6.4 (cont.) Combinations

Distinct elements
Order does not matter

Counting Subsets How many 3-elem. subsets of $\{1,2,3,4\}$?

A: 4 subsets $\{1,2,3\}$ $\{1,2,4\}$ $\{1,3,4\}$ $\{2,3,4\}$

Given n-elem set, how many k-elem subsets exist?

Binomial Coefficient Let $0 \le k \le n$, (k, n are integers)

The binomial coefficient $\binom{n}{k} = \dfrac{n!}{k! \cdot (n-k)!}$

Pronounced "n choose k"

Notation $\binom{n}{k} = C(n,k)$

$n! = n \cdot (n-1) \cdot (n-2) \cdots 1$

$4! = 4 \cdot \underbrace{3 \cdot 2 \cdot 1} = 4 \cdot 3!$

$3! = 3 \cdot 2 \cdot 1$

Ex 3-elem subsets of $\{1,2,3,4\}$

$$\binom{4}{3} = \frac{4!}{3!(4-3)!} = \frac{4!}{3! \cdot 1!} = \frac{4!}{3!} = \frac{4 \cdot \cancel{3!}}{\cancel{3!}} = 4$$

Ex Evaluate $\binom{20}{12}$

$$\binom{20}{12} = \frac{20!}{12! \cdot 8!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot \cancel{12!}}{\cancel{12!} \cdot 8!}$$

Ex Pick-6 lotto with 55 numbered balls

(Pick 6 distinct balls, no repeats)

Order balls chosen does ~~not~~ matter

$$\binom{55}{6} = \underline{C(55, 6)} = \frac{55!}{6! \cdot (55-6)!}$$

Look for Combination on calculator $= \dfrac{55!}{6! \cdot 49!}$

Ex 15 player pool. Select 5 player team.

(Not distinct positions)

$\binom{15}{5}$

Suppose Strongest player on team.

$\{\underline{S}, \_, \_, \_, \_\}$

14 players left $\binom{14}{4}$

4 pos to fill

Poker Hands 52 playing cards

↳ 13 values/ranks (Ace, 2, 3, . . .)
↳ 4 suits (Heart, Diamond, Spade, Club)

A poker hand is 5-card hand, where order does not matter.

a) How many poker hands? $\binom{52}{5}$

b) A full-house 3 cards of one rank, and 2 cards of a second rank.
How many full-houses with 3-queens and 2 10's?

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$\binom{4}{3} = 4$ ways to choose Queens

$\binom{4}{2} = \frac{4!}{2! \cdot 2!} = \frac{4 \cdot 3}{2} = 6$ ways of choosing 10's

$\binom{4}{3}\binom{4}{2} = 4 \cdot 6$ Full houses with 3 queens, 2 10's

10H, 10D   10D, 10S
10H, 10S   10D, 10C
10H, 10C   10S, 10C

c) Total # full houses

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R_1

$\binom{13}{1} = 13$ ways to sel. first rank (Q)

$\binom{13}{1}\binom{4}{3}$ of sel. 3 cards of first rank

R_2

$\binom{12}{1} = 12$ ways of sel. second rank (10's)

$\binom{4}{2}$ ways to sel. 2-cards of rank R_2

$\binom{12}{1}\binom{4}{2}$ ways of cards for second rank

~~Total # full houses~~

$$\underset{\text{Sel. } R_1}{\binom{13}{1}}\underset{\text{cards for } R_1}{\binom{4}{3}}\underset{\text{sel } R_2}{\binom{12}{1}}\underset{\text{Sel cards for } R_2}{\binom{4}{2}} = 13 \cdot 4 \cdot 12 \cdot 6$$