Math 170- Study Guide 7.1-7.3

Instructions: Answer all questions. Clearly justify your reasoning.

1 Section 7.1

Problem 1) Describe elements of the given sample space, as well as the elements of the specified event. Unless otherwise specified, assume that all coins and dice are distinguishable.

1. Two coins are tossed; the result is at most one tail.

   Answer: The sample space $S = \{(H, H), (H, T), (T, H), (T, T)\}$. Our event $E = \{(H, H), (H, T), (T, H)\}$.

2. Two coins are tossed; the result is one or more heads.
3. Two dice are rolled; the numbers add up to 5.
4. A letter is chosen at random from *Mozart*; the letter is not an a or M.

   Answer: The sample space is $S = \{M, o, z, a, r, t\}$. The event is $E = \{o, z, r, t\}$.

Problem 2) Suppose two distinguishable dice (one red, one green) are rolled. Consider the following events:

- A: The red die shows 1.
- B: The numbers add to 4.
- C: At least one of the numbers is 1.
- D: The numbers do not add to 11.

For the following problems, express the given event in symbols (that is, express the event in terms of A, B, C, or D, and the basic set operations of union, intersection, and complementation). Then determine the number of elements in each event.

1. The red die shows 1, and the numbers add to 4.
2. The red die shows 1, but the numbers do not add to 11.
3. The numbers do not add to 4.
4. At least one of the numbers is 1, or the numbers add up to 4.

Problem 3) Consider the following table, where the event is labeled with the row or column title (e.g., S is the event for Successful Authors).

	New Authors (N)	Published Authors (E)	Total
Successful (S)	5	25	30
Unsuccessful (U)	15	55	70
Total	20	80	100

Answer the following questions about the table.

(a) Describe the events $S \cup N$ and $S \cap N$ in words. Use the table to compute $n(S \cup N)$ and $n(S \cap N)$.

Answer: The event $S \cup N$ is the set of authors that are successful, new, or both. We note that $n(S \cup N) = 5 + 15 + 25 = 45$. The event $S \cap N$ is the set of authors that are both successful and new. We note that $n(S \cap N) = 5$.

(b) Which of the following pairs of events are mutually exclusive: N and E; N and S; S and E? Justify your answer.

Answer: We note that an author can be both new and successful, as well as established and successful. So N and S, and S and E are not mutually exclusive pairs of events. Now an author cannot be both new and established. So N and E are mutually exclusive.

(c) Which of the following pairs of events are mutually exclusive: U and E; U and S; S and N? Justify your answer.

(d) Describe $S \cap N'$ in words and find the number of elements $S \cap N'$ contains.

Problem 4) The seven contenders in the fifth horse race at Aqueduct on Debruary 17, 2002 were: Pipe Bomb, Expect a Ship, All Tht Magic, Electoral College, Celera, Cliff Glider, and Inca Halo. You are interested in the first three places (winner, second place, third place) for the race.

(a) A *finish* for the race consists of the first, second, and third place winners. Let S be the set of all possible finishes. Determine $n(S)$. Justify your answer. [**Hint:** Use the counting techniques from 6.4.]

(b) Let E be the event that Electoral College is in second or third place, and let F be the event that Celera is the winner. Express $E \cap F$ in words, and determine $n(E \cap F)$.

2 Section 7.2

Problem 5) Calculate the probability of the specified event E based on the relative frequencies.

(a) Eight hundred adults are polled, and 640 of them support universal healthcare coverage. Denote E as the event that an adult supports universal healthcare coverage.

Answer: $\Pr[E] = 640/800$.

(b) Eight hundred adults are polled, and 640 of them support universal healthcare coverage. Denote E as the event that an adult **does not** support universal healthcare coverage.

Answer: $\Pr[E'] = 1 - (640/800) = 160/800$.

(c) A die is rolled 60 times with the following results: $1, 2$, and 3 each come up 8 times; and $4, 5$, and 6 each come up 12 times. Let E be the event that the number which comes up is at most 4.

Problem 6) In a survey of Latin music downloads, 200 were regional, 130 were pop-rock, 45 were tropical, and 25 were urban. Calculate the following relative frequencies.

(a) That a music download was pop/rock.

(b) That a music download was neither tropical nor regional.

(c) That a music download was not regional.

Problem 7) A random sampling of chicken in supermarkets revealed that approximately 80% was contaminated with the organism *Campylobacter.* Of the contaminated chicken, 20% had the strain resistant to antibiotics. Construct a relative frequence distribution show the following outcomes for when a chicken is purchased at a supermarket:

- U: The chicken is not infected with Campylobacter.
- C: The chicken is infected with non-resistant Campylobacter.
- R: The chicken is infected with resistant Campylobacker.

3 Section 7.3

Problem 8) An experiment is given together with an event. Assuming that the coins and dice are both distinguishable and fair, compute the probabilities.

(a) Three coins are tossed. The result is at most one head.

(b) Three coins are tossed. The result is more tails than heads.

(c) Two dice are rolled; the numbers add to 5.

(d) Two dice are rolled; one of the numbers is even, and the other is odd.

Problem 9) Use the given information to find the indicated probability.

(a) $\Pr[A] = 0.1, \Pr[B] = 0.6, \Pr[A \cap B] = 0.05$. Find $\Pr[A \cup B]$.

(b) $A \cap B = \emptyset$, $\Pr[A] = 0.3, \Pr[A \cup B] = 0.4$. Find $\Pr[B]$.

(c) $\Pr[A] = 0.22$. Find $\Pr[A']$.

Problem 10) A die is weighted in such a way that each of $2, 4$, and 6 is twice as likely to come up as $1, 3$, and 5. Find the probability distribution. Then determine the probability of rolling less than 4.

Answer: Let Y denote the random variable associated with rolling the die. Let x denote the probability of rolling a 1. So we have that $x+2x+x+2x+x+2x = 9x = 1$. Thus, $x = 1/9$. So $\Pr[Y < 4] = x+2x+x = 4/9$.

Problem 11) A tetrahedral die has four faces, numbered $1 - 4$. Suppose the die is weighted such that rolling 1 is twice as likely as rolling 2; rolling 2 is twice as likely as rolling 3; and rolling 3 is twice as likely as rolling 4. Determine the probability distribution.