5.4 Fundamental Thm of Calculus

Ex $G(x) = \int_1^{x^2} \cos(t)\,dt$. **Want** $G'(x)$.

Let $F(t)$ be an anti-derivative of $\cos(t)$.
(That is, $F'(t) = \cos(t)$).

Now $G'(x) = \frac{d}{dx}\left[\int_1^{x^2} \cos(t)\,dt\right] = \frac{d}{dx}\left[F(x^2) - F(1)\right]$

Constant

$= \cos(x^2)\,2x - 0$

$\boxed{= 2x\cos(x^2)}$

Ex $G(x) = \int_{1+3x^2}^{4} \frac{1}{2+e^t}\,dt = -\int_4^{1+3x^2} \frac{1}{2+e^t}\,dt$

Let $F(t)$ be an anti-deriv. of $\frac{1}{2+e^t}$.

So $G'(x) = -\frac{d}{dx}\left[\int_4^{1+3x^2} \frac{1}{2+e^t}\,dt\right] = \frac{-d}{dx}\left[F(1+3x^2) - F(4)\right]$

$= -\left[\frac{1}{2+e^{1+3x^2}}(6x) - 0\right]$

$\boxed{= -6x\left(\frac{1}{2+e^{1+3x^2}}\right)}$

5.5 Integration by Substitution (u-sub)

Ex $\int (x^3+x)^{100}(3x^2+1)\,dx = \int u^{100}\,du$

$u = x^3+x$

$du = (3x^2+1)\,dx$

$(x^3+x)^{100}$ $\quad$ $(3x^2+1)\,dx$

$= \dfrac{u^{101}}{101} + C$

U-sub, indef integrals - Convert back to x.

$\boxed{= \dfrac{(x^3+x)^{101}}{101} + C}$

Ex $\int 2xe^{x^2}\,dx = \int e^u\,du = e^u + C$

$\boxed{= e^{x^2} + C}$

e^{x^2} $\quad$ $2x\,dx$

$u = x^2$

$du = 2x\,dx$

Ex $\int \frac{\ln(x)}{x}\,\cancel{dx} = \int \underbrace{\ln(x)}\underbrace{\left(\frac{1}{x}\right)\cancel{dx}}$

$u = \ln(x)$

$du = \frac{1}{x}\,\cancel{dx}$

$= \int u\,du = \frac{u^2}{2} + C$

$$\boxed{= \frac{(\ln(x))^2}{2} + C}$$

Ex $\int \sqrt{2x+1}\,\cancel{dx} = \frac{1}{2}\int \sqrt{u}\,du$

$u = 2x+1$

$du = 2\,\cancel{dx}$

$\sqrt{2x+1}\ (2dx)$

kills off

$= \frac{1}{\cancel{2}}\left(u^{3/2}\left(\frac{\cancel{2}}{3}\right)\right) + C$

$$\boxed{= \frac{1}{3}(2x+1)^{3/2} + C}$$

Ex $\displaystyle\int x^2 e^{x^3+1}\,dx = \frac{1}{3}\int e^u\,du$

$u = x^3+1$

$du = 3x^2\,dx$

$= \frac{1}{3}e^u + C$

$\boxed{= \frac{1}{3}e^{x^3+1} + C}$

Ex $\displaystyle\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$= -\int \frac{1}{u}\,du = -\ln(|u|) + C$

$\boxed{= -\ln(|\cos(x)|) + C}$

Ex $\displaystyle\int \cot(x)\,dx = \int \frac{\cos(x)}{\sin(x)}\,dx = \int \frac{1}{u}\,du$

$u = \sin(x)$

$du = \cos(x)\,dx$

$= \ln(|u|) + C$

$\boxed{= \ln(|\sin(x)|) + C}$

$$\int \sec(x) \cdot \frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\sec(x)(\sec(x)+\tan(x))}{\sec(x)+\tan(x)}\,dx$$

$$u = \sec(x) + \tan(x)$$

$$du = (\sec(x)\tan(x) + \sec^2(x))\,dx$$

$$= \int \frac{1}{u}\,du = \ln(|u|) + C$$

$$= \ln(|\sec(x) + \tan(x)|) + C$$