7.5 Conditional Probability

Idea We have two events: A, B.
If B occurred (or didn't occur),
what can we say about whether A occurred?

Notation $\Pr[A|B]$: probability of A occurring, given that B occurred.

$$\Pr[A|B] = \frac{\Pr[A \cap B]}{\Pr[B]}$$

Sample Space

If B occurs, pretend Sample space is B.

Ex Sample Space S is set of 52 playing cards in standard deck.
↳ X is event of drawing Ace
↳ Y is event of drawing Red card

$$\Pr[X|Y] = \frac{2}{26} = \Pr[X] = \frac{4}{52}$$

Def Let X and Y be events. We say that X, Y are *independent* if

$$\Pr[X|Y] = \Pr[X] \quad \left(\text{Similarly, } \Pr[Y|X] = \Pr[Y]\right)$$

Ex Suppose we toss two distinguishable 6-sided dice.

↳ X: Event that the dice added to 5

↳ Y: Event that second die rolled 2.

Determine $\Pr[X] = 4/36$

$$\Pr[X|Y] = \frac{1}{6}$$

$Y = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2)\}$

Q Are X and Y independent?

No. $\Pr[X] \neq \Pr[X|Y]$

$$4/36 \neq \frac{1}{6}$$

Ex ↳ 36% of families own a dog

↳ 30% of families own a cat

↳ 22% of families that own a dog also own cat.

$\Pr[\text{Dog}] = 0.36$ $\Pr[\text{Cat}] = 0.3$

$\Pr[\text{Cat} \mid \text{Dog}] = 0.22$

Goal Want $\Pr[\text{Dog} \mid \text{Cat}]$.

Recall $\Pr[\text{Cat} \mid \text{Dog}] = 0.22 = \dfrac{\Pr[\text{Cat} \cap \text{Dog}]}{\Pr[\text{Dog}] = 0.36}$

So $\Pr[\text{Cat} \cap \text{Dog}] = 0.22(0.36)$

$$\Pr[\text{Dog} \mid \text{Cat}] = \frac{\Pr[\text{Cat} \cap \text{Dog}]}{\Pr[\text{Cat}]} = \frac{0.22(0.36)}{0.3}$$