5.2 Graphical Methods for solving Linear Programs in 2 vars

A linear program is an optimization problem.
In two vars, we seek to either maximize or minimize a function $f(x,y)$ subject to linear constraints.

↳ Constraints are linear inequalities.

Ex
$$3x - 2y \geq 0$$
$$x + 2y \leq 10$$
$$x \geq 0$$

Ex maximize $x+y$

Constraints:

$2x + y \leq 12$ ($y \leq 12 - 2x$)

$x + 2y \leq 12$ ($y \leq 6 - \frac{x}{2}$)

$x \geq 0$

$y \geq 0$

max $x+y$ s.t. (such that/subject to)

$2x + y \leq 12$

$x + 2y \leq 12$

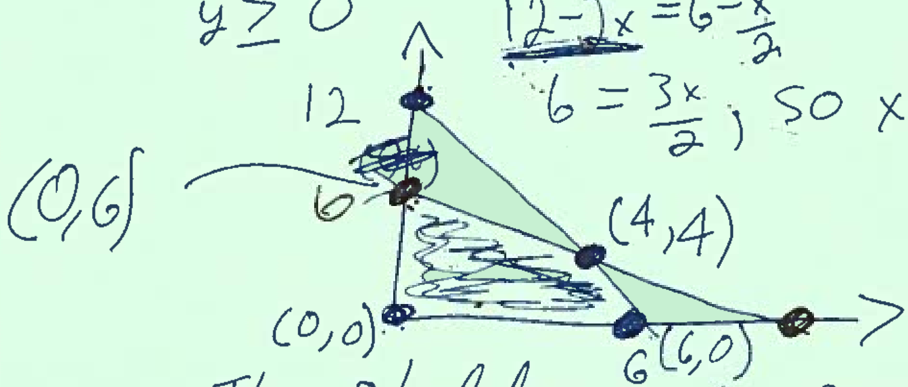
$x \geq 0$

$y \geq 0$

$12 - 2x = 6 - \frac{x}{2}$

$6 = \frac{3x}{2}$, so $x = 6\left(\frac{2}{3}\right) = 4$

$12 - 2(4) = 4$



The shaded region is search space/feasible region

Want to max $x+y$.

Point	$x+y$
$(0,0)$	0
$(0,6)$	$0+6=6$
$(6,0)$	$6+0=6$
$(4,4)$	$4+4=8$

So the largest value $x+y$ takes on in feasible region is 8.

The maximizer is $(4,4)$.

<u>Ex</u> Two types of Apple Juice

↳ Type X: 30 oz water, 2 oz Concentrate, profit of \$0.20/unit.

↳ Type Y: 20 oz water, 12 oz Concentrate, profit of \$0.30

↳ Constraints: 30,000 oz water, 3600 oz concentrate.

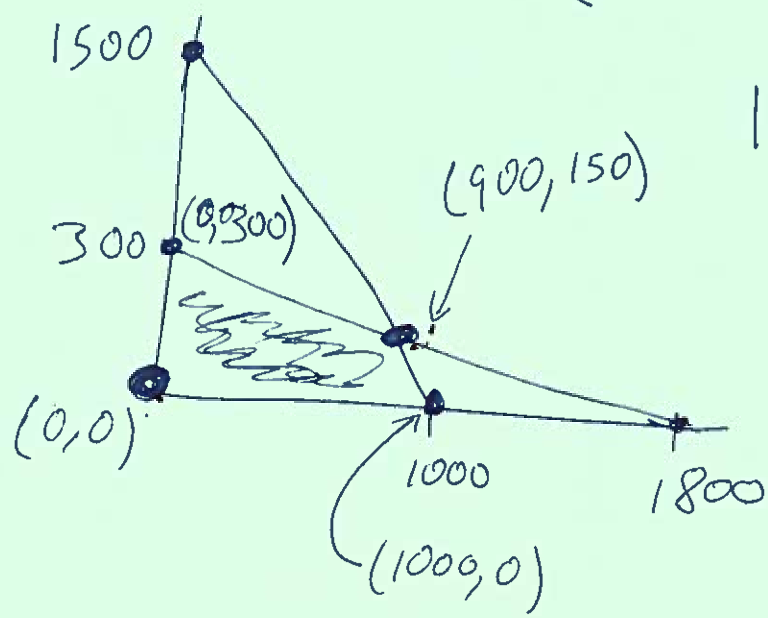
<u>Goal</u> Maximize Profit.

<u>Profit function</u>: $20x+30y$

Solve max $20x + 30y$ s.t.

$$30x + 20y \leq 30000 \quad \text{(Water Constraint)}$$
$$2x + 12y \leq 3600 \quad \text{(Concentrate)}$$
$$x \geq 0,$$
$$y \geq 0.$$

$$30x + 20y \leq 30{,}000 \quad \left(y \leq 1500 - \tfrac{3}{2}x\right)$$
$$2x + 12y \leq 3600 \quad \left(y \leq 300 - \tfrac{x}{6}\right)$$



$$1500 - \tfrac{3}{2}x = 0$$
$$1500 = \tfrac{3}{2}x$$
$$x = \tfrac{2}{3}(1500) = 1000$$

Set $1500 - \frac{3}{2}x = 300 - \frac{x}{6}$

$$1500 - \frac{9x}{6} = 300 - \frac{x}{6}$$

$$1200 = \frac{8}{6}x = \frac{4}{3}x$$

So $x = 900$ and $y = 1500 - \frac{3}{2}(900) = 150$

Recall We want to max $20x + 30y$

Point	$20x+30y$
$(0,0)$	0
$(0,300)$	$0 + 30(300) = 9000$
$(1000,0)$	$20(1000)+0 = 20,000$
$(900,150)$	$20(900) + 30(150) = 22,500$

So max value of $20x+30y$ in feasible region is 22,500. This occurs at $(900, 150)$

Sell 900 units type X
150 units of type Y

Profit \$225