7.4) Lotto Problems

Framework n numbered balls (so $\{1, 2, \ldots, n\}$)

↳ Pick $0 < k < n$ winning #'s (order not matter)

↳ What is the prob of match i of k winning #'s? (exactly)

Ex 50 balls, Pick-5

↳ So $\binom{50}{5}$ possible tickets

↳ Lotto Ticket is 5-elem subset of $\{1, \ldots, 50\}$

↳ Want # Lotto tickets that match 3 of 5 winners

5 winning #'s — $\binom{5}{3}$ ways of sel 3 winn. balls

45 losing #'s. — $\binom{45}{2}$ ways to pick 2 losing #s

$\binom{5}{3}\binom{45}{2}$ lotto tickets that match exactly 3 winning #'s.

↳ Prob of Match 3 winning #'s: $\frac{\#\text{ Tickets match 3 winners}}{\#\text{ Tickets in general}}$

$$\frac{\binom{5}{3}\binom{45}{2}}{\binom{50}{5}} = .00467$$

You can leave like 50

Fact Called Hypergeometric Distr.

Ex 407 balls, Pick 24

↳ What is prob of match 5 #s?

$$\frac{\binom{24}{5}\binom{383}{19}}{\binom{407}{24}}$$

Pick winners → $\binom{24}{5}\binom{383}{19}$ ← Pick losers

$\binom{407}{24}$ ← Total # tickets in general

Ex What is prob of match 3, 4, or 5 of 24 winners?

Prob of match 5

Prob of match 3: $\frac{\binom{24}{3}\binom{383}{21}}{\binom{407}{24}}$

Prob of match 4: $\frac{\binom{24}{4}\binom{383}{20}}{\binom{407}{24}}$

Prob of Match 3, 4, or 5 winners

$$\frac{\binom{24}{3}\binom{383}{21}}{\binom{407}{24}} + \frac{\binom{24}{4}\binom{383}{20}}{\binom{407}{24}} + \frac{\binom{24}{5}\binom{383}{19}}{\binom{407}{24}}$$

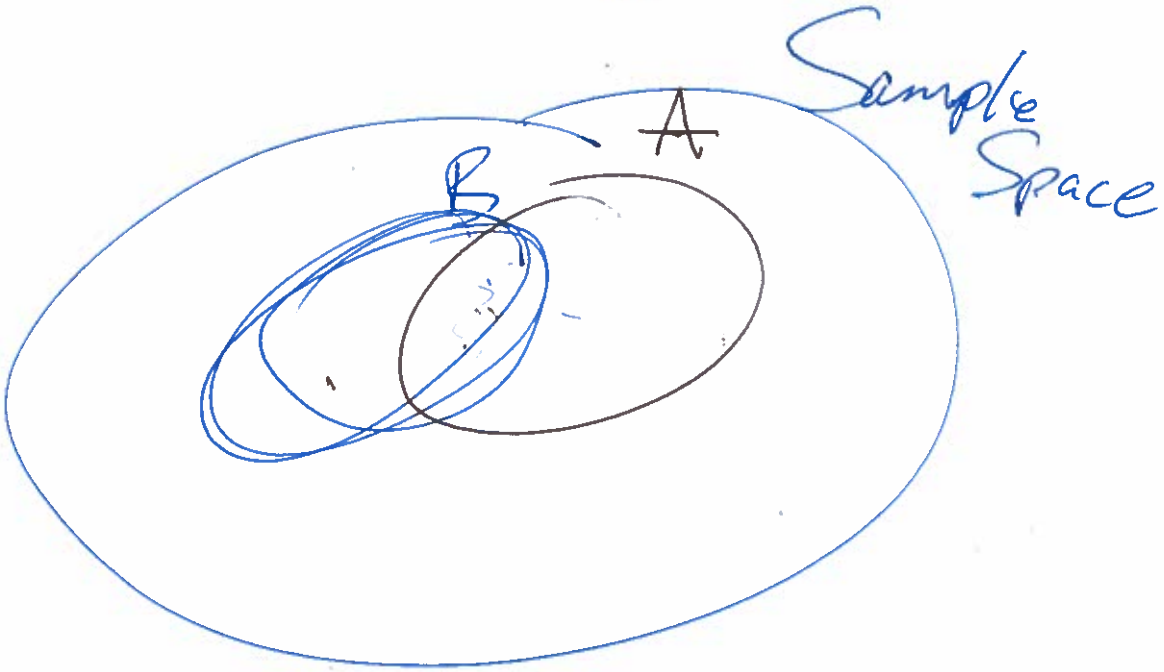
7.5 Conditional Prob

Idea Given additional info, can we better determine likelihood of event?

Notation Let A, B be events.

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{P(B)}$$

↑ "Prob of A, given B"



↳ Restricting to Y, we have 26 cards

↳ 2 Aces in Y

So $Pr(X|Y) = \frac{2}{26} = \frac{1}{13}$

↳ What is $Pr(X) = \frac{4}{52} = \frac{1}{13}$.

Def The events X and Y (from the sample space S) are **independent** if:

$$Pr(X|Y) = Pr(X).$$

X and Y are **dependent** otherwise.

Ex Deck of 52 playing cards

X event of sel Ace

Y event of sel Club/Spade

$$Pr(Y|X) = \frac{2}{4} = \frac{1}{2}$$

Restrict to Aces

What is prob of drawing A of Clubs or A of Spades

$$Pr(Y) = \frac{26}{52} = \frac{1}{2}$$

Pr 1 on SG

$$Pr(X|Y) = \frac{Pr(X \cap Y)}{Pr(Y)}$$

Ex Suppose toss 2 dist, 6-sided dice.

↳ Let X be the event that the dice added to 5

↳ Let Y be the event that the second die rolled 2.

↳ What is $Pr(X|Y)$?

~~Given Y: {(1,2), (2,2), (3,2), (4,2), (5,2), (6,2)}.~~

Given Y: {(1,2), (2,2), (3,2), (4,2), (5,2), (6,2)}

$$Pr(X|Y) = \frac{1}{6}$$

1 ← one roll that satisfies both X and Y

6 ← 6 rolls that satisfy Y

Ex Deck of 52 playing cards

↳ X be the event of sel. an Ace

↳ Y be the event of sel. Clubs or Spades

What is $Pr(X|Y)$?

7.5 Recall: Let A and B be events. Then $Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$

$Pr(A \cap B)$ vs. $Pr(A|B)$

$Pr(A \cap B)$: I pick someone at random and tell you nothing about them. What is prob that this person satisfies A and B?

$Pr(A|B)$: I pick someone at random who satisfies B. Knowing that, how likely is it they also satisfy A?

Ex 36% of families own dog

30% of families own cat

22% of families that own dog also own cat

$Pr(Dog) = 0.36$ $Pr(Cat) = 0.30$

22%: $Pr(Cat \cap Dog)$ or $Pr(Cat|Dog)$?

$Pr(Cat|Dog) = 0.22$

For intersection ($Pr(Cat \cap Dog)$) 22% of families own both cat and dog.

$Pr(Dog) = 0.36$

$Pr(Cat) = 0.3$

$Pr(Cat \mid Dog) = 0.22$

a) What is Prob that a family owns both Cat and dog?

unknown

Recall $Pr(Cat \mid Dog) = \dfrac{Pr(Cat \cap Dog)}{Pr(Dog) = 0.36}$

0.22

So $Pr(Cat \cap Dog) = 0.22 \cdot 0.36$

b) What is $Pr(Dog \mid Cat)$?

$$Pr(Dog \mid Cat) = \frac{Pr(Cat \cap Dog) = .22 \cdot .36}{Pr(Cat) = 0.3}$$

$$= \frac{.22 \cdot .36}{.3}$$

$.22 \cdot .36$ ← from (a)

$.3$ ← $Pr(Cat)$

Recall Let A, B are events. We say that A, B are independent if $Pr(A \mid B) = Pr(A)$

Ex Deck playing cards.

A: Selected 9 or 10

B: Suit is club or diamond

Are A and B independent?

$$Pr(A) = \frac{8}{52} = Pr(A|B) = \frac{4}{26}$$

So A and B are indep.