

July 3, 2022 - Assignment #2 - Estimating A Confidence Interval

July 3: Assignment: In a workplace of 1 lakh employees, "Large" & "X-Large" T-shirts need to be ordered for all employees. We do not have data for the entire population but only for a sample of 500 employees. Out of this sample, 200 employees wear "Large" T-shirt and 300 employees wear "X-Large" T-shirt. Construct a confidence interval for the number of t-shirts to be printed for all the employees.

What we know: (1) Sample size $n = 500$

(2) Sample proportion for Large T-shirt employees, let's call them, $P_L = 200/500 = 0.4 \text{ or } 40\%$.

(3) Sample proportion for X-Large T-shirt employees, let's call them $P_{XL} = 300/500 = 0.6 \text{ or } 60\%$.

Assumptions we make:

(1) The sample is truly random.

(2) Sampling distribution from this data is normally distributed.

(3) We select a 95% confidence interval and since we need both an upper fence and lower fence for our parameter estimates, we take a two-tailed test. Hence $Z_{C.L} = Z_{0.025} = 1.96$.

In order to calculate the standard error of the proportion we can use the formula:

$$\text{S.E.} = \sqrt{\frac{P_L * P_{XL}}{n}} = \sqrt{\frac{(0.4)(0.6)}{500}} = \sqrt{\frac{0.24}{500}} = 0.0219$$

CALCULATING A 95% Confidence Interval for Large T-shirt employees we use:

$$\begin{aligned}\text{Confidence Interval} &= P_L \pm Z_{C.L} * \text{S.E.} \\ &\text{of Large-Tshirts} \\ &= 0.4 \pm 1.96 * 0.0219 \\ &= 0.4 \pm 0.0429\end{aligned}$$

$$\begin{aligned}\text{Lower Fence for Large Shirt Employees} &= 0.4 - 0.0429 \\ &= 0.3571 \\ &\approx 35.71\%\end{aligned}$$

$$\text{Higher Fence for Large Shirt Employees} = 0.4 + 0.0429$$

$$\text{Large shirt employees} = 0.4429 \approx 44.29\%$$

Hence, out of 1 lakh employees, the confidence interval for Large T-shirts is between 35,710 and 44,290 employees.

Similarly, for X-Large T-shirts the calculation of C.I. will be :

$$\begin{aligned}P_{XL} \pm Z_{C.L} * S.E &= 0.6 \pm 1.96 * 0.0219 \\ &= 0.6 \pm 0.0429.\end{aligned}$$

$$\begin{aligned}\text{Lower Fence for X-Large Shirt employees} &= 0.6 - 0.0429 \\ &= 0.5571 \approx 55.71\%\end{aligned}$$

$$\begin{aligned}\text{Higher Fence for X-Large Shirt employees} &= 0.6 + 0.0429 \\ &= 0.6429 \approx 64.29\%\end{aligned}$$

Hence, out of 1 lakh employees the confidence interval for X-Large T-shirts is between 55,710 and 64,290 employees.

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