Experiment for HW 4 notebook

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1 Experimental data for Homework 4

Dhamma Kimpara and Sam Zhang

[1]:

```
%load_ext autoreload
%autoreload 2
```

[2]:

```
import generator as gen
import optimizer as opt
import numpy as np
import scipy

import matplotlib.pyplot as plt
%matplotlib inline
```

[3]:

```
kappa = 100
n = 20
d = 5
A = gen.generate_A(n, d)
alpha = 0.9
sigma0 = 1
sigma1 = 0.1
sigma2 = 2.0

tracking_errs0 = opt.gradient_descent_experiment(A, alpha, n, d, sigma0,␣
 ↪iters=50, projected=False)
tracking_errs1 = opt.gradient_descent_experiment(A, alpha, n, d, sigma1,␣
 ↪iters=50, projected=False)
tracking_errs2 = opt.gradient_descent_experiment(A, alpha, n, d, sigma2,␣
 ↪iters=50, projected=False)
tracking_errs_proj = opt.gradient_descent_experiment(A, alpha, n, d, sigma0,␣
 ↪iters=50, projected=True)
```

[4]:

```
rho = max([np.abs(1 - alpha/np.sqrt(kappa)), np.abs(1 - alpha)])
err0 = np.linalg.norm(tracking_errs0[0])
err1 = np.linalg.norm(tracking_errs1[0])
err2 = np.linalg.norm(tracking_errs2[0])
```

```
err_proj = np.linalg.norm(tracking_errs_proj[0])


bounds0 = [rho**t * err0 + sigma0 * (1 - rho**t)/(1-rho) for t in␣
 ↪range(len(tracking_errs0))]
bounds1 = [rho**t * err1 + sigma1 * (1 - rho**t)/(1-rho) for t in␣
 ↪range(len(tracking_errs1))]
bounds2 = [rho**t * err2 + sigma2 * (1 - rho**t)/(1-rho) for t in␣
 ↪range(len(tracking_errs2))]

bounds_proj = [rho**t * err_proj + sigma0 * (1 - rho**t)/(1-rho) for t in␣
 ↪range(len(tracking_errs_proj))]
```

1.0.1 Results for $\sigma = 1$

[5]:

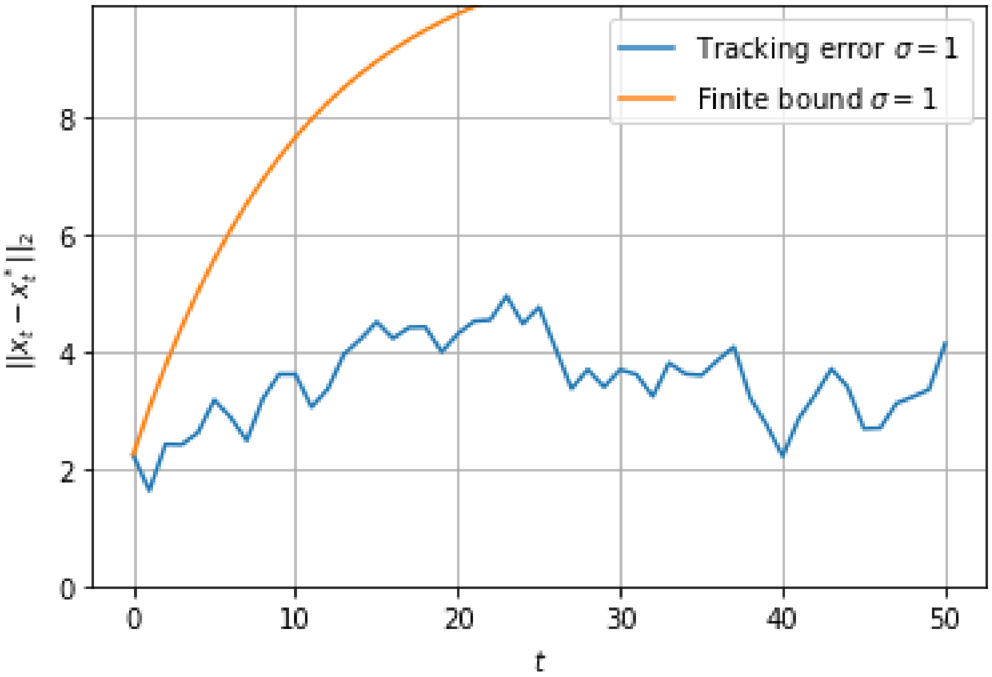
```
plt.plot(range(len(tracking_errs0)), tracking_errs0, label="Tracking error␣
 ↪$\sigma = 1$")
plt.plot(range(len(tracking_errs0)), bounds0, label="Finite bound $\sigma = 1$")

#plt.axhline(1/(1-rho), linestyle='--', label="Asymptotic bound")
plt.grid()
plt.ylim([0, max(tracking_errs0)*2])
plt.legend()

plt.xlabel("$t$")
plt.ylabel("$||x_t - x_t^*||_2$")
```

[5]:

```
Text(0, 0.5, '$||x_t - x_t^*||_2$')
```



1.0.2 Results for $\sigma = 0.1, \sigma = 2$

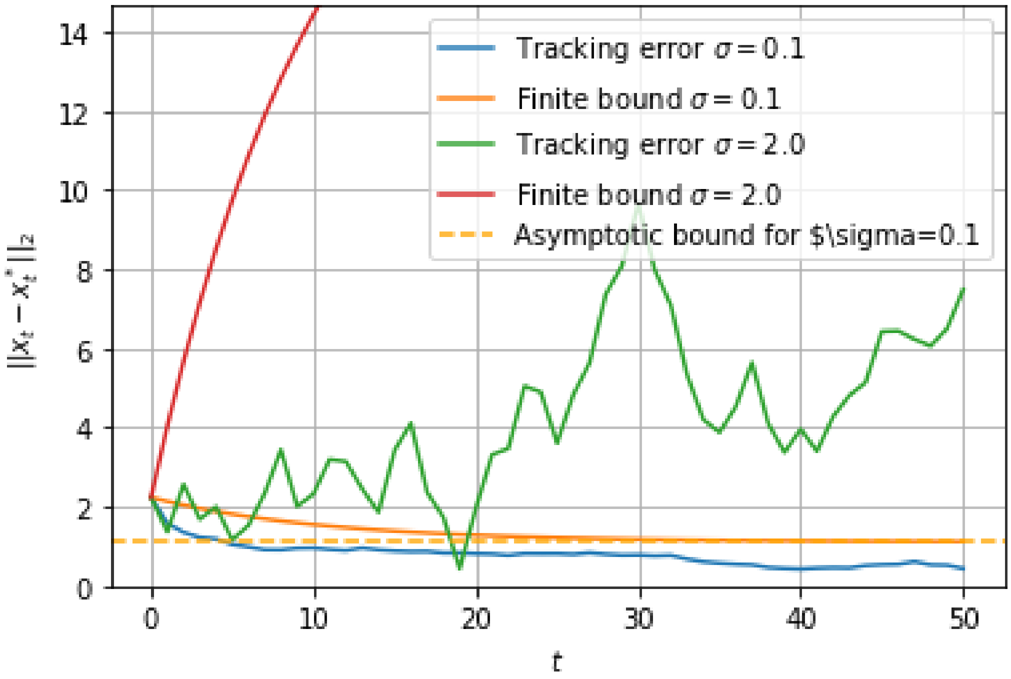
[10]:
```
plt.plot(range(len(tracking_errs1)), tracking_errs1, label="Tracking error␣
 ↪$\sigma = 0.1$")
plt.plot(range(len(tracking_errs1)), bounds1, label="Finite bound $\sigma = 0.
 ↪1$")

plt.plot(range(len(tracking_errs1)), tracking_errs2, label="Tracking error␣
 ↪$\sigma = 2.0$")
plt.plot(range(len(tracking_errs1)), bounds2, label="Finite bound $\sigma = 2.
 ↪0$")

plt.axhline(sigma1/(1-rho), linestyle='--', c='orange', label="Asymptotic bound␣
 ↪for $\sigma=0.1")
plt.grid()
plt.ylim([0, max(tracking_errs2)+5])
plt.legend()

plt.xlabel("$t$")
plt.ylabel("$||x_t - x_t^*||_2$")
```

[10]:
```
Text(0, 0.5, '$||x_t - x_t^*||_2$')
```

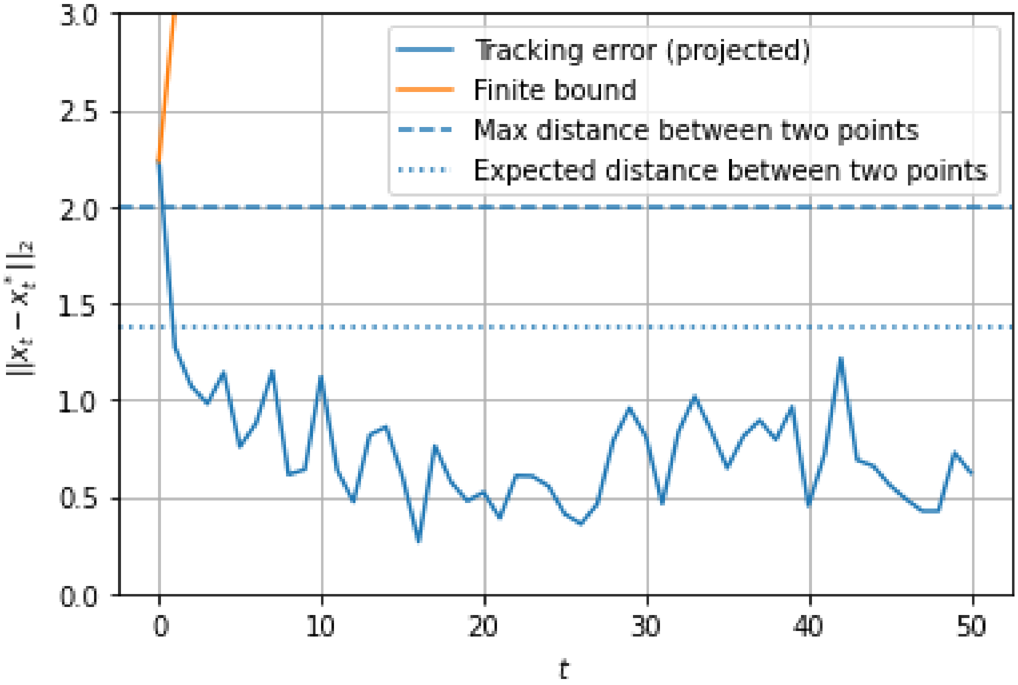


1.0.3 Results for projected GD

```
[7]: plt.plot(range(len(tracking_errs_proj)), tracking_errs_proj, label="Tracking error (projected)")
     plt.plot(range(len(tracking_errs_proj)), bounds_proj, label="Finite bound")
     #plt.axhline(1/(1-rho), linestyle='--', label="Asymptotic bound")
     plt.axhline(2, linestyle='--', label="Max distance between two points")
     plt.axhline(1.3796796759881553, linestyle=':', label="Expected distance between two points")
     plt.grid()
     plt.ylim([0, 3])
     plt.legend()

     plt.xlabel("$t$")
     plt.ylabel("$||x_t - x_t^*||_2$")
```

```
[7]: Text(0, 0.5, '$||x_t - x_t^*||_2$')
```


3.0
2.5
2.0
1.5
1.0
0.5
0.0
$||x_t - x_t^*||_2$
0
10
20
30
40
50
t
Tracking error (projected)
Finite bound
Max distance between two points
Expected distance between two points