Last Time Cartesian Product (3:55)

Thm (Rule of Product) Let A, B be finite sets. Then $n(A \times B) = n(A) \cdot n(B)$.

Def Let Σ be a finite set. We refer to Σ as an alphabet.

Ex $\Sigma = \{A, B, \dots, Z\}$

$\Sigma = \{0, 1\}$

k-letter alphabet: $\Sigma = \{0, 1, \dots, k-1\}$

Def Let Σ be an alphabet. A word or string $w = (w_1, w_2, w_3, \dots, w_k)$ of length k is an ordered sequence in $\Sigma^k = \underbrace{\Sigma \times \Sigma \times \cdots \times \Sigma}_{k\text{-times}}$.

So $w_1, w_2, \dots, w_k \in \Sigma$

Ex Let $\Sigma = \{0, 1\}$ be the binary alphabet.

$(0,1)$ is a binary string of length 2.

That is $(0,1) \in \Sigma \times \Sigma$

$(1,0,1) \in \Sigma \times \Sigma \times \Sigma$

Q How many binary strings of length 3?

Recall $\Sigma \times \Sigma \times \Sigma$ is set of binary strings of length 3, where $\Sigma = \{0,1\}$.

$$n(\underbrace{\Sigma \times \Sigma \times \Sigma}_{\text{Set of binary str. of length 3}}) = n(\Sigma) \cdot n(\Sigma) \cdot n(\Sigma) = 2 \cdot 2 \cdot 2 = 2^3$$

So there are 8 binary strings of length 3.

The binary strings of length 3 are:

$(0,0,0)$	$(1,0,0)$
$(0,0,1)$	$(1,0,1)$
$(0,1,0)$	$(1,1,0)$
$(0,1,1)$	$(1,1,1)$

On quiz or test: 2^3 is preferred to 8

Ex Given Σ, a five-letter alphabet.

How many strings of length 8 over Σ exist?

$$(\underset{5}{\_}, \underset{5}{\_}, \underset{5}{\_}, \underset{5}{\_}, \underset{5}{\_}, \underset{5}{\_}, \underset{5}{\_}, \underset{5}{\_})$$

$$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^8 \text{ strings}$$

Ex Red die
Green die

Both 6-sided

How many possible rolls do we have?

↳ Note The dice are distinguishable.

So $\begin{matrix}\text{Red}-3\\ \text{Green}-5\end{matrix} \neq \begin{matrix}\text{Red}-5\\ \text{Green}-3\end{matrix}$

$$(\underset{\text{Red}}{\_}, \underset{\text{Green}}{\_})$$

$$6 \cdot 6 = 36 \text{ possible rolls}$$

Ex Two 6-sided die (not distinguishable): $6+15=21$ rolls

↳ Same roll for both: 6

↳ Different rolls: $(\underset{6}{\_}, \underset{5}{\_}) = 30$. Divide $\frac{30}{2} = 15$ rolls

$(2,3) = (3,2)$ b/c die are not dist.

Def Let A and B be sets. We say that A is a subset of B (denoted $A \subseteq B$) if every element of A also belongs to B.

Ex $A = \{1,2,3\}$

$B = \{1,2,3,4,5\}$

So $A \subseteq B$

Let $C = \{2,3,5\}$

So $C \subseteq B$

Is $A \subseteq C$?

No $1 \in A$, but $1 \notin C$

6.2 Set Cardinality

Thm (Rule of Sum). Let A, B be finite sets, and suppose A and B share no common elements (ie, $A \cap B = \emptyset$). Then:

$$n(A \cup B) = n(A) + n(B)$$

Ex $A = \{a,b,c\}$, $B = \{d,e,f\}$, $A \cup B = \{a,b,c,d,e,f\}$

$$n(A \cup B) = 6 = \underbrace{n(A)}_{3} + \underbrace{n(B)}_{3}$$

Ex What if A, B are not disjoint?

$A = \{a, \underline{b, c, d}\}$

$B = \{\underline{b, c, d}, e, f\}$

$A \cup B = \{a, b, c, d, e, f\}$, $n(A \cup B) = 6$

$n(A \cup B) = n(A) + n(B) - \underline{n(A \cap B)}$

$6 = 4 + 5 - 3$

Instance of Principle of Inclusion-Exclusion (PIE)

Ex Amazon has 132,000 cookbooks.

↳ 20,000 were on regional cooking

↳ 5000 vegetarian

↳ 24000 on either vegetarian or regional

Q How many cookbooks were on neither vegetariarian nor regional?

A 132,000 − 24000

Q How many cookbooks are not both (veget and reg)?

↳ Start by det # both veget and reg.

Let A be set of reg cookbooks
B be set of veget cookbooks

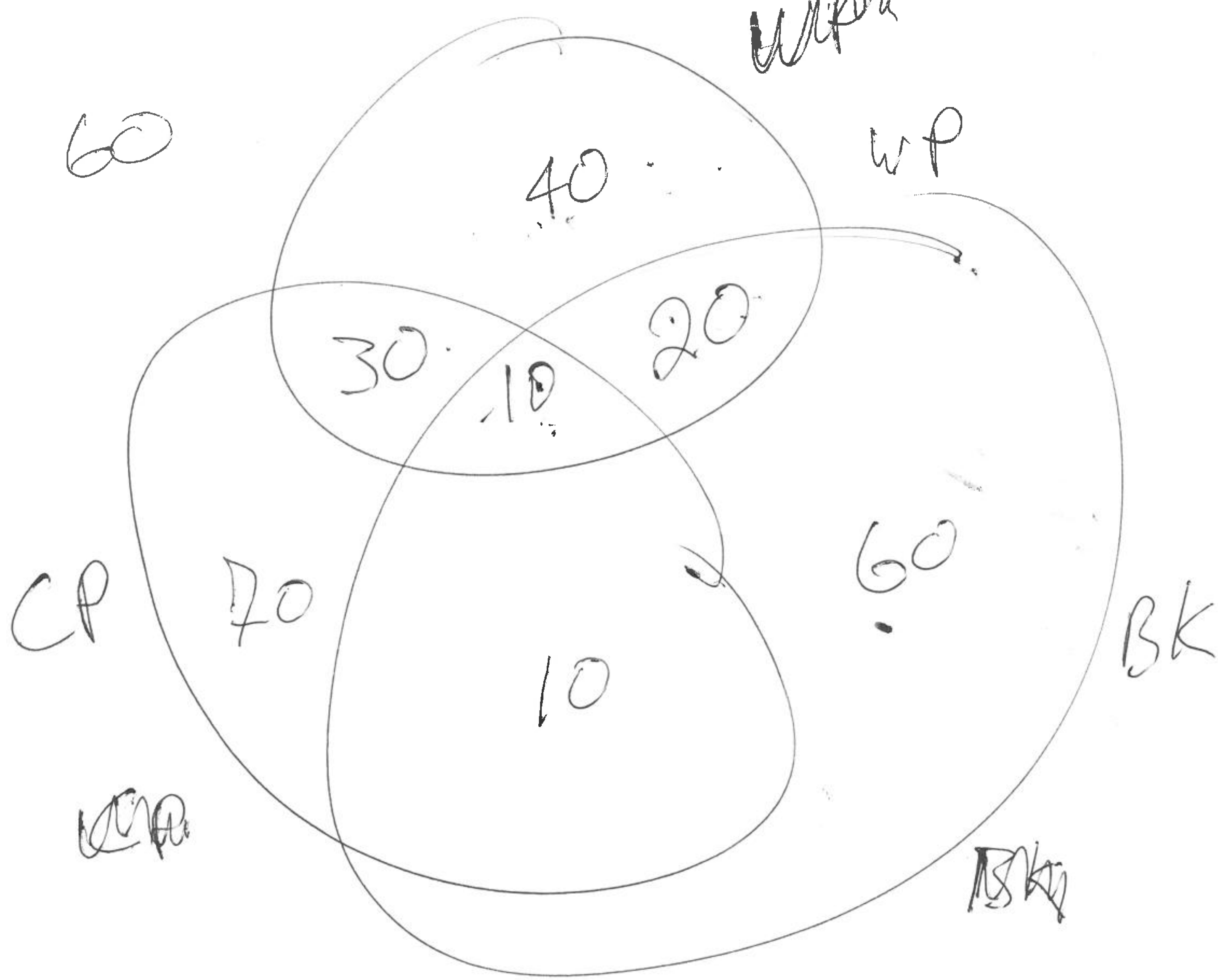
$n(A \cup B) = 24000$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$24000 = 20000 + 5000 - n(A \cap B)$
$n(A \cap B) = 20000 + 5000 - 24000 = 1000$

- So we have 1000 cookbooks that both veget/reg.

So: $132000 - 1000 = 131{,}000$ that not both veget/reg.

Ex 300 college students
↳ 100 War and Peace (WP)
↳ 120 Crime and Punishment (CP)
↳ 100 The Brothers Karamazov (BK)
↳ 40 read only WP
↳ 70 read WP but not BK
↳ 80 read BK but not CP
↳ Only 10 read All three



60
WP
CP
BK
40
30
10
20
70
10
60