4.1/4.2 Basic Matrix Algebra

↳ Matrix Addition
↳ Matrix Multiplication
↳ Scalar Multiplication

Matrix Addition Let A, B be two $n \times m$ matrices. Then $A + B$ is defined as follows.

(#rows, #cols)

$$(A+B)_{ij} = A_{ij} + B_{ij}$$

Ex
$$\begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} = \begin{bmatrix} 9 & 11 \\ 13 & 15 \end{bmatrix}$$

Ex
$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 5 & 7 \\ 11 & 13 & 17 \end{bmatrix} + \begin{bmatrix} 19 & 23 & 31 \\ 37 & 43 & 47 \\ 51 & 53 & 59 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 24 & 33 \\ 40 & 48 & 54 \\ 62 & 66 & 76 \end{bmatrix}$$

Ex $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 5 & 6 \\ 7 & 9 & 11 \end{bmatrix}$

Cannot Add the first matrix is 2x2, while the second matrix has dimensions 2x3.

Scalar Multiplication

Ex $4\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 12 & 20 \end{bmatrix}$

Ex $7\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 0 \\ 1 & 7 & 0 \\ 1 & 1 & 7 \end{bmatrix}$

Ex $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 0 \\ -1 & 7 & 0 \\ -1 & -1 & 7 \end{bmatrix}$

Matrix Multiplication

↳ Given A, an $n \times \underline{m}$ Matrix
B, an $\underline{m} \times k$ Matrix

Then AB is an $n \times k$ matrix

Ex $\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}_{2 \times 3} \underbrace{\begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}}_{3 \times 2} = \begin{bmatrix} 58 & 64 \\ \underline{139} & \underline{154} \end{bmatrix}$

$1 \cdot 7 + 2 \cdot 9 + 3 \cdot 11 = 58$

$1 \cdot 8 + 2 \cdot 10 + 3 \cdot 12 = 64$

$4 \cdot 7 + 5 \cdot 9 + 6 \cdot 11 = 139$

$154 = 4 \cdot 8 + 5 \cdot 10 + 6 \cdot 12$

Ex $\begin{bmatrix} 2 & 0 & -1 & 3 \\ 1 & -1 & 2 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 & -8 \\ 1 & 0 & 0 \\ 0 & 5 & 2 \\ -2 & 8 & -1 \end{bmatrix}$

$= \begin{bmatrix} -4 & 21 & -21 \\ 4 & -5 & -2 \end{bmatrix}$

$2(-8) + 0(0) + (-1)2$
$+ 3(-1) = -16 - 2 - 3 = -21$

Ex Not all matrices can be multiplied.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ ← Cannot pair these entries

Ex Matrix Multiplication DOES NOT COMMUTE!

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 0 \\ 5 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -2 & 1 \\ 10 & -2 \end{bmatrix} \qquad BA = \begin{bmatrix} 3 & -3 \\ 5 & -7 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$$

B A

$AB \neq BA$