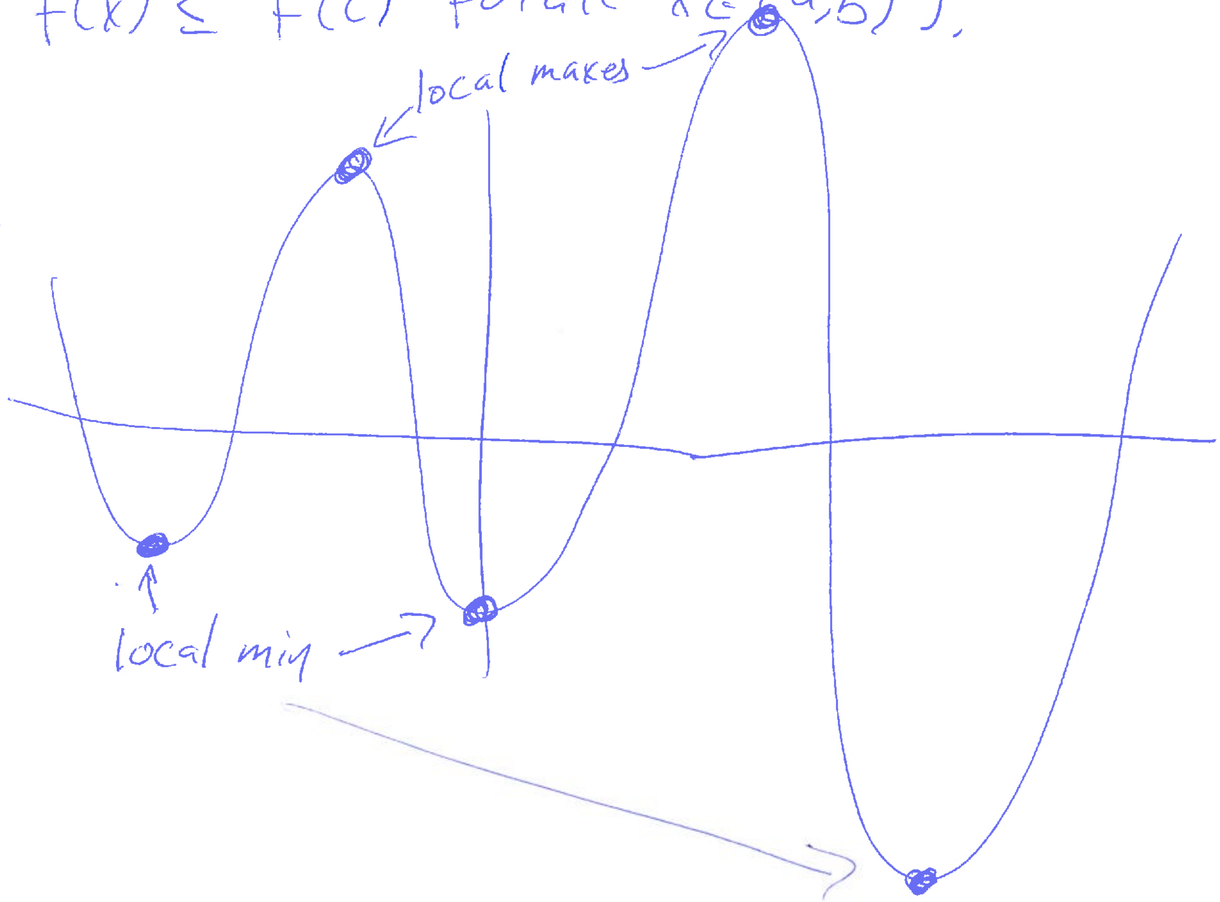
4.1 Extreme Values

Def A function $f(x)$ has a local min (resp. max) at the point $x=c$ in its domain D if there exists $(a,b) \subset D$ s.t. $c \in (a,b)$ and $f(x) \geq f(c)$ for all $x \in (a,b)$ (resp. $f(x) \leq f(c)$ for all $x \in (a,b)$).

Ex



Thm Suppose $f(x)$ is differentiable, and suppose $f(x)$ has local max/min @ $x=c$. If $f'(c)$ exists, then $f'(c)=0$.

First Derivative Test Find critical points (x-values where $f'(x)=0$ or $f'(x)$ is undefined) and check whether f' changes from $+$ to $-$ (local max), or f' changes from $-$ to $+$ (local min).

Ex Want local maxima/minima of

$f(x)=x^3-9x^2-48x+52$.

$f'(x)=3x^2-18x-48 = 3(x^2-6x-16)$

$= 3(x-8)(x+2)$

Crit pts $x=-2, 8$

$f'(x)$

$\underline{x<-2}$ $3(x-8)(x+2)$
$+ \quad - \quad -$

$\underline{-2<x<8}$ $3(x-8)(x+2)$
$+ \quad - \quad +$

$\underline{x>8}$ $+ \quad + \quad +$

As f' changes from $+$ to $-$ @ $x=-2$, $f(x)$ has local max @ $x=-2$, with max value
$f(-2)=(-2)^3-9(-2)^2-48(-2)+52$

As f' changes from $-$ to $+$ @ $x=8$, $f(x)$ has local min @ $x=8$.

Ex $f(x) = x^2 + 3x + 5$

Want Local Minima / Maxima

$f'(x) = 2x + 3 = 0$

Crit Pt $x = \frac{-3}{2}$

As f' changes from $-$ to $+$ @ $x = \frac{-3}{2}$, $f(x)$ has local min @ $x = \frac{-3}{2}$.

Min Value $f\left(\frac{-3}{2}\right) = \left(\frac{-3}{2}\right)^2 + 3\left(\frac{-3}{2}\right) + 5$

Ex Let a, b, c be fixed constants (with $a > 0$).

Let $f(x) = -ax^2 + bx + c$

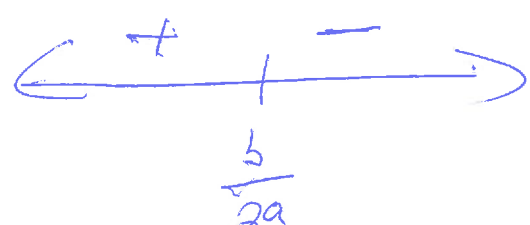
Find Local Maxima/Minima of $f(x)$.

$x = \frac{b-1}{2a}$

$f'(x) = -2ax + b = 0$

$b = 2ax$

Crit Pt $x = \frac{b}{2a}$



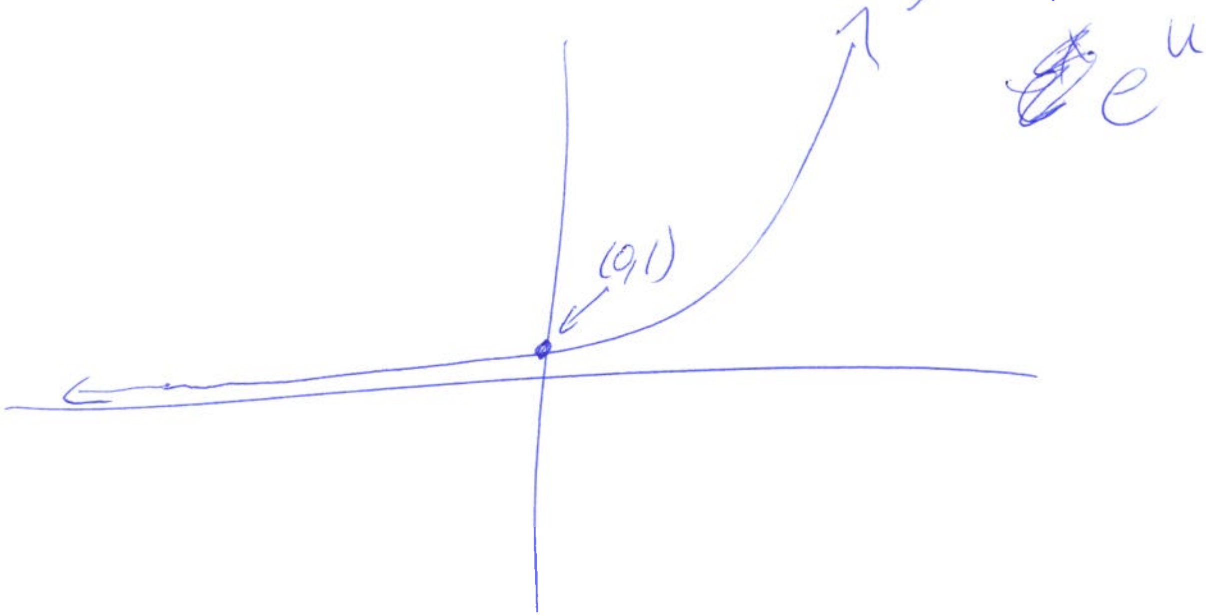
$f'\left(\frac{b-1}{2a}\right) = -2a\left(\frac{b-1}{2a}\right) + b = -(b-1) + b = 1 > 0$

$x = \frac{b+1}{2a}$, $f'\left(\frac{b+1}{2a}\right) = -2a\left(\frac{b+1}{2a}\right) + b$

$= -(b+1) + b = -1 < 0$

As f' changes from $+$ to $-$ @ $x = \frac{b}{2a}$, $f(x)$ has local max @ $x = \frac{b}{2a}$.

Recall Range of e^u: $(0, \infty) = \{x \in \mathbb{R} \mid x > 0\}$



Ex $f(x) = 10xe^{3-x^2}$, Find local maxima/minima

$$f'(x) = 10\left(e^{3-x^2} + x\left(-2xe^{3-x^2}\right)\right)$$

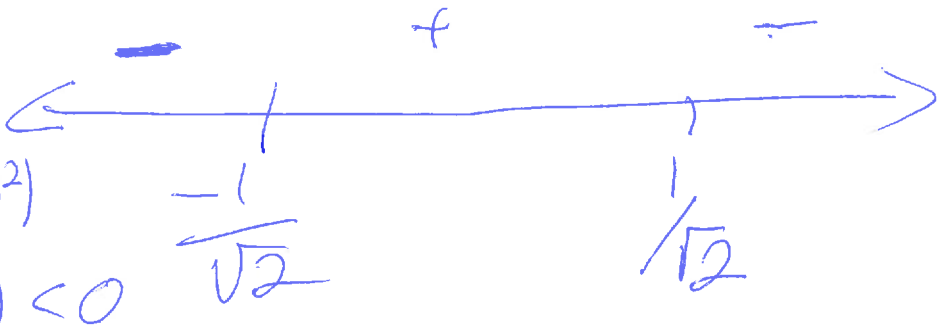
$$= \underbrace{10e^{3-x^2}}_{\text{Non-zero}}\left(1 - 2x^2\right) = 0$$

Check $1 - 2x^2 = 0$

$x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$ crit pts

Crit pts $x = \pm \frac{1}{\sqrt{2}}$

$f'(-2) = 10e^{3-4} \cdot (1-2(-2)^2)$
$= 10e^{-1}(1-8) < 0$
$\quad + \quad + \quad -$



$f'(0) = 10e^3(1-2\cdot 0^2) = 10e^3 > 0$

$f'(2) = 10e^{3-4}(1-2\cdot 2^2) = 10e^{-1}(1-8) < 0$

↳ As f' changes from − to + @ $x = \frac{-1}{\sqrt{2}}$, $f(x)$ has local min @ $x = \frac{-1}{\sqrt{2}}$

↳ As f' changes from + to − @ $x = \frac{1}{\sqrt{2}}$, $f(x)$ has local max @ $x = \frac{1}{\sqrt{2}}$

Def Let $f(x)$ be a function with domain D.
The absolute/global max (resp. min) is the largest (resp. smallest) y-value that $f(x)$ takes on, using x-values from D.

Ex $f(x) = x^2$ on $[-2, 1]$

Global max @ $x = -2$

Global min @ $x = 0$

Crit pt $x = 0$