5.2 Linear Programming (Optimization) (LP)

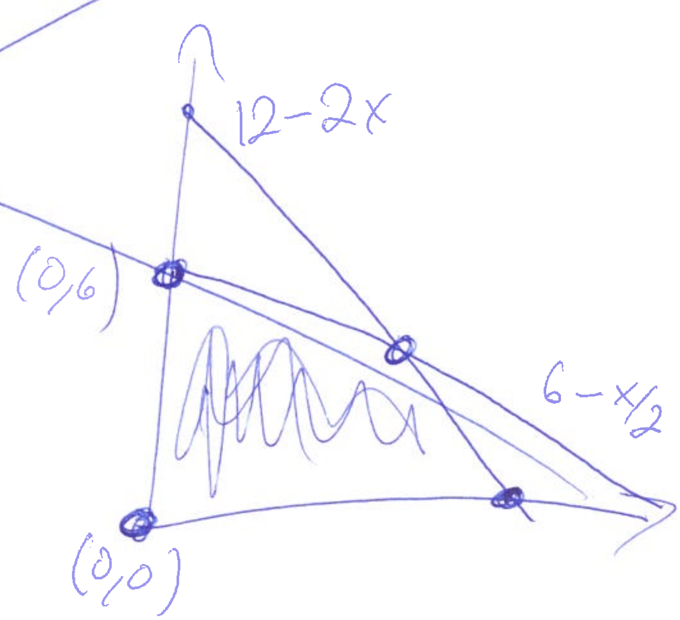
An **LP** is an optimization problem where the constraints are linear inequalities.

Constraints look like	and	NOT
$3x-2y \geq 5$		$x^2y > 0$
$x+2y \leq 0$		$x^3y \leq 14$
$x \geq 7$		

Ignore

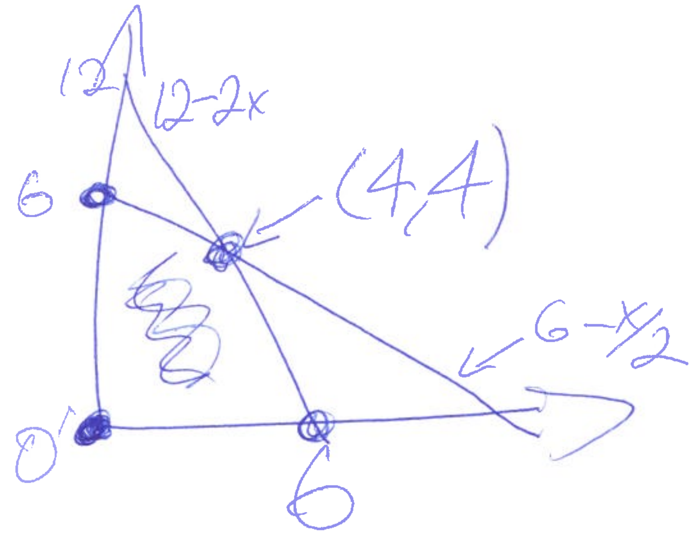
Ex max $x+y$ Subject to (s.t.)

$$3x-2y \geq 0$$
$$x+2y \leq 0$$
$$x \geq 0$$
$$y \geq 0$$



Ex max $x+y$ s.t.

$$2x+y \le 12$$
$$x+2y \le 12$$
$$x \ge 0$$
$$y \ge 0$$



$$0 = 12-2x \Rightarrow x=6$$

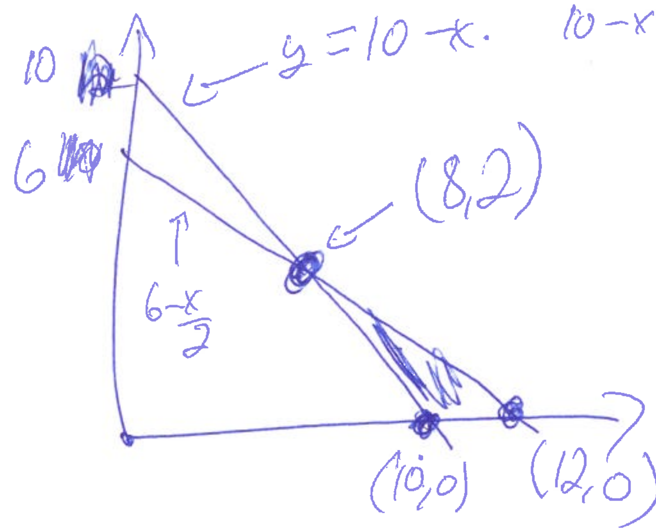
Pt	$x+y$
(0,0)	0
(0,6)	6
(6,0)	6
(4,4)	8 ← maximum value x+y takes on in feasible region

↑ maximizer

Ex min $2x+3y$ s.t.

$$x+y \geq 10$$
$$x+2y \leq 12$$
$$x \geq 0$$
$$y \geq 0$$

$10-x=0$



$$y = 10-x$$
$$y = 6-\frac{x}{2}$$

$$10-x = 6-\frac{x}{2}$$
$$4 = \frac{x}{2} \Rightarrow x=8$$
$$y = 10-8 = 2$$

minimizer

Pt	$2x+3y$
(10,0)	20 ← minimum
(12,0)	24
(8,2)	22

Ex Two types juice

↳ Type X: \$0.20/unit

30 oz water, 2 oz concentrate needed

↳ Type Y: \$0.30/unit

20 oz water, 12 oz concentrate

Objective Function $0.2x + 0.3y$ (profit function)

Told 30,000 oz water available

3600 oz concentrate available

LP $\max\ 0.2x + 0.3y$ s.t.

$$30x + 20y \le 30000 \quad \text{(Water)}$$

$$2x + 12y \le 3600 \quad \text{(Concentrate)}$$

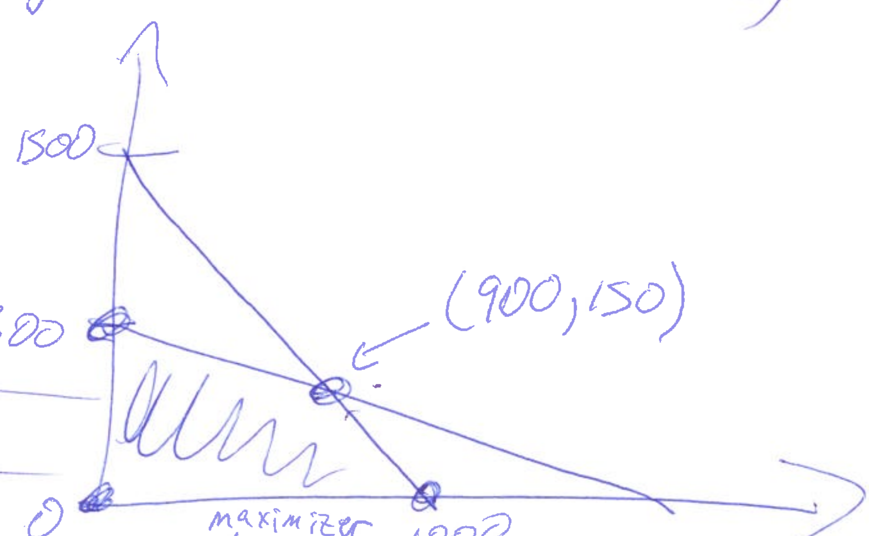
$$x \ge 0$$

$$y \ge 0$$

$1500 - \frac{3}{2}x$

$= 300 - \frac{x}{6}$

$1200 = \frac{8}{6}x$, so $x = 900$



Pt	$0.2x + 0.3y$
(0,0)	0
(0,300)	90
(1000,0)	200
maximizer ↓ (900,150)	225 ← maximum