Section 6.2 Solutions

1) $1-\sec x \csc x \tan x = 1 - \frac{1}{\cos x}\frac{1}{\sin x}\frac{\sin x}{\cos x} = 1 - \frac{1}{\cos^2 x} = 1-\sec^2 x$

$= -\tan^2 x.$ ✓

2) $\frac{\csc x - 1}{\cot x}\left(\frac{\csc x + 1}{\csc x + 1}\right) = \frac{\csc^2 x - 1}{\cot x(\csc x + 1)} = \frac{\cot^2 x}{\cot x(\csc x + 1)} = \frac{\cot x}{\csc x + 1}$ ✓

3) $\cos^2 x \tan^2 x = \cos^2 x \frac{\sin^2 x}{\cos^2 x} = \sin^2 x$ ✓

4) $\frac{\cos x \sin^2 x + \cos^3 x}{\sin x} = \frac{\cos x(1-\cos^2 x) + \cos^3 x}{\sin x} = \frac{\cos x - \cos^3 x + \cos^3 x}{\sin x}$

$= \frac{\cos x}{\sin x} = \cot x$ ✓

5) $\frac{1-\sin^2(-x)}{1-\sin(-x)} = \frac{1-\sin^2 x}{1+\sin x} = \frac{(1-\sin x)(1+\sin x)}{1+\sin x} = 1-\sin x$ ✓

6) $(1+\cot x)^2 - 2\cot x = 1 + 2\cot x + \cot^2 x - 2\cot x = 1 + \cot^2 x = \csc^2 x.$

- $\frac{1}{(1-\cos x)(1+\cos x)} = \frac{1}{1-\cos^2 x} = \frac{1}{\sin^2 x} = \csc^2 x.$

Thus $(1+\cot x)^2 - 2\cot x = \frac{1}{(1-\cos x)(1+\cos x)}$ ✓

7) $\frac{\csc x}{\cot x} - \frac{\cot x}{\csc x} = \frac{1/\sin x}{\cos x/\sin x} - \frac{\cos x/\sin x}{1/\sin x} = \frac{1}{\cos x} - \cos x = \frac{1-\cos^2 x}{\cos x}$

$= \frac{\sin^2 x}{\cos x} = \tan x \sin x = \frac{\tan x}{1/\sin x} = \frac{\tan x}{\csc x}$ ✓

8) $\frac{\sin x + \cos x}{\sin x - \cos x}\left(\frac{\sin x - \cos x}{\sin x - \cos x}\right) = \frac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x - 2\sin x \cos x}$

$= \frac{(1-\cos^2 x) - \cos^2 x}{1 - 2\sin x \cos x} = \frac{1 - 2\cos^2 x}{1 - 2\sin x \cos x}$ ✓

9) $\ln(\tan x) = \ln(\sin x) + \ln(\sec x) = \ln(\sin x \sec x)$

$= \ln\left(\frac{\sin x}{\cos x}\right) = \ln(\tan x)$ ✓

10) $-\ln|\sec x - \tan x| = \ln\left|\frac{1}{\sec x - \tan x}\right| = \ln\left|\frac{1}{1/\cos - \sin x/\cos x}\right|$

$= \ln\left|\frac{\cos x}{1 - \sin x}\right| = \ln\left|\frac{\cos x}{1-\sin x}\left(\frac{1+\sin x}{1+\sin x}\right)\right|$

$= \ln\left|\frac{\cos x(1+\sin x)}{1-\sin^2 x}\right| = \ln\left|\frac{\cos x(1+\sin x)}{\cos^2 x}\right|$

$= \ln\left|\frac{1+\sin x}{\cos x}\right| = \ln\left|\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right|$

$= \ln|\sec x + \tan x|$ ✓

11) $\frac{\sec x - \cos x}{\cos x} = \frac{\sec x}{\cos x} - \frac{\cos x}{\cos x} = \sec^2 x - 1 = \tan^2 x$ ✓

12) $\frac{1}{1-\sec x} + \frac{1}{1+\sec x} = \frac{1+\sec x + 1 - \sec x}{1 - \sec^2 x} = \frac{2}{-\tan^2 x} = -2\cot^2 x$ ✓

13). $\frac{1-\cos^2(-x)}{\sin(-x)} = \frac{1-\cos^2 x}{-\sin x} = \frac{\sin^2 x}{-\sin x} = -\sin x.$

• $\tan(-x)\cos(-x) = -\tan x \cos x = -\frac{\sin x}{\cos x}\cos x = -\sin x.$

So $\frac{1-\cos^2(-x)}{\sin(-x)} = \tan(-x)\cos(-x)$ ✓

14) $1-\csc x \sin^3 x = 1-\frac{\sin^3 x}{\sin x} = 1-\sin^2 x = \cos^2 x$ ✓

15) $\frac{-1}{\tan x - \sec x} = \frac{-1}{\frac{\sin x}{\cos x} - \frac{1}{\cos x}} = \frac{-\cos x}{\sin x - 1}\left(\frac{\sin x + 1}{\sin x + 1}\right) = \frac{-\cos x(\sin x + 1)}{\sin^2 x - 1}$

$= \frac{-\cos x(1+\sin x)}{-\cos^2 x} = \frac{1+\sin x}{\cos x}$ ✓

16) $\frac{1-\sin^2 x \csc^2 x + \sin^2 x}{\cos^2 x} = \frac{1-\frac{\sin^2 x}{\sin^2 x}+\sin^2 x}{\cos^2 x} = \frac{1-1+\sin^2 x}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x}$

$= \tan^2 x$ ✓

17) $\frac{4\sec^2 x + 4\sec x + 1}{2\sec x + 1} = \frac{(2\sec x + 1)^2}{2\sec x + 1} = 2\sec x + 1 = \frac{2}{\cos x} + 1$ ✓

18) $\frac{\csc x - 1}{\cot^2 x} = \frac{\frac{1}{\sin x} - 1}{\frac{\cos^2 x}{\sin^2 x}}\left(\frac{\sin^2 x}{\sin^2 x}\right) = \frac{\sin x(1-\sin x)}{\cos^2 x}\left(\frac{1+\sin x}{1+\sin x}\right)$

$= \frac{\sin x(1-\sin^2 x)}{\cos^2 x(1+\sin x)} = \frac{\sin x \cos^2 x}{\cos^2 x(1+\sin x)} = \frac{\sin x}{1+\sin x}$ ✓

19) $-\ln(\cos x) = \ln\left(\frac{1}{\cos x}\right) = \ln(\sec x)$ ✓

20) This is equivalent to

$$\csc x + \cot x = \frac{1}{\csc x - \cot x} \quad \text{or} \quad (\csc x + \cot x)(\csc x - \cot x) = 1$$

$$(\csc x + \cot x)(\csc x - \cot x) = \csc^2 x - \cot^2 x$$

$$= (1 + \cot^2 x) - \cot^2 x = 1 \quad ✓$$