

Ex 7.7 Markov Chains

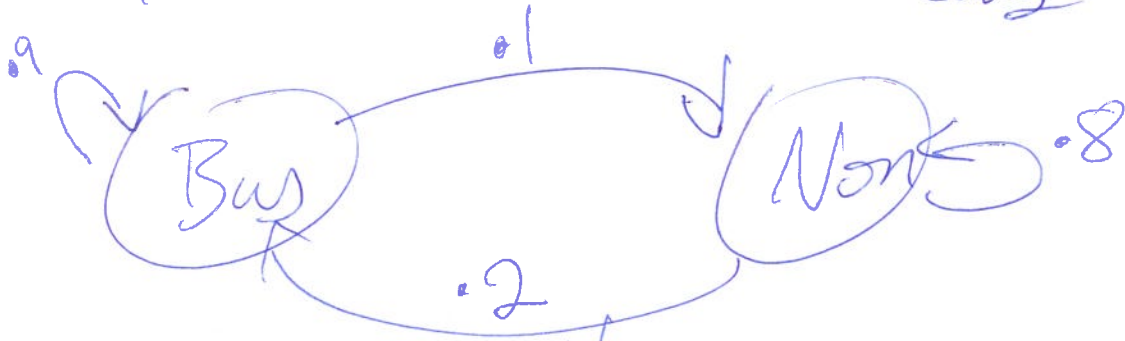
Ex Bus. Majors

- ↳ 90% remain
- ↳ 10% switch to non-bus

Non-Bus.

- ↳ 20% switch to bus.
- ↳ 80% remain non-bus

Find transition matrix and steady state.



B. $\begin{bmatrix} .9 & .1 \\ .2 & .8 \end{bmatrix}$ ← Trans. Matrix

N

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} .9 & .1 \\ .2 & .8 \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix}$$

$$\begin{aligned} .9x + .2y &= x \\ .1x + .8y &= y \\ x + y &= 1 \end{aligned} \quad \begin{aligned} &\rightarrow \begin{bmatrix} -.1x + .2y = 0 \\ .1x + .2y = 0 \end{bmatrix} \text{ Same line} \\ &\rightarrow x + y = 1 \end{aligned}$$

$$\begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 2/3 \\ 0 & 1 & 1/3 \end{bmatrix}$$

$$\boxed{[x \ y] = [2/3 \ 1/3]}$$

↑ steady state

Ex Gambler's Ruin

↳ You start w/ either \$0, \$10, \$20, \$30.

↳ Spin of lever costs \$10

↳ If you win, you have net gain of \$10 (50% chance)

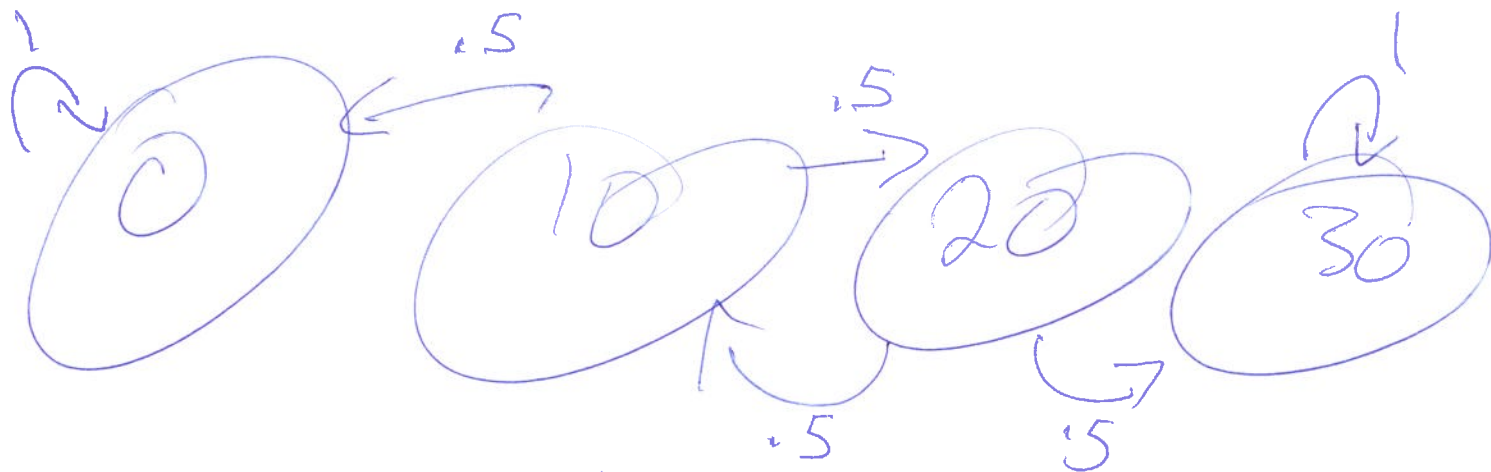
↳ If you lose you get nothing (50% chance).

↳ Stop when we have either \$0 or \$30

States Amt of \$ (in \$10 increments)

Find transition matrix and steady state (s)

[Note Any state vector has 4 components - $[a, b, c, d]$]



$$\begin{array}{c}
 0 \quad 1 \quad 2 \quad 3 \\
 \begin{bmatrix}
 0 & 1 & 0 & 0 & 0 \\
 1 & 0.5 & 0 & .5 & 0 \\
 2 & 0 & 0.5 & 0 & .5 \\
 3 & 0 & 0 & 0 & 1
 \end{bmatrix}
 \end{array}
 \leftarrow \text{trans matrix}$$

$$[a \ b \ c \ d] \begin{bmatrix}
 1 & 0 & 0 & 0 \\
 0.5 & 0 & .5 & 0 \\
 0 & .5 & 0 & .5 \\
 0 & 0 & 0 & 1
 \end{bmatrix} = [a \ b \ c \ d]$$

$$a + .5b = a \quad (\text{so } b = 0)$$

$$.5c = b \quad (\text{so } c = 0)$$

$$.5b = c \quad (\text{No new info})$$

$$.5c + d = d \quad (d = d, \text{ so new info})$$

$$a + b + c + d = 1$$

$$\hookrightarrow a + d = 1 \quad (\text{so } d = 1 - a)$$

Infinitely many
Steady states

$$[a \ 0 \ 0 \ 1-a]$$

where $0 \leq a \leq 1$