7.3 Probability is Similar to set cardinality (Counting)

Rule of Sum Let A, B be mutually exclusive events ($\Pr[A \cap B] = 0$). Then

$$\Pr[A \cup B] = \Pr[A] + \Pr[B]$$

Ex $\Pr[A] = 0.4$ A, B mutually exclusive

$\Pr[B] = 0.3$

$$\Pr[A \cup B] = \Pr[A] + \Pr[B]$$
$$= 0.4 + 0.3 = 0.7$$

Ex $\Pr[A] = 0.4$ $\Pr[A \cap B] = 0.1$

$\Pr[B] = 0.3$

$$\Pr[A \cup B] = \Pr[A] + \Pr[B] - \Pr[A \cap B]$$
$$= 0.4 + 0.3 - 0.1$$
$$= 0.6$$

Complements

↳ Sets U be a finite universal set

$A \subseteq U$.

$$n(A') = n(U) - n(A)$$

↳ Prob Sample Space S, Event $A \subseteq S$

$\Pr[S] = 1$ ← S need not be finite

$$\Pr[A'] = 1 - \Pr[A]$$

Ex $\Pr[A] = 0.3$

So $\Pr[A'] = 1 - 0.3 = 0.7$

7.4

Relative Frequency $= \dfrac{\#\text{objects of interest}}{\#\text{total possible objects}}$

Poker Hands

↳ $\binom{52}{5}$ possible hands

Ex One pair

↳ One rank occurs twice $\binom{13}{1}\binom{4}{2}$

↳ 3 additional ranks, each occur. once

$$\binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1} = \binom{12}{3}\binom{4}{1}^3$$

$$\Pr[\text{One Pair Hand}] = \frac{\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3}{\binom{52}{5}}$$

Lotto Problems

↳ n numbered balls $\{1, \ldots, n\}$
(no repeated numbers)

↳ Pick k of those #'s as winning #s.
(order does not matter)

↳ Want Prob of matching exactly i of k winners.

Ex 50 balls, Pick-5 (so each lotto ticket has 5 ~~winning~~ #s).

↳ $\binom{50}{5}$ possible tickets

↳ Want to match exactly 3 of 5 winners.

$$\Pr[\text{Match } 3] = \frac{\binom{5}{3}\binom{45}{2}}{\binom{50}{5}} \approx 0.00467$$

Ex 407 balls, Pick-24

↳ What is prob of match exactly 5 winners?

$$\frac{\binom{24}{5}\binom{383}{19}}{\binom{407}{24}}$$

↳ What is prob of match. exactly 3 winners, exactly 4 winners, or exactly 5 winners?

$$\frac{\binom{24}{3}\binom{383}{21} + \binom{24}{4}\binom{383}{20} + \binom{24}{5}\binom{383}{19}}{\binom{407}{24}}$$