# 7.7 Markov Chains

Ex Business Majors (10% Bus → Non-Bus, 90% rem bus)
Non-Business Majors (20% switch → bus, 80% stay)

Find transition matrix and steady state distr.

$$P = \begin{bmatrix} .9 & .1 \\ .2 & .8 \end{bmatrix}$$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} .9 & .1 \\ .2 & .8 \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix}$$

same line

$.9x + .2y = x \rightarrow -.1x + .2y = 0$
$.1x + .8y = y \rightarrow .1x - .2y = 0$
$x + y = 1 \qquad x + y = 1$

$$\left[\begin{array}{cc|c} .1 & -.2 & 0 \\ 1 & 1 & 1 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 2/3 \\ 0 & 1 & 1/3 \end{array}\right]$$

Steady-State $\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

Ex Gambler's Ruin:

↳ Costs $10.

↳ 50% winning $20 (net gain $10)

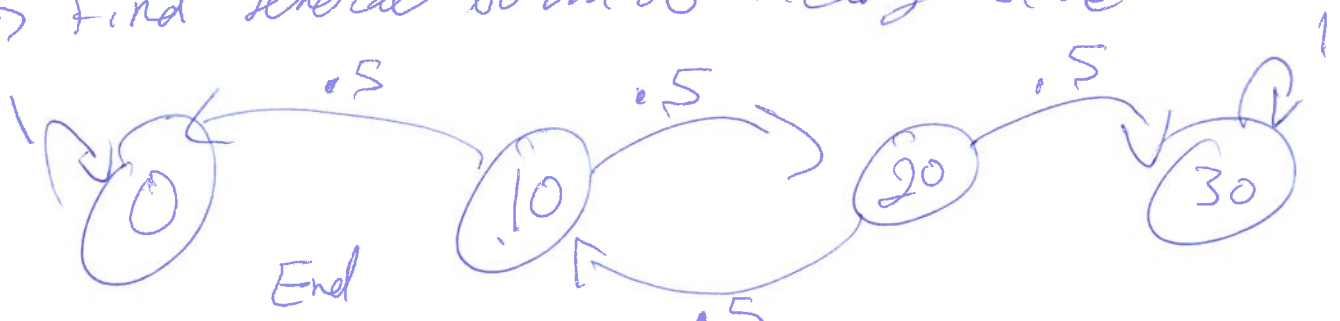
↳ 50% winning nothing

↳ You stop when you ~~win~~ have $30 or $0.

States: Amt of money in $10 increments

↳ Find trans matrix

↳ Find general form of steady state




| Start | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 |
| 2 | .5 | 0 | .5 | 0 |
| 3 | 0 | .5 | 0 | .5 |
| 4 | 0 | 0 | 0 | 1 |

← Transition matrix

$$\begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a, b, c, d \end{bmatrix}$$

$a + 0.5b = a \quad (\text{so } b = 0)$

$.5c = b \quad (c = 0)$

$.5b = c$

$.5c + d = d \quad (c = 0)$

$a + b + c + d = 1 \quad (\text{so } a + d = 1 \Rightarrow d = 1 - a)$

All Steady-State Vectors $\begin{bmatrix} a & 0 & 0 & 1-a \end{bmatrix}$

$(0 \leq a \leq 1)$