Study Guide 3.7-3.9

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Instructions: Complete the following problems. Justify all your answers in complete sentences, where appropriate.

1 Sections 3.7

Problem 1) Find $\frac{dy}{dx}$ for the following curves.

(a) $y^2 = x^3 + \cos(x^2 y)$

(b) $e^{2x} = \sin(x + 3y)$

(c) $\exp(x^2 y) = 2x + 2y$ (where $\exp(u)$ is the function mapping $u \mapsto e^u$)

(d) $x^3 = \frac{2x - y}{x + 3y}$

Problem 2) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the following curves. You should start by finding $\frac{dy}{dx}$ first.

(a) $x^2 + y^2 = r^2$ (where $r > 0$ is some constant).

(b) $2\sqrt{y} = x - y$

(c) $y^2 = e^{x^2} + 2x$

Problem 3) For each of the following, **(i)** verify that the point is on the curve; **(ii)** find the equation of the line tangent to the point on the curve; and **(iii)** find the line perpendicular to the tangent line at the point on the curve (we call this perpendicular line the **normal line**).

(a) $f(x) = \sin(x)$ at $x = -\pi/3$ (find the corresponding y-value).

(b) $x^2 + xy - y^2 = 1$, at $(2, 3)$

(c) $2xy + \pi \sin(y) = 2\pi$ at $(1, \pi/2)$

(d) $x^2 \cos(y) - \sin(y) = 0$ at $(0, \pi)$

2 Sections 3.8 and 3.9

Problem 4) Suppose that $f(x)$ has interval I as its domain, and that $f'(x) \neq 0$ on I. Suppose $f(x)$ is one-to-one on I. Then f^{-1} is differentiable on every point in the range of $f(x)$. Show that:

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}.$$

Problem 5) Evaluate $(f^{-1})'(a)$, for each of the following functions $f(x)$ and points $x = a$.

(a) $f(x) = 2x + 3$, with $a = -1$.

(b) $f(x) = x/5 + 7$, with $a = -1$

(c) $f(x) = 2x^2$, with $a = 5$.

Problem 6) Derive $f'(x)$ for each of the following functions.

(a) $f(x) = \sin^{-1}(x)$

(b) $f(x) = \cos^{-1}(x)$

(c) $f(x) = \tan^{-1}(x)$

Problem 7) Using logarithmic differentiation, determine the derivative for each of the following functions.

(a) $f(x) = \sqrt{x(x+1)}$

(b) $f(x) = \sqrt{t/(t+1)}$

(c) $f(\theta) = \tan(\theta)\sqrt{2\theta+1}$

(d) $f(x) = \sqrt{\dfrac{(x+1)^{10}}{(2x+1)^5}}$

(e) $f(x) = x^x$.

Problem 8) Let $f(x) = \ln(x)$. Recall that $f'(x) = 1/x$ and $f'(1) = 1$. Using these facts, show that:

$$\lim_{x\to 0}(1+x)^{1/x} = e.$$