Instructions: This quiz is closed book, closed note, and an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places.

1. A landscape architect wants to enclose a 3000 square foot rectangular region. She will use shrubs costing \$20/ft along three sides and fencing costing \$10/ft along the fourth side. What is the minimum cost to enclose such a region?

(a) **(2 pts)** Determine the function to minimize, as well as the constraints.

Answer: We seek to solve:

$$\begin{aligned} \min\ & 20(2w + \ell) + 10\ell \text{ s.t.} \\ & \ell w = 3000 \\ & \ell, w > 0 \end{aligned}$$

So the function we wish to minimize is $20(2w + \ell) + 10\ell$. Our constraints are:

- Area: $\ell w = 3000$.
- $\ell, w > 0$.

(b) **(3 pts)** Construct the corresponding optimization problem in one variable.

Answer: From part (a), we have the constraint that $\ell w = 3000$. Solving for ℓ, we have that $\ell = 3000/w$. So we have the following optimization problem in one variable:

$$\begin{aligned} \min\ & 20\left(2w + \frac{3000}{w}\right) + 10 \cdot \frac{3000}{w} \text{ s.t.} \\ & w > 0 \end{aligned}$$

(c) **(4 pts)** Determine the global minimizer (this will be either the length or width of the rectangle, based on your answer in part (b)). Justify your answer.

Answer: Denote $f(w) := 20\left(2w + \frac{3000}{w}\right) + 10 \cdot \frac{3000}{w}$. So $f'(w) = 40 - \dfrac{90000}{w^2}$. Solving for our critical point, we have:

$$\begin{aligned} 40 - \frac{90000}{w^2} &= 0 \\ 40 &= \frac{90000}{w^2} \\ w^2 &= \frac{90000}{40} \\ w &= \pm\sqrt{\frac{90000}{40}} \end{aligned}$$

We note that $w = 0$ is also a critical point, as $f'(w)$ is undefined at $w = 0$.

As $w > 0$, we only consider the critical point $w = \sqrt{\frac{90000}{40}}$. We now verify that $f(w)$ has a maximum at $w = \sqrt{\frac{90000}{40}}$. We note that $f''(W) = \frac{180000}{w^3}$. So $f''(\sqrt{\frac{90000}{40}}) > 0$. Thus, by the second derivative test, $f(w)$ has a local minimum at $w = \sqrt{\frac{90000}{40}}$. As $w > 0$, we have no endpoints to check. Furthermore, $f(w)$ has no other critical points greater than 0. So $f(w)$ has a global minimum at $w = \sqrt{\frac{90000}{40}}$.

(d) **(1 pt)** Determine the minimum cost.

Answer: We evaluate $f\left(\sqrt{\frac{90000}{40}}\right) = \$3794.73$.