Ch.2 (2.2) Evaluating Limits

Approach Plug in the x-value. Do we run into problems?

Ex $\lim_{x\to 3} (2x+5) = 2(3)+5 = 11$

Ex $\lim_{x\to 5} \frac{x^2-1}{x-1} = \frac{5^2-1}{5-1} = \frac{24}{4} = 6.$

Ex $\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = \lim_{x\to 1} (x+1) = \boxed{2}$

Ex $\lim_{x\to 5} \frac{x-5}{x^2-25} = \lim_{x\to 5} \frac{\cancel{(x-5)}}{\cancel{(x-5)}(x+5)} = \lim_{x\to 5} \left(\frac{1}{x+5}\right) = \boxed{\frac{1}{10}}$

Ex $\lim_{t\to -1} \frac{t^2+3t+2}{t^2-t-2} = \lim_{t\to -1} \frac{(t+2)\cancel{(t+1)}}{(t-2)\cancel{(t+1)}}$

$= \lim_{t\to -1} \frac{t+2}{t-2} = \frac{-1+2}{-1-2} = \boxed{\frac{-1}{3}}$

Ex $\lim_{x\to 0} \frac{\sqrt{x^2+100}-10}{x^2} = \lim_{x\to 0}\left[\left(\frac{\sqrt{x^2+100}-10}{x^2}\right)\left(\frac{\sqrt{x^2+100}+10}{\sqrt{x^2+100}+10}\right)\right]$

$= \lim_{x\to 0} \frac{(x^2+100)-100}{x^2(\sqrt{x^2+100}+10)} = \lim_{x\to 0} \frac{\cancel{x^2}}{\cancel{x^2}(\sqrt{x^2+100}+10)}$

$= \lim_{x\to 0} \frac{1}{\sqrt{x^2+100}+10} = \frac{1}{\sqrt{100}+10} = \boxed{\frac{1}{20}}$

Ex $\lim_{h\to 0} \frac{\sqrt{5h+4}-2}{h} = \lim_{h\to 0}\left[\left(\frac{\sqrt{5h+4}-2}{h}\right)\left(\frac{\sqrt{5h+4}+2}{\sqrt{5h+4}+2}\right)\right]$

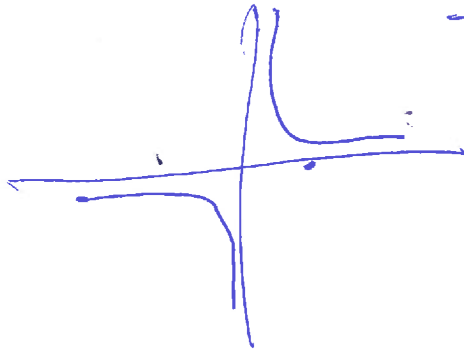
$= \lim_{h\to 0} \frac{(5h+4)-4}{h(\sqrt{5h+4}+2)} = \lim_{h\to 0} \frac{5\cancel{h}}{\cancel{h}(\sqrt{5h+4}+2)}$

$= \lim_{h\to 0} \frac{5}{\sqrt{5h+4}+2} = \frac{5}{\sqrt{4}+2} = \frac{5}{4}$

Try $\underbrace{(\sqrt{5h+4}-2)(\sqrt{5h+4}+2)}_{\text{Foil}} = (5h+4)-4$

Not all limits exist

Ex $\lim_{x \to 0} \frac{1}{x}$



$\frac{1}{x}$

Observe: As $x \to 0$ from the right, $\frac{1}{x} \to \infty$

$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

↳ As $x \to 0$ from the left, $\frac{1}{x} \to -\infty$

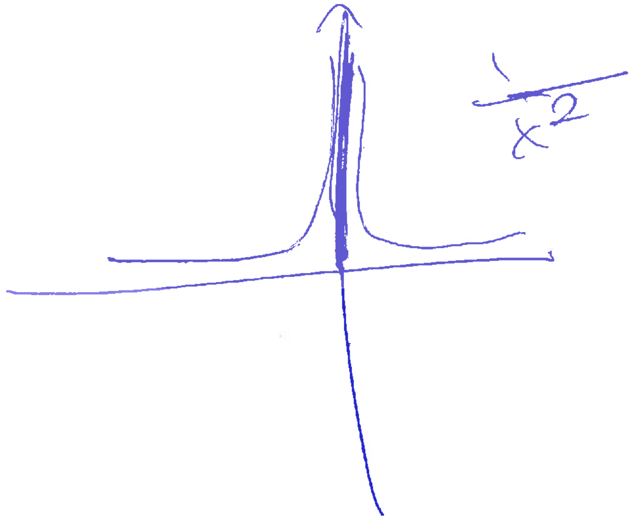
$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

In order for limit to exist (ie, $\lim_{x \to 0} \frac{1}{x}$) limit must exist coming from all directions and must be same

As $\lim_{x \to 0^+} \frac{1}{x} = \infty$ and $\lim_{x \to 0^-} \frac{1}{x} = -\infty$,

$\lim_{x \to 0} \frac{1}{x}$ does not exist (DNE, but don't write "= DNE")

$\underline{\text{Ex}}$ $\lim_{x \to 0} \frac{1}{x^2}$



$\lim_{x \to 0^+} \frac{1}{x^2} = \infty$

$\lim_{x \to 0^-} \frac{1}{x^2} = \infty$

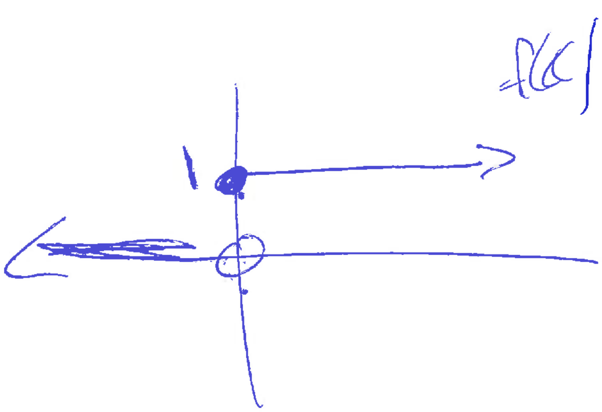
So $\lim_{x \to 0} \frac{1}{x^2} = \infty$

$\underline{\text{Ex}}$ $f(x) = \begin{cases} 0 : x < 0 \\ 1 : x \geq 0 \end{cases}$

$\lim_{x \to 0} f(x)$ $\boxed{\text{DNE}}$

$\lim_{x \to 0^-} f(x) = 0$

$\lim_{x \to 0^+} f(x) = 1$



How to compute $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x^2}\right)$?

Thm (Squeeze Thm) Suppose we have $f(x)$, $g(x)$, $h(x)$, and let $c \in \mathbb{R}$. Suppose that $f(x) \le g(x) \le h(x)$ for all x in some interval containing c. If:

$$\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L,$$

then $\lim_{x \to c} g(x) = L$.

Ex $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x^2}\right)$

Want Bound $x^2 \cos\left(\frac{1}{x^2}\right)$

$$-1 \le \cos\left(\frac{1}{x^2}\right) \le +1 \quad \text{(Range of cos)}$$

$$-x^2 \le x^2 \cos\left(\frac{1}{x^2}\right) \le x^2$$

$\hookrightarrow \lim_{x \to 0} (-x^2) = 0 \qquad \lim_{x \to 0} x^2 = 0$

By Squeeze Thm, $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x^2}\right) = 0$