Exercises for the lecture course Algebraic Topology II – Sheet 12

University of Bonn, summer term 2025

Exercise 45. Let X be a connected finite CW-complex with finite fundamental group. Prove or disprove that $\pi_n(X)$ is finitely generated for all $n \geq 1$.

Exercise 46. Let A be a finitely generated abelian group and $n \in \mathbb{Z}^{\geq 1}$. Prove or disprove:

(a) Suppose that A has an element of infinite order. Then there exists $d \in \mathbb{Z}^{\geq 2}$ such that $H_i(K(A,n);\mathbb{Q}) = \{0\}$ holds for every for $i > d$ if and only if n is odd;

(b) $H_*(K(\mathbb{Z}/2,n);\mathbb{Z})$ is nontrivial in infinitely many degrees;

(c) We have $H_i(K(A,n);\mathbb{Q}) = \{0\}$ for every $i \geq 1$ if and only if A is finite;

(d) We have $H_i(K(A,n);\mathbb{Z}) = \{0\}$ for every $i \geq 1$ if and only if A is trivial;

(e) $H_i(K(A,n))$ is finitely generated for all $i \in \mathbb{Z}^{\geq 0}$.

Exercise 47. Decide which of the following Serre classes in $\mathbb{Z}\text{-}\mathsf{Mod}$ are Serre ideals: $\mathbb{Z}\text{-}\mathsf{Tors}_p$, $\mathbb{Z}\text{-}\mathsf{Tors}$, $\mathbb{Z}\text{-}\mathsf{Mod}_{\mathrm{fg}}$, $R\text{-}\mathsf{Mod}_{\mathrm{fin}}$.

Exercise 48. State all the results which are presented in the script about $\pi_n(S^k)$ and the stable stems π_n^s for $n \in \mathbb{Z}^{\geq 0}$ and $k \in \mathbb{Z}^{\geq 1}$.

[0]Hand-in Monday 07.07.