

# Math 122- Final Exam Review

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## 1 Exam 1

**Problem 5, Section 1.4.** Suppose we have a market for apples, where the quantity supplied is given by the function  $S(p) = 4p - 50$  and the quantity demanded is given by the function  $Q(p) = 300 - 6p$ .

- (a) **(10 pts)** Find the equilibrium price and quantity. Show all work.
- (b) **(10 pts)** Now suppose a 5% tax is imposed on the **producer**. Determine the new equilibrium price and quantity.

**(10 pts) Problem 6, Section 1.7.** You invest \$3000 in an account with 6% interest, compounded monthly. Determine the number of months it will take for your account to accrue \$7000.

**Problem 7, Section 1.7.** Suppose the population of Kenya was 19.5 million in 1985 and 39.0 million in 2010.

- (a) **(10 pts).** Assuming the population increases exponentially, find a formula for the population of Kenya as a function of time (where  $t$  is the number of years since 1980; this is not a typo).
- (b) **(5 pts)** Using your answer to part (a), determine the year in which the population of Kenya will be 45.0 million. [If you get a fractional year, round down].

**(15 pts) Problem 9, Sections 3.1-3.2.** Differentiate the following functions.

- (a)  $f(x) = 3x^5 + 5x^4 + 2x^3 + \frac{1}{x} + x^{-1}$
- (b)  $g(x) = e^x + \ln(x) + 3 \cdot 5^x$
- (c)  $h(x) = 4x^2 + 5 \cdot 2^x + 4\log_5(x)$
- (d)  $k(x) = \frac{1}{\sqrt{x}} + \sqrt[3]{x}$
- (e)  $m(x) = 3$

## 2 Exam 2

**(15 pts) Problem 1, Sections 3.3-3.4.** Differentiate the following functions.

- (a)  $\frac{x+1}{x-5}$
- (b)  $\ln(12e^{-0.5x})$ .
- (c)  $2^{3^x}$ .

**(10 pts) Problem 2, Sections 3.3-3.4.** Suppose  $f(1) = 2$  and  $f'(1) = -3$ . If  $g(x) = \sqrt{f(x)}$ , determine  $g'(1)$ .

**(10 pts) Problem 3, Section 2.1.** Graph a differentiable function that satisfies **ALL** of the following properties.

- $f(x)$  is increasing whenever  $x < -5$ .
- $f(-5) < 0$  and  $f'(-5) = 0$ .
- $f(0) > 0$  and  $f'(0) < 0$ .
- $f(5) > 0$  and  $f'(5) > 0$ .

**(10 pts) Problem 8, Section 4.1.** Find the constants  $a, b$  such that the minimum for the parabola  $f(x) = x^2 + ax + b$  is at the point  $(3, 9)$ .

**Problem 9, Section 2.4.** Let  $f(x) = e^{-3x}$ .

- (5 pts)** Determine all intervals where  $f(x)$  is increasing; all intervals where  $f(x)$  is decreasing; and all intervals where  $f(x)$  is neither increasing nor decreasing.
- (10 pts)** Determine all intervals where  $f(x)$  is concave-up; all intervals where  $f(x)$  is concave-down; and all intervals where  $f(x)$  is neither concave-up nor concave-down.

### 3 Exam 3

**(10 pts) Problem 1, Section 4.2.** Determine all points of inflection of  $f(x) = x^3 - 2x^2 - 3x + 4$ .

**(15 pts) Problem 3, Section 4.4.** A landscape architect wants to enclose a 2000 square foot rectangular region. She will use shrubs costing \$30/ft along three sides and fencing costing \$20/ft along the fourth side. Determine the minimum cost.

**(15 pts) Problem 4, Section 4.4.** A farmer uses  $x$  lb of fertilizer per acre, at a cost of \$2/lb. The farmer has a revenue of  $R = 700 - 400e^{-x/100}$  dollars per acre. Determine the amount of fertilizer that should be applied per acre to maximize profit.

**(15 pts) Problem 5, Sections 6.2-6.3, 6.6.** Evaluate the following integrals.

(a)  $\int 2\pi r \, dr$

(b)  $\int \left( \frac{2}{3t} + t^{-3} + 3e^t + \ln(3)3^t \right) dt$

(c)  $\int_{-1}^1 x^7 \, dx$

**(20 pts) Problem 6, Section 6.6.** Evaluate the following integrals.

(a)  $\int \frac{\ln(x)}{x} \, dx.$

(b)  $\int_1^{15} \frac{1}{\sqrt{t+1}} \, dt$

**(10 pts) Problem 8, Section 5.4.** Water is pumped out of a holding tank at a rate of  $5 - 5e^{-0.12t}$  litres per minute, where  $t$  is the number of minutes since the pump started. If the holding tank contains 1200 litres of water when the pump is started, how many litres does it contain an hour later?