

5.5

Last time  $\int \sec(x) dx = \int \sec(x) \left( \frac{\sec(x) + \tan(x)}{\sec(x) + \tan(x)} \right) dx$

$$= \ln(|\sec(x) + \tan(x)|) + C$$

Ex  $\int \csc(x) dx = \int \csc(x) \left( \frac{\csc(x) + \cot(x)}{\csc(x) + \cot(x)} \right) dx$

$$u = \csc(x) + \cot(x)$$

$$du = (-\csc(x)\cot(x) - \csc^2(x)) dx$$

$$du = -(\csc(x)\cot(x) + \csc^2(x)) dx$$

$$\Rightarrow \int \frac{1}{u} du = -\ln(|u|) + C$$

$$= -\ln(|\csc(x) + \cot(x)|) + C$$

$$\underline{\text{Ex}} \quad \int x \sqrt{x+1} \, dx = \int (u-1) \sqrt{u} \, du$$

$u = x+1$   
 $du = dx$

$\begin{matrix} & & \uparrow & & \uparrow \\ & x & & \sqrt{x+1} & \end{matrix}$

$$= \int (u^{3/2} - u^{1/2}) \, du = \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{5} (x+1)^{5/2} - \frac{2}{3} (x+1)^{3/2} + C$$

$$\underline{\text{Ex}} \quad \int 3x \sqrt[4]{x-2} \, dx = 3 \int (u+2) u^{1/4} \, du$$

$u = x-2$   
 $du = dx$

$$= 3 \int (u^{5/4} + 2u^{1/4}) \, du$$

$$= 3 \left( u^{9/4} \left( \frac{4}{9} \right) + 2 \left( \frac{4}{5} \right) u^{5/4} \right) + C$$

$$= 3 \left( (x-2)^{9/4} \cdot \frac{4}{9} + \frac{8}{5} (x-2)^{5/4} \right) + C$$

## U-Sub (Definite Integrals)

$$\underline{\text{Ex}} \int_1^3 2x(x^2+5)^{100} dx = \int_{u(1)=6}^{u(3)=14} u^{100} du = \frac{u^{101}}{101} \Big|_6^{14}$$

$$u = x^2 + 5$$

$$du = 2x dx$$

$$= \frac{14^{101}}{101} - \frac{6^{101}}{101}$$

$$\underline{\text{Ex}} \int_e^{e^2} \frac{1}{x \ln(x)} dx = \int_1^2 \frac{1}{u} du = \ln(|u|) \Big|_1^2$$

$$u = \ln(x)$$

$$du = \frac{1}{x} dx$$

$$= \ln(2) - \ln(1)$$

$$= \ln(2)$$

Ex

$$\int_4^9 \frac{e^{\sqrt{y}}}{\sqrt{y}} dy = \int_4^9 e^{y^{1/2}} \cdot y^{-1/2} dy$$

$$u = \sqrt{y} = y^{1/2}$$

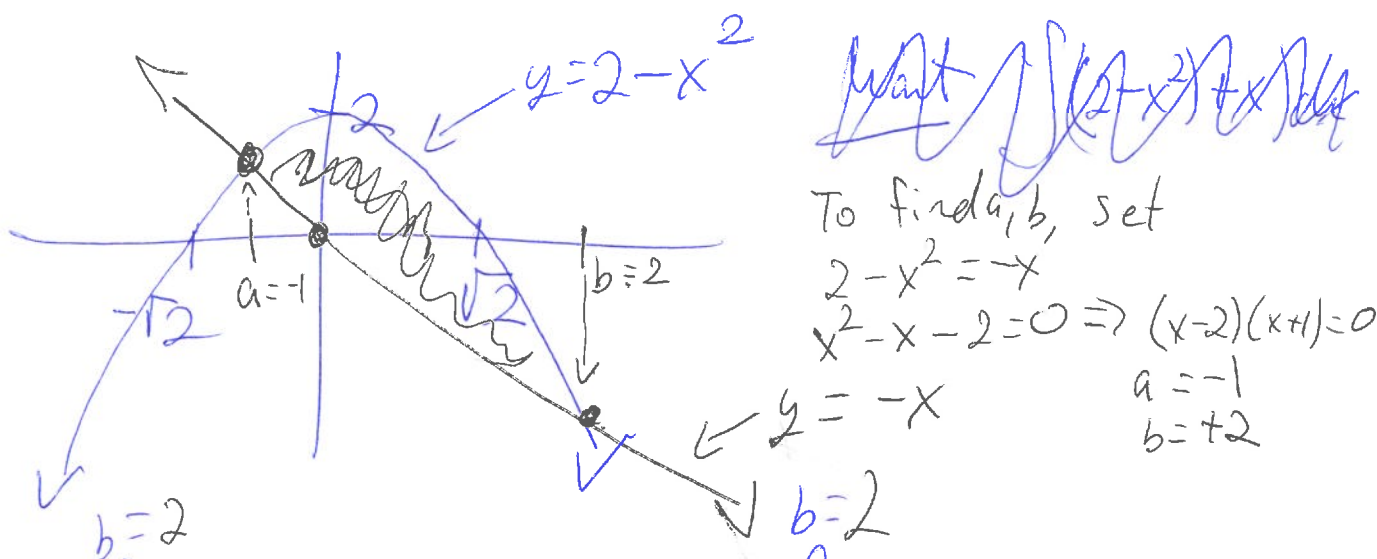
$$du = \frac{1}{2} y^{-1/2} dy$$

$$= 2 \int_2^3 e^u du = 2e^u \Big|_2^3$$

$$= 2e^3 - 2e^2$$

## 5.6 Area btwn curves

Ex Want area btwn  $y = -x$  and  $y = 2 - x^2$



Want

$$\int_{a=-1}^b \int (2 - x^2 - (-x)) dx = \int_{a=-1}^b (2 - x^2 + x) dx$$

$$= \left( 2x - \frac{x^3}{3} + \frac{x^2}{2} \right) \Big|_{-1}^2 = \left( 4 - \frac{8}{3} + \frac{4}{2} \right) - \left( -2 + \frac{1}{3} + \frac{1}{2} \right)$$

$$= 9/2$$

Ex Find area btwn  $y = \underline{2x^2 + 10}$  and  $y = \underline{4x + 16}$

$$b = 3$$

$$\int_a^b (4x + 16 - 2x^2 - 10) dx = \frac{64}{3}$$

$$a = -1$$

$$2x^2 + 10 = 4x + 16$$

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 0$$

$$2(x-3)(x+1) = 0$$

$$x = 3, -1$$

$$b \quad a$$