* Construction of CFG from PDA.

- ~~We~~ If PDA is given then we can construct CFG for given PDA. such that $L(G) = L(M)$

Steps

1] Variables of CFG will be of the form.

a) $S \rightarrow$ start symbol

b) $[pXq]$ where p & $q \in Q$ & $X \in \Gamma$

$\therefore$ if $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, \phi)$

then

$G = (V, T, P, S)$ where

$V = \{S, [pXq] \mid p, q \in Q \text{ \& } X \in \Gamma\}$

eg. ~~Let~~ If $Q = \{q_0, q_1\}$ & $\Gamma = \{a, b, Z_0\}$ then set of variables in corr. CFG is given by,

1). S

2) $[q_0 a q_0]$, $[q_0 a q_1]$, $[q_1 a q_0]$ $[q_1 a q_1]$

3) $[q_0 b q_0]$, $[q_0 b q_1]$, $[q_1 b q_0]$ $[q_1 b q_1]$

4) $[q_0 Z_0 q_0]$, $[q_0 Z_0 q_1]$ $[q_1 Z_0 q_0]$ $[q_1 Z_0 q_1]$

Step-II **set of productions for the equivalent CFG.**

2)

a) Productions for the start symbol S.

$S \rightarrow [q_0 Z q_i]$ For each $q_i \in Q$, Z is start sym

Z - top symbol.

$q_0 \rightarrow$ initial state of PDA.

b) Prod^n For each transition of the form

$\delta(q_i, a, B) \Rightarrow (q_j, C)$

where,

$q_i, q_j \in Q$

$a \in (\Sigma \cup \epsilon)$

$B, C \in (\Gamma \cup \epsilon)$

Productions For each $q \in Q$:

$[q_i B q] \rightarrow a [q_j C q]$

c) For each transition of the form

$\delta(q_i, a, B) \Rightarrow (q_j, C_1 C_2)$

where;

$q_i, q_j \in Q$

$a \in (\Sigma \cup \epsilon)$

$B, C_1, C_2 \in \Gamma$

then For each P_1 & $P_2 \in Q$ we add the Prod^n

$[q_i B P_1] \rightarrow a [q_j C_1 P_2] [P_2 C_2 P_1]$

eg. 2 Give the CFG generating the lang. accepted by the following PDA.

$M = (\{q_0, q_1\}, \{0,1\}, \{z_0, x\}, \delta, q_0, z_0, \phi)$ where δ is given below:

$\delta(q_0, 1, z_0) = \{(q_0, xz_0)\}$

$\delta(q_0, 1, x) = \{(q_0, xx)\}$, $\delta(q_0, 0, x) = \{(q_1, x)\}$

$\delta(q_0, \epsilon, z_0) = \{(q_0, \epsilon)\}$, $\delta(q_1, 1, x) = \{(q_1, \epsilon)\}$

$\delta(q_1, 0, z_0) = \{(q_0, z_0)\}$

→

Step-I : Add productions for the start symbol.

$S \rightarrow \underbrace{[q_0 z_0 q_0]}_{A} \mid \underbrace{[q_0 z_0 q_1]}_{B}$

Step 2 :- prodns for $\delta(q_0, 1, z_0) = (q_0, xz_0)$

$[q_0 z_0 q_0] \rightarrow 1 \cdot [q_0 x q_1] [q_1 z_0 q_0]$

$[q_0 z_0 q_0] \rightarrow 1 \cdot [q_0 x q_0] [q_0 z_0 q_0]$

$[q_0 z_0 q_1] \rightarrow 1 \cdot [q_0 x q_0] [q_0 z_0 q_1]$

$[q_0 z_0 q_1] \rightarrow 1 \cdot [q_0 x q_1] [q_1 z_0 q_1]$

step-II prod^n for $\delta(q_0, 1, x) = \{(q_0, xx)\}$

$P_1 = q_0, P_2 = q_1$

$[q_0 \, x \, q_0] \rightarrow 1 \cdot [q_0 \, x \, q_0] \, [q_0 \, x \, q_0]$ $P_1 = q_0 \;\; P_2 = q_0$

$[q_0 \, x \, q_0] \rightarrow 1 \cdot [q_0 \, x \, q_1] \, [q_1 \, x \, q_0]$ $P_1 = q_1 \;\; P_2 = q_0$

$P_1 = q_1 \;\; P_2 \rightarrow q_1$

$[q_0 \, x \, q_1] \rightarrow 1 \cdot [q_0 \, x \, q_0] \, [q_0 \, x \, q_1]$

$[q_0 \, x \, q_1] \rightarrow 1 \cdot [q_0 \, x \, q_1] \, [q_1 \, x \, q_1]$

Step III for $\delta(q_0, 0, x) = (q_1, x)$

$q = q_0, q_1$

$[q_0 \, x \, q_0] \rightarrow 0 \cdot [q_1 \, x \, q_0]$

$[q_0 \, x \, q_1] \rightarrow 0 \, [q_1 \, x \, q_1]$

Step-IV $\delta(q_0, \epsilon, z_0) = (q_0, \epsilon)$

$[q_0 \, z_0 \, q_0] \rightarrow \epsilon$

step-V $\delta(q_1, 1, x) \rightarrow (q_1, \epsilon)$

$[q_1 \, x \, q_1] \rightarrow 1$

Step-VI $\delta(q_1, 0, z_0) \rightarrow (q_0, z_0)$

$[q_1 \, z_0 \, q_1] \rightarrow 0 \cdot [q_0 \, z_0 \, q_1]$

$[q_1 \, z_0 \, q_0] \rightarrow 0 \, [q_0 \, z_0 \, q_0]$