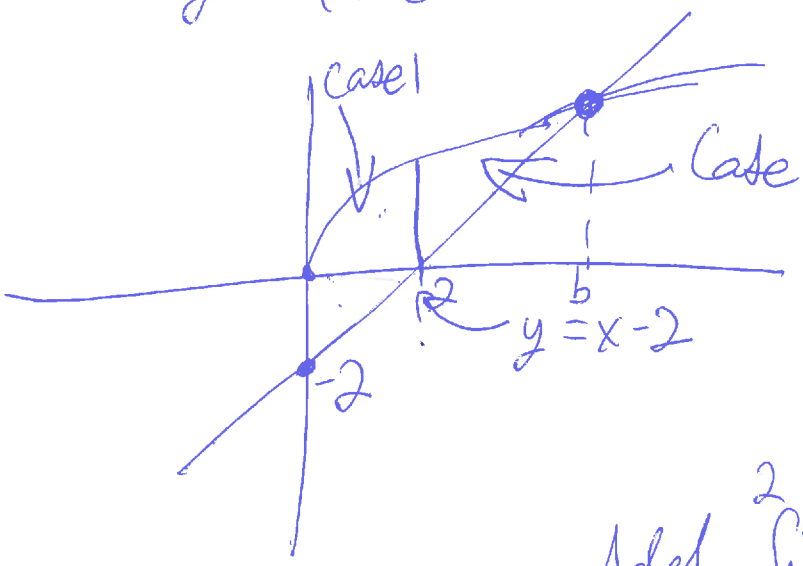
5.6 Want region enclosed btwn $y=\sqrt{x}$ and $y=x-2$. Also & 1st quadrant.



Case 1 $\int_0^2 \sqrt{x}\,dx$

Case 2 $\int_2^4 \left|(\sqrt{x}-(x-2)\right|dx$

Add $\int_0^2 \sqrt{x}\,dx + \int_2^4 (\sqrt{x}-x+2)\,dx$

$= \frac{10}{3}$

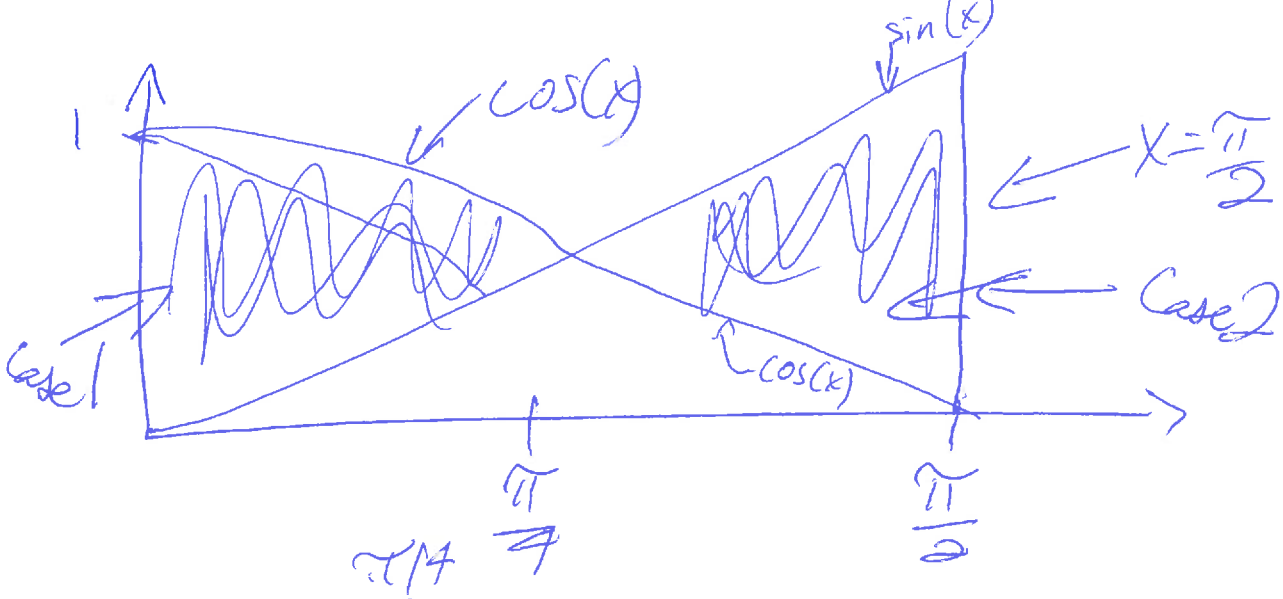
Set $\sqrt{x} = x-2$

$x = (x-2)^2 = x^2-4x+4$

$x^2-5x+4=0 \Rightarrow x=4,1.$

Only $x=4$ satisfies: $\sqrt{x}=x-2$

Ex Area btwn y-axis, $x=\frac{\pi}{2}$, $y=\cos(x)$, $y=\sin(x)$



Case 1 $\int_0^{\pi/4} (\cos(x) - \sin(x))\,dx = (\sin(x)+\cos(x))\Big|_0^{\frac{\pi}{4}}$

$= \left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right) - (0+1)$

$= \sqrt{2}-1$

Case 2 $\int_{\pi/4}^{\pi/2} (\sin(x) - \cos(x))\,dx$

$= (-\cos(x)-\sin(x))\Big|_{\frac{\pi}{4}}^{\pi/2} = (0-1) - \left(\frac{-\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right)$

$= \frac{2\sqrt{2}}{2} - 1 = \sqrt{2}-1$

Area $2(\sqrt{2}-1)$

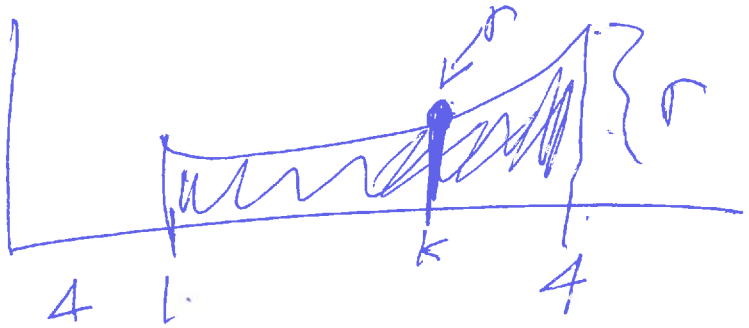
6.2 Volumes of Solids

Area of Circle $= \pi r^2$

Method of Disks $\int_a^b \pi (r(x))^2 \, dx$

$r(x)$ is our curve

Ex Vol of solid obtained by rotating region from: $x=1$, $x=4$, $y = x^2 - 4x + 5$ around x-axis.



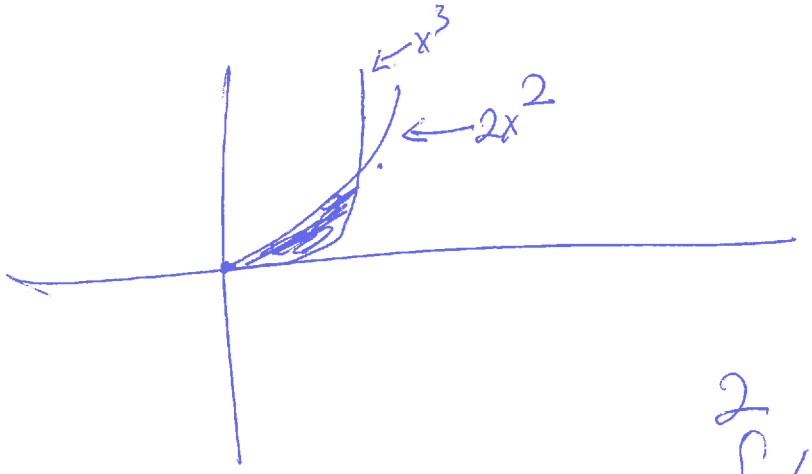
$$\pi \int_1^4 (x^2 - 4x + 5 - 0)^2 \, dx$$

$x^2 - 4x + 5 - 0$: dist btwn $(x^2 - 4x + 5)$ and $x = 0$

$$= \pi \int_1^4 (x^4 - 8x^3 + 26x^2 - 40x + 25) \, dx = \frac{78\pi}{5}$$

Volume of solid

Ex Volume of Solid obtained by rotating region btwn $y=2x^2$ and $y=x^3$ around x-axis.



Bounds Set $x^3=2x^2$

$x^3-2x^2=0$

$x^2(x-2)=0$

$x=0,2.$

Vol rotating $2x^2$ $\pi\int_0^2 (2x^2-0)^2\,dx$ ← dist from $2x^2$ to x-axis

$= \pi\int_0^2 4x^4\,dx = \frac{4\pi}{5}x^5\Big|_0^2 = \frac{4\pi}{5}(2^5) = \frac{\pi 2^7}{5}$

Vol rotating x^3 $\pi\int_0^2 (x^3-0)^2\,dx = \pi\int_0^2 x^6\,dx$

$= \frac{\pi}{7}x^7\Big|_0^2 = \frac{\pi\cdot 2^7}{7}$

Subtract $\frac{2^7\pi}{5} - \frac{2^7\pi}{7} = 2^7\cdot\pi\left(\frac{1}{5}-\frac{1}{7}\right)$

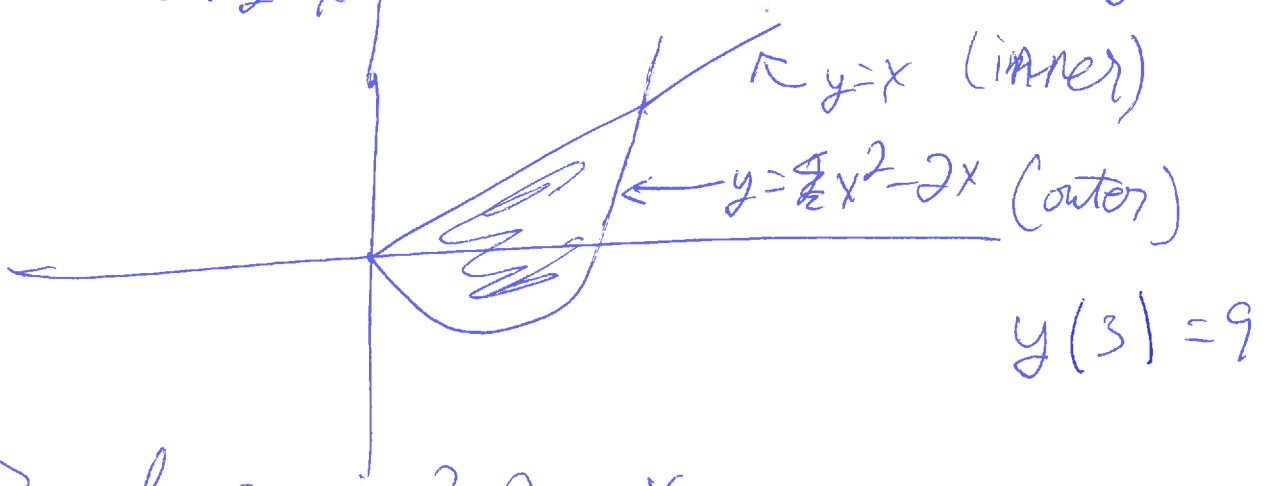
↑ Volume of filled in solid

Volume of region we remove by hollowing out

Ex Vol of solid, rotated around $y=4$

↳ $y=x^2-2x$

↳ $y=x$



$y(3)=9-6=3$

Bounds Set $x^2-2x=x$

$x^2-3x=0 \Rightarrow x=0,3$

Vol Rotating x^2-2x $\pi\int_0^3\left(4-(x^2-2x)\right)^2dx$

Vol Rotating $y=x$ $\pi\int_0^3(4-x)^2dx$

Vol of Solid $\pi\int_0^3\left(4-(x^2-2x)\right)^2dx-\pi\int_0^3(4-x)^2dx$

$=\frac{153\pi}{5}$