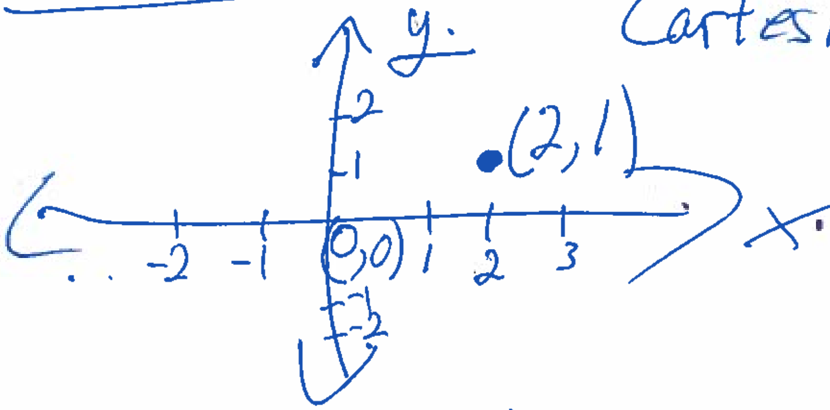
Cartesian Product



Cartesian plane

$(2, 1)$

x-coord

y-coord

$(2,1)$ vs. $(1,2)$

$\mathbb{R} \times \mathbb{R} = \{ (x,y) \mid x \in \mathbb{R}, y \in \mathbb{R} \}$

set of real numbers

x is an element of $\mathbb{R}$

x is a real number

x belongs to the set of real #'s

Cartesian Product

Let A and B be sets.

The Cartesian Product

$$A \times B = \{ (a,b) \mid a \in A,\ b \in B \}$$

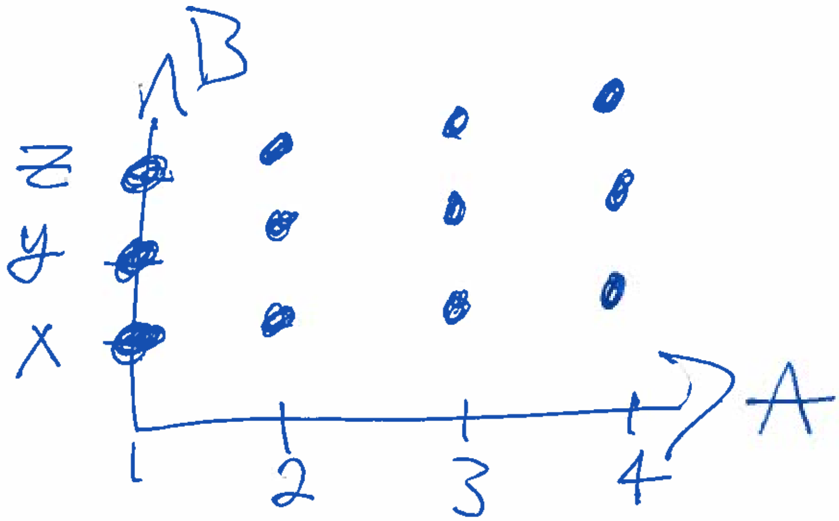
Ex Let $A = \{1, 2, 3, 4\}$

$B = \{x, y, z\}$

$$A \times B = \{ (1,x), (1,y), (1,z), (2,x), (2,y), (2,z), (3,x), (3,y), (3,z), (4,x), (4,y), (4,z) \}$$

$$\boxed{n(A \times B) = n(A) * n(B) = 12}$$

Rule of Product



<u>Ex</u> Suppose we roll two distinguishable 6-sided dice, one red and one green. What is the sample space (or set of possible outcomes)?

$\{(1,1), (1,2), (1,3), \ldots \}$

$R = \{1,2,3,4,5,6\}$

$G = \{1,2,3,4,5,6\}$

$R \times G$ is the sample space

<u>Ex</u> How many binary strings of length 5 are there?

↳ 00000

10101

✗ 21111

$(\_\_, \_\_, \_\_, \_\_, \_\_)$

$$\underbrace{\{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\}}$$

$= \{0,1\}^5$ (This is the set of binary strings of length 5)

$$n(\{0,1\}^5) = n(\{0,1\}) \cdot n(\{0,1\}) \cdot n(\{0,1\}) \cdot n(\{0,1\}) \cdot n(\{0,1\}) = 2^5 = 32$$

Ex ~~#~~ A SSN is a sequence of 7 digits, where each digit is drawn $\{0, 1, \ldots, 9\}$.

a) How many SSNs?

$$(\_\_, \_\_, \_\_, \_\_, \_\_, \_\_, \_\_)$$
$$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$$

Subset Let A and B be sets. We say that A is a subset of B if every element of A is contained in B.

Ex Let $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$, $C = \{1, 2, 4\}$.

So A is a subset of B ($A \subseteq B$)

$C \subseteq B$

$A \nsubseteq C$ (A is not a subset of C)

$C \nsubseteq A$

6.2 Set Cardinality

Rule of Product Let A and B be finite sets,

$$n(A \times B) = n(A) \cdot n(B)$$

Rule of Sum For finite, disjoint sets A and B,

$$n(A \cup B) = n(A) + n(B).$$

Ex Let $A = \{1, 2, 3, 4\}$, $B = \{6, 7, 8\}$

$$n(A \cup B) = n(A) + n(B)$$
$$= 4 + 3 = 7$$

Question What if A and B are not disjoint?

Ex Let $A = \{a, b, c, d\}$, $B = \{b, c, d, e, f\}$

$$A \cup B = \{a, b, c, d, e, f\}$$

$$n(A \cup B) = 6 = n(A) + n(B) - n(A \cap B)$$
$$4 + 5 - 3$$

This counts b, c, d

Note $A \cap B = \{b, c, d\}$

Ex Amazon has 132,000 cookbooks.

↳ 20,000 were regional cookbooks,

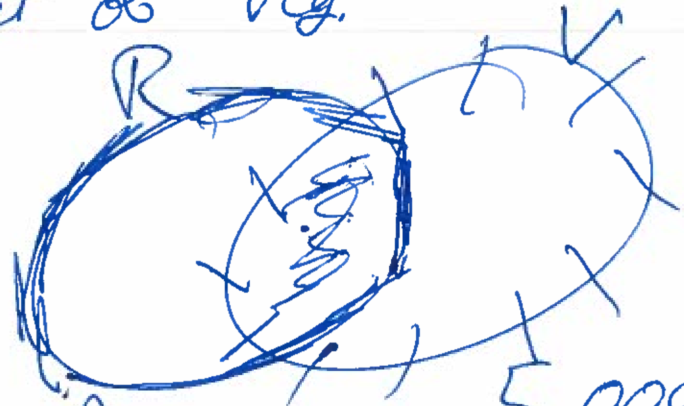
↳ 5,000 were on vegetarian recipes

↳ 24,000 were on regional or vegetarian (or both)

Q How many of these 132,000 cookbooks were on ~~neither~~ both regional ~~or~~ and vegetarian cooking?

Let R be the Set of reg.

V be the Set of veg.



$n(R) = 20{,}000$

$n(V) = 5{,}000$

$n(R \cup V) = 24{,}000$

$n(R \cup V) = n(R) + n(V) - n(R \cap V)$ regional & veget. counted

$24{,}000 = 20{,}000 + 5{,}000$

What we want

$24{,}000 = 20{,}000 + 5{,}000 - \underbrace{n(R \cap V)}_{1000}$

So there are 1,000 cookbooks that are <u>both</u> reg. and veget.