Each player X has <u>strict</u> preferences over Y players, though an X player may prefer to remain single than match w/ Y player.

↳ Same holds for Y players.

<u>Ex</u> (Prefs listed Most to least Pref)

x_1: y_2, y_1

x_2: y_1, y_2, y_3

x_3: y_1, y_2

y_1: x_1, x_3, x_2

y_2: x_2, x_1, x_3

y_3: x_1, x_3, x_2

<u>Matching</u> (Not stable)

$x_1 - y_1$

$x_2 - y_2$

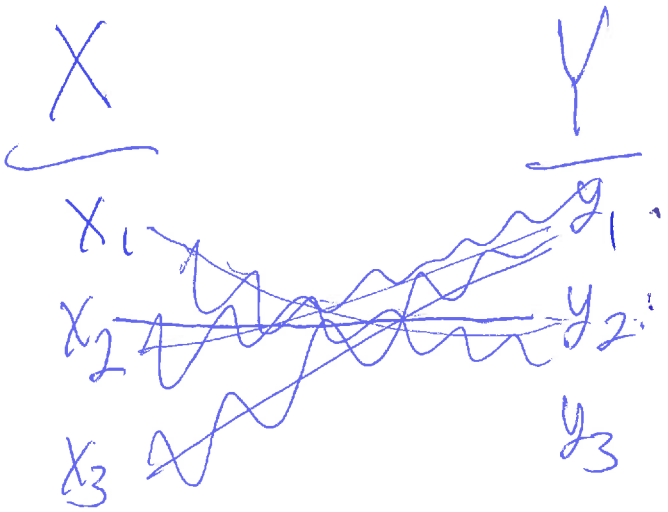
$x_3 - y_3$

<u>Stable Matching</u>

$x_1 - y_1$	x_3
$x_2 - y_2$	y_3

Gale-Shapley Algorithm

Input Two disjoint sets of Players X, Y
Each player's preference list
Proposer Set: X
Acceptor Set: Y.



~~x_3 y_1~~
$x_2 - y_2$
$x_1 - y_1$
x_3
y_3

∈ Stable Matching

Try x_3, x_2, x_1 (order of proposal)

X
x_1
x_2
x_3.

Y
y_1
y_2.
y_3

~~x_3 y_1~~

$x_2 - y_2$.
$x_1 - y_1$
x_3
y_3

Thm Every ordering of proposers yields same stable marriage. (Ordering should be same for duration of algorithm).

Ex **Firms**

F_1	F_2	F_3	F_4
W_3	W_1	W_1	W_3
W_1	W_3	W_3	W_1
	W_4		W_4
	W_2		W_2

Workers

W_1	W_2	W_3	W_4
F_1	F_1	F_3	F_4
F_3	F_4	F_1	F_1
	F_2	F_4	F_3
	F_3		

Firms Propose

$F_1 - W_3$

$F_2 - W_2$

$F_3 - W_1$

$F_4 - W_4$

Workers Propose

$F_1 - W_1$

$F_2 - W_2$

$F_3 - W_3$

$F_4 - W_4$