Gale-Shapley Algorithm

Input Two disjoint sets of players X, Y; and strict preferences for each player.

↳ Identify proposer set: X

↳ Identify acceptor set: Y

Stable Matching

$x_1 - y_1$

$x_2 - y_2$

x_3

y_3

Try x_3, x_2, x_1

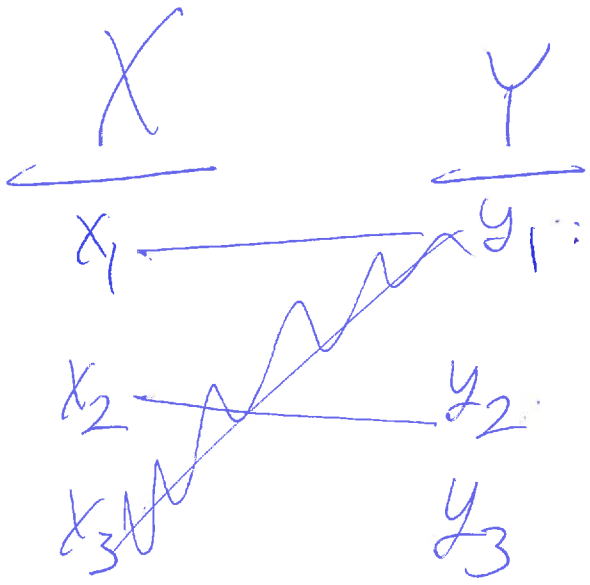
Stable Matching

$x_1 - y_1$

$x_2 - y_2$

x_3

y_3



Thm The order in which the proposers propose does not matter. (Though order should stay same for entire algorithm.)

Stable-Marriage Problem

Two disjoint sets of players
↳ $X = \{x_1, \ldots, x_n\}$
$Y = \{y_1, \ldots, y_n\}$

The X players can only be matched with Y players
The Y players can only be matched with X players
Everyone can be single, and everyone starts single.
↳ Each X player has strict preferences over Y players, though an X player may prefer to remain single than match w/ Y players.
↳ Same holds for Y-players, though Y-player preferences look at X-players.

Ex (Prefs listed Most to least preferred)

- x_1: y_2, y_1
- x_2: y_1, y_2, y_3
- x_3: y_1, y_2
- y_1: x_1, x_3, x_2
- y_2: x_2, x_1, x_3
- y_3: x_1, x_3, x_2

Matching (Not stable)

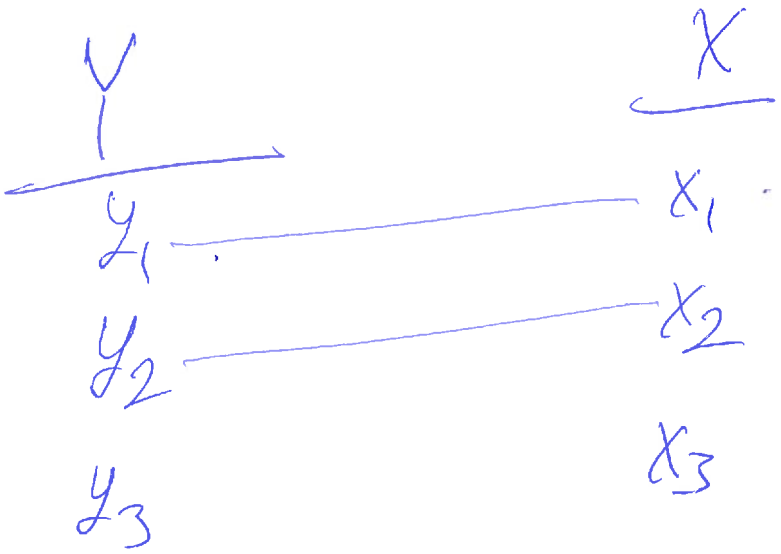
$x_1 - y_1$
$x_2 - y_2$
$x_3 - y_3$

b/c x_3 would rather be single than match w/ y_3.

Stable Matching

$x_1 - y_1$	x_3
$x_2 - y_2$	y_3

Ex What is stable Matching if Y players propose?



Stable Matching

$x_1 - y_1$

$x_2 - y_2$

x_3

y_3

Ex

Firms

F_1	F_2	F_3	F_4
	W_1	W_1	W_3
W_3	W_3	W_3	W_1
W_1	W_4		W_4
	W_2		W_2

Workers

W_1	W_2	W_3	W_4
F_1	F_1	F_3	F_4
F_3	F_4	F_1	F_1
	F_2	F_4	F_3
	F_3		

Firms propose

$F_1 - W_3$ (Better for F_1, F_3)

$F_2 - W_2$

$F_3 - W_1$

$F_4 - W_4$

Workers Propose

$W_1 - F_1$ (Better for W_1, W_3)

$W_2 - F_2$

$W_3 - F_3$

$W_4 - F_4$