

Ch. 7

7.1 Sample Spaces and Events.

Def A sample space S is the set of all possible outcomes (ie, a universal set).

Ex Toss coin. Sample Space $S = \{H, T\}$

Roll 6-sided die. Sample Space $S = \{1, 2, 3, 4, 5, 6\}$

Def Let S be a sample space.
An event $E \subseteq S$.

Ex Roll 6-sided die, event roll even #
 $E = \{2, 4, 6\}$.

Def Let S be a sample space, and let $E, F \subseteq S$ be events. We say that E and F are mutually exclusive if $E \cap F = \emptyset$.

Ex Roll 6-sided die.

$$E = \{2, 4, 6\} \quad F = \{1, 3, 5\}$$

E and F are mutually exclusive events.

7.2 Relative Frequency.

Idea Let S be a Finite Sample Space, and let $E \subseteq S$ be an event.
Then: $\Pr[E] = \frac{n(E)}{n(S)}$.

Ex Roll 6-sided die 10 times

$\hookrightarrow 1$ (x3) $\hookrightarrow 4$ (x2)
 $\hookrightarrow 2$ (x3) $\hookrightarrow 5$ (x1)
 $\hookrightarrow 3$ (x0) $\hookrightarrow 6$ (x1)

x	1	2	3	4	5	6
$\Pr(x)$	$\frac{3}{10}$	$\frac{3}{10}$	0	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Check $\frac{3}{10} + \frac{3}{10} + 0 + \frac{2}{10} + \frac{1}{10} + \frac{1}{10} = \frac{10}{10} = 1 \checkmark$

Note Assuming each outcome equally likely. (Fair die, fair coins, etc.)

Ex Suppose fair coin is tossed 3 times.
What is $\text{Prob}[1H]$?

$$\underbrace{\quad \quad \quad}_3 \quad \quad \quad \swarrow \begin{array}{l} \text{Size of} \\ \text{Sample space} \end{array}$$
$$2 \times 2 \times 2 = 2^3 = 8 \text{ possible tosses}$$

Pick pos for H in $\binom{3}{1} = 3$ ways

$$\text{Pr}[1H] = \frac{3}{8}$$

Ex Roll 2 distinguishable dice:
one red, one green.

↳ 36 possible outcomes

↳ What is prob of rolling 2 and 3?

(2, 3) or (3, 2)

$$\text{Prob}[\text{Roll 2 and 3}] = \frac{2}{36}$$

Ex Roll 2 indistinguishable dice.

↳ Determine size of Sample space

↳ Case Outcomes on dice are distinct

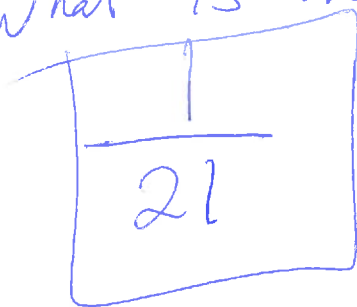
$$(1,2) = (2,1) \quad \{ \overline{6}, \overline{5} \} = 15 \text{ rolls}$$

→ 2

↳ Case Outcomes are same
6 rolls

Size of Sample Space $15 + 6 = 21$

What is Prob of rolling 2 and 3?



7.3 Ex Weighted 6-sided die, Prob of rolling 1, 2, 3, 4, or 5 is same.
Prob of rolling 6 is 3x that of rolling 1.

Det. Prob. for each outcome.

$$x_1 = \text{Pr}[1] \quad \text{Pr}[6] = 3x$$

$$\text{Pr}[2] = x$$

$$x + x + x + x + x + 3x = 1$$
$$8x = 1 \Rightarrow x = \frac{1}{8}$$