4.4

Recall 2nd Derivative Test

↳ $f(x)$ twice diff @ $x=c$

↳ $f(x)$ has crit pt @ $x=c$

If:

↳ $f''(c) > 0$, then $f(x)$ has local min @ $x=c$.

↳ $f''(c) < 0$, then $f(x)$ has local max @ $x=c$

↳ $f''(c) = 0$, go do 1st Deriv Test

Ex Find local minima/maxima of

↳ $f(x) = x^3$

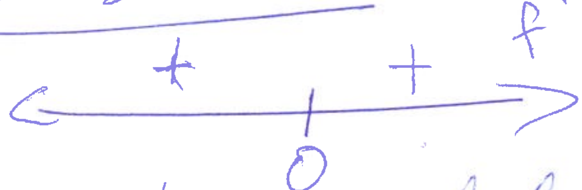
$f'(x) = 3x^2$, so crit pt @ $x=0$

$f''(x) = 6x$, $f''(0) = 0$

↳ $f(x) = x^4$

Local Min @ $x=0$.

Do First Deriv Test



So $f(x)=x^3$ has no local minima/maxima.

4.5 L'Hopital's Rule

Motivation Want to evaluate $\lim_{x \to \infty} \frac{x^2}{2^x}$. $\left(\frac{\infty}{\infty}\right)$

Thm (L'Hopital's Rule) Suppose $f(x)$, $g(x)$ are differentiable on some open interval I, except perhaps at some point $x = c \in I$. Suppose that

↳ $L := \lim_{x \to c} f(x) = \lim_{x \to c} g(x)$, $L = 0, \pm\infty$

↳ $g'(x) \neq 0$ on I (except perhaps at c)

↳ $\lim_{x \to c} \frac{f'(x)}{g'(x)}$ exists

Then $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$

Ex $\lim_{x \to \infty} \frac{x}{2^x}$ (Indeterminate Form: $\frac{\infty}{\infty}$)

$$= \lim_{x \to \infty} \frac{1}{\ln(2) 2^x} = 0.$$

Ex $\lim_{x \to 0} \frac{\sin(x)}{x}$ (Indet. Form: $\frac{0}{0}$)

$$= \lim_{x \to 0} \frac{\cos(x)}{1} = 1$$

Ex $\lim_{x\to 0} \frac{1-\cos(x)}{x}$ (Indet form: $\frac{0}{0}$)

$= \lim_{x\to 0} \frac{\sin(x)}{1} = 0.$

Ex $\lim_{x\to\infty} \frac{x^2}{2^x}$ (IF: $\frac{\infty}{\infty}$) ← Show this on quiz/exam

$= \lim_{x\to\infty} \frac{2x}{\ln(2)2^x}$ (IF: $\frac{\infty}{\infty}$)

$= \lim_{x\to\infty} \frac{2}{(\ln(2))^2 2^x} = 0$

$\left(\frac{a}{b/c}\right) = c \cdot \frac{a}{b}$

$\frac{\left(\frac{a}{b}\right)}{c} = \frac{1}{c}\left(\frac{a}{b}\right)$

Indet form: $0 \cdot \infty$

Ex $\lim_{x\to 0^+} \sqrt{x}\ln(x)$ (IF: $0\cdot(-\infty)$) $\frac{1}{\sqrt{x}}$

$= \lim_{x\to 0^+} \frac{\ln(x)}{1/\sqrt{x}}$ (IF: $\frac{-\infty}{\infty}$)

$= \lim_{x\to 0^+} \frac{1/x}{-\frac{1}{2}x^{-3/2}} = \lim_{x\to 0^+} \frac{1}{x}(x^{3/2})(-2) = \lim_{x\to 0^+} -2\sqrt{x}$

$= 0.$

Ex $$\lim_{x\to 0} x\sin\left(\frac{1}{x}\right) = \lim_{h\to 0^+} \frac{1}{h}\sin(h) = 1$$

$$= \lim_{x\to 0} \frac{\sin\left(\frac{1}{x}\right)}{1/x} =$$

Take $h = \frac{1}{x}$

Indet Form $\infty - \infty$

Ex $$\lim_{x\to 0^+}\left(\frac{1}{\sin(x)} - \frac{1}{x}\right) \quad (\text{IF: } \infty - \infty)$$

$$= \lim_{x\to 0^+}\left(\frac{x - \sin(x)}{x\sin(x)}\right) \quad \left(\text{IF: } \frac{0}{0}\right)$$

$$= \lim_{x\to 0^+}\frac{1-\cos(x)}{\sin(x) + x\cos(x)} \quad \left(\text{IF: } \frac{0}{0}\right).$$

$$= \lim_{x\to 0^+}\frac{\sin(x)}{\cos(x) + (\cos(x) - x\sin(x))} = \lim_{x\to 0^+}\frac{\sin(x)}{2\cos(x) - x\sin(x)}$$

$$= \frac{0}{2} = 0.$$