eg. 1 Convert PDA to CFG. PDA is given by

$M = (\{P, q\}, \{0,1\}, \{x, Z\}, \delta, q, Z)$, transition f^n δ is defined by:

$\delta(q, 1, Z) = \{(q, xZ)\}$
$\delta(q, 1, x) = \{(q, xx)\}$
$\delta(q, \epsilon, x) = \{(q, \epsilon)\}$
$\delta(q, 0, x) = \{(P, x)\}$
$\delta(P, 1, x) = \{(P, \epsilon)\}$

$\delta(P, 0, Z) = \{(q, Z)\}$

→ Step I Find set of variables V of G.

a) Start symbol S

b) $\underset{A}{[p^x p]}, \underset{B}{[p^x q]}, \underset{C}{[q^x p]}, \underset{D}{[q^x q]}$

$\underset{E}{[p^Z p]}, \underset{F}{[p^Z q]}, \underset{G}{[q^Z p]}, \underset{H}{[q^Z q]}$

Step-II Find set of productions P of G.

1) Start symbol productions

$S \rightarrow [q^Z p] \mid [q^Z q]$

2) Prod^n for $\delta(q, 1, Z) = (q, xZ)$

$\therefore$ $\cancel{\delta(q_i, a, B) \Rightarrow (q_j, C) \rightarrow [q_i{}^B q] = a[q_j{}^C q]}$ $q \in Q$.

$\cancel{[q^Z p] = P \cdot L}$

$\delta(q_i, a, B) \Rightarrow (q_j, C_1 C_2) \rightarrow [q_i{}^B P_1] = a \cdot [q_j{}^{C_1} P_2][P_2{}^{C_2} P_1]$

$P_1 \rightarrow P \quad P_2 = q.$

$\therefore [q^Z p] \Rightarrow 1 [q^x q][q^Z p]$ $\quad P_1 \rightarrow P, \ P_2 - q$

$[q^Z q] \rightarrow [q^x p][p^Z q]$ $\quad P_1 \rightarrow q, \ P_2 \rightarrow p$

(2)

eg.3 ~~Let~~ Find PDA for G.

$S \to aABB \mid aAA, \quad A \to aBB \mid a, \quad B \to bBB \mid A$.

$\to G = (\{S, A, B\}, \{a, b\}, \{\cancel{a, b}\} P, S)$

equi. M for G is.

$M = (\{q_0, q_1\}, \{a, b\}, \{S, A, B, a, b, z_0\}, z_0, q_0, \delta, \phi)$

'δ' is given by

step-I Place S on top of the stack.

$\delta(q_0, \epsilon, z_0) \Rightarrow (q_1, S z_0)$

step-II trans. for all var. $A \in V$.

$\delta(q_1, \epsilon, S) = \{(q_1, aABB), (q_1, aAA)\}$

$\delta(q_1, \epsilon, A) = \{(q_1, aBB), (q_1, a)\}$

$\delta(q_1, \epsilon, B) = \{(q_1, bBB), (q_1, A)\}$

step-III trans. for all terminal $T \in V$

$\delta(q_1, a, a) = (q_1, \epsilon)$

$\delta(q_1, b, b) = (q_1, \epsilon)$

Trans. Table

state	input	stack sym.	move
q_0	ϵ	z_0	$(q_1, S z_0)$
q_1	ϵ	S	$(q_1, aABB), (q_1, aAA)$
q_1	ϵ	A	(q_1, aBB) (q_1, a)
q_1	ϵ	B	(q_1, bBB) (q_1, A)
q_1	ϵ	z_0	(q_1, ϵ)

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consider string aabaaaaa

$(q_0, aabaaaaa, z_0) \vdash (q_1, aabaaaaa, Sz_0)$

$\vdash (q_1, aabaaaaa, aABBz_0)$ $S \to aABB$

$\vdash (q_1, abaaaaa, ABBz_0)$

$\vdash (q_1, abaaaaa, aBBBBz_0)$ $A \to aBB$

$\vdash (q_1, baaaaa, BBBBz_0)$ $(q,a,a) = (q_1, \epsilon)$

$\vdash (q_1, baaaaa, bBBBBBz_0)$ $B \to bBB$

$\vdash (q_1, aaaaa, BBBBBz_0)$

$\vdash (q_1, aaaaa, ABBBBz_0)$ $B \to A$

$\vdash (q_1, aaaaa, aBBBBz_0)$ $A \to a$

$\vdots$

$\vdash (q_1, \epsilon, z_0) \vdash (q_1, \epsilon, \epsilon)$

★ construction of CFG from PDA.

- IF PDA is given then we can construct CFG for given PDA. such that

$L(G) = L(M)$

steps

1] Variables of CFG will be of the form.

a) $S \to$ start symbol

b) $[pxq]$ where $p + q \in Q$ & $x \in \Gamma$

$\therefore$ if $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, \phi)$

then

$G = (V, T, P, S)$ where

$V = \{S, [p x q] \mid p, q \in Q \text{ \& } x \in \Gamma\}$

eg. ~~Let~~ If $Q = \{q_0, q_1\}$ & $T = \{a, b, z_0\}$ then set of variables in corr. CFG is given by,

1). S
2) $[q_0 a q_0]$, $[q_0 a q_1]$, $[q_1 a q_0]$ $[q_1 a q_1]$
3) $[q_0 b q_0]$, $[q_0 b q_1]$, $[q_1 b q_0]$ $[q_1 b q_1]$
4) $[q_0 z_0 q_0]$, $[q_0 z_0 q_1]$ $[q_1 z_0 q_0]$ $[q_1 z_0 q_1]$

Step-II <u>set of productions for the equivalent CFG</u>

a) Productions for the start symbol S.

$S \rightarrow [q_0 \, z \, q_i]$ For each $q_i \in Q$, Z is ~~start sym~~
Z - top symbol.
$q_0 \rightarrow$ initial state of PDA.

b) Prodn For each transition of the form
$\delta(q_i, a, B) \Rightarrow (q_j, C)$
where,
$q_i, q_j \in Q$
$a \in (\Sigma \cup \epsilon)$
$B, C \in (T \cup \epsilon)$

Productions for each $q \in Q$:
$[q_i \, B \, q] \rightarrow a [q_j \, C \, q]$

c) For each transition of the form
$\delta(q_i, a, B) \Rightarrow (q_j, C_1 C_2)$
where;
$q_i, q_j \in Q$
$a \in (\Sigma \cup \epsilon)$
$B, C_1, C_2 \in T$
then for each P_1 & $P_2 \in Q$ we add the prodn
$[q_i \, B \, P_1] \rightarrow a [q_j \, C_1 \, P_2] [P_2 \, C_2 \, P_1]$

* ## construction of CFG from PDA.

$L(G) = L(M)$

ie

$[q\,z\,p] \rightarrow 1\,[q\,x\,p]\,[p\,z\,p]$ $\quad P_1 \rightarrow p, \quad P_2 \rightarrow q$

$[q\,z\,q] \rightarrow 1\,[q\,x\,q]\,[q\,z\,q]$ $\quad P_1 \rightarrow q \quad P_2 \rightarrow q$

3) Prodn for $\delta(q, 1, x) = \{q, xx\}$

$[q\,x\,p] \Rightarrow 1\,[q\,x\,q]\,[q\,x\,p]$

$[q\,x\,p] \rightarrow 1\,[q\,x\,p]\,[p\,x\,p]$

$[q\,x\,q] \rightarrow 1\,[q\,x\,p]\,[p\,x\,q]$

$[q\,x\,q] \rightarrow 1\,[q\,x\,q]\,[q\,x\,q]$

4) Add prodn. For $\delta(q, \epsilon, x) \rightarrow (q, \epsilon)$

$[q\,x\,q] \rightarrow \epsilon$

5) Add prodn For $\delta(q, 0, x) = (p, x)$

$[q\,x\,p] \rightarrow 0\,[p\,x\,p]$

$[q\,x\,q] \rightarrow 0\,[p\,x\,q]$

6) Add Prodn for $\delta(p, 1, x) \Rightarrow (p, \epsilon)$

$[p\,x\,p] \rightarrow 1\,[p\,\epsilon\,p] \rightarrow 1$

7) Add prodn for $\delta(p, 0, z) \Rightarrow (q, z)$

$[p\,z\,p] \rightarrow 0\,[q\,z\,p]$

$[p\,z\,q] \rightarrow 0\,[q\,z\,q]$

Rename variables as in step - I (b)
rewrite prod^n for renamed variables

$S \rightarrow G \mid H$

eg. 2 Give the CFG generating the lang. accepted by the following PDA.

$M = (\{q_0, q_1\}, \{0, 1\}, \{z_0, x\}, \delta, q_0, z_0, \phi)$ where δ is given below:

$\delta(q_0, 1, z_0) = \{(q_0, xz_0)\}$
$\delta(q_0, 1, x) = \{(q_0, xx)\}$, $\delta(q_0, 0, x) = \{(q_1, x)\}$
$\delta(q_0, \epsilon, z_0) = \{(q_0, \epsilon)\}$, $\delta(q_1, 1, x) = \{(q_1, \epsilon)\}$
$\delta(q_1, 0, z_0) = \{(q_0, z_0)\}$

→ step-I : Add productions for the start symbol.

$S \rightarrow \underbrace{[q_0 \, z_0 \, q_0]}_{A} \mid \underbrace{[q_0 \, z \, q_1]}_{B}$

Step 2 :- prod^{n}s for $\delta(q_0, 1, z_0) = (q_0, xz_0)$

$[q_0 \, z_0 \, q_0] \rightarrow 1 \cdot [q_0 \, x \, q_1] [q_1 \, z_0 \, q_0]$.

$[q_0 \, z_0 \, q_0] \rightarrow 1 \cdot [q_0 \, x \, q_0] [q_0 \, z_0 \, q_0]$

$[q_0 \, z_0 \, q_1] \rightarrow 1 \cdot [q_0 \, x \, q_0] [q_0 \, z_0 \, q_1]$

$[q_0 \, z_0 \, q_1] \rightarrow 1 \cdot [q_0 \, x \, q_1] [q_1 \, z_0 \, q_1]$

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step-II prodⁿ for. $\delta(q_0, 1, x) = \{(q_0, xx)\}$

$P_1 = q_0, P_2 = q_1$

$[q_0 x q_0] \rightarrow 1 \cdot [q_0 x q_0][q_0 x q_0]$ ← $P_1 = q_0 \quad P_2 = q_0$

$P_1 = q_1 \quad P_2 = q_0$

$[q_0 x q_0] \rightarrow 1 \cdot [q_0 x q_1][q_1 x q_0]$

$P_1 = q_1 \quad P_2 = q_1$

$[q_0 x q_1] \rightarrow 1 \cdot [q_0 x q_0][q_0 x q_1]$

$[q_0 x q_1] \rightarrow 1 \cdot [q_0 x q_1][q_1 x q_1]$

Step III for $\delta(q_0, 0, x) = (q_1, x)$

$q = q_0, q_1$

$[q_0 x q_0] \rightarrow 0 \cdot [q_1 x q_0]$

$[q_0 x q_1] \rightarrow 0 [q_1 x q_1]$

Step-IV $\delta(q_0, \epsilon, z_0) = (q_0, \epsilon)$

$[q_0 z_0 q_0] \rightarrow \epsilon$

step-V $\delta(q_1, 1, x) \rightarrow (q_1, \epsilon)$

$[q_1 x q_1] \rightarrow 1$

Step-VI $\delta(q_1, 0, z_0) \rightarrow (q_0, z_0)$

$[q_1 z_0 q_1] \rightarrow 0 \cdot [q_0 z_0 q_1]$

$[q_1 z_0 q_0] \rightarrow 0 [q_0 z_0 q_0]$