Study Guide 3.1-3.5

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Instructions: Complete the following problems. Justify all your answers in complete sentences, where appropriate.

1 Sections 3.1-3.2

You may freely use the following theorem, without proof.

Theorem 1.1. $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$.

Problem 1) Using Theorem 1.1, prove that:

$$\lim_{x \to 0} \frac{1 - \cos(x)}{x} = 0.$$

Problem 2) Use the limit definition to determine the derivative of the following functions.

(a) $f(x) = \sqrt{x+3}$

(b) $f(x) = \sin(x)$

(c) $f(x) = \cos(x)$

(d) $f(x) = \frac{1}{x}$

(e) $f(x) = \frac{x}{x-1}$

(f) $f(x) = x^2$

(g) $f(x) = x^3$

(h) $f(x) = x^n$, where n is a positive integer. [**Hint:** Use the Binomial Theorem, which you learned in recitation on 2/14.]

2 Sections 3.3-3.5

Problem 3) Evaluate the following derivatives.

(a) $4 - x^2$

(b) $\frac{1}{t^2}$.

(c) $\frac{1-z}{2z}$.

(d) $\sqrt[5]{x}$.

(e) $2x^5 + e^x$.

(f) $f(x) = x^2(x^3 + 5)$. [Do this in two ways- (1) by using the product rule; and (2) by multiplying through. Do you get the same result? Should you?]

(g) $f(x) = (2x - 1)(3x + 2)$.

(h) $f(t) = te^{-2t}$

(i) $f(t) = \dfrac{t}{e^{2t}}$

(j) $g(x) = 5x \cdot \exp(x^2)$. [Note: $\exp(u) = e^u$].

(k) $R(q) = 3qe^{-q}$.

(l) $f(z) = \sqrt{z}e^{-z}$

(m) $w(y) = \dfrac{3y + y^2}{5 + y}$

(n) $z(t) = \dfrac{1 - t}{1 + t}$

(o) $y(z) = \dfrac{1 + x}{\ln(x)}$

(p) $f(x) = (x + 1)^{99}$

(q) $f(x) = (x^3 + x^2)^{-99}$

(r) $f(x) = \sqrt[6]{x^3 + 1}$

(s) $f(x) = \sqrt{2 + \sqrt{x}}$

(t) $3\sec(x) - 10\cot(x)$

(u) $5\sin(x)\cos(x) + 4\csc(x)$

(v) $\dfrac{\sin(t)}{3 - 2\cos(t)}$.

Problem 4) Suppose that $f(x)$ is differentiable at the point $x = c$. Prove that $f(x)$ is continuous at $x = c$. [**Hint:** Show that $\lim_{h \to 0} f(c + h) = f(c)$. To start, observe that $f(c + h) = f(c) + (f(c + h) - f(c))$.]

Problem 5) You may assume only the following:

- The power rule for derivatives holds for positive integers. That is, if n is a positive integer, then the derivative of $f(x) = x^n$ is nx^{n-1}.
- The quotient rule for derivatives.

Prove the following statement. If n is a positive integer, then the derivative of $f(x) = x^{-n}$ is $f'(x) = -nx^{-n-1}$. [**Hint:** How can you write x^{-n} in a way that suggests using the quotient rule?]