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Ex $f(z) = \sqrt{25 - z^2}$

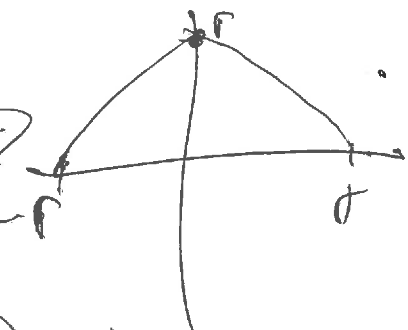
Domain: $[-5, 5]$

Recall $x^2 + y^2 = r^2$ is circle of radius r centered at $(0,0)$

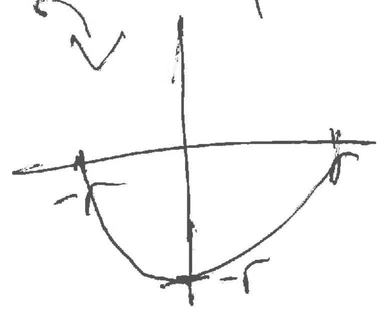
Solving for y: $y^2 = r^2 - x^2$

$y = \pm\sqrt{r^2 - x^2}$

$y = \sqrt{r^2 - x^2}$ (upper half)



$y = -\sqrt{r^2 - x^2}$ (lower half)



$y = \sqrt{25 - z^2}$, the range $[0, 5]$.

Def We say that $f(x)$ is <u>even</u> if $f(-x) = f(x)$ for all x.

Ex $f(x) = x^2$ is even. Observe $f(-x) = (-x)^2 = x^2 = f(x)$

$f(x) = x^2 + 1$ is even. Observe $f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x)$

Def We say that $f(x)$ is <u>odd</u> if $f(-x) = -f(x)$

Ex $f(x) = x$ is odd. <u>Observe</u> $f(-x) = -x = -f(x)$.

$f(x) = x^3$ is odd. Observe $f(-x) = (-x)^3 = (-1)^3 x^3 = -x^3$
$= -f(x)$.

$f(x) = x+1$ is <u>not</u> odd. $f(-x) = -x+1 \neq -f(x)$
$-f(x) = -(x+1) = -x-1$

Ex $f(x) = 0$ is the <u>unique</u> function that is both even and odd.

<u>Even</u>: $f(-x) = 0 = f(x)$

<u>Odd</u> $f(-x) = 0 = -0 = -f(x)$

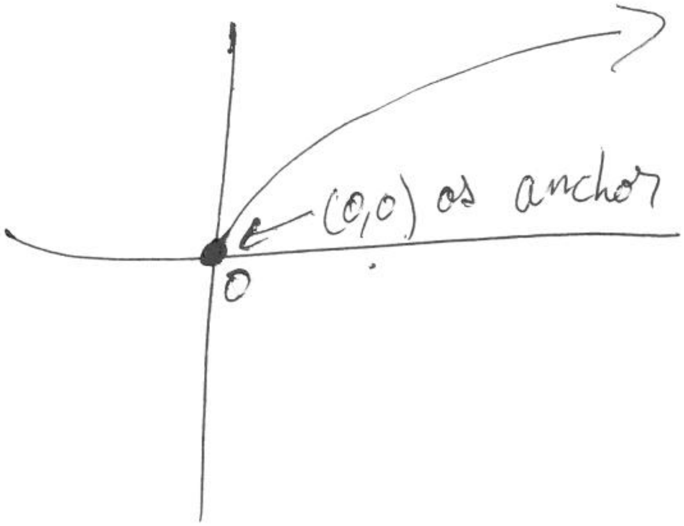
<u>Uniqueness</u> Exercise.

1.2 Function Transformations

Idea Start with parent function (eg, $\sqrt{x}$, x^2, $|x|$, etc.) and want to graph functions like $\sqrt{x-3}$, $(x+2)^2+5$, etc.

• **Ex** Want to graph $f(x)=\sqrt{x-3}$.

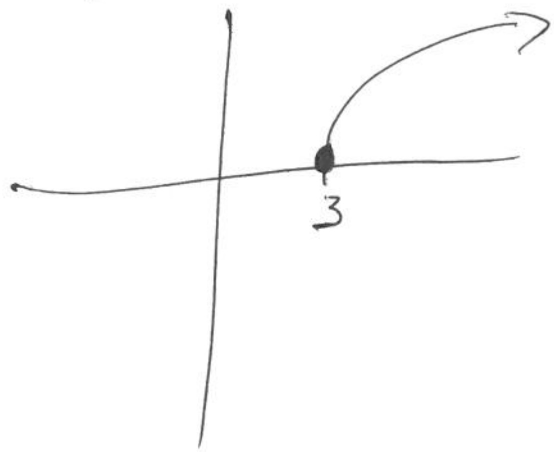
Parent: $\sqrt{x}$



Q $f(x)=\sqrt{x-3}=0$?

At $x=3$, $\sqrt{3-3}=0$

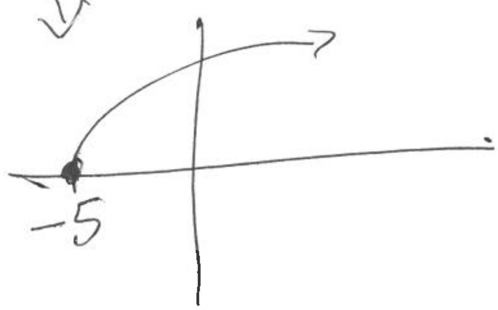
$f(x)=\sqrt{x-3}$



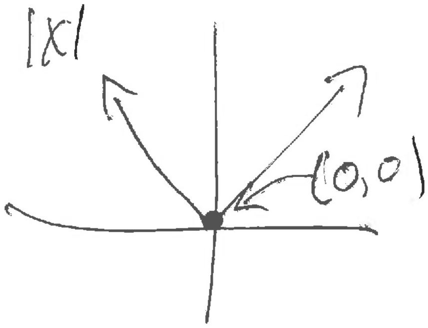
$g(x)=\sqrt{x+5}$

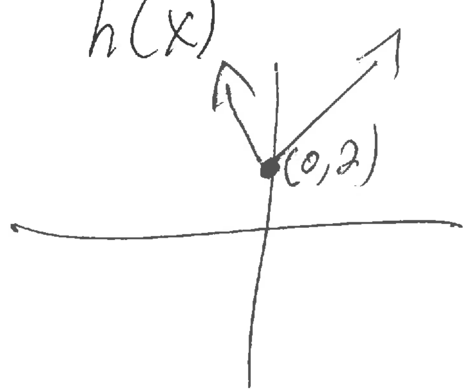
$x+5=0$

$x=-5$



Ex $h(x)=|x|+2$

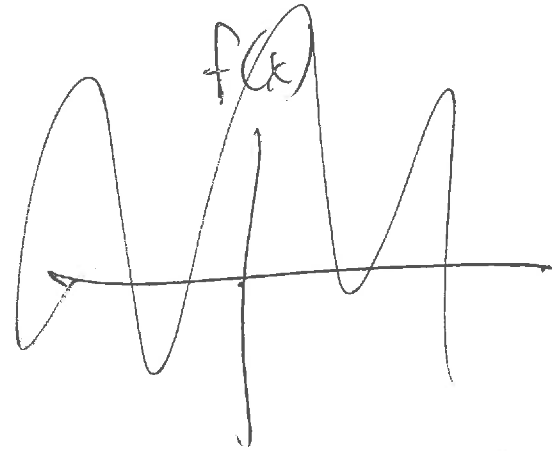




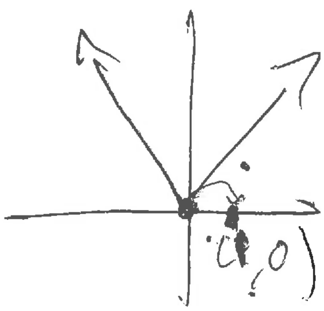
Try $f(x) = -|x-1|+3$

$g(x) = -\sqrt{x}$

$h(x) = \sqrt{-x}$

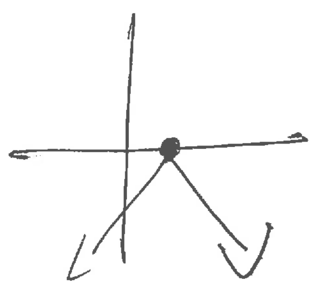


$|x|$

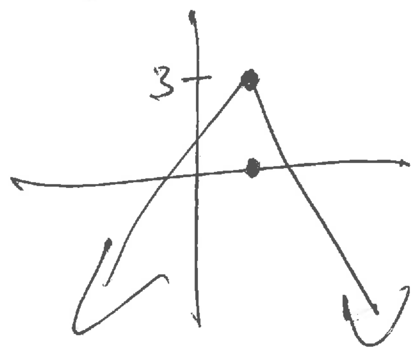


$f(x) = -|x-1|+3$

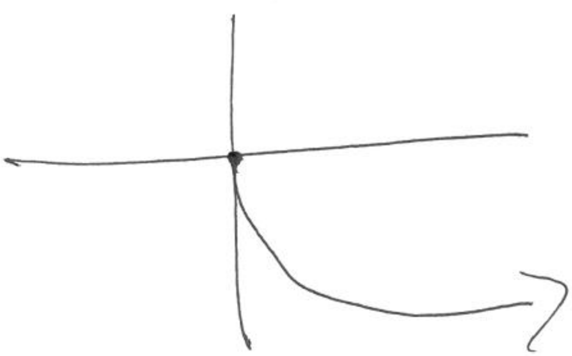
$-|x-1|$



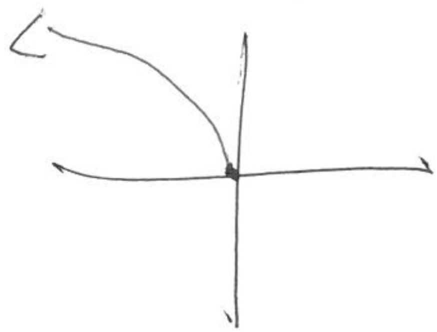
$-|x-1|+3$



$g(x) = -\sqrt{x}$

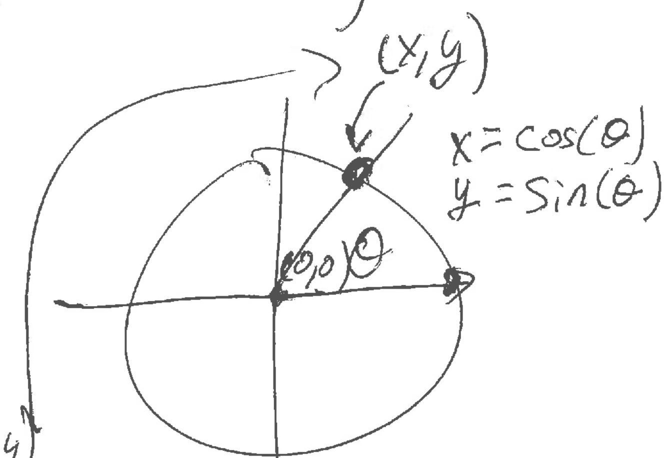


$h(x) = \sqrt{-x}$



1.3 Trig Functions

Assuming
- ↳ Know unit circle
- ↳ Know sin, cos, tan, csc, sec, cot and can evaluate along unit circle
- ↳ Know right triangle trig (SOHCAHTOA)



Pythagorean Identities

↳ $\cos^2(\theta) + \sin^2(\theta) = 1$

Distance from (0,0) to (x,y)

Divide by $\cos^2(\theta)$:

$$1 + \frac{\sin^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)}$$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

Divide by $\sin^2(\theta)$:

$$\frac{\cos^2(\theta)}{\sin^2(\theta)} + 1 = \frac{1}{\sin^2(\theta)}$$

$$\cot^2(\theta) + 1 = \csc^2(\theta)$$

Angle-Sum Formulas (Memorize)

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$
$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

Ex Evaluate $\sin(\pi/12)$

Observe: $\frac{\pi}{4} - \frac{\pi}{6} = \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{12}$

$$\sin\left(\frac{\pi}{12}\right) = \sin(\pi/4 - \pi/6) =$$
$$\sin(\pi/4)\cos(\pi/6) - \cos(\pi/4)\sin(\pi/6)$$
$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

Claim $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

Pf $\sin(2\theta) = \sin(\theta + \theta)$
$$= \sin(\theta)\cos(\theta) + \cos(\theta)\sin(\theta)$$
$$= 2\sin(\theta)\cos(\theta). \quad \square$$

Claim $\cos(2\theta) = 2\cos^2(\theta) - 1$

Pf $\cos(2\theta) = \cos(\theta + \theta)$
$$= \cos(\theta)\cos(\theta) - \sin(\theta)\sin(\theta)$$
$$= \cos^2(\theta) - \sin^2(\theta)$$

Recall $\sin^2(\theta) + \cos^2(\theta) = 1$; so
$$\sin^2(\theta) = 1 - \cos^2(\theta)$$

So: $\cos^2(\theta) - \sin^2(\theta)$

$$= \cos^2(\theta) - (1 - \cos^2(\theta))$$
$$= \cos^2(\theta) - 1 + \cos^2(\theta)$$
$$= 2\cos^2(\theta) - 1. \quad \square$$

<u>Claim</u> $\cos(\theta) = \pm\sqrt{\frac{1+\cos(2\theta)}{2}}$ (+ or − depending on which quadrat θ is in).

<u>Pf</u> By the previous claim, we have: $\cos(2\theta) = 2\cos^2(\theta) - 1$.

So: $2\cos^2(\theta) = \cos(2\theta) + 1$

$$\cos^2(\theta) = \frac{\cos(2\theta)+1}{2}$$

$$\cos(\theta) = \pm\sqrt{\frac{\cos(2\theta)+1}{2}}. \quad \square$$

<u>Ex</u> $\cos(\pi/8) = \sqrt{\frac{\cos(2\cdot\frac{\pi}{8})+1}{2}} = \sqrt{\frac{\cos(\frac{\pi}{4})+1}{2}}$

$$= \sqrt{\frac{1}{2}\left(\frac{\sqrt{2}}{2}+1\right)}$$