Instructions: This quiz is closed book, closed note, and an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places.

(6 pts) 1) Find **all** the local maxima and local minima of the differentiable function $f(x)$, given its derivative:

$$f'(x) = x(x+5)^2(x+4)^3(x-3).$$

Clearly justify, in complete sentences, why the relevant points are maxima or minima. For your convenience, I have included the table for the first derivative test below:

	$(x+5)^2$	$(x+4)^3$	x	$(x-3)$	$f'(x)$
$x < -5$	+	-	-	-	-
$-5 < x < -4$	+	-	-	-	-
$-4 < x < 0$	+	+	-	-	+
$0 < x < 3$	+	+	+	-	-
$x > 3$	+	+	+	+	+

Answer: We fill in a cell in the table with + if the given factor is positive on the specified interval, and we fill the cell with $-$ if the factor is negative on the specified interval. So for example, $(x+4)^3 < 0$ if $x < -5$ (to see this, take $x = -6$ as an example), so we fill in the corresponding cell with $-$.

Now the factors of a given row are multiplied together. So for the first row $x < -5$, we have that:

- $(x+5)^2$ is positive,
- $(x+4)^3$ is negative,
- x is negative, and
- $(x-3)$ is negative.

Thus, $f'(x)$ is negative when $x < -5$. The logic to fill in the remaining rows is similar.

Now to identify a local min, we look at critical points where $f'(x)$ changes from $-$ to $+$. This occurs at $x = -4$ and $x = 3$. Similarly, to identify a local max, we look at critical points where $f'(x)$ changes from $+$ to $-$. This occurs at $x = 0$. Note that $x = -5$ is **neither** a local max nor a local min, as $f'(x)$ is $-$ to the left and to the right of $x = -5$.

Common Error: Note that local maxima and minimal occur at critical points, such as $x = -5$. A number of folks indicated that a local maximum or local minimum occurred, for instance, at the interval $-5 < x < -4$, which is **not correct.**

A number of folks also tried to evaluate $f'(x)$ at specific numbers. The point of the table is to avoid dealing with large numbers. Is $(x+5)$ positive or negative when $x < -5$? Is $(x-3)$ positive or negative when $x > 3$? These are the questions you should be asking.

(4 pts) 2) Find the constants a, b such that the parabola $f(x) = x^2 + ax + b$ has a minimum at $(3, 4)$.

Answer: We first note that $f'(x) = 2x + a$. Now as $f(x)$ has a local minimum at $(3, 4)$, we have that $f'(3) = 2(3) + a = 0$. So $a = -6$. So we have that $f(x) = x^2 - 6x + b$. Next, we use the fact that $f(3) = 4$ to solve for b. Thus:

$$f(2) = 3^2 - 6(3) + b = 4.$$

Solving for b, we have that, $b = 13$.