Ex $\displaystyle\int_0^{\pi/4} \sqrt{1+\cos(4x)}\,dx$

Recall $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$

$= \cos^2(\theta) - (1-\cos^2(\theta))$

$= 2\cos^2(\theta) - 1$

So: $2\cos^2(\theta) = 1+\cos(2\theta)$

$2\cos^2(2x) = 1+\cos(4x)$

$$\int_0^{\pi/4} \sqrt{1+\cos(4x)}\,dx = \int_0^{\pi/4} \sqrt{2\cos^2(2x)}\,dx$$

$$= \int_0^{\pi/4} \sqrt{2}\cdot\sqrt{\cos^2(2x)}\,dx = \sqrt{2}\int_0^{\pi/4} \sqrt{\cos^2(2x)}\,dx$$

$(ab)^{1/2} = a^{1/2}b^{1/2}$

$$= \sqrt{2}\int_0^{\pi/4} |\cos(2x)|\,dx = \sqrt{2}\int_0^{\pi/4} \cos(2x)\,dx$$

$u = 2x$

$du = 2dx$

$$= \frac{\sqrt{2}}{2}\int_0^{\pi/2} \cos(u)\,du = \frac{\sqrt{2}}{2}\sin(u)\Big|_0^{\pi/2}$$

$$= \frac{\sqrt{2}}{2}(1-0) = \frac{\sqrt{2}}{2}$$

Ex $\displaystyle\int_0^{\pi/3} \sqrt{1-\underset{\cos}{\cancel{\sin}}(6x)}\,dx$

Hint $\cos(2\theta) = \underline{\ ?\ }$ (In terms of $\sin^2(\theta)$)

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$
$$= (1-\sin^2(\theta)) - \sin^2(\theta)$$
$$= 1 - 2\sin^2(\theta)$$

$$\cos(2\theta) = 1 - 2\sin^2(\theta)$$
$$2\sin^2(\theta) = 1 - \cos(2\theta)$$

So: $2\sin^2(3x) = 1-\cos(6x)$

$$\int_0^{\pi/3} \sqrt{1-\cos(6x)}\,dx = \int_0^{\pi/3} \sqrt{2\sin^2(3x)}\,dx$$

$$= \sqrt{2}\int_0^{\pi/3} \sqrt{\sin^2(3x)}\,dx = \sqrt{2}\int_0^{\pi/3} |\sin(3x)|\,dx$$

$$= \sqrt{2}\int_0^{\pi/3} \sin(3x)\,dx = \frac{\sqrt{2}}{3}\int_0^{\pi} \sin(u)\,du$$

$u = 3x$
$du = 3dx$

$$= \frac{\sqrt{2}}{3}\left(-\cos(u)\right)\Big|_0^{\pi}$$

$$= \frac{\sqrt{2}}{3}\left(-(-1) - (-1)\right) = \frac{\sqrt{2}}{3}(1+1) = \frac{2\sqrt{2}}{3}$$

Pythagorean Identities

$$\sin^2(x) + \cos^2(x) = 1$$

Divide by $\cos^2(x)$: $$\frac{\sin^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$\tan^2(x) + 1 = \sec^2(x).$$

Divide by $\sin^2(x)$: $$\frac{\sin^2(x)}{\sin^2(x)} + \frac{\cos^2(x)}{\sin^2(x)} = \frac{1}{\sin^2(x)}$$

$$1 + \cot^2(x) = \csc^2(x)$$

Ex $$\int \tan^4(x)\,dx = \int \tan^2(x)\,\underline{\tan^2(x)}\,dx$$

$$= \int \tan^2(x)\left(\sec^2(x) - 1\right)dx$$

$$= \int \tan^2(x)\sec^2(x)\,dx - \int \tan^2(x)\,dx$$

$$= \int \tan^2(x)\sec^2(x)\,dx - \underbrace{\int \left(\sec^2(x) - 1\right)dx}_{-\left(\tan(x) - x\right) + C}$$

$$= \int u^2\,du$$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$$= \boxed{\frac{u^3}{3}} - (\tan(x) - x) + C$$

$$\boxed{= \frac{\tan^3(x)}{3} - \tan(x) + x + C}$$

Ex $\displaystyle\int \tan^4(x)\sec^4(x)\,dx = \int \tan^4(x)\underline{\underline{\sec^2(x)}}\sec^2(x)\,dx$

Pythag I

$$= \int \tan^4(x)\left(1+\tan^2(x)\right)\sec^2(x)\,dx$$

$$= \int \left(\tan^4(x)\sec^2(x) + \tan^6(x)\sec^2(x)\right)dx$$

$$= \int \tan^4(x)\sec^2(x)\,dx + \int \tan^6(x)\sec^2(x)\,dx$$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

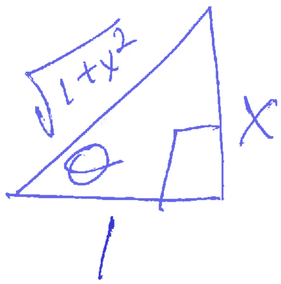
$$= \int u^4\,du + \int u^6\,du = \frac{u^5}{5} + \frac{u^7}{7} + C$$

$$\boxed{= \frac{\tan^5(x)}{5} + \frac{\tan^7(x)}{7} + C}$$

Ex Want algebraic expression for

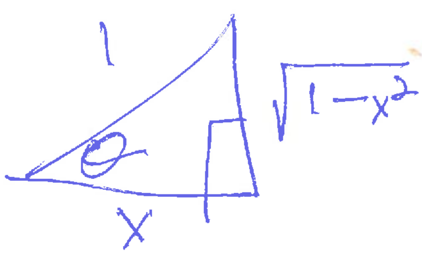
$$\boxed{\sin(\tan^{-1}(x)) = \sin(\theta) = \frac{x}{\sqrt{1+x^2}}}$$

$\theta = \tan^{-1}(x)$ s.t. $\tan(\theta) = \frac{x}{1}$



Ex $\sin(\cos^{-1}(x)) = \sin(\theta) = \sqrt{1-x^2}$

$\theta = \cos^{-1}(x)$ s.t. $\cos(\theta) = \frac{x}{1}$



Ex $\sec(\sin^{-1}(x)) = \boxed{\sec(\theta) = \frac{1}{\sqrt{1-x^2}}}$

$\theta = \sin^{-1}(x)$ s.t. $\sin(\theta) = \frac{x}{1}$