4.2 MVT

Cor 1 Let $a, b \in \mathbb{R}$ be fixed constants, with $a < b$. Suppose $f'(x) = 0$ for all $x \in (a,b)$. Then for all $x \in (a,b)$, $f(x) = C$ for some constant C.

Cor 2 Let $a, b \in \mathbb{R}$ be fixed. Suppose that $f(x), g(x)$ satisfy: $f'(x) = g'(x)$ for all $x \in (a,b)$. Then $f(x) = g(x) + C$.

Ex $f(x) = x^2$, $g(x) = x^2 + 13$

$f'(x) = 2x$, $g'(x) = 2x$ ← Start w/ this i.e.

Pf Let $h(x) = f(x) - g(x)$. As $f(x), g(x)$ are differentiable on (a,b) and $f' = g'$ on (a,b), then $h'(x)$ exists and $h'(x) = 0$ on (a,b). So by Cor 1, $h(x) = C_1$, for some C_1 (on (a,b)). So $h(x) = C = f(x) - g(x)$ we have ~~that~~ $f(x) = g(x) + C$.

$\square$

Ex Wat function $h(x)$ with $h'(x) = -\sin(x)$ and $h(0) = 2$.

$h(x) = \cos(x) + C$

Use $h(0) = 2 = \cos(0) + C$

$2 = 1 + C$

$C = 1$

$h(x) = \cos(x) + 1$

Ex Want $h(x)$, with $h'(x) = e^{2x}$, $h(0) = \frac{3}{2}$.

$h(x) = \frac{1}{2}e^{2x} + C$ $\qquad \frac{d}{dx}\frac{1}{2}e^{2x} = 2e^{2x} \cdot \frac{1}{2}$

$h(0) = \frac{1}{2}e^{0} + c = \frac{3}{2}$

$\frac{1}{2} + c = \frac{3}{2} \Rightarrow c = 1$

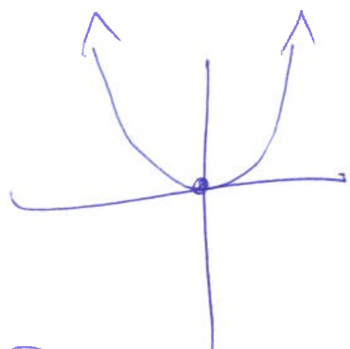
$\boxed{h(x) = \frac{1}{2}e^{2x} + 1}$

4.4 Concavity

Def Let $f(x)$ be a function that is twice differentiable @ $x=c$. We say that:

- $f(x)$ is <u>concave up</u> (CCU) @ $x=c$ if $f''(c)>0$.
- $f(x)$ is <u>concave down</u> (CCD) @ $x=c$ if $f''(c)<0$.

Ex $f(x)=x^2$

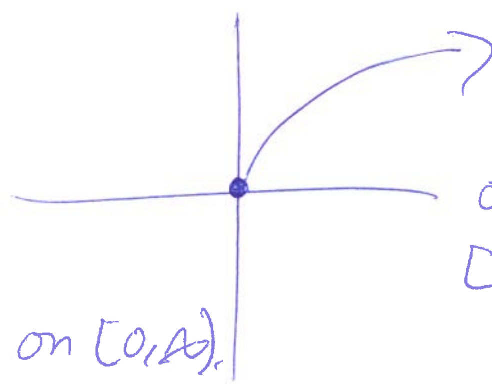


$f(x)=x^2$ is CCU everywhere

$f'(x)=2x$

$f''(x)=2>0$. So

Ex $f(x)=\sqrt{x}=x^{1/2}$



$f(x)=\sqrt{x}$ is CCD on its domain of $[0,\infty)$.

$f'(x)=\frac{1}{2}x^{-1/2}$

$f''(x)=\frac{-1}{4}x^{-3/2}<0$ on $[0,\infty)$.

So $f(x)=x^{1/2}$ is CCD on $[0,\infty)$.

Ex Let $f(x) = x^3$. Determine all x-values where $f(x)$ is CCU, and all x-values where $f(x)$ is CCD.

$f''(x) = 6x$

↳ $f(x)$ is CCU whenever $f''(x) = 6x > 0$. So $f(x)$ is CCU on $(0, \infty)$.

↳ $f(x)$ is CCD whenever $f''(x) = 6x < 0$. So $f(x)$ is CCD on $(-\infty, 0)$.

$f''(0) = 0$. Observe f'' changes from $-$ to $+$ @ $x = 0$.

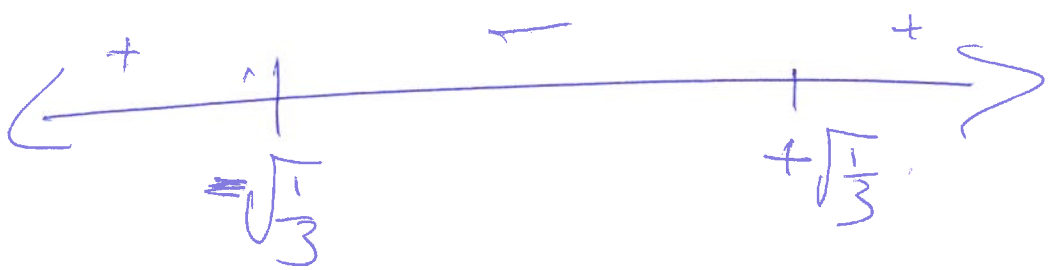
Def Let $f(x)$ be a function, and suppose $f(x)$ is twice differentiable @ $x = c$. We say that $f(x)$ has a <u>point of inflection</u> @ $x = c$ if f'' changes from $-$ to $+$, or $+$ to $-$ @ $x = c$.

Ex Find POI for $f(x) = x^4 - 2x^2$

$$f''(x) = 12x^2 - 4 = 0$$

$$4(3x^2 - 1) = 0 \Rightarrow x = \pm\sqrt{\tfrac{1}{3}}$$

Need To determine @ which pts f'' changes sign



E.g. $f''(-1) = 4(3\cdot(-1)^2 - 1) = 4(3-1) > 0$

$f''(0) = 4(0-1) < 0$

$f''(1) = 4(3\cdot 1 - 1) = 4(3-1) > 0$

As f'' changes from:

↳ + to − @ $x = -\sqrt{\frac{1}{3}}$, $f(x)$ has POI @ $x = -\sqrt{\frac{1}{3}}$

↳ − to + @ $x = \sqrt{\frac{1}{3}}$, $f(x)$ has POI @ $x = \sqrt{\frac{1}{3}}$.

Second Derivative Test Let $f(x)$ be twice diff., and suppose that $f(x)$ has crit pt. @ $x=c$.

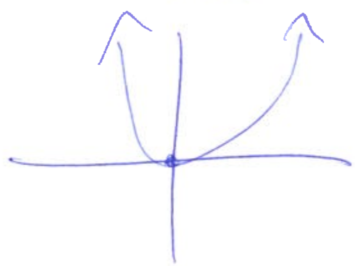
If:

↳ $f''(c) > 0$, then $f(x)$ has local min @ $x=c$.

↳ $f''(c) < 0$, then $f(x)$ has local max @ $x=c$.

↳ otherwise, if $f''(c) = 0$, go do First Deriv Test.

Ex $f(x) = x^2 + bx + c$ (where b, c are constants)



$f'(x) = 2x + b$

$f''(x) = 2 > 0$

↳ So $f(x)$ has local min @ crit pt $\left(x = \frac{-b}{2}\right)$

Ex $f(x) = -x^2 + bx + c$

$f''(x) = -2 < 0$, so $f(x)$ has local max @ crit pt $\left(x = \frac{b}{2}\right)$