Math 122 Section 4.2-4.4 Study Guide

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1 Section 4.2

Second Derivative Test:

- **Given:** A function $f(x)$ that is twice differentiable (that is, both $f'(x)$ and $f''(x)$ exist).
- **Goal:** Find the local maxima and local minima of $f(x)$.
- **Approach:**
  - Find the critical points of $f(x)$. Recall that the critical points are the x-values where $f'(x) = 0$ or $f'(x)$ is undefined.
  - If c is a critical point and $f''(c) > 0$, then $f(x)$ has a local minimum at $x = c$.
  - If c is a critical point and $f''(c) < 0$, then $f(x)$ has a local maximum at $x = c$.
  - If c is a critical point and $f''(c) = 0$, then the Second Derivative Test provides no information, and you must go back and apply the First Derivative Test.

Problem 1) Find all local maxima, local minima, and points of inflection for the following functions.

- $f(x) = x^4 - 4x^3$.
- $f(x) = -x^3 + 3x^2 + 5$.
- $f(x) = x + \dfrac{4}{x}$.
- $f(x) = x^3$.
- $f(x) = 3x^5 - 5x^3$.

Problem 2) Let $f(x) = x^3 + bx^2 + cx + d$, where b, c, and d are unknown constants. Suppose that there is a critical point at $x = 2$, and an inflection point at $(1, 4)$. Determine b, c, and d.

2 Section 4.3

Problem 3) Find the global maximum and minimum values of the following functions. Unless otherwise speci ed, assume the domain is $\mathbb{R}$.

- $f(x) = -x^2 - 4x - 5$
- $f(x) = x + x^{-1}$, for $x > 0$.
- $f(x) = xe^{-x}$
- $f(x) = e^{3x} - e^{2x}$
- $f(x) = x - \ln(x)$, for $x > 0$.
- $f(x) = x^3 - 3x^2$ on $[-1, 3]$
- $f(x) = x^3 - 3x^2 - 9x + 15$ on $[-5, 4]$.

Problem 4) An individual seeks to encolse a rectangular 1000 square foot plot of land. The fence for the first three sides costs \$22 per foot, and the fence for the remaining sides costs \$13 per foot. Determine the minimum cost to enclose the fence.

Problem 5) Determine the minimum value that $x + y$ takes on, given that $xy = 324$ and $x, y > 0$.

Problem 6) Determine the minimum value that $x + 2y$ takes on, given that $x^2y = 10$ and $x, y > 0$.

3 Section 4.4

Problem 7) At a price of \$10 per ticket, a musical theater group can fill every seat in the theater, which has a capacity of 1300. For every additional dollar charged, the number of people buying tickets decreases by 50. What ticket price maximizes revenue? You must use Calculus techniques, and **not** the guess-and-check method.

Problem 8) A farmer uses x lb of fertilizer per acre, at a cost of \$2/lb. The farmer has a revenue of $R = 700 - 400e^{-x/100}$ dollars per acre. Determine the amount of fertilizer that should be applied per acre to maximize profit.

Problem 9) A firm sells a good at \$10 per unit. The cost of producing the good is given by $C(q) = 2q^2 + 5$. What is the quantity the firm should produce to maximize profit?

Problem 10) A landscape architect plans to enclose a 3000 square foot rectangular region. She will use shrubs costing \$45 per foot along three sides, and fencing costing \$20 per foot along the fourth side. What is the minimum cost the architect will incur?