

Building a Winner in the Modern NFL

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1 Introduction

The National Football League (NFL) is a \$15 billion dollar business that dwarfs its competitors both in terms of financial might and nation-wide interest. With so much money and power at stake it comes as no surprise that the NFL is hypercompetitive. A common moniker for the league is “Not For Long”, and with good reason: outside of notable outliers such as the New England Patriots, success in the NFL is often fleeting. Teams that appear to be on an upward trajectory one year often completely flame out the next.

Many rules are in place to encourage parity. For example, the first selection in the NFL draft is “rewarded” to the team with the worst record from the prior season, and teams that win their division one year must play their 4 “wildcard” games in the upcoming season against teams that also won their division. Additionally, teams that are driven by young talent must eventually pay market value to retain that talent. When NFL players are drafted, they are signed to relatively low-paying contracts, even at the very top-end of the draft. When these top picks pan out as projected, teams are receiving high-end production at bargain rates. However, first year players (known as rookies) are signed to a maximum contract length of 5 years. Once this rookie contract expires, these players will demand contracts that matches their production, making it impossible to keep every high-performing player due to the existence of a league-wide salary cap. The salary cap places a limit on how much money each team can spend annually on player salaries. With limited resources to spend, teams must make difficult decisions that are crucial to future success.

The purpose of this study is to analyze roster-based decision making of NFL teams in order to identify trends between these decisions and winning football games. Should teams be spending more money on offensive positions vs defensive positions? Which positions should teams be spending their draft capital on? Is it more beneficial to trade draft picks for players or the other way around? Do teams that spend more on free agents outperform teams that extend the contracts of their own players instead? These are a few of the questions that this study aims to answer.

2 About the Data

All salary cap, transactional and draft data used in this study was either web-scraped or manually compiled from Spotrac, an online source that provides salary cap and contract details for all major US sports. Historical roster age data was taken from phillyvoice.com, while the response variable, regular season win percentage, was taken from NFL.com. The web-scraped data was cleaned using the stringr package and all data was eventually joined into a single data frame.

The data spans a period of 9 seasons (2013-2021) for all 32 teams for a total of 288 observations. The amount a team spent on player salaries in total, was normalized to 2021

spending figures based on mean salary spend for each season. The raw data from Spotrac included a breakdown of total spending numbers for each of 9 different positional groups (*Figure 1* for summary and definitions). The positional spend variables were transformed into proportions of total team spend to isolate allocation of salary resources on each position from the previously mentioned total salary spend variable.

The raw draft data spanned 2010-2021, a period beginning 3 years prior to the rest of the data. The decision to include this data was made because draft picks often take time to develop and are often best evaluated over a period of many years. Since the average rookie contract is about 4 years, the decision was made to examine each team's last 4 drafts (including the draft in the current year).

Values were assigned to each draft pick a team made over this period using a modified version of the renowned Jimmy Johnson draft chart (*Figure 1*). The chart provides an estimate of relative value for each pick number in the draft. For a given observation, total draft pick value for each of a team's past 4 drafts was computed. From there, the percent of this value spent on each of the 9 positional groups was calculated, which yielded the final positional proportion variables.

Free agency is the period that begins 6 weeks after the prior season has ended in which players that are out of contract, called "free agents", seek new contracts either from their former team or another team. The Free agency spend variable represents the total amount of money a team spent that season on free agent contracts. This data was normalized to 2021 values based on the mean spend for each year. Extension spending, or the amount of money added to players contracts who are already under contract with a team, was also added to the final data-frame and normalized to 2021 values. Note that there is a natural trade-off between extending current players' contracts and spending money on players in free agency.

Although trading for players in the NFL is less prevalent and often less impactful than it is in baseball or basketball, it can still be a useful method to build a competitive team. One can look no further than the 2021 LA Rams who won the super bowl primarily because of the players they traded for throughout the season. Since trades in the NFL are often made in exchange for draft compensation rather than for another player, trading players out for draft compensation can also be a shrewd way to build a team. Therefore, both the number of players traded in and out of a team were included in the final data set.

Average roster age was the last variable included in the final dataset in hopes of trying to identify whether experienced, older rosters or more athletic, less injury-prone younger rosters tend to be more successful.

3 Exploratory Data Analysis

Figure 3 shows a histogram for the response variable, regular season win percentage (RSWP). The distribution of the histogram is approximately normal, centered around .500, and thus no transformation was made.

Due to the large number of variables in the dataset, the predictor variables were initially examined in three separate groups: 1) salary spend proportions by position, 2) draft capital spend proportions by position, and 3) remaining variables, including: total salary spend, free agency spend, extension spend, number of players traded to, number players traded from, and average roster age.

The salary proportions for each position were generally approximately normal and therefore none of the variables were transformed (*Figure 4*). One issue with using proportional data in a linear model is that since the sum of the variables equals one by definition, the regression coefficients will yield NAs for the last variable if every variable is used. More advanced techniques such as mixture models were researched and examined, but the decision was made to treat the least impactful position (Special Teams) as a null case and leave it out of the linear model. *Figure 5* shows the summary of the linear model predicting RSWP using the remaining 8 positional salary proportions. The linear model yields a negative adjusted R^2 with none of the predictor significant at even the $\alpha = .10$ level.

The draft capital spend by position variables were generally right skewed with many entries equal to zero (*Figure 6*). Transformation analysis of the variables generally showed that a log transformation was most appropriate (*Figure 7*). To account for the entries equal to zero, each variable was transformed as: $\text{variable} = \log(\text{variable} + .01)$. Given that this set of variables also represented proportions that sum to 1, the ST variable was again left out in the linear model (*Figure 8*). While the model only yielded an adjusted R^2 of .025, the proportion of draft capital spent on linebackers was statistically significant (*Figure 9*).

Although all but one of the position-related variables were statistically significant, *Figure 10* shows an interesting visualization of these variables.

Total normalized spend appeared to be approximately normally distributed and thus was not transformed (*Figure 11*). Normalized free agency spend had a right-skewed distribution and transformation analysis showed a square-root transform to be most appropriate (*Figure 12*). The normalized extension spend variable was widely distributed with many entries equal to zero in some of the earlier years. After further research, some of the 2014 entries which were deemed to be incomplete. The decision was made to turn the extension spend variable into a categorical variable with 4 levels based on percentiles. Levels were assigned intra-year and then grouped together for each year, with zero entries in 2014 randomly assigned ranks and then assigned quartiles based on this rank.

The histogram for average roster age showed an approximately normal distribution (*Figure 13*), while both the `traded_to` variable and `traded_from` variable were right-skewed. Transformation analysis showed a square-root transform to be most appropriate for both the `traded_to` and `traded_from` variables.

Finally, a linear model was created in the form of:

$$\text{rswp} \sim \text{norm_total_spend} + \text{norm_fa_spend}^{.5} + \text{ext_spend_rank} + \text{avg_age} + \text{traded_to}^{.5} + \text{traded_from}^{.5} \text{ (Figures 14-15).}$$

This model yielded an R^2 of .14 with `norm_total_spend` and `traded_to^.5` statistically significant and `norm_total_spend` and `avg_age` yielding p-values of .11.

4 Data Analysis

Formal Data Analysis was done using both R and C . The data was modeled using both a linear model and a GAM model that was fit with kernel regression. C was used to help calculate the confidence intervals.

4.1 Linear Modeling

A final linear model was created using a top-down strategy. The initial linear model tested was:

$$\text{rswp} \sim \log(\text{lb} + .01) + \text{norm_total_spend} + \text{traded_to}^{\wedge}.5 + \text{norm_fa_spend}^{\wedge}.5 + \text{avg_age}$$

(See Figure 16).

The first three variables showed statistical significance at $\alpha = .01$, while `norm_fa_spend^.5` showed statistical significance at $\alpha = .10$. Average age did not show statistical significance and thus was dropped from the model. A second model was run dropping the `avg_age` variable (Figure 17). This model yielded an R^2 of .171 with the first three variables significant at $\alpha = .01$ and `norm_fa_spend^.5` significant at $\alpha = .10$. A third model was tested dropping `norm_fa_spend^.5` which yielded an R^2 of .165.

Since the R^2 only decreased by .06 after removing `norm_fa_spend^.5` and the variable was significant only at the $\alpha = .10$ level, it was a toss-up on whether to keep the variable in the final model. Since the R^2 of the final model was relatively low to begin with, the decision was made to include `norm_fa_spend^.5` in the final model:

$$\text{rswp} \sim \log(\text{lb} + .01) + \text{norm_total_spend} + \text{traded_to}^{\wedge}.5 + \text{norm_fa_spend}^{\wedge}.5$$

4.2 GAM Model fit using Kernel Regression

The data was also fit to a GAM model using kernel regression. The same four variables used in the final linear model were used in the GAM model. Kernel regression was done in R using the `ksmooth` function while confidence intervals were computed using C.

Note that some points on the kernel regression plots fall outside the confidence intervals calculated in C, particularly in the first smoothing using the `norm_total_spend` variable. The reason for this is that the confidence intervals are computed based on evenly spaced points in the variable, versus k-smooth using the observations directly. Upon examination of the variable there are very few points on the left-hand side of the plot, meaning each point is highly influential in the kernel regression estimate. This resulted in a very jagged estimate. Increasing the bandwidth did not do much to hinder this issue so results were left as-is.

After the four kernel smoothings, the final GAM model yielded an RMSE of .1609 versus a null model standard deviation of .1919. The ratio $\text{RMSE}/\text{SD}_{\text{null}} = .1616$ which is lower than the adjusted R^2 of the linear model.

Kernel regression of the 4 variables with confidence intervals are shown in *Figure 18*. The histogram and qq plot of the final residuals supports the assumption of normality (*Figure 19*).

4.3 Outlier Analysis

Potential outliers were examined using Studentized Residuals (*Figure 20*). Since none of the observations had a studentized residual $> |3|$, no observations were identified as outliers and the dataset was kept as-is.

5 Interpreting Results and Conclusions

Although both the linear and GAM models yielded similar results, the linear model is preferred as the better model due to its simplicity and easy interpretability.

Based on the data, salary allocation by positional groups is highly efficient which is not surprising given the amount of money NFL franchises have poured into data analytics over the past decade. Although draft capital allocation generally shows similar results, there does seem to be inefficiency related to drafting linebackers. The linear model coefficient for spending draft capital on linebackers is positive indicating that teams may want to invest more of their draft capital in the position. The general conclusion from the positional data is that acquiring talent at all positions is important and that teams can have success spending money and draft capital on any positional group.

It is slightly surprising that normalized total salary spend is as highly statistically significant with regular season win percentage as it is. The difference between the 25th and 75th percentile of `norm_total_spend` is only about 14%. In other sports, such as baseball, this difference is much larger and thus a highly significant correlation with win percentage would not be as surprising. Another interesting note is that the regression coefficient for `norm_fa_spend^0.5` is negative. This indicates that spending significant money in free agency far from guarantees success and often actually inhibits winning games. A much better strategy would be to trade draft picks for established players that are likely on contracts that are below the value they would receive on the open free agency market.

Although the positional variables do not show much correlation with regular season win percentage, it is beneficial to be able to rule out these variables in the model. Instead, future studies can focus on other variables such as college statistics (for draft picks) and NFL production (for free agents) to improve the current model. Interaction effects between positional allocation variables and other variables may also be worth looking into.

| Position | Brief Description |
|---------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Quarterback (QB) | Has the ball in his hands on every offensive play. Primarily responsible for throwing the ball to advance field position and score points. |
| Running back (RB) | Primarily responsible for running with the ball on running plays. Also can receive the ball on passing plays or protect the quarterback on passing plays. |
| Wide Receiver (WR) | Primarily responsible for catching the ball from the quarterback. There are usually 2-4 wide receivers on the field at one time. |
| Tight End (TE) | A hybrid position. Primarily responsible for both blocking defenders on running plays and receiving passes on passing plays. |
| Offensive Line (OL) | Primarily responsible for blocking defenders on both running and passing plays. There are usually 5 OL on the field at one time. |
| Defensive Line (DL) | Primarily responsible for stopping the running back on run plays and pressuring the quarterback on passing plays. There are usually 4 DL on the field at one time. |
| Linebacker (LB) | LBs play behind the DL and are primarily responsible for tackling the running back on run plays and preventing WRs and TEs from catching the ball on passing plays. There are usually 2-3 LBs on the field at one time. |
| Secondary (SEC) | Primarily responsible from covering the other team's pass catchers on passing plays. There are usually 4-5 SEC on the field at one time. |
| Special Teams (ST) | Specialists are responsible for kicking, punting and returning the football on special teams plays. |

Figure 1 - Positional Group Descriptions

| NFL DRAFT TRADE VALUE CHART | | | | | | | | | | | | | |
|-----------------------------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|
| Round 1 | | Round 2 | | Round 3 | | Round 4 | | Round 5 | | Round 6 | | Round 7 | |
| pick | value | pick | value | pick | value | pick | value | pick | value | pick | value | pick | value |
| 1 | 3000 | 33 | 580 | 65 | 265 | 97 | 112 | 129 | 43 | 161 | 27 | 193 | 14 |
| 2 | 2600 | 34 | 560 | 66 | 260 | 98 | 108 | 130 | 42 | 162 | 26 | 194 | 13 |
| 3 | 2200 | 35 | 550 | 67 | 255 | 99 | 104 | 131 | 41 | 163 | 26 | 195 | 13 |
| 4 | 1800 | 36 | 540 | 68 | 250 | 100 | 100 | 132 | 40 | 164 | 25 | 196 | 13 |
| 5 | 1700 | 37 | 530 | 69 | 245 | 101 | 96 | 133 | 39 | 165 | 25 | 197 | 12 |
| 6 | 1600 | 38 | 520 | 70 | 240 | 102 | 92 | 134 | 39 | 166 | 25 | 198 | 12 |
| 7 | 1500 | 39 | 510 | 71 | 235 | 103 | 88 | 135 | 38 | 167 | 24 | 199 | 11 |
| 8 | 1400 | 40 | 500 | 72 | 230 | 104 | 86 | 136 | 38 | 168 | 24 | 200 | 11 |
| 9 | 1350 | 41 | 490 | 73 | 225 | 105 | 84 | 137 | 37 | 169 | 23 | 201 | 11 |
| 10 | 1300 | 42 | 480 | 74 | 220 | 106 | 82 | 138 | 37 | 170 | 23 | 202 | 10 |
| 11 | 1250 | 43 | 470 | 75 | 215 | 107 | 80 | 139 | 36 | 171 | 23 | 203 | 10 |
| 12 | 1200 | 44 | 460 | 76 | 210 | 108 | 78 | 140 | 36 | 172 | 22 | 204 | 9 |
| 13 | 1150 | 45 | 450 | 77 | 205 | 109 | 76 | 141 | 35 | 173 | 22 | 205 | 9 |
| 14 | 1100 | 46 | 440 | 78 | 200 | 110 | 74 | 142 | 35 | 174 | 21 | 206 | 9 |
| 15 | 1050 | 47 | 430 | 79 | 195 | 111 | 72 | 143 | 34 | 175 | 21 | 207 | 8 |
| 16 | 1000 | 48 | 420 | 80 | 190 | 112 | 70 | 144 | 34 | 176 | 21 | 208 | 8 |
| 17 | 950 | 49 | 410 | 81 | 185 | 113 | 68 | 145 | 33 | 177 | 20 | 209 | 7 |
| 18 | 900 | 50 | 400 | 82 | 180 | 114 | 66 | 146 | 33 | 178 | 20 | 210 | 7 |
| 19 | 875 | 51 | 390 | 83 | 175 | 115 | 64 | 147 | 32 | 179 | 19 | 211 | 7 |
| 20 | 850 | 52 | 380 | 84 | 170 | 116 | 62 | 148 | 32 | 180 | 19 | 212 | 6 |
| 21 | 800 | 53 | 370 | 85 | 165 | 117 | 60 | 149 | 31 | 181 | 19 | 213 | 6 |
| 22 | 780 | 54 | 360 | 86 | 160 | 118 | 58 | 150 | 31 | 182 | 18 | 214 | 5 |
| 23 | 760 | 55 | 350 | 87 | 155 | 119 | 56 | 151 | 31 | 183 | 18 | 215 | 5 |
| 24 | 740 | 56 | 340 | 88 | 150 | 120 | 54 | 152 | 30 | 184 | 17 | 216 | 5 |
| 25 | 720 | 57 | 330 | 89 | 145 | 121 | 52 | 153 | 30 | 185 | 17 | 217 | 4 |
| 26 | 700 | 58 | 320 | 90 | 140 | 122 | 50 | 154 | 29 | 186 | 17 | 218 | 4 |
| 27 | 680 | 59 | 310 | 91 | 136 | 123 | 49 | 155 | 29 | 187 | 16 | 219 | 3 |
| 28 | 660 | 60 | 300 | 92 | 132 | 124 | 48 | 156 | 29 | 188 | 16 | 220 | 3 |
| 29 | 640 | 61 | 292 | 93 | 128 | 125 | 47 | 157 | 28 | 189 | 15 | 221 | 3 |
| 30 | 620 | 62 | 284 | 94 | 124 | 126 | 46 | 158 | 28 | 190 | 15 | 222 | 2 |
| 31 | 600 | 63 | 276 | 95 | 120 | 127 | 45 | 159 | 27 | 191 | 15 | 223 | 2 |
| 32 | 590 | 64 | 270 | 96 | 116 | 128 | 44 | 160 | 27 | 192 | 14 | 224 | 2 |

Figure 2: Jimmy Johnson NFL draft pick value chart

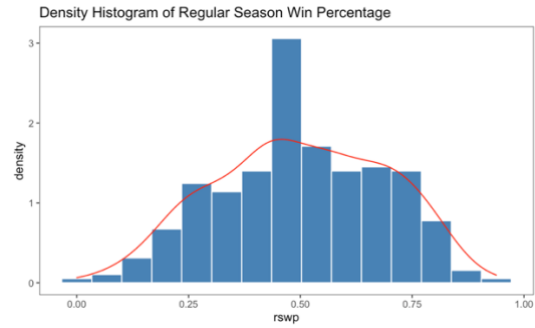


Figure 3: Histogram with Density of RSWP

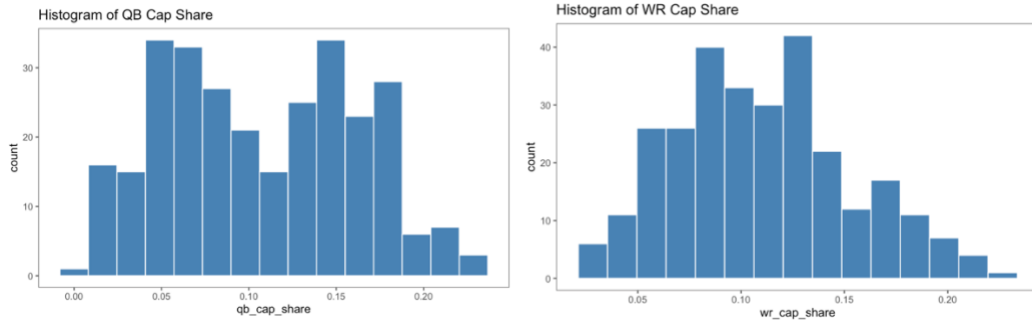


Figure 4: Positional Salary Proportions Histograms

```
Call:
lm(formula = rswp ~ qb_cap_share + rb_cap_share + wr_cap_share +
    te_cap_share + ol_cap_share + dl_cap_share + lb_cap_share +
    sec_cap_share, data = df)

Residuals:
    Min       1Q   Median       3Q      Max
-0.42518 -0.12923 -0.00643  0.15004  0.43048

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   1.1915     0.8239   1.446   0.149
qb_cap_share  -0.5039     0.8733  -0.577   0.564
rb_cap_share  -1.0720     1.0247  -1.046   0.296
wr_cap_share  -0.6625     0.8683  -0.763   0.446
te_cap_share  -0.3631     1.0114  -0.359   0.720
ol_cap_share  -0.9977     0.8508  -1.173   0.242
dl_cap_share  -0.6492     0.8389  -0.774   0.440
lb_cap_share  -0.6515     0.8722  -0.747   0.456
sec_cap_share -0.6892     0.8892  -0.775   0.439

Residual standard error: 0.1927 on 279 degrees of freedom
Multiple R-squared:  0.0195,    Adjusted R-squared:  -0.008612
F-statistic: 0.6937 on 8 and 279 DF,  p-value: 0.6971
```

Figure 5: Linear Model Summary for Positional Salary Proportion Variables

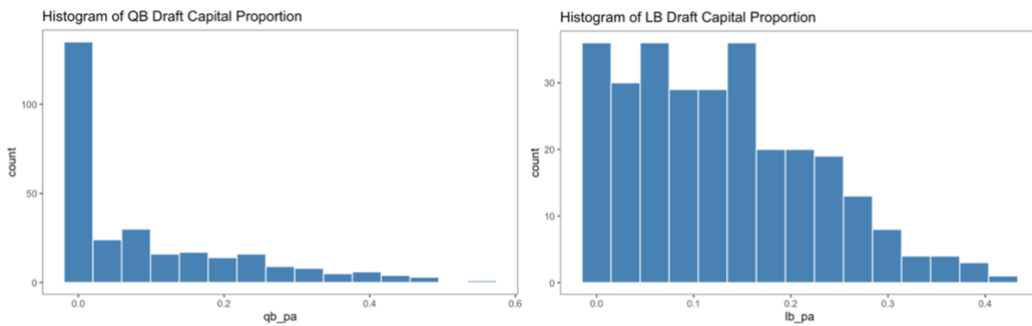


Figure 6: Draft Capital Proportion by Position Histograms

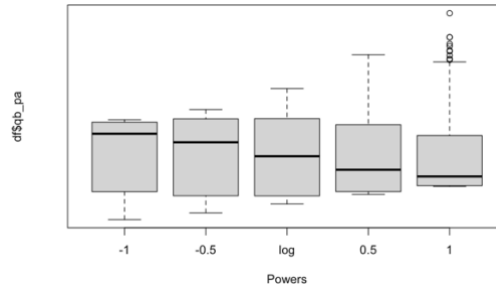


Figure 7: Transformation Analysis for QB Draft Capital Proportion

```

Null:
lm(formula = rswp ~ log(qb_pa + 0.01) + log(rb_pa + 0.01) + log(wr_pa + 0.01) + log(te_pa + 0.01) + log(col_pa + 0.01) + log(dl_pa + 0.01) + log(lb_pa + 0.01) + log(sec_pa + 0.01), data = df)

Residuals:
    Min       1Q   Median       3Q      Max
-0.47254 -0.13602 -0.00629  0.14638  0.40762

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    0.675803   0.218972   3.086  0.00223 **
log(qb_pa + 0.01)  0.004633   0.012539   0.369  0.71207
log(rb_pa + 0.01) -0.003616   0.014101  -0.256  0.79778
log(wr_pa + 0.01) -0.012525   0.017109  -0.732  0.46474
log(te_pa + 0.01)  0.017010   0.014187   1.199  0.23156
log(col_pa + 0.01)  0.003628   0.022354   0.162  0.87120
log(dl_pa + 0.01) -0.001258   0.020723  -0.061  0.95163
log(lb_pa + 0.01)  0.043481   0.015693   2.771  0.00597 **
log(sec_pa + 0.01)  0.022085   0.021987   1.004  0.31603
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1895 on 279 degrees of freedom
Multiple R-squared:  0.05196, Adjusted R-squared:  0.02478
F-statistic: 1.911 on 8 and 279 DF, p-value: 0.05826

```

Figure 8: LM Summary of Draft Capital Spend Proportions

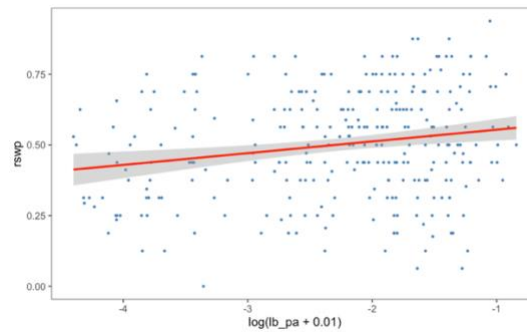


Figure 9: RSWP vs. log(LB_PA + .01)

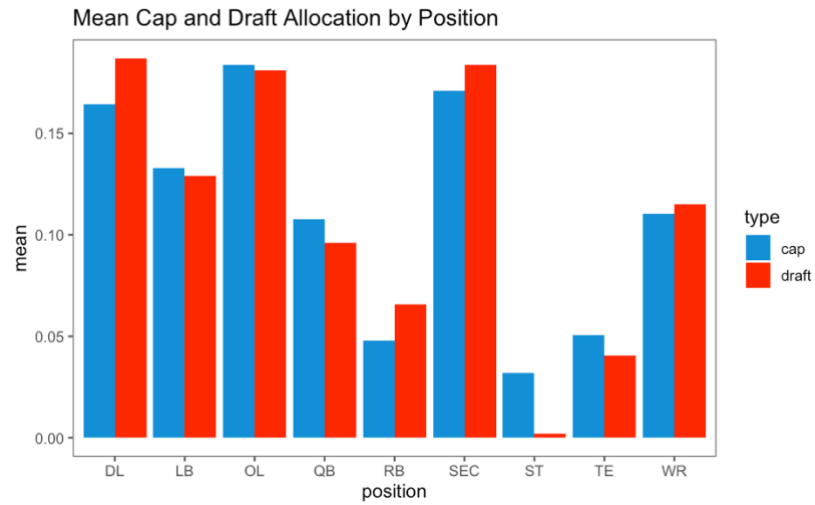


Figure 10: Summary of Positional Variables

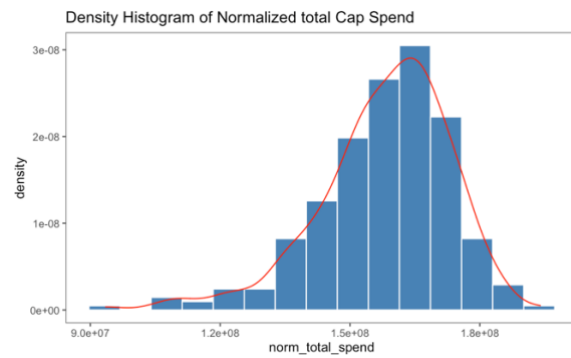


Figure 11: Histogram Normalized Total Cap Spend

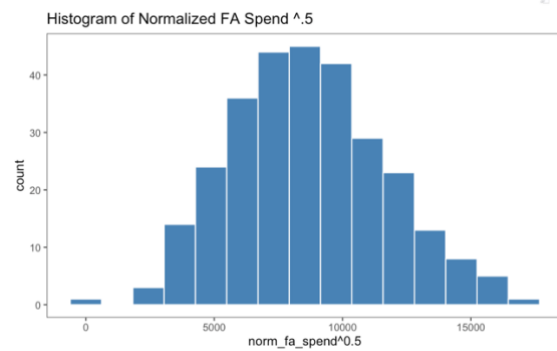


Figure 12: Histogram Normalized FA Spend^{.5}

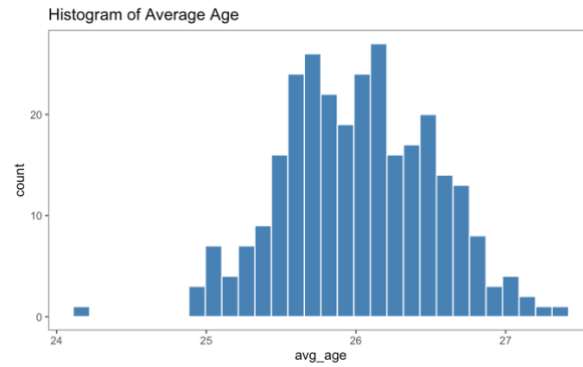


Figure 13: Histogram Average Age Variable

```
Call:
lm(formula = rswp ~ norm_total_spend + I(norm_fa_spend^0.5) +
  ext_spend_rank + avg_age + I(traded_to^0.5) + I(traded_from^0.5),
  data = df)

Residuals:
    Min       1Q   Median       3Q      Max
-0.44350 -0.12930 -0.00481  0.13455  0.44931

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -1.026e+00  5.626e-01  -1.824   0.0692
norm_total_spend  4.374e-09  7.288e-10   6.002 6.04e-09 ***
I(norm_fa_spend^0.5) -5.707e-06  3.638e-06  -1.569   0.1178
ext_spend_rank2  -2.844e-02  2.978e-02  -0.955   0.3404
ext_spend_rank3  -2.709e-02  2.990e-02  -0.906   0.3656
ext_spend_rank4  -2.777e-02  3.067e-02  -0.906   0.3659
avg_age       3.342e-02  2.128e-02   1.570   0.1175
I(traded_to^0.5)  4.223e-02  1.784e-02   2.367   0.0186 *
I(traded_from^0.5) -4.232e-03  1.754e-02  -0.241   0.8095
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1779 on 279 degrees of freedom
Multiple R-squared:  0.1645, Adjusted R-squared:  0.1405
F-statistic: 6.864 on 8 and 279 DF, p-value: 3.055e-08
```

Figure 14: Linear Model for Remaining Variables

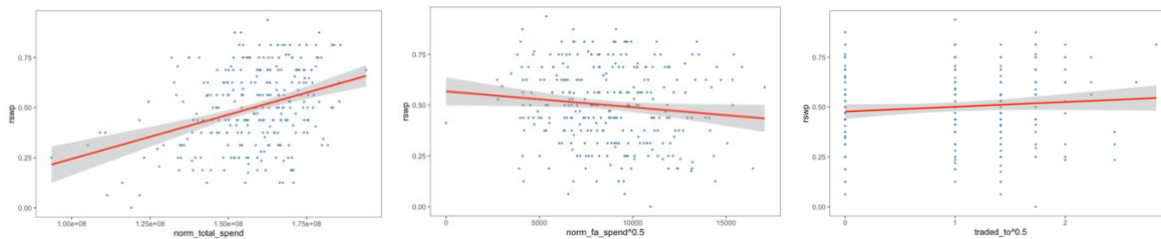


Figure 15: Scatterplots of Significant and Nearly Significant Variables

```
Call:
lm(formula = rswp ~ norm_total_spend + log(lb_pa + 0.01) + I(norm_fa_spend^0.5) +
  I(traded_to^0.5) + avg_age, data = df)

Residuals:
    Min       1Q   Median       3Q      Max
-0.47601 -0.13334 -0.00096  0.13479  0.41583

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -8.548e-01  5.490e-01  -1.557   0.12056
norm_total_spend  4.374e-09  6.869e-10   6.368 7.74e-10 ***
log(lb_pa + 0.01)  3.583e-02  1.159e-02   3.091  0.00219 **
I(norm_fa_spend^0.5) -5.926e-06  3.514e-06  -1.686   0.09281 .
I(traded_to^0.5)  4.197e-02  1.482e-02   2.831  0.00497 **
avg_age       2.912e-02  2.081e-02   1.399   0.16278
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1745 on 282 degrees of freedom
Multiple R-squared:  0.1879, Adjusted R-squared:  0.1735
F-statistic: 13.05 on 5 and 282 DF, p-value: 1.961e-11
```

Figure 16: Summary of Initial Final Linear Model

```
Call:
lm(formula = rswp ~ norm_total_spend + log(lb_pa + 0.01) + I(norm_fa_spend^0.5) +
  I(traded_to^0.5), data = df)

Residuals:
    Min       1Q   Median       3Q      Max
-0.47191 -0.12255 -0.00394  0.13255  0.41140

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -1.058e-01  1.221e-01  -0.866   0.38725
norm_total_spend  4.449e-09  6.859e-10   6.486 3.91e-10 ***
log(lb_pa + 0.01)  3.723e-02  1.157e-02   3.219  0.00144 **
I(norm_fa_spend^0.5) -5.890e-06  3.520e-06  -1.673  0.09538 .
I(traded_to^0.5)  4.145e-02  1.484e-02   2.792  0.00559 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1748 on 283 degrees of freedom
Multiple R-squared:  0.1822,    Adjusted R-squared:  0.1707
F-statistic: 15.77 on 4 and 283 DF,  p-value: 1.159e-11
```

Figure 17: Summary of Final Linear Model

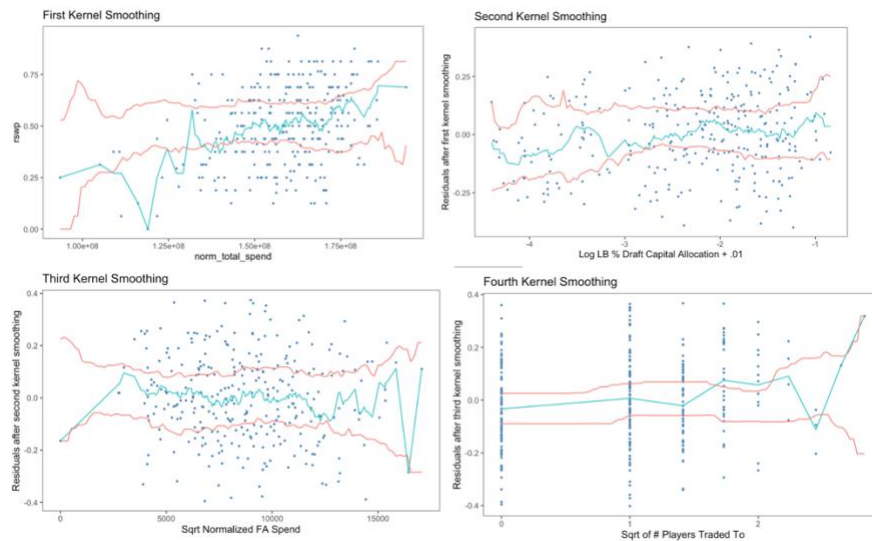


Figure 18: Kernel Regression with Confidence Intervals

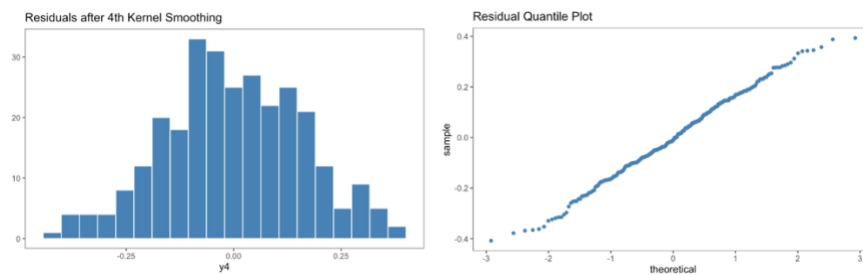


Figure 19: Residual Plots

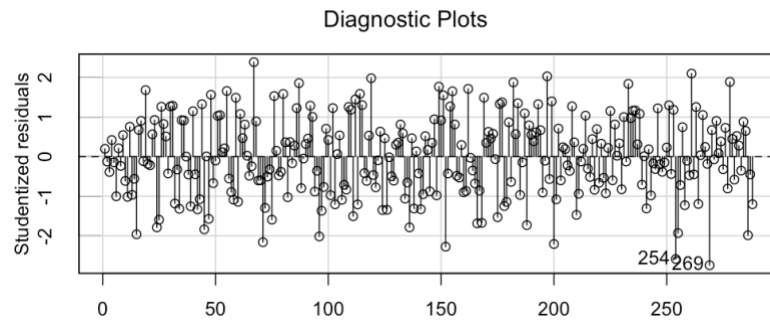


Figure 20: Influence Plot: Cook's Distances and Y-hat values