7.5 Conditional Probability

Idea We have two events A, B

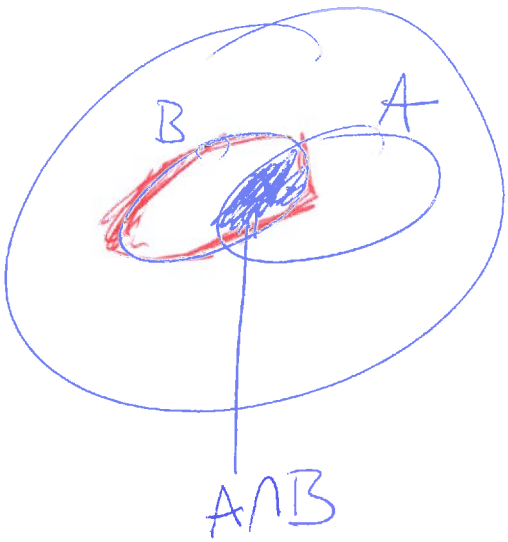
If B occurred (or didn't occur), what can we say about whether A occurred?

Notation $\Pr[A|B]$: Prob of A occurring, given that B occurred

↖ vertical line

NOT Fraction

$$\Pr[A|B] = \frac{\Pr[A \cap B]}{\Pr[B]}$$



Sample Space

If B occurred, pretend B is sample space

Ex Sample Space S is set of 52 playing cards in standard deck

↳ X: Event of drawing Ace

↳ Y: Event of drawing Red Card

$$Pr[X] = \frac{4}{52} = Pr[X|Y] = \frac{2}{26}$$

Def Let X and Y be events. We say that X and Y are independent if $Pr[X|Y] = Pr[X]$ (similarly $Pr[Y|X] = Pr[Y]$).

Ex Suppose we toss two distinguishable 6-sided dice.

↳ X be event that the dice add to 5

↳ Y be event that second die rolled 2.

Determine $Pr[X] = \frac{4}{36}$

$$Pr[X|Y] = \frac{1}{6}$$

$Y = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2)\}$.

Note $\Pr[X] \neq \Pr[X|Y]$, X and Y are not independent

$\frac{4}{36} \neq \frac{1}{6}$

Ex ↳ 36% families own dog

↳ 30% families own cat

↳ 22% families that own a dog also own a cat.

$\Pr[\text{Dog}] = 0.36, \Pr[\text{Cat}] = 0.3$

$\Pr[\text{Cat} | \text{Dog}] = 0.22$

Recall $\Pr[A|B] = \frac{\Pr[A \cap B]}{\Pr[B]}$

Q what is $\Pr[\text{Cat} \cap \text{Dog}]$?

$\Pr[\text{Cat}|\text{Dog}] = 0.22 = \frac{\Pr[\text{Cat} \cap \text{Dog}]}{\Pr[\text{Dog}] = 0.36}$

So $\Pr[\text{Cat} \cap \text{Dog}] = 0.22(0.36)$

Q Determine $\Pr[\text{Dog}|\text{Cat}]$.

$\Pr[\text{Dog}|\text{Cat}] = \frac{\Pr[\text{Cat} \cap \text{Dog}]}{\Pr[\text{Cat}]} = \frac{0.22(0.36)}{0.3}$