6.4 Permutations and Combinations

5:20

Def Let S be a finite set. A permutation π is a one-to-one function from S to S.

Ex # ways to assign 6 distinct books to 6 children, each child gets exactly one book.

Book	Children
B_1	C_1
B_2	C_2
B_3	C_3
B_4	C_4
B_5	C_5
B_6	C_6

$$\boxed{6\cdot5\cdot4\cdot3\cdot2\cdot1}$$

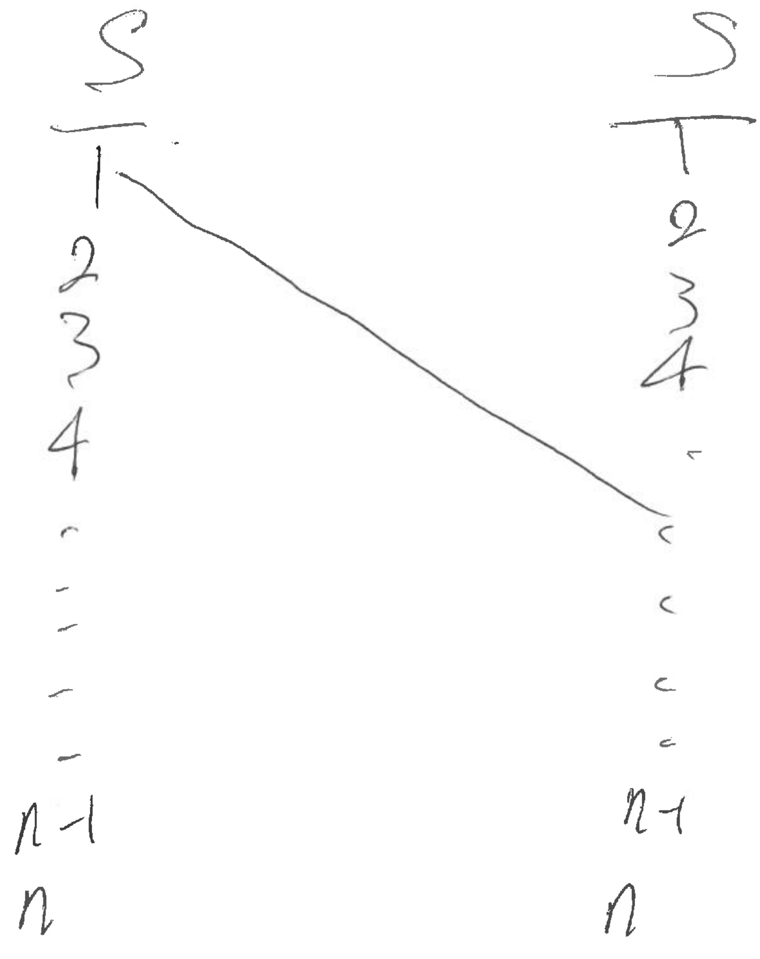
Def Let $n \geq 0$ be an integer. Define

$$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1, \quad (n>0)$$

$$0! := 1$$

So we have $6!$ ways of assigning 6 distinct books to 6 distinct children, where each child gets one book.

Thm Let S be an n-elem set. There are $n!$ permutations on S.



$$n! = n(n-1)(n-2)\cdots 2\cdot 1$$

Q Why is $0! = 1$?

Ex How many ways can 4 distinct songs be played in sequence?

A $\boxed{4!}$

↳ $4 \cdot 3 \cdot 2 \cdot 1$

↳ 24

Ex 3 drivers, 3 distinct cars

How many ways can we match drivers to cars?

A $3!$

Ex 5 distinct books, 7 children

Each child gets at most one book

Books	Children
	C_1
B_1	C_2
	C_3
B_2	C_4
B_3	C_5
B_4	C_6
B_5	C_7

$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$

<u>Restricted Permution</u> Let ~~...~~ $n \geq k \geq 0$ be integers,

$$P(n,k) = \frac{n!}{(n-k)!}$$

↳ Counts # ways of assigning k-books to n children. Each child gets at most 1 book

$$P(7,5) = \frac{7!}{(7-5)!} = \frac{7!}{2!} = \frac{7\cdot6\cdot5\cdot4\cdot3\cdot\cancel{2\cdot1}}{\cancel{2\cdot1}}$$

$$= 7\cdot6\cdot5\cdot4\cdot3$$

On Quiz / Exam:

↳ $P(7,5)$ ↳ $\frac{7!}{(7-5)!}$

↳ $7\cdot6\cdot5\cdot4\cdot3$ ↳ $\frac{7!}{2!}$

<u>Ex</u> 10 Companies, apply to exactly 6, <u>order matters</u>.

How many ways to apply?

<u>A</u> $P(10,6)$ (~~...~~ 10 dist companies
6 dist slots
order matters)

· <u>Combinations</u> Distinct elements, order does not matter.

↳ How many 1-elem subsets of $\{1,2,3,4\}$?

$\{1\}, \{2\}, \{3\}, \{4\}$

So 4 1-elem subsets of $\{1,2,3,4\}$.

↳ How many 2-elem subsets of $\{1,2,3,4\}$?

$\{1,2\}, \{1,3\}, \{1,4\}$

$\{2,3\}, \{2,4\}, \{3,4\}$

6 2-elem subsets of $\{1,2,3,4\}$

<u>Def</u> Let $n \geq k \geq 0$ be integers. The binomial coefficient $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$

<u>Ex</u> $\binom{4}{1} = \frac{4!}{1!\,(4-1)!} = \frac{4!}{3!} = \frac{4\cdot 3\cdot 2\cdot 1}{3\cdot 2\cdot 1} = 4.$

The binomial coefficient $\binom{n}{k}$ counts the # of k-elem subsets on an n-elem sets.

<u>Ex</u> $\binom{4}{2} = \frac{4!}{2!\cdot 2!} = \frac{4\cdot 3\cdot 2\cdot 1}{2\cdot 2} = 6$

May see $\binom{n}{k}$ or $C(n,k)$