5.2

Ex Loan budget of \$25 M

↳ Condos 12% rate

↳ lend at least \$10M for condos

↳ Low-Income @ 10% rate

↳ $\frac{1}{3}$ or more of total loans to low income

↳ Constraint $\frac{2}{3}y \geq \frac{1}{3}x$

x : Condos

y : Low income

$$-\frac{1}{3}x + \frac{2}{3}y \geq 0$$

$$\max \; 0.12x + 0.1y \quad \text{s.t.}$$

$$x + y \leq 25$$

$$x \geq 10$$

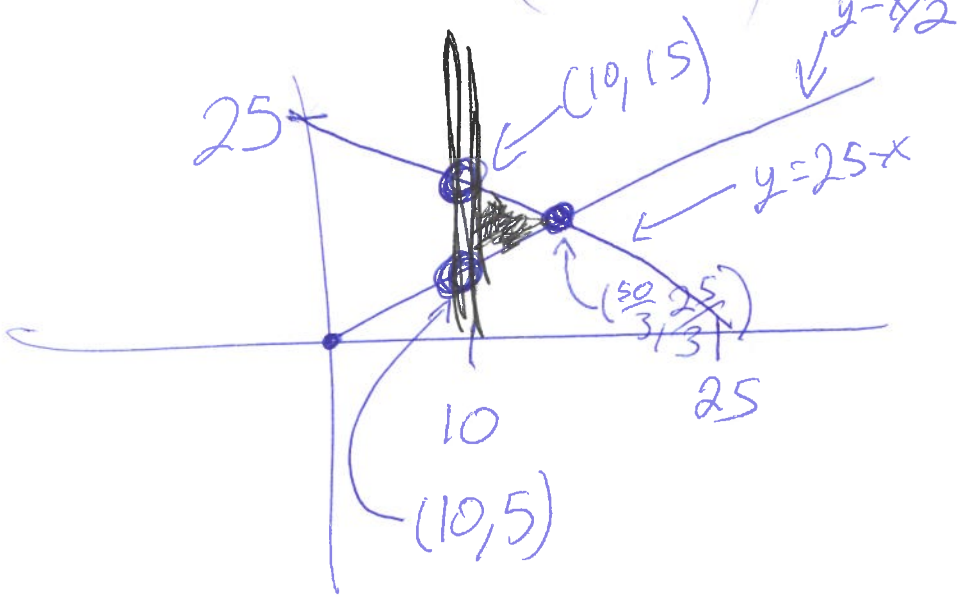
$$-\frac{1}{3}x + \frac{2}{3}y \geq 0 \quad \text{(Third of loans for low-inc)}$$

$$y \geq 0$$

$x + y \leq 25 \quad (y \leq 25 - x)$

$-\frac{1}{3}x + \frac{2}{3}y \geq 0 \quad \left(y \geq \frac{x}{2}\right)$

$(x \geq 10)$



Set

$25 - x = \frac{x}{2}$

$25 = \frac{3x}{2}$

$x = \frac{50}{3}$

$y = \frac{25}{3}$

Pt	$.12x + .1y$
$(10, 5)$	1.7
$(10, 15)$	2.7
$\left(\frac{50}{3}, \frac{25}{3}\right)$	2.833

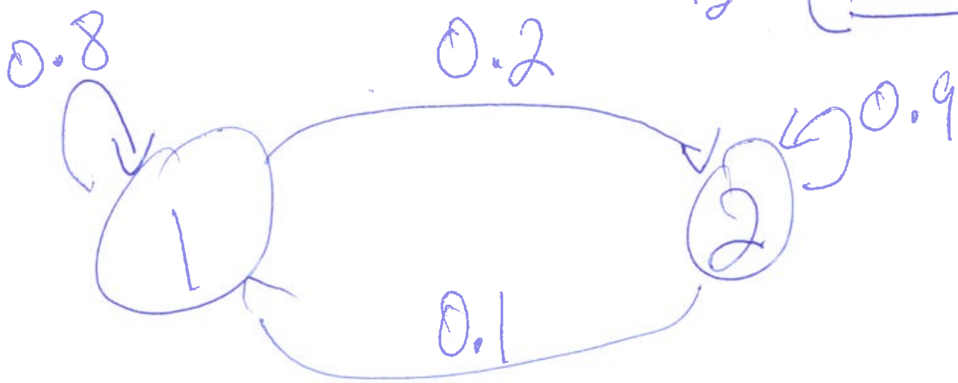
Maximizer

Maximum profit

7.7 Markov Chains

Def A Markov Chain has a finite set of states $[n] = \{1, 2, \ldots, n\}$, and for every $i, j \in [n]$, a transition probability p_{ij}. [Note $i = j$ is possible].

Ex



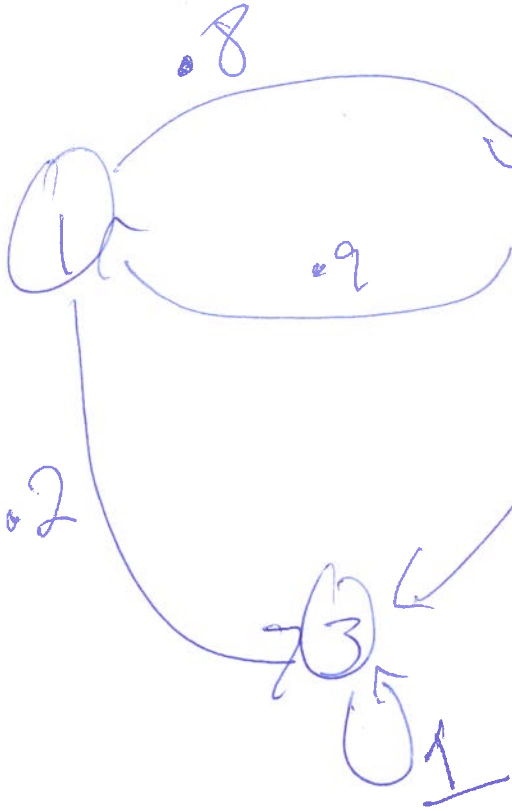
Transition Matrix

$$\text{Start}\ \begin{array}{c|cc} & 1 & \text{End}\quad 2 \\ \hline 1 & 0.8 & 0.2 \\ 2 & 0.1 & 0.9 \end{array}$$

Q What is sum of given row?

A 1 (probabilities add to 1)

Ex



$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & .8 & .2 \\ .9 & 0 & .1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Ex $P = \begin{bmatrix} 0 & 1 \\ .5 & .5 \end{bmatrix}$ (Given)

$v_0 = [.2, \ .8]$ (Initial distr.)

One Time Step $\quad V_1 = V_0 P = [.2 \quad .8]\begin{bmatrix} 0 & 1 \\ .5 & .5 \end{bmatrix}$

$$= [.4 \quad .6]$$

Second Time Step $\quad V_2 = V_1 P = (V_0 P) P = V_0 (P \cdot P)$

$$= V_0 P^2$$

$$= [.3 \quad .7]$$

$V_3 = V_0 P^3 = [.35 \quad .65]$

$V_k = V_0 P^k$

Q If we run a MC for a large number of time steps, does MC force V_0 to steady-state distribution?

Def A steady state vector $\vec{v}$ for MC satisfies $\vec{v} P = \vec{v}$ (where P is trans matrix).

For 2x2 case: $[x \quad y] P = [x \quad y]$

Ex $P = \begin{bmatrix} .8 & .2 \\ .1 & .9 \end{bmatrix}$ Want steady-state vector

$$[x \quad y] \begin{bmatrix} .8 & .2 \\ .1 & .9 \end{bmatrix} = [x \quad y]$$

$.8x + .1y = x$ → $-.2x + .1y = 0$

$.2x + .9y = y$ → $.2x - .1y = 0$

$x + y = 1$ → $x + y = 1$

Setup $\left[\begin{array}{cc|c} .2 & -.1 & 0 \\ 1 & 1 & 1 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} \end{array}\right]$

Steady State $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

Ex $P = \begin{bmatrix} .2 & .8 \\ .4 & .6 \end{bmatrix}$

Find Steady-state distr.

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} .2 & .8 \\ .4 & .6 \end{bmatrix} = \begin{bmatrix} x, & y \end{bmatrix}$$

Same line

$$.2x + \cancel{.8y}\ .4y = x \longrightarrow -.8x + .4y = 0$$

$$.8x + .6y = y \longrightarrow .8x - .4y = 0$$

$$x + y = 1 \qquad\qquad x + y = 1$$

$$\left[\begin{array}{cc|c} .8 & -.4 & 0 \\ 1 & 1 & 1 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{3} \\ 0 & 1 & 2/3 \end{array}\right]$$

Steady State $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix}$