Math 141 Study Guide: Sections 1.5-1.6, 2.2

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Instructions: Answer all questions. Show all work and justify all your answers in **complete sentences.**

1 Section 1.5-1.6 (Exponentials and Logarithms)

Problem 1) Simplify the Following Expressions. You should leave the numbers in exponential form (e.g., 2^7). If the expression is an exponential, simplify so there is only one base. If there is a logarithm in your answer, none of the logarithms should be decomposable using the rules of logs.

(a) $2^5 \cdot 2^7$

(b) $(3^3 \cdot 9^5)^{-2}$

(c) $(5^2)^4 \cdot 25^2 \cdot 125^{-5}$

(d) $\ln\left(3x^4 y^{-7}\right)$

(e) $\ln\left(x\sqrt{y^2+z^2}\right)$

(f) $\ln\left(\dfrac{x-4}{y^2\sqrt[5]{z}}\right)$

Problem 2) Write each expression as a single logarithm. Justify each step with the appropriate rule of logarithm.

(a) $2\ln(x) + 5\ln(y) - \frac{1}{2}\ln(z)$

(b) $3\ln(t+5) - 4\ln(t) - 2\ln(s-1)$

Problem 3) Find an exponential function between each pair of points.

(a) $(0,5)$ and $(2,9)$

(b) $(2,2)$ and $(3,4)$

Problem 4) If \$3000 is invested at 4% compounded monthly, then how long (in months) would it take for the investment to double?

Problem 5) Suppose \$3000 is invested at 4% compounded *continuously* instead. How long (in days) would it take for the investment to double?

Problem 6) The population of Kenya was 18.9 million in 1984 and 46.1 million in 2015. Assume the population increases exponentially at a continuous rate.

(a) Suppose $P(t)$ is the population of Kenya, where t is the number of years since 1984. Find a formula for $P(t)$.

(b) Suppose $P(t)$ is the population of Kenya, where t is the number of years **since 1980**. Find a formula for $P(t)$.

2 Section 1.6- Inverse Functions

Problem 7) Find the inverse function for each of the following.

(a) $f(x) = \dfrac{2x+1}{x-3}$.

(b) $f(x) = \sqrt[3]{x+5} + 7$

(c) $f(x) = x^2 - 2x + 1$, restricting to the domain $[1, \infty)$.

Problem 8) For each of the following, state whether the function is one-to-one. If the function $f(x)$ is not one-to-one, find two distinct x-values x_1, x_2 such that $f(x_1) = f(x_2)$.

(a) $f(x) = x^4$, on the domain $[0, \infty)$.

(b) $f(x) = x^4$, on the domain $\mathbb{R}$.

(c) $f(x) = \sin(x)$.

(d) $f(x) = \cos(x)$.

(e) $f(x) = \tan(x)$.

(f) $f(x) = x^3$.

Problem 9) Evaluate the following.

(a) $\sin^{-1}(0)$

(b) $\sin^{-1}(1/2)$

(c) $\sin^{-1}(\sqrt{3}/2)$

(d) $\sin^{-1}(-\sqrt{2}/2)$

(e) $\cos^{-1}(0)$

(f) $\cos^{-1}(1/2)$

(g) $\cos^{-1}(\sqrt{3}/2)$

(h) $\cos^{-1}(-\sqrt{2}/2)$

(i) $\tan^{-1}(1)$

(j) $\tan^{-1}(\sqrt{3})$

(k) $\csc^{-1}(2)$

(l) $\csc^{-1}(\sqrt{2})$

(m) $\sec^{-1}(2)$

(n) $\sec^{-1}(2\sqrt{3}/3)$

3 Section 2.2- Evaluating Limits Algebraically

Problem 10) Evaluate the following limits algebraically. Show all work. If the limit does not exist, write **the limit does not exist** or **the limit DNE**, and not **the limit equals DNE**. Do **not** use L'Hospital's rule.

(a) $\lim_{x\to -7}(2x+5)$

(b) $\lim_{x\to -2}(x^3-2x^2+4x+8)$

(c) $\lim_{x\to -3}(5-y)^{4/3}$

(d) $\lim_{h\to 0}\dfrac{\sqrt{3h+1}-1}{h}$

(e) $\lim_{x\to -3}\dfrac{x+3}{x^2+4x+3}$

(f) $\lim_{y\to 0}\dfrac{5y^3+8y^2}{3y^4-16y^2}$

(g) $\lim_{u\to 1}\dfrac{u^4-1}{u^3-1}$

(h) $\lim_{x\to 1}\dfrac{x-1}{\sqrt{x+3}-2}$

(i) $\lim_{x\to -1}\dfrac{\sqrt{x^2+8}-3}{x+1}$.

(j) $\lim_{t\to 0}\dfrac{5t}{|t|}$.

(k) $\lim_{t\to 1}\sqrt{x-3}$.

Problem 11) Suppose that $3+2x \le f(x) \le x-1$ for all x. Determine $\lim_{x\to -4} f(x)$.

Problem 12) Use the Squeeze Theorem to evaluate the following limits.

(a) $\lim_{x\to 0} x^4\cos\left(\dfrac{3}{x}\right)$

(b) $\lim_{x\to 0} x\sin\left(\dfrac{1}{x}\right)$

Problem 13) Do problems 1-4 in Section 2.2 of the course textbook for practice evaluating limits graphically.