8.2 Binomial Distribution

↳ n trials
↳ Each trial has binary outcome (eg. H or T, Success or Failure, etc.)
↳ Success w/ prob p
↳ Fail w/ prob $1-p$

Want $\Pr[X = k \text{ successes}] = \binom{n}{k} p^k (1-p)^{n-k}$

- p^k: prob of success in each of k slots
- $\binom{n}{k}$: Pick pos for Success
- $(1-p)^{n-k}$: Prob remaining slots are Failures

Ex Toss weighted coin 5 times
↳ 60% chance of H on given toss
↳ 40% chance of T on given toss

Want $\Pr[X = 3 \text{ H}] = \binom{5}{3} (0.6)(0.6)(0.6)(0.4)(0.4)$

- $\binom{5}{3}$: Pick H pos
- $(0.6)(0.6)(0.6)$: Prob that 3 selected pos have H

T H H T T
0.4 0.4

$$\Pr[X = 3 \text{ H}] = \binom{5}{3} (0.6)^3 (0.4)^2$$

- $\binom{n}{k}$
- p^k
- $(1-p)^{n-k}$

Q $Pr[X \leq 3\ H] =$

↳ 0.6 prob of H on given toss

↳ 0.4 prob T on given toss

↳ Toss 5 times

$$\binom{5}{0}(.6)^0(.4)^5 + \binom{5}{1}(.6)^1(.4)^4 + \binom{5}{2}(.6)^2(.4)^3 + \binom{5}{3}(.6)^3(.4)^2 = Pr[X \leq 3\ H]$$

$Pr[X = 3\ H] = 0.3456$

↳ binompdf(n trials, p prob success, k successes)

$= \binom{n}{k} p^k (1-p)^{n-k}$

2nd + vars

$Pr[X \leq 3\ H] = 0.66304$

↳ binomcdf(n trials, p prob success, at most k succ)

$= \sum_{i=0}^{k} Pr[X = i \text{ succ}]$

Ex $n = 100$ trials

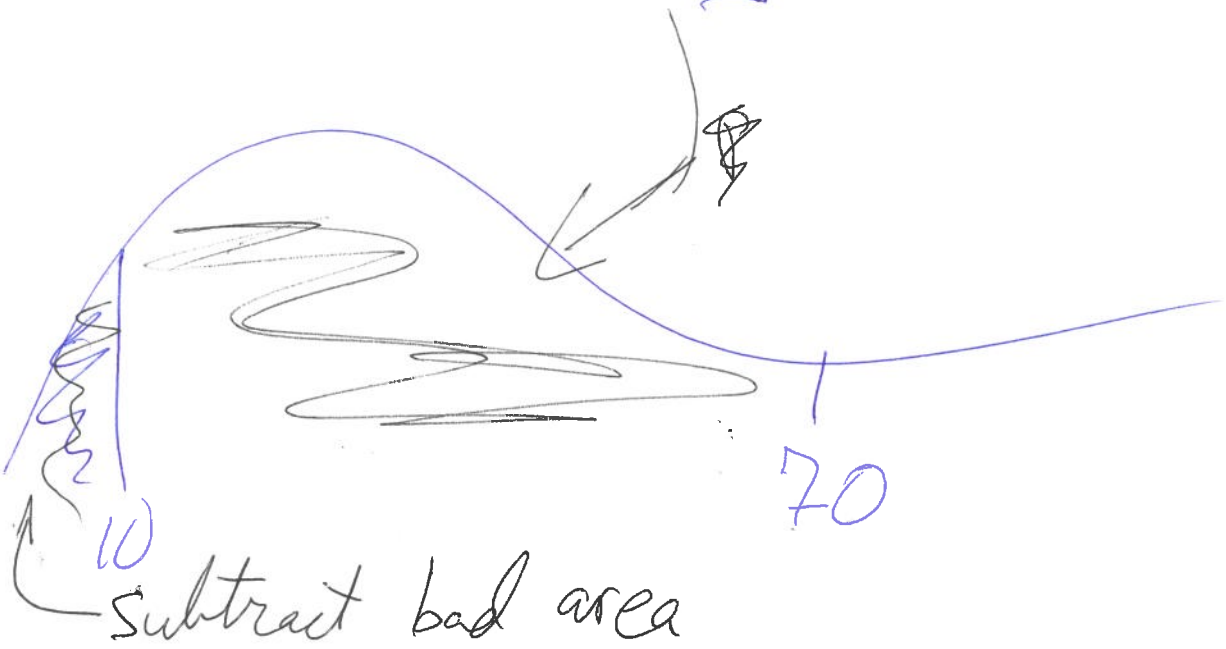
$p = 0.6$

Want # succ btwn 10 and 70

$Pr[X \leq 70] = \text{binomcdf}(100, .6, 70)$

$- Pr[X \leq 9] = \text{binomcdf}(100, .6, 9)$

$= Pr[X \geq 10 \text{ and } X \leq 70]$



8.3 Measures of Central Tendancy.

Sample Mean Given sample $x_1, x_2, \ldots, x_n$,
the sample mean $\bar{x} = \frac{1}{n}(x_1 + x_2 + \cdots + x_n)$

Ex 1, 2, 3, 4, 5, $\bar{x} = \frac{1}{5}(1+2+3+4+5) = 3$

Median Given Sample $x_1, \ldots, x_n$. The median is: (sorted in ascending order ↑)
↳ Middle element (if n is odd)
↳ Avg of two middle elems (if n is even)

Ex $-3, -1, \textcircled{2}, 2, 4$
↑ Median

Ex $-1, 0, 2, 5, 6, 6$ Median: $\left(\frac{5+2}{2}\right) = 3.5$

Expected Value Let X be a finite random variable.

The expected value of X, $E[X] = \sum_{i=1}^{n} i \cdot Pr[X=i]$ ← times

Ex Fair six-sided die

$$E[X] = \frac{1}{6}(1+2+3+4+5+6) = 3.5$$

Ex

X	-1	0	4	5
$Pr[X]$	0.3	0.5	0.1	0.1

$$E[X] = -1(0.3) + 4(0.1) + 5(0.1) + 0(0.5)$$
$$= 0.6$$

Fact For Binomial Random Variable X (X has binom. dist.),

$E[X] = np$ where n is # trials, p Prob success

Ex $n=20$, $p=0.7$

$$E[X] = 20(.7) = 14$$