Math 122- 4.1 Example

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Example: Find the maxima and minima for $h(t) = 10t\text{exp}(3 - t^2)$.

Solution: We employ the first derivative test.

- **Step 1:** We find the critical points of $h(t)$. To do this, we first differentiate $h(t)$:

$$\begin{aligned} h'(t) &= 10\text{exp}(3 - t^2) + 10t(-2t)\text{exp}(3 - t^2) \\ &= 10\text{exp}(3 - t^2)\Big(1 - 2t^2\Big) \end{aligned}$$

  We note that $h'(t)$ is defined for all $t \in \mathbb{R}$. So the critical points occur precisely when $h'(t) = 0$. Now observe that:

$$10\text{exp}(3 - t^2) > 0 \text{ for all } t \in \mathbb{R}$$

  So $h'(t) = 0$ if and only if $1 - 2t^2 = 0$. Solving for t:

$$\begin{aligned} 1 - 2t^2 &= 0 \\ 1 &= 2t^2 \\ t &= \pm\frac{1}{\sqrt{2}} \end{aligned}$$

  So our critical points are $t = -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$.

- **Step 2:** Now that we have our critical points, we build our table to look for sign changes in $h'(t)$ (i.e., when $h'(t)$ changes from positive to negative, or negative to positive).

	$10\text{exp}(3 - t^2)$	$1 - 2t^2$	$h'(t)$
$x < -\frac{1}{\sqrt{2}}$			
$-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$			
$x > \frac{1}{\sqrt{2}}$			

  Now recall that $10\text{exp}(3-t^2) > 0$ for all $t \in \mathbb{R}$. So we fill in + on the column corresponding to $10\text{exp}(3-t^2)$ to indicate this:

	$10\text{exp}(3 - t^2)$	$1 - 2t^2$	$h'(t)$
$t < -\frac{1}{\sqrt{2}}$	+		
$-\frac{1}{\sqrt{2}} < t < \frac{1}{\sqrt{2}}$	+		
$t > \frac{1}{\sqrt{2}}$	+		

Now if $t < -\frac{1}{\sqrt{2}}$, we have that $1 - 2t^2 < 0$. It may be helpful to pick an example to illustrate this. As $-\sqrt{1}\sqrt{2}$ is our smallest critical point, any point to the left of $-\frac{1}{\sqrt{2}}$ will serve this purpose. Let us pick $x = -10$. We have that:

$$1 - 2(-10)^2 = 1 - 200 < 0$$

So we fill in a $-$ in the cell corresponding to $t < -\frac{1}{\sqrt{2}}$ under the column for $1 - 2t^2$. As $10\exp(3 - t^2) > 0$ and $1 - 2t^2 < 0$ for $t < -\frac{1}{\sqrt{2}}$, we have that $h'(t) < 0$ when $t < -\frac{1}{\sqrt{2}}$.

	$10\exp(3 - t^2)$	$1 - 2t^2$	$h'(t)$
$t < -\frac{1}{\sqrt{2}}$	+	-	-
$-\frac{1}{\sqrt{2}} < t < \frac{1}{\sqrt{2}}$	+		
$t > \frac{1}{\sqrt{2}}$	+		

By similar analysis, we have that $1 - 2t^2 > 0$ when:

$$-\frac{1}{\sqrt{2}} < t < \frac{1}{\sqrt{2}}$$

And $1 - 2t^2 < 0$ when $t > \frac{1}{\sqrt{2}}$. So we update the table accordingly:

	$10\exp(3 - t^2)$	$1 - 2t^2$	$h'(t)$
$t < -\frac{1}{\sqrt{2}}$	+	-	-
$-\frac{1}{\sqrt{2}} < t < \frac{1}{\sqrt{2}}$	+	+	+
$t > \frac{1}{\sqrt{2}}$	+	-	-

Now we see that $h'(t)$ changes from negative to positive at $t = -\frac{1}{\sqrt{2}}$. So by the First Derivative Test $h(t)$ has a local minimum at $t = -\frac{1}{\sqrt{2}}$.

Similarly, $h'(t)$ changes from positive to negative at $t = \frac{1}{\sqrt{2}}$. So by the First Derivatve Test, $h(t)$ has a local maximum at $t = \frac{1}{\sqrt{2}}$.