# Math 122 Sections 6.4 Example

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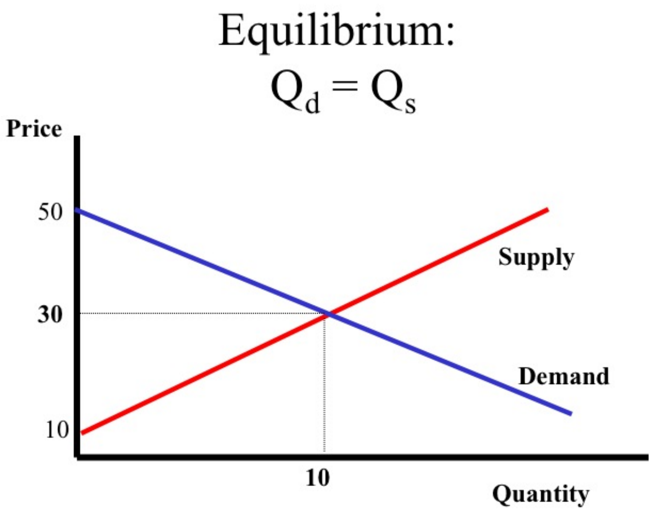
April 12, 2018

Recall the market equilibrium model. We have:

- A producer supply function $S(p)$, where $p$ is the price and $S(p)$ is the amount of the good supplied.
- A consumer demand function $Q(p)$, where $p$ is the price and $Q(p)$ is the amount of good demanded.

The *equilibrium price* is the value $p^*$ such that $Q(p) = S(p)$, and the equilibrium quantity $q^*$ is the value of $Q(p^*)$ (or equivocally, $S(p^*)$, as $Q(p^*) = S(p^*)$).

When graphing the supply and demand functions, the $x$-axis is labeled with quantity and the $y$-axis is labeled with price. Below is an example of such a graph.




There are a couple ways to compute the producer and consumer surplus. Note that:

- The *consumer surplus* is the area under the demand curve and above the line $y = p^*$.
- The *producer surplus* is the area above the supply curve and below the line $y = p^*$.
- Suppose we have the *inverse demand function* $p(q)$. Then the consumer surplus is given by:

$$\int_0^{q^*} p(q)\,dq - p^*q^*$$

  Equivocally, let $p_1 = p(0)$ be the intercept of the demand function along the $y$-axis. Then consumer surplus is given by:

$$\int_{p^*}^{p_1} Q(p)\,dp.$$

- Suppose we have the *inverse supply function* $S^{-1}(q)$. Then the producer surplus is given by:

$$p^*q^* - \int_0^{q^*} S^{-1}(q)\,dq.$$

Equivocally, let $p_0 = S^{-1}(0)$ by the intercept of the demand function along the $y$-axis. Then the producer surplus is given by:

$$\int_{p_0}^{p^*} S(p)\, dp.$$

**Example 1:** Suppose that $Q(p) = 100 - 2p$ and $S(p) = 3p - 50$. We wish to compute the Producer Surplus and Consumer Surplus. We do so first using the approach in class, integrating with respect to price ($dp$).

- **Step 1:** We find the equilibrium price and quantity. To do so (Recall Section 1.4), we set $Q(p) = S(p)$. So:

$$\begin{aligned} 100 - 2p &= 3p - 50 \\ 150 &= 5p \\ p^* &= 30 \end{aligned}$$

  And so $q^* = Q(p^*) = Q(30) = 40$.

- **Step 2:** We find the $y$-intercepts of the supply and demand functions. To do so, we solve for $p$ for both $Q(p)$ and $S(p)$.

$$\begin{aligned} Q(p) : p &= 50 - \frac{q}{2} \\ S(p) : p &= \frac{50}{3} + \frac{q}{3} \end{aligned}$$

  So $p_1$, the y-intercept of the demand function, is 50. Similarly, the $y$-intercept of the supply function is $p_0 = \frac{50}{3}$.

- **Step 3:** Compute the consumer surplus and producer surplus.

  **Consumer Surplus:**

$$\int_{p^*}^{p_1} Q(p)\, dp = \int_{30}^{50} (100 - 2p)\, dp = 400$$

  **Producer Surplus:**

$$\int_{p_0}^{p^*} S(p)\, dp = \int_{50/3}^{30} (3p - 50)\, dp = \frac{800}{3}$$

**Example 2:** We re-work Example 1, to demonstrate the alternative formulas for computing producer surplus and consumer surplus.

- **Step 1:** We find the equilibrium price and quantity. To do so (Recall Section 1.4), we set $Q(p) = S(p)$. So:

$$\begin{aligned} 100 - 2p &= 3p - 50 \\ 150 &= 5p \\ p^* &= 30 \end{aligned}$$

  And so $q^* = Q(p^*) = Q(30) = 40$.

- **Step 2:** We solve for the inverse supply and inverse demand functions. These are the functions being integrated. To do so, we solve for $p$ for both $Q(p)$ and $S(p)$.

$$\begin{aligned} Q(p) : p &= 50 - \frac{q}{2} \\ S(p) : p &= \frac{50}{3} + \frac{q}{3} \end{aligned}$$

- **Step 3:** We now compute the consumer surplus and producer surplus.

  **Consumer Surplus:**

$$\int_0^{q^*} \left(50 - \frac{q}{2}\right)\, dq - p^*q^* = \int_0^{40} \left(50 - \frac{q}{2}\right)\, dq - 30(40) = 400.$$

  **Producer Surplus:**

$$p^*q^* - \int_0^{q^*} \left(\frac{50}{3} + \frac{q}{3}\right)\, dq = 30(40) - \int_0^{40} \left(\frac{50}{3} - \frac{q}{3}\right)\, dq = \frac{800}{3}.$$

**Remark 1:** In many cases, the supply and demand functions will be linear. So it is possible to determine the producer surplus and consumer surplus by calculating the areas of the appropriate triangles. You are free to adopt this approach, if you would like.

**Remark 2:** In cases where you are given the supply and demand functions in forms such as $p = 30 - q^2$, the approach outlined in Example 2 will make the integration a bit easier. Of course, you are welcome to use either approach, so long as your work is correct.