

Verifying Identities

So far we have seen the following trigonometric identities:

$\cot x = \frac{1}{\tan x}$	$\sec x = \frac{1}{\cos x}$	$\csc x = \frac{1}{\sin x}$
$\sin^2 x + \cos^2 x = 1$	$\tan^2 x + 1 = \sec^2 x$	$1 + \cot^2 x = \csc^2 x$
$\sin(-x) = -\sin(x)$	$\cos(-x) = \cos(x)$	$\csc(-x) = -\csc(x)$
$\tan(-x) = -\tan(x)$	$\sec(-x) = \sec(x)$	$\cot(-x) = -\cot(x)$
	$\tan x = \frac{\sin x}{\cos x}$	

When trying to verify an identity, we have talked about the following strategy:

1. **Work on one side** of the equation (usually the more complicated side), keeping in mind the expression on the other side is your goal.
2. Some expressions can be simplified quickly if they are **rewritten in terms of sines and cosines** only.
3. To convert one rational expression into another, **multiply** the numerator and denominator of the first **by either the numerator or the denominator** of the desired expression.
4. If the numerator of a rational expression is a sum or difference, **convert the rational expression into a sum or difference** of two rational expressions.
5. If a sum or difference of two rational expressions occurs on one side of the equation, then **find a common denominator** and combine them into one rational expression.

Verify the following identities:

$$1. 1 - \sec x \csc x \tan x = -\tan^2 x$$

$$11. \frac{\sec x - \cos x}{\cos x} = \tan^2 x$$

$$2. \frac{\csc x - 1}{\cot x} = \frac{\cot x}{\csc x + 1}$$

$$12. -2 \cot^2 x = \frac{1}{1 - \sec x} + \frac{1}{1 + \sec x}$$

$$3. \cos^2 x \tan^2 x = \sin^2 x$$

$$13. \frac{1 - \cos^2(-x)}{\sin(-x)} = \tan(-x) \cos(-x)$$

$$4. \frac{\cos x \sin^2 x + \cos^3 x}{\sin x} = \cot x$$

$$14. 1 - \csc x \sin^3 x = \cos^2 x$$

$$5. 1 - \sin x = \frac{1 - \sin^2(-x)}{1 - \sin(-x)}$$

$$15. \frac{-1}{\tan x - \sec x} = \frac{1 + \sin x}{\cos x}$$

$$6. (1 + \cot x)^2 - 2 \cot x = \frac{1}{(1 - \cos x)(1 + \cos x)}$$

$$16. \tan^2 x = \frac{1 - \sin^2 x \csc^2 x + \sin^2 x}{\cos^2 x}$$

$$7. \frac{\csc x}{\cot x} - \frac{\cot x}{\csc x} = \frac{\tan x}{\csc x}$$

$$17. \frac{4 \sec^2 x + 4 \sec x + 1}{2 \sec x + 1} = \frac{2}{\cos x} + 1$$

$$8. \frac{1 - 2 \cos^2 x}{1 - 2 \cos x \sin x} = \frac{\sin x + \cos x}{\sin x - \cos x}$$

$$18. \frac{\sin x}{\sin x + 1} = \frac{\csc x - 1}{\cot^2 x}$$

$$9. \ln(\tan x) = \ln(\sin x) + \ln(\sec x)$$

$$19. \ln(\sec x) = -\ln(\cos x)$$

$$10. \ln|\sec x + \tan x| = -\ln|\sec x - \tan x|$$

$$20. \ln|\csc x + \cot x| = -\ln|\csc x - \cot x|$$