Math 170- Worksheet 6.1-6.2 (Selected Solutions)

Problem 1) Let $U = \{0, 1, 2, \ldots, 10\}$ be the universal set. $A = \{0, 2, 4, 6, 8\}, B = \{1, 3, 5, 7\}$ and $C = \{2, 8, 4\}$. For each of the following sets, (i) list the elements and (ii) determine the cardinality of the given set.

(a) $A \cup B$

(b) $A \cap C$

(c) $B \cap C$

(d) $A \cap A'$

(e) $A \cap (B \cup C)$

Solution: We note that $B \cup C = \{1, 2, 3, 4, 5, 7, 8\}$. Now the elements A shares with $B \cup C$ include $2, 4, 8$. So $A \cap (B \cup C) = \{2, 4, 8\}$. We have that $n(A \cap (B \cup C)) = 3$.

(f) $A \cap B \cap C$

Solution: We first remark that the intersection operation is associative. That is, $A \cap (B \cap C) = (A \cap B) \cap C$. It does not matter if you evaluate $A \cap B$ first or $A \cap C$ first. We note that $A \cap B = \emptyset$. So $(A \cap B) \cap C = \emptyset \cap C = \emptyset$. So $n(A \cap B \cap C) = 0$.

You may also arrive at this conclusion by observing that $A \cap B \cap C$ contains precisely the elements that belong to A and belong to B and belong to C.

Problem 2) Let $U = \{1, 2, 3, 4, 5\}$ be our universal set. Let $A = \{1, 2, 3\}$ and $B = \{2, 4\}$. For each of the following sets, (i) list the elements and (ii) determine the cardinality of the given set.

(a) $A \times B \times A$

Solution: The set $A \times B \times A = \{(x, y, z) | x \in A, y \in B, \text{ and } z \in A\}$. Listing out the ordered triples, we have:

$$\begin{aligned} A \times B \times A = \{&(1,2,1),(1,2,2),(1,2,3),(1,4,1),(1,4,2),(1,4,3),\\ &(2,2,1),(2,2,2),(2,2,3),(2,4,1),(2,4,2),(2,4,3),\\ &(3,2,1),(3,2,2),(3,2,3),(3,4,1),(3,4,2),(3,4,3)\}. \end{aligned}$$

We note that $n(A \times B \times A) = n(A) \cdot n(B) \cdot n(C) = 3 \cdot 2 \cdot 3 = 18$.

(b) $(A \times B) \cap (B \times B)$

(c) $(A \cap B) \times A'$

Problem 3) Let S be the set of outcomes when two distinguishable 6-sided dice are rolled. Let $E \subseteq S$ be the set in which at least one die shows an even number, and let $F \subseteq S$ be the set of outcomes in which at least one die shows an odd number. List the elements in each of the following subsets of S.

(a) E'

(b) $E \cup F$

(c) $E' \cup F'$

(d) $(E \cap F)'$

(e) $E' \cap F'$

Remark: The set of outcomes for rolling a single die is $\{1, 2, 3, 4, 5, 6\}$. So $S = \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}$. In particular, note that E and F consist of *ordered pairs.* Start by listing out the elements in E and F if you are struggling with this problem. On a quiz or exam, the listing problems will be much shorter.Given the somewhat tedious nature of this problem, I will **not** discuss it during review time in class. However, I will be happy to discuss this with you in office hours.

Problem 4) In November 2011, a Google search for "asteroid" yielded 25 million hits. A search for "comet" on Google yielded 93.5 million hits. A search for both "asteroid" and "comet" yielded 3.1 million hits. How many hits contained "asteroid", "comet", or both?

Problem 5) The dining hall offers a total of 14 desserts, of which 8 have ice cream as a main ingredient and 9 have fruit as a main ingredient. Assuming that all of them have either ice cream, fruit, or both a a main ingredient, how many have both?

Solution: Let D be the set of desserts. Let I be the set of desserts with ice cream, and let F be the set of desserts with fruit. By the Principle of Inclusion-Exclusion, we have that:

$$n(D) = n(I) + n(F) - n(I \cap F).$$

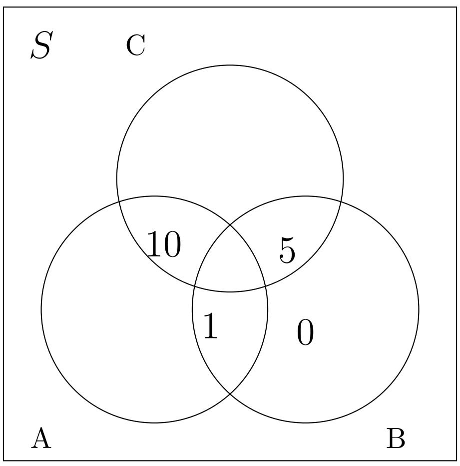
So $14 = 8 + 9 - n(I \cap F)$. Solving for $n(I \cap F)$, we have that $n(I \cap F) = 3$.

Problem 6) In a study of Tibetan children, a total of 1556 children were examined. Of these, 615 had cavities. Of the 1313 children living in non-urban areas, 504 had cavities.

(a) How many children living in urban areas had cavities?

(b) How many children living in urban areas did not have cavities?

Problem 7) Using the information given, complete the following venn diagram: $|A| = 16, |B| = 11, |C| = 30$, and $|S| = 40$. [**Note:** S is the universal set.]



Problem 8) Of the 4700 students at Medium Suburban College (MSC), 50 play soccer, 60 play lacrosse, and 96 play football. Only 4 students play both soccer and locrosse, 6 play soccer and football, and 16 play both lacrosse and football. No students play all three sports.

(a) Set up and complete the venn diagram.

(b) How many students play no sports?

(c) How many students play only soccer?