6.2 Volumes of Solids

↳ Disk Method $\int_a^b \pi (r(x))^2 \, dx$

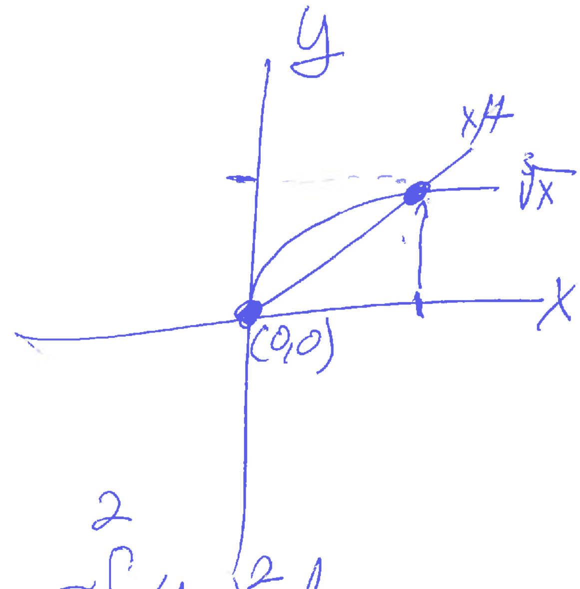
__Ex__ Vol of solid bounded by:

↳ $y = \sqrt[3]{x}$ $(x = y^3)$

$y = \frac{x}{4}$ $(x = 4y)$

x-axis

Rotated about y-axis



__Set__ $y^3 = 4y$

$y^3 - 4y = 0$

$y(y^2 - 4) = 0$

$y = 0, \cancel{-2}, 2$

__Vol 4y__ $\pi \int_0^2 (4y)^2 \, dy$

__Vol y^3:__ $\pi \int_0^2 (y^3)^2 \, dy$

__Vol Solid__ $\pi \int_0^2 (4y)^2 \, dy - \pi \cdot \int_0^2 (y^3)^2 \, dy$

$= \pi \int_0^2 [(4y)^2 - (y^3)^2] \, dy$

$= \frac{512\pi}{21}$

Ex Region bounded by

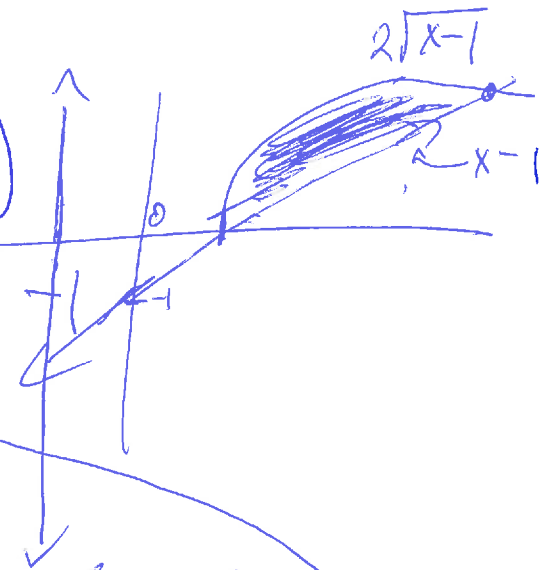
↳ $y = 2\sqrt{x-1}$ $\left(x = \frac{y^2}{4}+1\right)$

$y = x-1$ $(x = y+1)$

x-axis

Rotate around $x = -1$

So integrate wrt y



Set $\frac{y^2}{4}+1 = y+1$

$\frac{y^2}{4} = y$

$\frac{y^2}{4} - y = 0$

$y^2 - 4y = 0$

$y(y-4) = 0$

$y = 0, 4$

Vol $y+1$ $\pi\int_0^4 (y+1+1)^2\,dy$

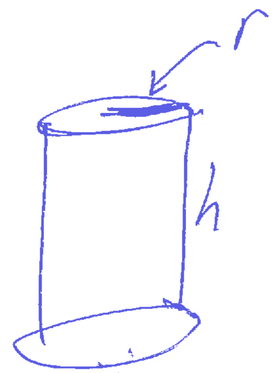
Vol $\frac{y^2}{4}+1$ $\pi\int_0^4 \left(\frac{y^2}{4}+1+1\right)^2 dy$

$$\text{Vol} = \pi\int_0^4 (y+2)^2\,dy - \pi\int_0^4 \left(\frac{y^2}{4}+2\right)^2 dy$$

$$= \frac{96\pi}{5}$$

Cylinder Method

Vol Cylinder $= 2\pi r h$



Ex $y = (x-1)(x-3)^2$

bound above x-axis

Rotate about y-axis

(integrate wrt x)

$$\text{Vol} = 2\pi \int_1^3 \underbrace{x}_{\text{radius}} \underbrace{\left[(x-1)(x-3)^2\right]}_{\text{height}} dx = \frac{24\pi}{5}$$

Ex region bounded by $y = \sqrt[3]{x}$, $x = 8$ and x-axis.

(integrate wrt y)

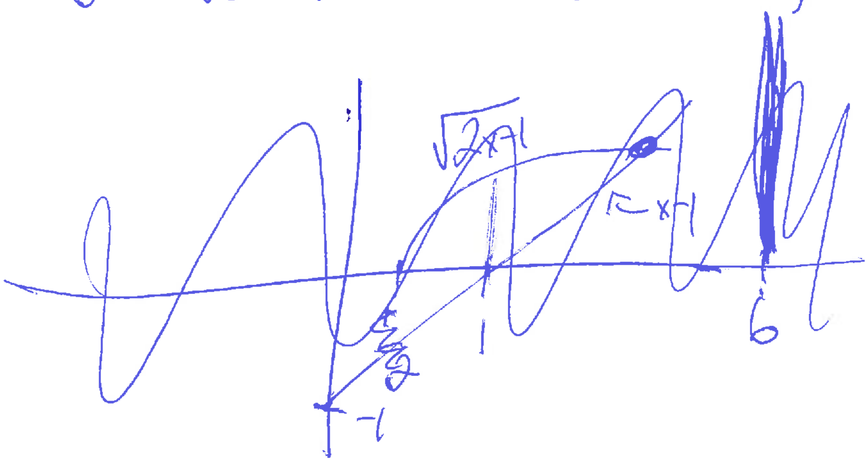
Rotate about x-axis

$x = y^3$

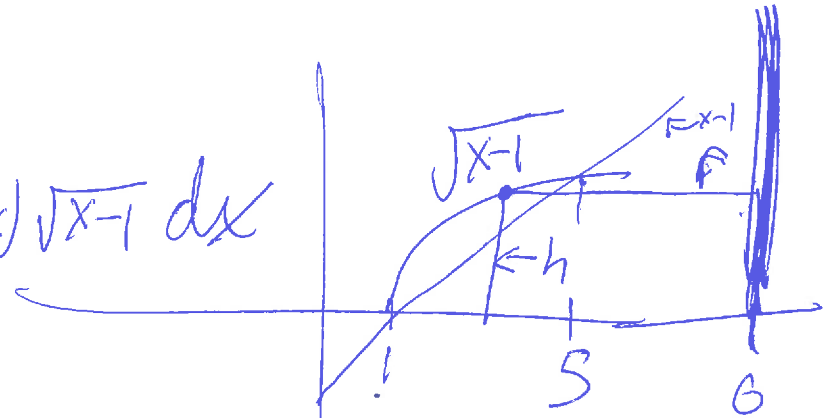
$8 = y^3 \Rightarrow y = 2$

$$2\pi \int_0^2 \underbrace{y}_{\text{radius}} \overbrace{(8 - y^3)}^{\text{ht}} dy = \frac{96\pi}{5}$$

Ex $y=\sqrt{2x-1}$ and $y=x-1$, rotate about $x=6$.



Vol $\sqrt{2x-1}$ $= 2\pi \int_1^5 (6-x)\sqrt{x-1}\,dx$



Vol $x-1$ $2\pi \int_1^5 (6-x)(x-1)\,dx$

Total Vol $2\pi \int_1^5 (6-x)\sqrt{x-1}\,dx - 2\pi \int_1^5 (6-x)(x-1)\,dx$

$= \frac{272\pi}{15}$