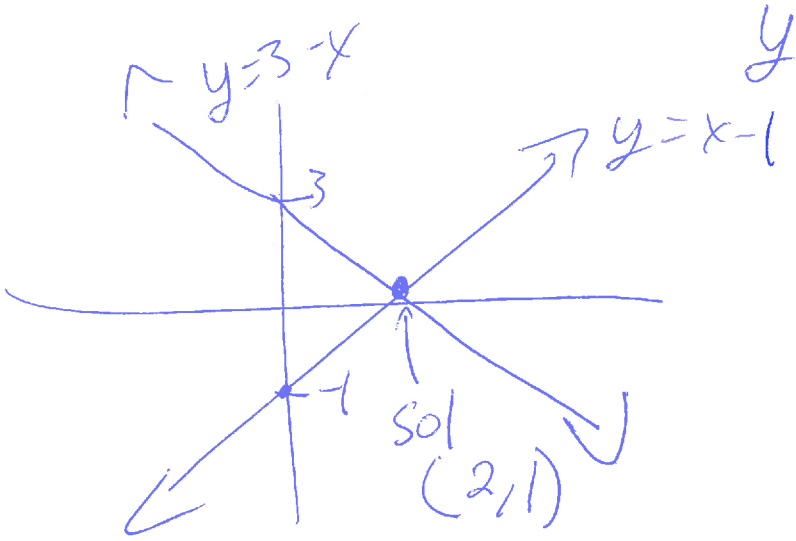
Ch. 3 Solving Systems of Equations.

3.1 Ex $x+y=3$ $(y=3-x)$

$+\ x-y=1$ $(y=x-1)$

$2x=4 \Rightarrow x=2$

$y=1$] Sol



Solutions for Systems of Equations

Case 1 Exactly one solution (ie, unique sol)

↳ Ex $x+y=3$ Unique Sol: (2, 1)

$x-y=1$

Case 2 No solutions (Parallel Lines)

Ex $y=x+1$

$y=x+3$

Ex $y = x+1$ $\quad -x + y = 1$ So: $-x+y=1$

$y = x+3$ $\quad -x + y = 3$ $\quad +\ x-y=-3$

$0x+0y=2$

$0 = 2$

Not true.

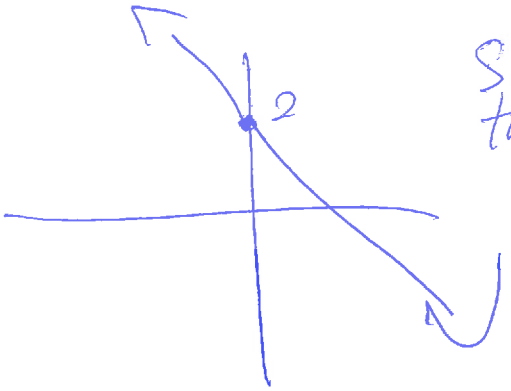
So no sols.

Case 3 Infinitely Many Sols.

Ex $x+y=2$ $\quad (y = 2-x)$

$2x+2y=4$ $\quad (2y = 4-2x,$ so $y = 2-x)$

Sol Set $= \{(x,y) \mid y = 2-x\}$



Solutions are on the line

3.2/3.3 Using Matrices to Solve Systems of Equations

Ex $2x - y = 3$
$-x + 2y = -4$

Show

$$\left[\begin{array}{cc|c} 2 & -1 & 3 \\ -1 & 2 & -4 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 2/3 \\ 0 & 1 & -5/3 \end{array}\right]$$

$x = \frac{2}{3}$, $y = \frac{-5}{3}$

Calculator

2nd Matrix
Edit
Go Home
2nd Matrix
↳ Math
↳ rref
2nd Matrix
Select your Matrix
Math → Frac

Check

↳ $2x - y = 3$ ✓
$-x + 2y = -4$ ✓

$2\left(\frac{2}{3}\right) - \left(\frac{-5}{3}\right) = \frac{4}{3} + \frac{5}{3} = \frac{9}{3} = 3$

$\frac{-2}{3} + 2\left(\frac{-5}{3}\right) = \frac{-2}{3} - \frac{10}{3} = \frac{-12}{3} = -4$

Ex $-\frac{2}{3}x + \frac{1}{2}y = -3$

$\frac{1}{4}x - y = \frac{11}{4}$

$$\left[\begin{array}{cc|c} -2/3 & 1/2 & -3 \\ 1/4 & -1 & 11/4 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array}\right]$$

$x = 3,\ y = -2$

Ex $2x + y + 3z = 1$

$4x + 2y + 4z = 4$

$x + 2y + z = 4$

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 4 & 2 & 4 & 4 \\ 1 & 2 & 1 & 4 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$x = 1$

$y = 2$

$z = -1$

Ex $2x+y+3z=1$

$4x+2y+4z=4$

$x+2y+z=4$

Ex $x+y+z=1$

$\frac{1}{4}x-\frac{1}{2}y+\frac{3}{4}z=0$

$x+7y-3z=3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1/4 & -1/2 & 3/4 & 0 \\ 1 & 7 & -3 & 3 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 5/3 & 2/3 \\ 0 & 1 & -2/3 & 1/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Infinitely many solutions

Ex

$$x+y+z=1$$
$$2x-y+z=0$$
$$4x+y+3z=3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -1 & 1 & 0 \\ 4 & 1 & 3 & 3 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 2/3 & 0 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

False $0x+0y+0z=1$

No solutions

Ex Purchase Planes for 4800 person cap.

Plane A 320 pass, costs $200 M

Plane B 250 pass, costs $125 M

Plane C 275 pass, costs $200 M

Given Costs $3,100 M

Given Twice as many Plane C as Plane B.

$c = 2b \Rightarrow 2b - c = 0$

System

$0a + 2b - c = 0$ (2x C's as b's)

$320a + 250b + 275c = 4800$ (Capacity)

$200a + 125b + 200c = 3100$ (Cost)

$$\begin{bmatrix} 0 & 2 & -1 & 0 \\ 320 & 250 & 275 & 4800 \\ 200 & 125 & 200 & 3100 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 8 \end{bmatrix}$$

$a = 5, \quad b = 4, \quad c = 8$