Instructions: This quiz is closed book, closed note, and an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places.

1. (5 pts) Solve the following system of equations. If you use a matrix approach, please clearly specify the initial matrix, the reduced matrix, and how you are using the reduced matrix to obtain a solution. If there is no solution, clearly specify this.

$$x + 2y - z + w = 30$$
$$2x - z + 2w = 30$$
$$x + 3y + 3z - 4w = 2$$
$$2x - 9y + w = 4$$

Answer: We set up our matrix as follows (note that the order in which the rows are listed does not matter):

$$\begin{pmatrix} 1 & 2 & -1 & 1 & 30 \\ 2 & 0 & -1 & 2 & 30 \\ 1 & 3 & 3 & -4 & 2 \\ 2 & -9 & 0 & 1 & 4 \end{pmatrix}$$

Row reducing our matrix, we obtain:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 16 \\ 0 & 1 & 0 & 0 & \frac{12}{7} \\ 0 & 0 & 1 & 0 & -\frac{162}{7} \\ 0 & 0 & 0 & 0 & -\frac{88}{7} \end{pmatrix}$$

So $x = 16, y = \dfrac{12}{7}, z = -\dfrac{162}{7}, w = -\dfrac{88}{7}$.

2. (5 pts) Suppose you purchase three stocks. The prices and dividend yields are given below:

- **Company A.** Price- \$16 per share, with a dividend of 7%.
- **Company B.** Price- \$56 per share, with a dividend of 2%.
- **Company C.** Price- \$80 per share, with a dividend of 2%.

Suppose you invested \$8400 in the above stocks at the given prices, and that you expected to earn \$248 in dividends. If you purchased a total of 200 shares, how many shares of each stock did you purchase?

Answer: Let a denote the number of shares of Company A, b denote the number of shares of Company B, and c denote the number of shares of Company C. We have the following system of equations:

- **Shares:** $a + b + c = 200$
- **Budget:** $16a + 56b + 80c = 8400$
- **Dividends:** $16(0.07)a + 56(0.02)b + 80(0.02)c = 248$.

We set up our matrix as follows (note that the order in which the rows are listed does not matter):

$$\begin{pmatrix} 1 & 1 & 1 & 200 \\ 16 & 56 & 80 & 8400 \\ 0.07 & 0.02 & 0.02 & 248 \end{pmatrix}$$

Row reducing our matrix, we obtain:

$$\begin{pmatrix} 1 & 0 & 0 & 100 \\ 0 & 1 & 0 & 50 \\ 0 & 0 & 1 & 50 \end{pmatrix}$$

So we purchased 100 shares of Company A, 50 shares of Company B, and 50 shares of Company C.