7.6 Bayes' Law

Recall Let A, B be events. Then:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$\Pr(A \cap B)$:

$\Pr(B|A)\Pr(A)$

How likely is it that event A occurs? I.e., how likely is it that we end up?

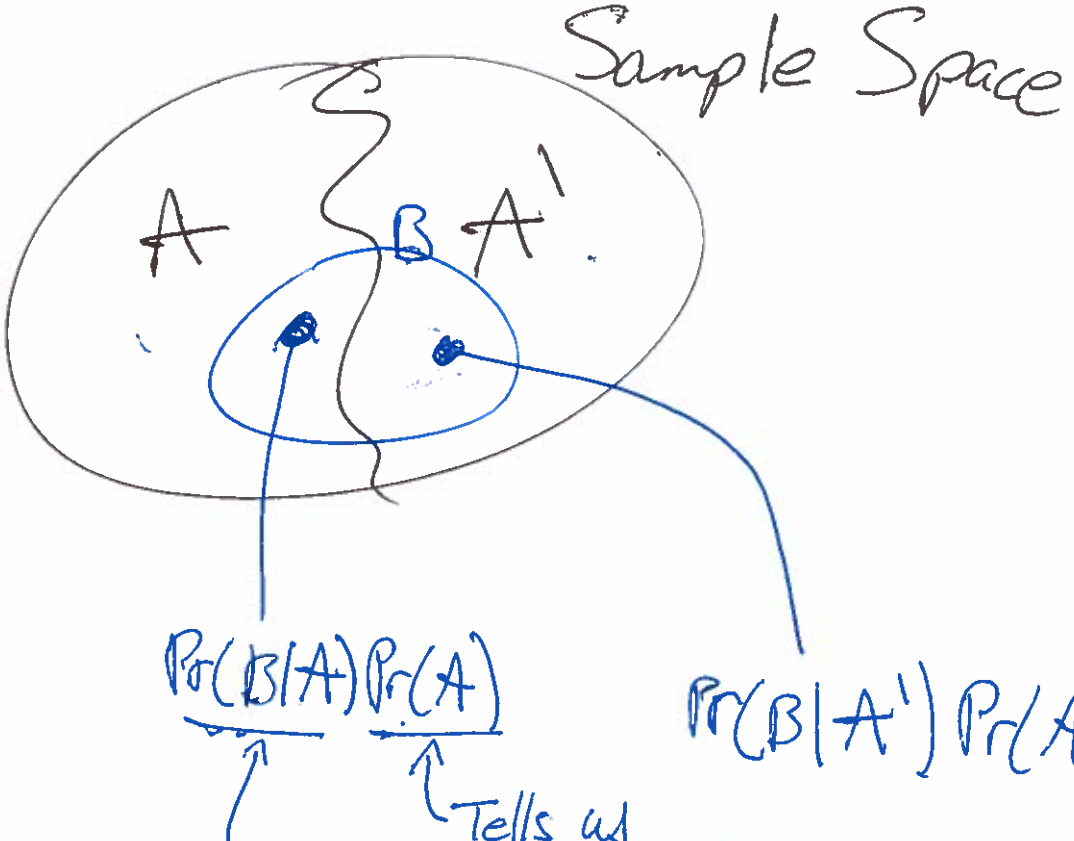
Given that A occurred, how likely is it that B occurred?

$$\Pr(A \cap B) = \Pr(B|A)\Pr(A)$$

So $$\Pr(A|B) = \frac{\Pr(B|A)\Pr(A)}{\Pr(B)}$$

Observe ~~$\Pr(B) \quad \Pr(A|B)$~~.

$$\Pr(B) = \Pr(B|A)\Pr(A) + \Pr(B|A')\Pr(A')$$



$$\Pr(B) = \Pr(B|A)\Pr(A) + \Pr(B|A')\Pr(A')$$

Bayes' Law $$\Pr(A|B) = \frac{\Pr(B|A)\Pr(A)}{\Pr(B|A)\Pr(A) + \Pr(B|A')\Pr(A')}$$.

Ex Test (Outcomes Positive, Negative)

↳ Has 0.95 prob of giving positive result when person has disease ($Pr(Pos|Dis)$)

↳ Has 0.1 probability of giving pos. result when person does not have disease

↳ Prob of disease is 0.005

Events

↳ Test Pos: $Pr(Pos) = \overbrace{Pr(Pos|Dis)Pr(Dis)}^{\text{Actual/Truthful Pos.}} + Pr(No\,Dis)Pr(Pos|Not\,Dis)$

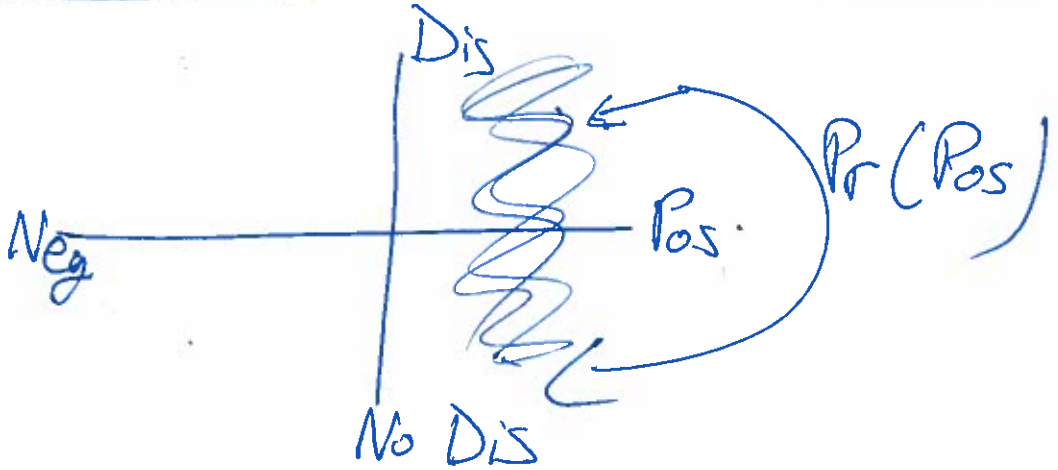
↳ Test Neg

↳ Have Disease: $Prob(Disease) = 0.005$

↳ Not Have Disease: $Prob(No\,Disease) = 0.995$

$$Pr(Pos) = Pr(Pos|Dis)Pr(Dis) + Pr(Pos|No\,Dis)Pr(No\,Dis)$$

$$= 0.95 \cdot .005 + 0.1 \cdot 0.995$$



$$Pr(Neg) = 1 - Pr(Pos)$$

b) $\Pr(Dis \mid Pos)$

Recall $\Pr(Pos \mid Dis) = 0.95$.

$\Pr(Dis) = 0.005$

$\Pr(Pos) = 0.95(.005) + 0.1(0.995)$

By Bayes' Law

$$\Pr(Dis \mid Pos) = \frac{\Pr(Pos \mid Dis)\Pr(Dis)}{\Pr(Pos)}$$

$$\Pr(Dis \mid Pos) = \frac{0.95(.005)}{0.95(.005) + 0.1(0.995)}$$

↑ You can leave as-is on quizzes/exams

$Pr(\text{No Dis} \mid \text{Neg})$

↳ Recall: $Pr(\text{No Dis}) = 0.995$

$Pr(\text{Neg}) = 1 - [0.95(0.005) + 0.1(0.995)]$

$Pr(\text{Neg} \mid \text{No Dis})$

(i) Determine $Pr(\text{Neg} \mid \text{No Disease})$

↳ Recall $Pr(\text{Pos} \mid \text{Disease}) = 0.95$

↳ $Pr(\text{Pos} \mid \text{No Disease}) = 0.1$

So $Pr(\text{Neg} \mid \text{No Dis}) = 0.9$

(ii) $Pr(\text{No Dis} \mid \text{Neg})$ = Bayes' Law

$$= \frac{Pr(\text{Neg} \mid \text{No Dis}) \cdot Pr(\text{No Dis})}{Pr(\text{Neg})}$$

↳ $Pr(\text{No Dis}) = 0.995$

↳ $Pr(\text{Neg}) = 1 - [Pr(\text{Pos})]$

↳ $Pr(\text{Neg} \mid \text{No Dis}) = 0.9$

$$= \frac{0.9(0.995)}{1 - Pr(\text{Pos})}$$

Fill in the #

Ex Test for Swine Flu

↳ $Pr(\text{Swine Flu}) = \frac{1}{10,000}$

~~↳ $Pr(\text{Swine Flu} \mid \text{Pos}) = 0.99$~~

~~↳ $Pr(\text{No Swine Flu})$~~

↳ $Pr(\text{Pos} \mid \text{Swine Flu}) = 0.99$

↳ $Pr(\text{Pos} \mid \text{No Swine Flu}) = 0.01$.

Determine $Pr(\text{Swine} \mid \text{Pos})$

By Bayes' Law $Pr(\text{Swine} \mid \text{Pos}) = \frac{Pr(\text{Pos} \mid \text{Swine})\, Pr(\text{Swine})}{Pr(\text{Swine})\, Pr(\text{Pos} \mid \text{Swine}) + Pr(\text{Pos} \mid \text{NS})\, Pr(\text{NS})}$

$= \frac{0.99\left(\frac{1}{10,000}\right)}{Pr(\text{Pos} \mid \text{Swine})\, Pr(\text{Swine}) + Pr(\text{Pos} \mid \text{NS})\, Pr(\text{NS})}$

$0.99 \cdot \frac{1}{10000} \qquad 0.01\left(\frac{9999}{10,000}\right)$

So $Pr(\text{Swine} \mid \text{Pos}) = \frac{(.99/10,000)}{(.99/10,000) + \left(\frac{9999}{10000}\right) \cdot .01}$