Math 170 Sections 7.5-7.6 Study Guide

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1 Section 7.5

Problem 1) In a certain community, 36% of families own a dog, 30% own a cat, and 22% of families that own a dog also own a cat.

(a) What is the probability that a family owns both a cat and a dog?

Answer: We note that $\Pr[\text{Dog}] = 0.36$, $\Pr[\text{Cat}] = 0.3$, and $\Pr[\text{Cat}|\text{Dog}] = 0.22$. By Bayes' Law, we have that:

$$\Pr[\text{Cat}|\text{Dog}] = \frac{\Pr[\text{Cat} \cap \text{Dog}]}{\Pr[\text{Dog}]}.$$

So we have that:

$$0.22 = \frac{\Pr[\text{Cat} \cap \text{Dog}]}{0.36} \implies \Pr[\text{Cat} \cap \text{Dog}] = 0.22(0.36).$$

(b) What is the probability that a family owns a dog, given that it owns a cat?

Problem 2) Suppose we draw a single card at random from a standard deck of 52 playing cards. Let A be the event that the card is a Diamond Card; let B be the event that the card is Red; and let C be the event that the card is a Jack.

(a) Are A and B independent? Justify your answer.

(b) Are B and C independent? Justify your answer.

Problem 3) Suppose a fair coin is tossed three times. What is the probability of tossing two Heads, given that the first toss results in Heads?

Problem 4) Suppose two distinguishable, fair, 6-sided dice are rolled. Let X denote the result of the first die, and Y denote the result of the second die.

(a) Determine $\Pr[X + Y = 8 | X = 3]$.

Answer: If $X = 3$, we have 6 rolls to consider: $\{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)\}$. Out of these, only the outcome $(3, 5)$ has entries that add up to 8. So $\Pr[X + Y = 8|X = 3] = 1/6$.

(b) Determine $\Pr[X + Y \text{ is odd } | X = 3]$.

2 Section 7.6

Note: We recall Bayes' Law for your convenience. Given events A and B, we have that:

$$\Pr[B|A] = \frac{\Pr[A|B] \cdot \Pr[B]}{\Pr[A]}.$$

In particular, we may write:

$$\Pr[A] = \Pr[A|B] \cdot \Pr[B] + \Pr[A|\overline{B}]\Pr[\overline{B}].$$

Problem 5) You go to the doctor regarding an ingrown toenail. The doctor selects you at random to have a blood test for swine flu, which affects 1 in $10,000$ people. The test is 99% accurate; that is, the probability of a false positive is 1%. The probability of a false negative is 0%. What is the probability you have the swine flu, given that you test positive?

Answer: We have the following:

- $\Pr[\text{Flu}] = .0001$.
- $\Pr[\text{Pos}| \sim \text{Flu}] = 0.01$
- $\Pr[\text{Pos}|\text{Flu}] = 1$.

We want to know $\Pr[\text{Flu}|\text{Pos}]$. By Bayes' Law, we have that:

$$\begin{aligned}\Pr[\text{Flu}|\text{Pos}] &= \frac{\Pr[\text{Pos}|\text{Flu}] \cdot \Pr[\text{Flu}]}{[\text{Pos}|\text{Flu}] \cdot \Pr[\text{Flu}] + \Pr[\text{Pos}| \sim \text{Flu}] \cdot \Pr[\sim \text{Flu}]} \\ &= \frac{1 \cdot 0.0001}{1 \cdot 0.0001 + 0.01 \cdot 0.9999}.\end{aligned}$$

Problem 6) In society, 1% of people have liver cancer. For a test T, 90% of people that have cancer test positive. For people who do not have cancer, 8% of people have false positives. What is the probability that someone has cancer if they test positive?