4.1 Let $f(x) = 10x(2-\ln(x))$ on $[1, e^2]$.

Find: Find global extrema (Global max/min).

$$f'(x) = 10(2-\ln(x)) + 10x\left(0-\frac{1}{x}\right) = 10 - 10\ln(x) = 0$$

$$10 = 10\ln(x)$$

$$\ln(x) = 1 \Rightarrow x = e \quad \text{(Crit pt)}$$

As f' changes from + to − @ $x=e$, $f(x)$ has local max @ $x=e$.

Recall $f(x) = 10x(2-\ln(x))$, on $[1, e^2]$

↳ Local Max @ $x=e$, with $f(e) = 10e(2-\ln(e)) = 10e$

↳ Endpts $f(1) = 10(2-\ln(1)) = 10(2) = 20$

$f(e^2) = 10e^2(2-\ln(e^2)) = 10e^2(2-2) = 0$

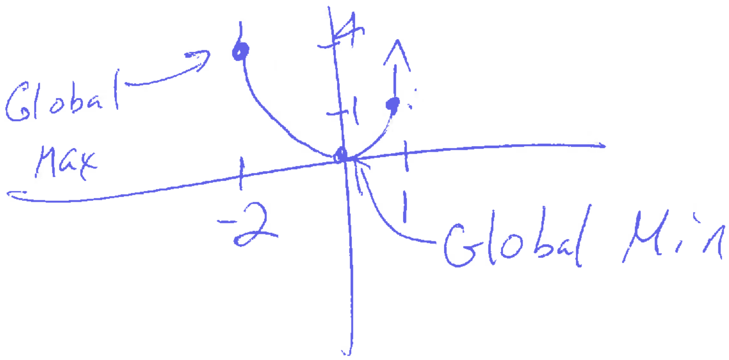
Global Max $x=e$, with $f(e) = 10e$

Global Min $x=e^2$, with $f(e^2) = 0$.

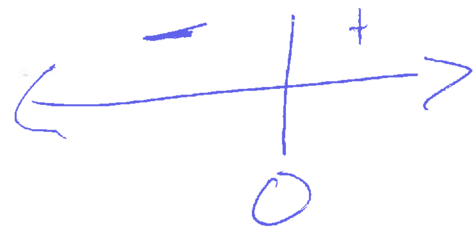
Ex $f(x) = x^2$, $[-2, 1]$

$f'(x) = 2x = 0$

Crit Pt $x = 0$



f'



Local Min @ $x=0$, $f(0)=0$

Endpts

↳ $x=-2$, $f(-2)=4$

↳ $x=1$, $f(1)=1$

As f' changes from $-$ to $+$ @ $x=0$, $f(x)$ has local min @ $x=0$.

Global Max @ $x=-2$, w/ $f(-2)=4$

Global Min @ $x=0$, w/ $f(0)=0$

$V = \frac{1}{3}\pi r^2 h$

Given

Ex Smallest value $x+y$ takes on s.t. $xy = 324$ ← constraint $(x, y > 0)$.

So $y = \frac{324}{x}$. Thus: min $\left(x + \frac{324}{x}\right)$

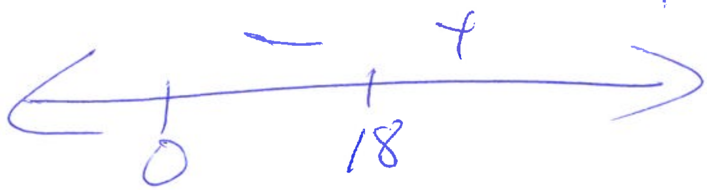
Let $f(x) = x + \frac{324}{x}$, so $f'(x) = 1 - 324x^{-2} = 0$

$1 = \frac{324}{x^2}$

$x^2 = 324 \Rightarrow x = \pm\sqrt{324}$
$= \pm 18$

Crit Pts $x = \pm 18, 0$

As f' changes from $-$ to $+$ @ $x=18$, $f(x)$ has local ~~max~~ min @ $x=18$.



$f'(x) = 1 - \frac{324}{x^2}$

$= 1 - \frac{18^2}{x^2}$

$f'(17) = 1 - \left(\frac{18}{17}\right)^2 < 0$

$f'(19) = 1 - \left(\frac{18}{19}\right)^2 > 0$

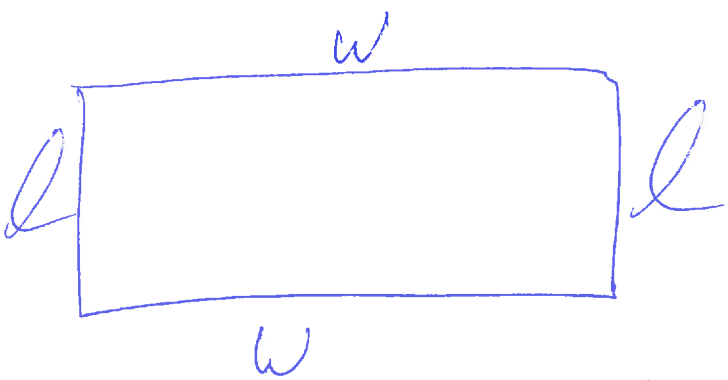
No endpts

Local Min @ $x=18$

Global Min @ $x=18$

$f(18) = 18 + \frac{18^2}{18} = 36$

Ex Enclose $1000\ ft^2$ rectangular plot



Cost of fence for 3 sides is \$2/ft

Cost of shrubs for last side is \$1/ft

Want to minimize cost (weighted perimeter)

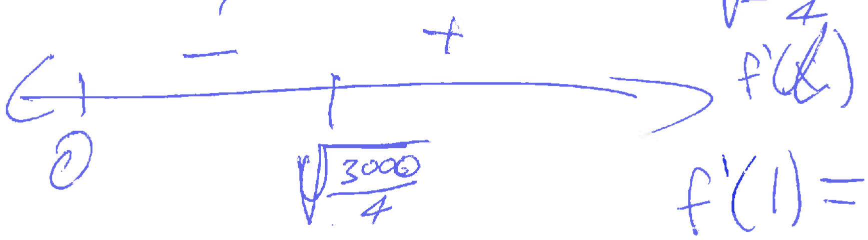
$$\min \ 2(2l + w) + w \quad \text{s.t.}$$
$$lw = 1000 \quad \text{(Area)}$$
$$l, w > 0$$

$$w = \frac{1000}{l}, \text{ so } f(l) = 4l + \frac{3000}{l}$$

$$f'(l) = 4 - \frac{3000}{l^2} = 0$$

$$4 = \frac{3000}{l^2} \Rightarrow l^2 = \frac{3000}{4}$$
$$l = \pm\sqrt{\frac{3000}{4}}$$

Since $l > 0$, restrict $l = \sqrt{\frac{3000}{4}}$



$$f'(1) = 4 - \frac{3000}{1^2} < 0$$

$$f'(3000) = 4 - \frac{3000}{(3000)^2} = 4 - \frac{1}{3000} > 0$$

Local Min @ $l = \sqrt{\frac{3000}{4}}$

No endpts (as $l, w > 0$)

Min Cost: $4\left(\sqrt{\frac{3000}{4}}\right) + \frac{3000}{\left(\sqrt{\frac{3000}{4}}\right)}$