Math 141 Study Guide: Section 2.3

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Instructions: Answer all questions. Show all work and justify all your answers in **complete sentences.**

1 Section 2.3.

Problem 1) State the definition of a limit in terms of δ and ϵ. [**Note:** This will be a problem on Exam 1 and the Final Exam.]

Problem 2) For each of the following, evaluate the limit. Then for the given ϵ, find a corresponding δ satisfying the $\delta - \epsilon$ definition of the limit.

(a) $\lim_{x \to 4}(x+1)$, with $\epsilon = 0.01$.

(b) $\lim_{x \to -2}(2x-2)$, with $\epsilon = 0.02$.

(c) $\lim_{x \to 12} \sqrt{16-x}$, $\epsilon = 2$.

(d) $\lim_{x \to 3} x^2$, $\epsilon = 1$.

(e) $\lim_{x \to 4} \frac{1}{x}$, $\epsilon = 1$.

(f) $\lim_{x \to -5} \frac{x^2+6x+5}{x+5}$, $\epsilon = 1$. [**Note:** We will cover an example similar to this problem in class on Wednesday.]

Problem 3) Suppose that $\lim_{x \to c} f(x) = L$. Now suppose that you are given $\epsilon > 0$, and that $\delta > 0$ satisfies the definition of the limit. That is, if $0 < |x - c| < \delta$, then $|f(x) - L| < \epsilon$.

(a) Could we have chosen $\delta/2$ instead, to satisfy the definition of a limit? Justify your reasoning.

(b) Would a bigger δ be guaranteed to work? If so, justify. If not, clearly explain your reasoning. [**Hint:** Consider the example from class $\lim_{x \to 10} \sqrt{19-x}$. For $\epsilon = 1$, we found that $\delta = 5$ would work. Could we have picked a larger δ?]