7.6 Bayes' Law

Recall $\Pr[A|B] = \dfrac{\Pr[A \cap B]}{\Pr[B]}$

Observe $\Pr[A \cap B] = \Pr[B|A]\Pr[A]$
$= \Pr[A|B]\Pr[B]$

Observe $\Pr[B] = \Pr[B|A]\Pr[A] + \Pr[B|A']\Pr[A']$

Bayes' Law $\Pr[A|B] = \dfrac{\Pr[B|A]\Pr[A]}{\Pr[B|A]\Pr[A] + \Pr[B|A']\Pr[A']}$

↳ Numerator $\Pr[A \cap B]$

↳ Denominator $\Pr[B]$

Ex Test for Disease (Outcomes Pos, Neg)

↳ Has 0.95 prob giving pos result when person has disease. $Pr[Pos | Dis] = 0.95$

↳ Has 0.1 prob giving pos. result when person does not have disease. $Pr[Pos | \sim Dis] = 0.1$

($\sim Dis = Dis'$)

↳ $Prob[Dis] = 0.005$, $Pr[\sim Dis] = 0.995$

Want $Pr[Dis | Pos] = \dfrac{Pr[Pos | Dis]\, Pr[Dis]}{Pr[Pos|Dis]Pr[Dis] + Pr[Pos|\sim Dis]Pr[\sim Dis]}$

$$= \frac{0.95(0.005)}{0.95(0.005) + 0.1(0.995)}$$

Ex Test for Dis

↳ $Pr[Dis] = \frac{1}{10,000}$

↳ $Pr[Pos|Dis] = 0.99$

↳ $Pr[Pos|\sim Dis] = 0.01$

Want $Pr[Dis|Pos] = \frac{0.99\left(\frac{1}{10,000}\right)}{0.99\left(\frac{1}{10,000}\right) + .01\left(\frac{9999}{10000}\right)}$