Study Guide 4.1-4.2

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Instructions: Complete the following problems. Justify all your answers in complete sentences, where appropriate.

1 Section 4.1

Problem 1) Find the global maximum and minimum of the following functions, if they exist. If they do not exist, explain why (recall that global extrema occur at critical points and endpoints, provided the domain has any endpoints).

(a) $f(x) = \dfrac{6}{x^2+2}$, on the domain $(-1, 1)$.

(b) $f(x) = |x|$, on the domain $(-\infty, \infty)$.

(c) $f(t) = t + \dfrac{1}{t}$, on the domain $(-\infty, \infty)$.

(d) $f(t) = t - \ln(t)$, on the domaint $(0, \infty)$.

(e) $f(x) = e^{3x} - e^{2x}$, on the domain $(-\infty, \infty)$.

(f) $f(x) = xe^{-x}$, on the domain $(-\infty, \infty)$.

Problem 2) Determine the global minimum and global maximum of the following functions.

(a) $f(x) = \dfrac{2}{3}x - 5$, on the domain $[-2, 3]$.

(b) $f(x) = x^2 - 1$, on the domain $[-1, 2]$.

(c) $f(x) = 4 - x^3$, on the domain $[-2, 1]$.

(d) $f(x) = \sqrt{4 - x^2}$, on the domain $[-2, 1]$.

(e) $f(\theta) = \tan(\theta)$, on the domain $\left[-\dfrac{\pi}{3}, \dfrac{\pi}{4}\right]$.

Problem 3) Determine the minimum value that $x + y$ takes on, given that $xy = 324$ and $x, y > 0$.

Problem 4) Determine the minimum value of $x + 2y$, given that $x^2y = 10$ and $x, y > 0$.

Problem 5) Find the constants a, b such that the parabola $f(x) = x^2 + ax + b$ has a minimum at $(3, 5)$.

Problem 6) Find **all** the maxima and minima of the differentiable function $f(x)$, given its derivative:

$$f'(x) = x^2(x+5)(x+4)(x-3)$$

Clearly justify, in complete sentences, why the relevant points are maxima or minima. For your convenience, I have included the table for the first derivative test below. In each cell, indicate if the factor is positive or negative on the given interval.

	$(x+5)$	$(x+4)$	x^2	$(x-3)$	$f'(x)$
$x < -5$					
$-5 < x < -4$					
$-4 < x < 0$					
$0 < x < 3$					
$x > 3$					

Problem 7) At a price of \$10 per ticket, a musical theater group can fill every seat in the theater, which has a capacity of 1300. For every additional dollar charged, the number of people buying tickets decreases by50. What ticket price maximizes revenue? [**Note:** Revenue is price $\times$ quantity.]

Problem 8) A farmer uses x lb of fertilizer per acre, at a cost of \$2/lb. The farmer has a revenue of $R = 700 - 400e^{-x/100}$ dollars per acre. Determine the amount of fertilizer that should be applied per acre to maximize profit. [**Note:** Profit = Revenue − Cost.]

2 Section 4.2

Problem 9) State the Mean Value Theorem precisely. [**Note**: This will almost certainly be on Exam 3.]

Problem 10) Show that if a car accelerating from 0 velocity takes 8 sec. to go 352 ft, then at some point in time between 0 and 8 seconds, the car was traveling at a velocity of 44 ft/sec.

Problem 11) A marathoner ran the 26.2 mile marathon in 2.2 hours. Show that at least twice, the marathoner was running at exactly 11 miles/hour, assuming the initial and final velocities are zero.

Problem 12) Find a function $f(x)$ such that $f'(x) = -\sin(x)$ and $f(0) = 3$.

Problem 13) Find a function $f(x)$ with derivative $f'(x) = e^{2x}$ and $f(0) = \frac{3}{2}$.

Problem 14) Consider the function $f(x) = x^4 + 3x + 1$, on the interval $[-2, -1]$. Our goal is to show that $f(x)$ has exactly one zero on the interval $[-2, 1]$. We will break this problem down into the following parts.

(a) First, explain why $f(x) = x^4 + 3x + 1$ has at least one zero on $[-2, -1]$. [**Hint:** Appeal to the IVT.]

(b) What is $f'(x)$? Determine all intervals where $f' > 0$, all intervals where $f' < 0$, and all intervals where $f' = 0$.

(c) Argue that if $f(x)$ has two zeros on $[-2, -1]$, then $f'(x) = 0$ at some x-value on $[-2, -1]$. In light of your answer in part (b), can this happen? Conclude that $f(x)$ has a unique zero on $[-2, -1]$.

Problem 15) We wish to generalize the result of Problem 14 for arbitrary polynomials. Let:

$$f(x) = x^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0.$$

Suppose that $f(x)$ has zeros at c_0 and c_1. That is, $f(c_0) = f(c_1) = 0$. Prove that there exists $x_0 \in (c_0, c_1)$ such that $f'(x_0) = 0$.