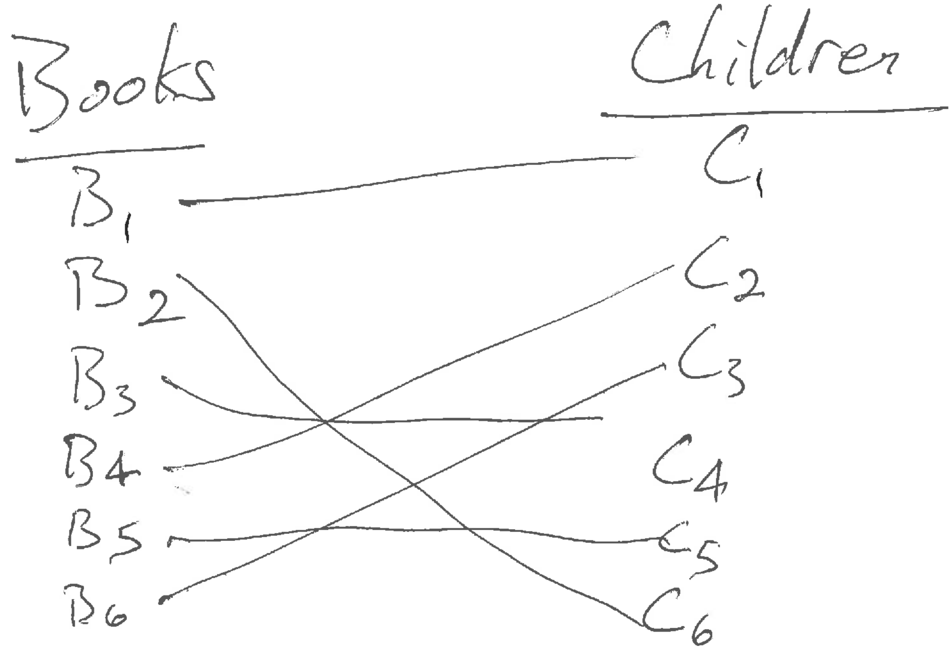
6.4 Permutations and Combinations

Def A permutation π on a finite set S ~~It~~ is a one-to-one function from S to itself.

Ex 6 ~~Be~~ distinct books, 6 children, each child gets exactly one book



$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ ways of giving books to children

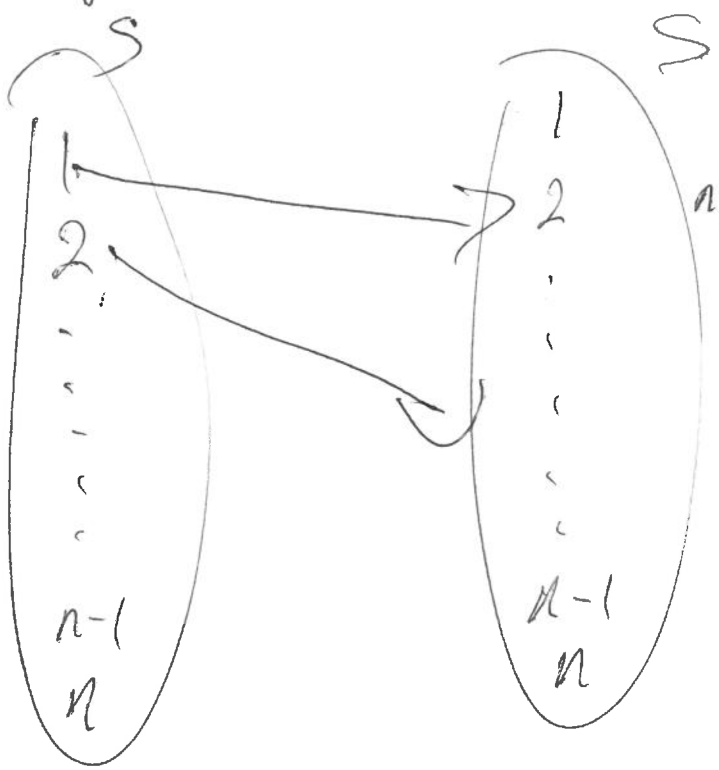
Note This 720 possible ways

Def Let $n \geq 0$ integer. The factorial

$$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$$

Pronounced "n"-factorial.

Thm Let S is an n-element set, then there are $n!$ permutations of S.



$$n! = \underbrace{n}_{\text{map } 1} \cdot \underbrace{(n-1)}_{\text{map } 2} \cdot (n-2) \cdots \cdot 1$$

Q: What is $0!$?

A $0! = 1$ (There is one permutation of $\emptyset$)

$f: \emptyset \to \emptyset$

Ex How many ways can 4 distinguishable songs be played in sequence?

A $\boxed{4!}$

Ex How many ways can we match 3 distinct cars w/ 3 drivers?

A $\boxed{3!}$

Restricted Permutation

↳ **Idea** 5 distinct books, 7 children, each child gets at most one book

Want Count # ways to assign books to children

~~Children~~ Books	Children
	C_1
B_1	C_2
B_2	C_3
B_3	$\vdots$
B_4	
B_5	C_7

$$7\cdot 6\cdot 5\cdot 4\cdot 3$$

Restricted Permutation $P(n,k) = \dfrac{n!}{(n-k)!}$

↳ Assign k books to n children
Each child gets at most one book

Ex $P(7,5) = \dfrac{7!}{(7-5)!} = \dfrac{7!}{2!} = \dfrac{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot \cancel{2\cdot 1}}{\cancel{2\cdot 1}}$

$= 7\cdot 6\cdot 5\cdot 4\cdot 3$

↳ Assign 5 books to 7 children
There are:

↳ $P(7,5)$ ↳ $7\cdot 6\cdot 5\cdot 4\cdot 3$

↳ $\dfrac{7!}{(7-5)!} = \dfrac{7!}{2!}$

Ex 10 companies, apply to exactly 6.
Need to do so in some order.

How many ways to apply to companies?

A $P(10,6)$

Slots	Comp.
S_1	C_1
	C_2
	C_3
S_2	$\vdots$
S_3	
S_4	
S_5	
S_6	C_8
	C_9
	C_{10}

$$\boxed{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}$$

$$= P(10,6) = \frac{10!}{(10-6)!} = \frac{10!}{4!}$$

<u>Combination</u> Count distinct, unordered selections.

↳ Counting subsets.

<u>Ex</u> How many 1-elem subsets of $\{1,2,3,4\}$?

↳ $\{1\}, \{2\}, \{3\}, \{4\}$

<u>A</u> 4 1-elem subsets of $\{1,2,3,4\}$

<u>Def</u> Let $n \geq k \geq 0$ be non-negative integers. Define the binomial coefficient $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$.

Do <u>not</u> write ~~as the~~ with horizontal line

<u>Rmk</u> $\binom{n}{k}$ counts the # of k-element subsets of an n-elem set

<u>Ex</u> $$\binom{4}{1} = \frac{4!}{1!\,(4-1)!} = \frac{4!}{3!} = \frac{4\cdot 3\cdot 2\cdot 1}{3\cdot 2\cdot 1} = 4$$

There are 4 1-elem subsets of $\{1,2,3,4\}$.