Math 170- Final Exam Information

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1 Final Exam

Recall the grade scale from the syllabus:

- HW Average: 15%
- Quiz Average: 15%
- Best Midterm: 20%
- Middle Midterm: 15%
- Worst Midterm: 10%
- Final Exam: 25%.

The following changes will be made.

1. Students who are happy with their grades prior to the final exam may elect to keep their grade and not take the final exam. **If you intend to take the final exam, you must let me know in writing by December 11 at 11:59 PM.**

2. Students who take the final exam will be graded under either the scheme on the syllabus or the following scheme, whichever yields the higher grade.

   - HW Average: 20%
   - Quiz Average: 20%
   - Best Exam: 26.67%
   - Middle Exam: 20%
   - Worst Exam: 13.33%

   Note: This scheme is the same as the following:

$$\frac{1}{0.75}\left(HW * 0.15 + \text{Quiz} * 0.15 + \text{Best Exam} * 0.2 + \text{Middle Exam} * 0.15 + \text{Worst Exam} * 0.1\right).$$

2 Last Week of Class

In looking at the schedule, I anticipate we will have covered everything by November 27. Exam 3 is scheduled for November 29, and December 6 will be used for Final Exam review. This leaves December 4 as a tentative free day. We have some options as to what to cover.

- **Ch. 8 Random Variables:** We would cover the Binomial and Geometric distributions, with some example problems. The benefit of this option would be additional preparation for your Probability and Statistics course. There would be one optional WebAssign assignment, as well as an optional bonus quiz on December 6. Effectively, I would drop your lowest homework and an additional quiz. So anyone who chooses not to do the homework or take the quiz would not be affected. On the other hand, this would allow you to replace your lowest homework and quiz grades respectively.

- **Graph Theory:** To quote Bud Brown, "Graph Theory is a subject whose deceptive simplicity masks its vast applicability." Informally, a graph is an object used to model relations amongst various objects. The applications are quite numerous, including:
  - The efficient storage of chemicals (graph coloring).
  - Optimal Assignments (matchings).
  - Distribution networks, such as routing traffic or oil (network flows).
  - Navigation (e.g., finding the shortest path to drive).
  - Determining which roads to plow in a snow storm to ensure everyone can get between any two destinations (minimum spanning trees).

  Graph theory is a very visual and tangible subject. We will be drawing lots of diagrams. Additionally, we will examine problems whose setups are easy to explain to middle school students, yet still provide significant depth. For example, consider the graph coloring problem. A *coloring* of a graph is an assignment colors to the vertices, so that no two adjacent vertices have the same color. The goal of the graph coloring problem is to determine the minimum number of colors needed to color the vertices of the graph.

  The *Peterson Graph* is pictured below. So for instance, vertices a and j are not adjacent, so they can receive the same color. However, vertices a and b are adjacent (as they are connected by an edge/line), and so they must receive different colors. Can you determine the minimum number of colors needed to color the Peterson graph?

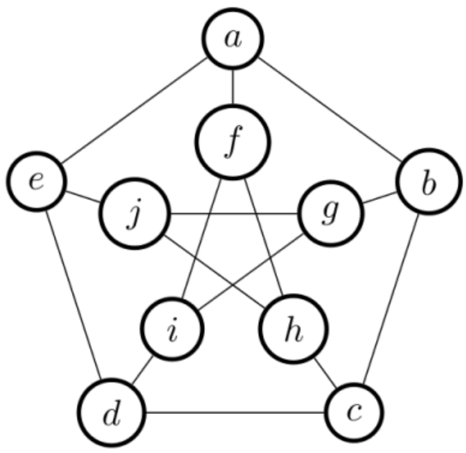

Figure 1- Peterson Graph.

  Despite how easy it is to introduce the graph coloring problem, it is something that has stumped even computers. While determining the minimum number of required colors is easy for some graphs, it is infeasible (even for computers) to determine this number for arbitrary graphs, a result that dates back to 1972 (the precise notion of hardness is NP-Completeness; see Richard Karp's paper *Reducibility Among Combinatorial Problems*).

  Assessments: I will hand out a problem set in class on December 4. It will be due on December 7 by 4:00 PM to my mailbox on the fourth floor of LeConte. Problems will have point values. Any points you earn will be added to your best exam. If we cover graph theory, you should expect a couple graph theory questions on the final exam. I will make every effort to grade your work by December 10, so that you can have feedback.

- **Game Theory:** Game Theory provides a mathematical framework for the study of competitive and cooperative behavior where the agents are rational (or exhibit a degree of bounded rationality). Economics, the social sciences, the life sciences, and computer science frequently employ game theory to model behavior, such as in evolutionary biology (evolutionary game theory), artificial intelligence, the formation of networked structures based on certain incentives (economic and social networks), designing election systems (social choice theory), and auctions.

  We will examine (finite) one-stage, simultaneous-play games of perfect information, known as normal form games, as well as the solution concept known as a Nash equilibrium. Informally, a Nash equilibrium is a selection of strategies such that no individual player wishes they had chosen a different (better) strategy (with respect to their own payoff). In particular, we will identify pure-strategies Nash equilibria in normal form games, as well as compute mixed-strategies Nash equilibria.

Let's examine an example of a normal form game, the standard Prisoner's Dilemma.

Example 1 (Prisoner's Dilemma). In this game, the police have two accomplices of a crime in separate rooms. They are each offered a deal: implicate the other prisoner and earn a reduced sentence if the other player remains silent. If both players remain silent, they each end up in jail for two years. If both players implicate each other, they each go to jail for five year.

Formally, we have two players $N = \{1, 2\}$. Each player has the strategy set $S_i = \{\text{Quiet}, \text{Fink}\}$. If both players play Quiet, they each go to jail for 2 years (denoted by the $(-2, -2)$ payoff in the top cell); and if both play Fink, they each go to jail for 5 years. If one player plays Quiet and the other Fink, they go to jail for 10 years and 1 year respectively.

We represent the normal form game using the following matrix known as a **payoff matrix**. Player 1's strategies are on the left-side while Player 2's strategies are on the top of the matrix. Each cell represents the payoffs of the form $(u_1(s_1, s_2), u_2(s_1, s_2))$ for the selected strategies $s_1 \in S_1$ and $s_2 \in S_2$.

At first glance, it appears that it is best for both players to stay Quiet. Upon further inspection, we see this is not the case. If one player stays Quiet, the other player benefits by choosing Fink instead. The player who chose Fink earns only 1 year in prison rather than 2 years, while the person who remained Quiet gets 10 years in prison rather than 2 years. So (Quiet, Quiet) is **not** a Nash equilibrium. By similar argument, (Fink, Quiet) and (Quiet, Fink) are not Nash equilibria. Observe now that if both players choose to Fink, no single player wishes they had chosen to remain Quiet instead. So (Fink, Fink) is our Nash equilibrium; no player can unilaterally deviate and improve their outcome.

		Player 2	
		Quiet	Fink
Player 1	Quiet	$-2, -2$	$-10, -1$
	Fink	$-1, -10$	$-5, -5$

Assessment: You will be assessed in the manner specified in the Graph Theory section.

Remark: As a matter of personal taste, I am partial towards spending a day on graph theory. Out of the three subjects, this is the one I think you all will enjoy the most. Also, the difficulties of (our coverage of) the three subjects will be equal.