[illegible]

6.1 - Sets and Set operations

<u>Def</u> A <u>set</u> is a collection of distinct elements (so no repeated elems). The order in which the elems are listed does not matter.

<u>Ex</u> $\mathbb{R}$ the set of real numbers

$\mathbb{N} = \{0, 1, 2, \ldots\}$ the set of natural numbers

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ the set of integers

<u>Ex</u> The set $\{1, 2, 3\}$.

<u>Note</u> $\{1, 2, 3\} = \{3, 1, 2\} = \{2, 3, 1\}$

<u>Important</u> Use <u>curly braces</u> to denote ~~elem~~ a set.

<u>Membership</u>

<u>Ex</u> $S = \{1, 2, 3\}$

$1 \in S$ (1 is an element of S)

$5 \notin S$ (5 is <u>not</u> an element of S)

Def Let S be a finite set. The *cardinality* of S is the number of elements it contains. We denote the cardinality of S as $n(S)$ (book's notation) or $|S|$

Ex $S = \{1, 2, 8\}$

$n(S) = 3$

$|S| = 3$

Def The *empty set* is the set with no elements. We denote the empty set as $\emptyset$.

Set Operations

Def Let A, B be sets. The *union* of A and B is: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Ex $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

Ex $\{1,2\} \cup \{1,3\} = \{1,2,3\}$

$$n(\{1,2,3\}) = n(\{1,2\}) + n(\{1,3\}) - 1$$

$$3 = 2 + 2 - \underbrace{1}_{n(\{1\})}$$

Inclusion - Exclusion

Def Let A, B be sets. The intersection of A and B is: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Ex $A = \{1,2,3\}$, $B = \{1,3,5\}$

$$A \cap B = \{1,3\}$$

Ex $\{1,2\} \cap \{4\} = \emptyset$

Def We say that the set $\mathcal{U}$ is our <u>universal set</u> if all sets in the given context are subsets of $\mathcal{U}$. (ie, all sets have their elements drawn only from $\mathcal{U}$).

Def Let $\mathcal{U}$ be our universal set, and let A be a subset of $\mathcal{U}$. The <u>complement</u> of A, denoted $A' = \{x \in \mathcal{U} \mid x \notin A\}$.

Ex $\mathcal{U} = \{a, b, c, d, e\}$

$A = \{a, b\}$

$A' = \{c, d, e\}$

<u>Observe</u> $|A| + |A'| = |\mathcal{U}|$

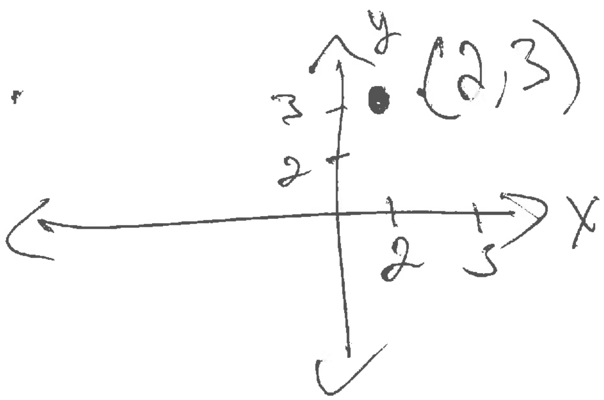
<u>Remark</u> The complement of A is denoted:

(i) A' (by the book).

(ii) A^c

(iii) $\overline{A}$

Ex Let $\mathcal{U}$ be our universal set. So $\mathcal{U}' = \emptyset$

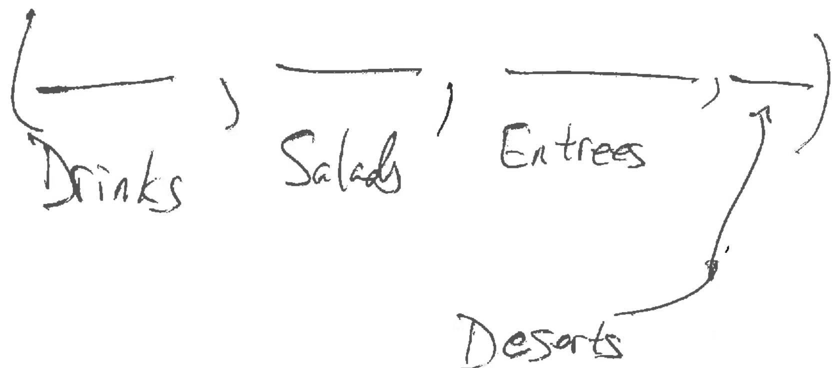


Q Is $(2,3) = (3,2)$?

A No. The order matters

Ex We go out to eat. Choices of:

(i) Drinks
(ii) Salads
(iii) Entrees
(iv) Deserts



Motivation for Cartesian Products

Def Let A, B be sets. The Cartesian product

$$A \times B = \{(a,b) \mid a \in A,\ b \in B\}$$

Ex $\mathbb{R} \times \mathbb{R}$ is the Cartesian plane

x-axis   y-axis

Ex $A = \{1, 2, 3\}$

$B = \{x, y\}$

$A \times B = \{(1,x), (1,y),$
$(2,x), (2,y),$
$(3,x), (3,y)\}$

Q Is $(y,1) \in A \times B$?

No. $(1,y) \neq (y,1)$

Q Is $(y,1) \in B \times A$?

A yes - $y \in B$
$1 \in A$

$A = \{1,2,3\}$, $B = \{x,y\}$

Q How is $n(A \times B)$ related to $n(A)$ and $n(B)$?

$n(A \times B) = 6$

$n(A) = 3$

$n(B) = 2$

So $n(A \times B) = n(A) \cdot n(B)$