Math 170- Worksheet 6.3-6.4

Instructions: Answer all questions. Clearly justify your reasoning.

1 Section 6.3

Problem 1) When Baskin-Robbins was founded in 1945, it made 31 different flavors of ice cream. If you had a choice of having your ice cream in a cone, a cup, or a sundae, how many different desserts could you have?

Answer: We can view a dessert offering as an ordered pair (x, y) where x represents the flavor and y represents whether we are getting a cone, cup, or sundae. There are 31 choices for x and 3 choices for y. The Rule of Product tells us that there are $31 \cdot 3 = 93$ possible desserts.

Problem 2) Determine the number of binary strings of length 8. Do the same for ternary strings (strings over the alphabet $\Sigma = \{0, 1, 2\}$).

Problem 3) While selecting candy for students in his class, Professor Murphy must choose between gummy candy and licorice nibs. Gummy candy comes in three sizes, while packets of licorice nibs come in two sizes. If he chooses gummy candy, he must select gummy bears, gummy worms, or gummy dinos. If he chooses licorice nibs, he must choose between black and red. How many choices does he have?

Problem 4) How many seven-digit phone numbers do not begin with one of the prefixes: $1, 911, 411$, or 555?

Problem 5) A Social-Security Number (SSN) is a sequence of 9 digits.

(a) How many SSNs are possible?

(b) How many SSNs begin with either 023 or 003?

Answer: Let S_1 be the set of social security numbers beginning with 023, and let S_2 be the set of social security numbers beginning with 003. We are interested in $n(S_1 \cup S_2)$. Observe that S_1 and S_2 share no common elements (that is, $S_1 \cap S_2 = \emptyset$). So $n(S_1 \cup S_2) = n(S_1) + n(S_2)$, by the rule of sum. We now determine $n(S_1)$ and $n(S_2)$.

- Note that for an SSN in S_1, the first three digits are fixed: $(0, 2, 3, \_, \_, \_, \_, \_, \_)$. This leaves six remaining slots. There are ten digits in our alphabet ($\{0, 1, \ldots, 9\}$), so there are 10^6 possible SSNs that begin with 023. Thus, $n(S_1) = 10^6$.
- By similar argument, $n(S_2) = 10^6$.

Thus, $n(S_1 \cup S_2) = 2 \cdot 10^6$.

(c) How many SSNs are possible if no two adjacent digits are permitted? [**Note:** So $235 - 93 - 2345$ is permissible, but $126 - 67 - 8189$ is not permissible as there are two consecutive 6's.]

2 Section 6.4

2.1 Permutations and Combinations

Problem 6) How many five-letter sequences are possible using b, o, g, e, y once each?

Problem 7) How many unordered sets are there that contain 3 objects from the set $\{1, 2, \ldots, 7\}$?

Answer: There are $\binom{7}{3}$ such subsets.

Problem 8) How many ordered sequences are there that contain 3 objects from the set $\{1, 2, \ldots, 7\}$, assuming that repeated elements are not allowed?

Answer: $P(7, 3) = 7!/(7-3)! = 7!/4!$.

Problem 9) How many ways are there to pick a 7-person basketball team from 20 possible players? How many teams if the weakest player and strongest player must be on the team?

Answer: (a) $\binom{20}{7}$; (b) $\binom{18}{5}$.

Problem 10) How many ways are there to distribute six different books among 12 children if no child gets more than one book?
Answer: $P(12, 6) = 12!/(12-6)! = \binom{12}{6}6!$.

2.2 Poker Hands

Problem 11) A *three of a kind* is a 5-card poker hand with three cards of the same rank and two additional cards of two different ranks (different from both each other and the first three cards).

(a) How many three of a kind hands are there if we have three Queens?

(b) In general, how many three of a kind hands are there?

Problem 12) Determine the probabilities of drawing the following poker hands.

(a) A *Royal Flush* consists of ranks 10, J, Q, K, A, all of the same suit.

(b) A *Straight Flush* consists of five cards with values in a row, all of the same suit. An Ace may be considered as the highest rank (so Ace immediately follows king) or lowest rank (so Ace immediately precedes 2), but not both. So for example, A, 2, 3, 4, 5 (with all cards of the same suit) is a straight flush, but Q, K, A, 2, 3 is **not** a straight flush.

(c) A *Straight* consists of five cards with consecutive ranks, not all of the same suit. Just as in a straight flush, the Ace may be considered as the highest rank or lowest rank, but not both.

Hint: Start by determining the number of five-card hands where the ranks are consecutive. Then subtract out the number of such hands where they all have the same suit.

(d) A *Flush* consists of five cards, all of the same suit.

Answer: We begin by selecting the suit. There are 4 suits and we want only one, so there are $\binom{4}{1} = 4$ ways of selecting the suit. Now there are 13 cards in our fixed suit, and we want five cards. The order in which we select the cards does not matter, so there are $\binom{13}{5}$ ways of selecting the cards (given that we have fixed the suit). So by the rule of product, we have $\binom{4}{1}\binom{13}{5}$ flushes.

(e) A *Four of a Kind* consists of four cards of one rank, and a fifth card of a different rank.

(f) A *Full House* consists of three cards of one value, and two cards whose values are the same as each other but different than the first three cards. (E.g., three Queens and two 10's).

Answer: See course notes.

(g) A *Two Pair* consists of two pairs, one pair of the same value and a second pair of a different value. The fifth card has yet a different value. (E.g., two 10's, two Queen's, and a King).

(h) A *One Pair* consists of a pair of one value, and then three additional cards each of a different value. (E.g., 10, 10, Ace, Jack, Queen).