

# A code generator for ODE-based models

—

 -package rodeo

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# Outline

Introduction

Objectives

Concepts

Minimalistic example

Specific features & limitations

Applications

Summary

Hands-on part

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**Introduction**

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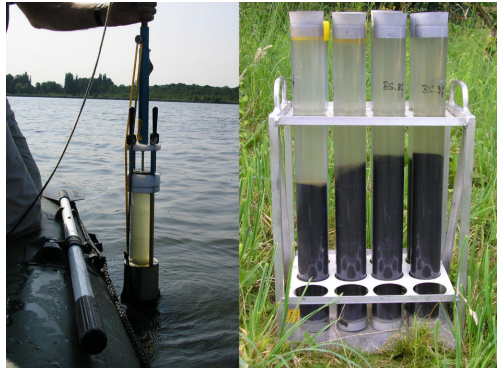
- ▶ Lake eutrophication
- ▶ Flood management
- ▶ Operational runoff forecasting
- ▶ Early diagenesis of lake sediments
- ▶ Fate of antibiotic resistant bacteria



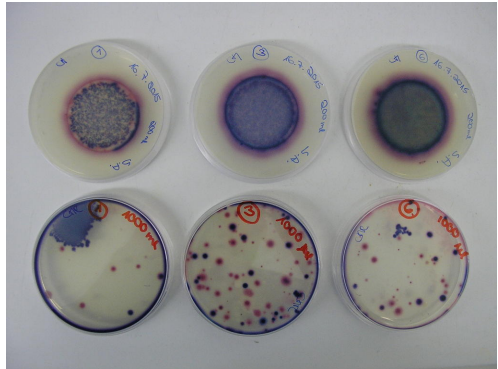
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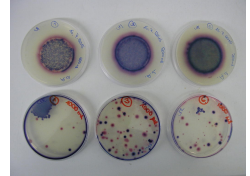
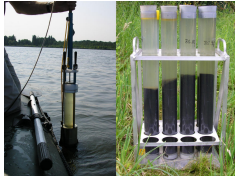
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→ Several years of model/software development



- ▶ Best way to learn modeling is via model development.
- ▶ 'Monolithic codes' are hard to extend.
- ▶ Rising interest in structural uncertainty  
→ Need for Re-implementations

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Hands-on part

- ▶ Often incomplete or outdated
- ▶ Mistakes in published equations
- ▶ Source code alone not sufficient

→ Embedded / automatic documentation

- ▶ Implementation in specific language / framework
- ▶ Impedes collaborative development
- ▶ Software undergoes aging

→ True portability

→ Equations to be separated from source code

# Handling of large arrays

- ▶ Access by index: Hard to read / maintain
- ▶ Access by name: Slow

→ Combine the two options

- ▶ Interpreted code is convenient but relatively slow
- ▶ Need for high-performance (Optimization, Uncertainty, ...)

→ Use compiled code sections

- ▶ Repeated evaluation wastes time
- ▶ Code is difficult to maintain

- Use proper notation to reduce redundancies
- Let the compiler eliminate them

- ▶ Effort for users
- ▶ Individual pre-/post-processors
- ▶ Impedes coupling of models

→ Unified interface



- ▶ Built-in documentation
- ▶ True portability
- ▶ Save & fast array access
- ▶ Compiled code sections
- ▶ Less redundancies
- ▶ Unified interface

→ rodeo is one attempt, among others, to achieve this

- Models built on simultaneous ODE

$$\frac{d}{dt} Y_1 = f(\text{time}, Y, \text{parameters})$$

...

$$\frac{d}{dt} Y_n = f(\text{time}, Y, \text{parameters})$$

- Numerical integration or steady-state estimation

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## (1) Use of a table-based standard notation for ODE

- ▶ Built-in documentation
- ▶ Less redundancies
- ▶ Unified interface

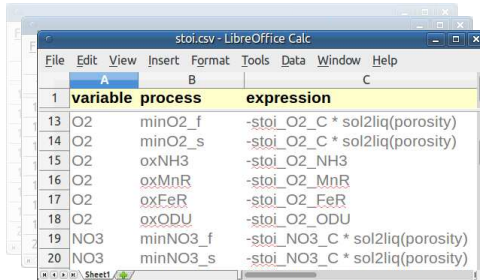
## (1) Use of a table-based standard notation for ODE

- ▶ Built-in documentation
- ▶ Less redundancies
- ▶ Unified interface

## (2) Automatic code generation

- ▶ Save & fast array access
- ▶ Use of compiled code
- ▶ Portability

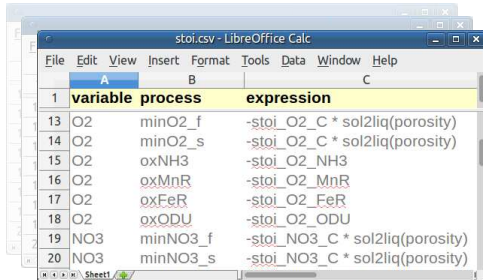
### Table-based model definition



	A	B	C
1	variable	process	expression
13	O2	minO2_f	-stoi_O2_C * sol2liq(porosity)
14	O2	minO2_s	-stoi_O2_C * sol2liq(porosity)
15	O2	oxNH3	-stoi_O2_NH3
16	O2	oxMnR	-stoi_O2_MnR
17	O2	oxFeR	-stoi_O2_FeR
18	O2	oxODU	-stoi_O2_ODU
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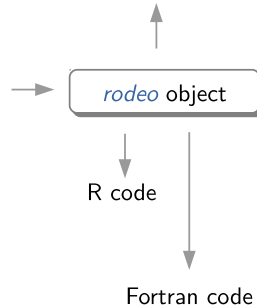
→ *rodeo* object

## Table-based model definition

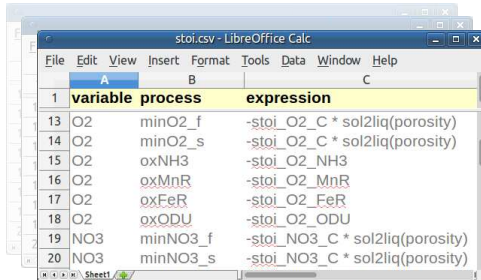


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Components of *tex*  
or *html* documents



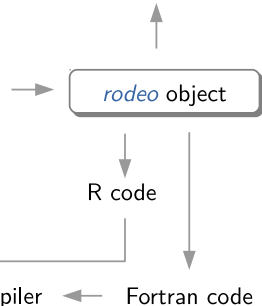
## Table-based model definition



The screenshot shows a LibreOffice Calc spreadsheet with the following data:

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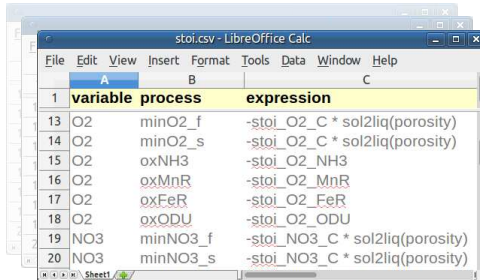
Numerical solvers  
*deSolve*, *rootSolve*

Compiler

Fortran code



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Components of *tex*  
or *html* documents

*rodeo* object

R code

Numerical solvers

*deSolve*, *rootSolve*

Compiler

Fortran code

States & rates

Initial values, parameters

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**Minimalistic example**

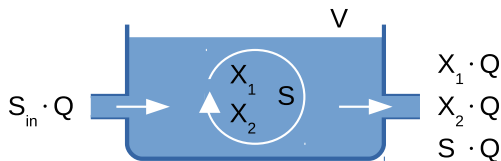
Specific features & limitations

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Summary

Hands-on part

- ▶ Mixed reactor with constant volume  $V$  and flow rate  $Q$
- ▶ Two species ( $X_1$ ,  $X_2$ ) competing for dissolved resource  $S$

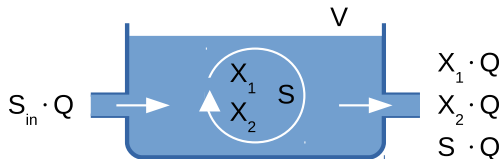


$$\frac{d}{dt}X_1 = r_1 \cdot X_1 \cdot \frac{S}{S + h_1} - X_1 \cdot \frac{Q}{V}$$

Growth  
Im/Export

$$\frac{d}{dt}X_2 = r_2 \cdot X_2 \cdot \frac{S}{S + h_2} - X_2 \cdot \frac{Q}{V}$$

$$\frac{d}{dt}S = -c_1 \cdot r_1 \cdot X_1 \cdot \frac{S}{S + h_1} - c_2 \cdot r_2 \cdot X_2 \cdot \frac{S}{S + h_2} + (S_{in} - S) \cdot \frac{Q}{V}$$



$$\frac{d}{dt}X_1 = r_1 \cdot X_1 \cdot \frac{S}{S + h_1} - X_1 \cdot \frac{Q}{V}$$

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$$\frac{d}{dt} \begin{bmatrix} X_1 \\ X_2 \\ S \end{bmatrix} = \begin{bmatrix} 1 & 0 & -X_1 \\ 0 & 1 & -X_2 \\ -c_1 & -c_2 & S_{in} - S \end{bmatrix} \cdot \begin{bmatrix} r_1 \cdot X_1 \cdot S/(S + h_1) \\ r_2 \cdot X_2 \cdot S/(S + h_2) \\ Q/V \end{bmatrix}$$

$$\frac{d}{dt} \begin{bmatrix} X_1 \\ X_2 \\ S \end{bmatrix} = \begin{bmatrix} 1 & 0 & -X_1 \\ 0 & 1 & -X_2 \\ -c_1 & -c_2 & S_{in} - S \end{bmatrix} \cdot \begin{bmatrix} r_1 \cdot X_1 \cdot S / (S + h_1) \\ r_2 \cdot X_2 \cdot S / (S + h_2) \\ Q/V \end{bmatrix}$$

	A	B	C	D
1	<b>name</b>	<b>unit</b>	<b>description</b>	<b>expression</b>
2	growthX1	cells/ml/h	growth of X1	$r_1 \cdot X_1 \cdot \text{monod}(S, h_1)$
3	growthX2	cells/ml/h	growth of X2	$r_2 \cdot X_2 \cdot \text{monod}(S, h_2)$
4	flushing	1/h	flushing	$Q / V$

## Table of stoichiometric factors

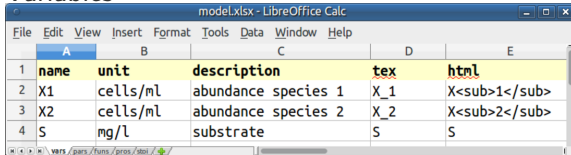
$$\frac{d}{dt} \begin{bmatrix} X_1 \\ X_2 \\ S \end{bmatrix} = \begin{bmatrix} 1 & 0 & -X_1 \\ 0 & 1 & -X_2 \\ -c_1 & -c_2 & S_{in} - S \end{bmatrix} \cdot \begin{bmatrix} r_1 \cdot X_1 \cdot S / (S + h_1) \\ r_2 \cdot X_2 \cdot S / (S + h_2) \\ Q/V \end{bmatrix}$$

model.xlsx - LibreOffice Calc

	G	H	I	J	K
1	<u>proc.</u> \ var.	X1	X2	S	
2	growthX1	1	0	-c1	
3	growthX2	0	1	-c2	
4	flushing	-X1	-X2	Sin - S	
5					
6					
7					
8					

vars / pars / funs / proc / **stoi**

## Variables

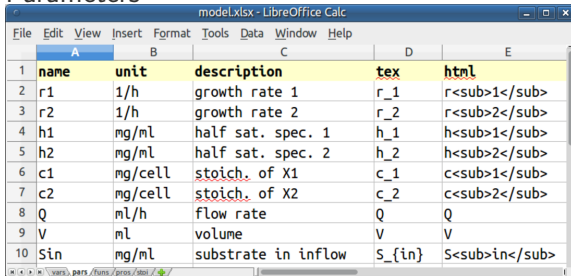


model.xlsx - LibreOffice Calc

	A	B	C	D	E
1	name	unit	description	tex	html
2	X1	cells/ml	abundance species 1	X_1	X <sub>1</sub>
3	X2	cells/ml	abundance species 2	X_2	X <sub>2</sub>
4	S	mg/l	substrate	S	S

vars / pars / funs / gros / sto /

## Parameters



model.xlsx - LibreOffice Calc

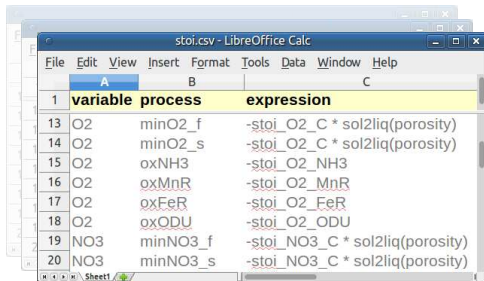
	A	B	C	D	E
1	name	unit	description	tex	html
2	r1	1/h	growth rate 1	r_1	r <sub>1</sub>
3	r2	1/h	growth rate 2	r_2	r <sub>2</sub>
4	h1	mg/ml	half sat. spec. 1	h_1	h <sub>1</sub>
5	h2	mg/ml	half sat. spec. 2	h_2	h <sub>2</sub>
6	c1	mg/cell	stoich. of X1	c_1	c <sub>1</sub>
7	c2	mg/cell	stoich. of X2	c_2	c <sub>2</sub>
8	Q	ml/h	flow rate	Q	Q
9	V	ml	volume	V	V
10	S <sub>in</sub>	mg/ml	substrate in inflow	S <sub>in</sub>	S <sub>in</sub>

vars / pars / funs / gros / sto /

+ Functions



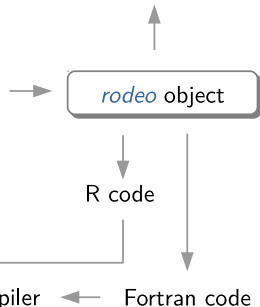
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Components of *tex*  
or *html* documents

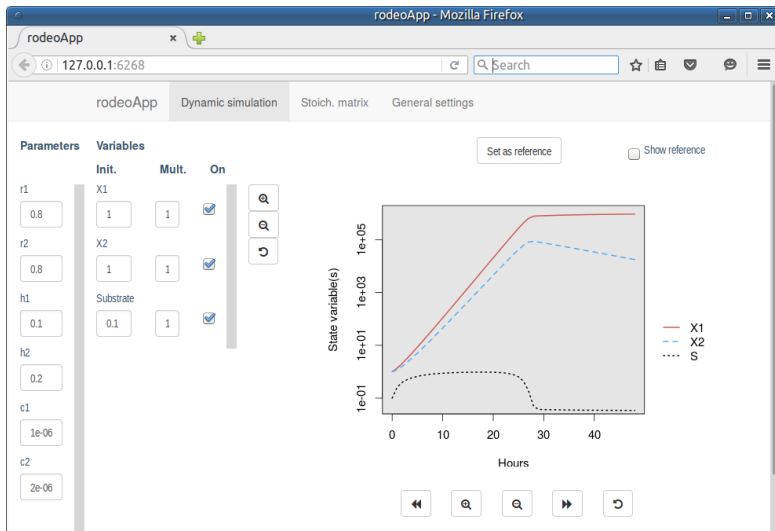


States & rates      Initial values, parameters

# Minimalistic example

## Auto-generated GUI

34



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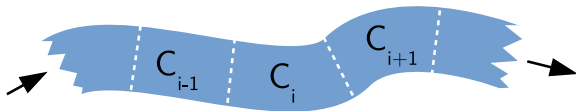
Hands-on part

$$\frac{\partial c}{\partial t} = \underbrace{D \cdot \frac{\partial^2 c}{\partial x^2}}_{\text{Dispersion}} - \underbrace{u \cdot \frac{\partial c}{\partial x}}_{\text{Advection}} + \underbrace{R}_{\text{Reactions}}$$

$$\frac{\partial c}{\partial t} = \underbrace{D \cdot \frac{\partial^2 c}{\partial x^2}}_{\text{Dispersion}} - \underbrace{u \cdot \frac{\partial c}{\partial x}}_{\text{Advection}} + \underbrace{R}_{\text{Reactions}}$$

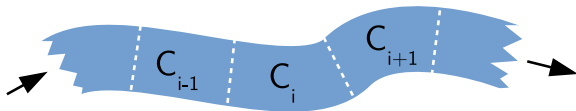
Method-of-lines

$$\frac{dc_i}{dt} = D \cdot \frac{(c_{i+1} - c_i) - (c_i - c_{i-1}))}{\Delta x^2} - u \cdot \frac{c_i - c_{i-1}}{\Delta x} + R_i$$



Method-of-lines

$$\frac{dc_i}{dt} = D \cdot \frac{(c_{i+1} - c_i) - (c_i - c_{i-1}))}{\Delta x^2} - u \cdot \frac{c_i - c_{i-1}}{\Delta x} + R_i$$



Method-of-lines

$$\frac{dc_i}{dt} = D \cdot \frac{(c_{i+1} - c_i) - (c_i - c_{i-1})}{\Delta x^2} - u \cdot \frac{c_i - c_{i-1}}{\Delta x} + R_i$$

Function-like syntax to access adjacent cells, e.g.

`u / dx * ( c - left(c) )`

`foo(time)` can appear in right hand side expressions

Actual functions must be defined



**Analytical**



**Interpolation**

- Use `approxFun` in R-based models
- Use `rodeo`-generated Fortran code



- ▶ No forced documentation for user-function arguments
- ▶ No built-in support for 2D or 3D models
- ▶ Generated code uses a Fortran 2008 feature

CRAN Package Check Results for Package [rodeo](#)

Last updated on 2016-04-28 06:47:39.

Flavor	Version	T <sub>install</sub>	T <sub>check</sub>	T <sub>total</sub>	Status	Flags
<a href="#">r-devel-linux-x86_64-debian-gcc</a>	0.3	1.18	18.93	20.11	OK	
<a href="#">r-devel-linux-x86_64-fedora-clang</a>	0.3			34.64	OK	
<a href="#">r-devel-linux-x86_64-fedora-gcc</a>	0.3			22.28	OK	
<a href="#">r-devel-osx-x86_64-clang</a>	0.3			37.30	OK	
<a href="#">r-devel-windows-ix86+x86_64</a>	0.3	5.00	59.00	64.00	OK	
<a href="#">r-patched-linux-x86_64</a>	0.3	1.19	18.58	19.77	OK	
<a href="#">r-patched-solaris-sparc</a>	0.3			184.30	WARN	
<a href="#">r-patched-solaris-x86</a>	0.3			41.20	WARN	
<a href="#">r-release-linux-x86_64</a>	0.3	1.36	21.82	23.18	OK	
<a href="#">r-release-osx-x86_64-mavericks</a>	0.3				OK	
<a href="#">r-release-windows-ix86+x86_64</a>	0.3	5.00	72.00	77.00	OK	
<a href="#">r-oldrel-windows-ix86+x86_64</a>	0.3	5.00	82.00	87.00	OK	

**WARN:** Compiler doesn't implement pointer initialization yet

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## Existing

- ▶ Lake ecology (0D)
- ▶ Sediment diagenesis
- ▶ Dynamics of E. coli
- ▶ Prey-predator systems

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- ▶ Lake ecology (0D)
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### Planned

- ▶ Lake ecology (1D)
- ▶ Activated sludge model

### Existing

- ▶ Lake ecology (0D)
- ▶ Sediment diagenesis
- ▶ Dynamics of *E. coli*
- ▶ Prey-predator systems

### Planned

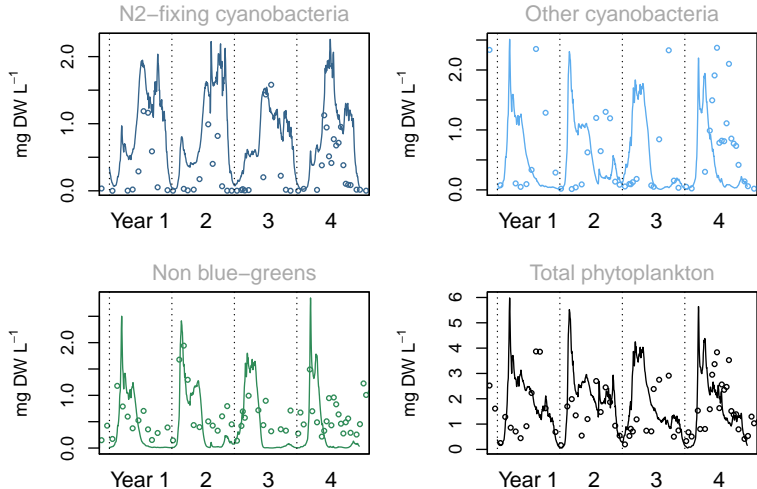
- ▶ Lake ecology (1D)
- ▶ Activated sludge model

- ▶ Heavily based on BELAMO
- ▶ Applied to a shallow lake,  $1.3 \text{ km}^2$ ,  $z_{mean} 2.1 \text{ m}$

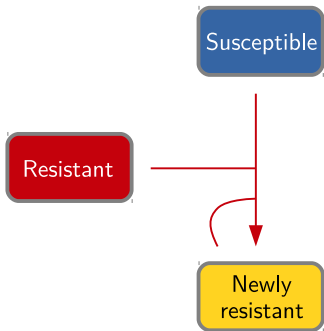


Th. Petzoldt

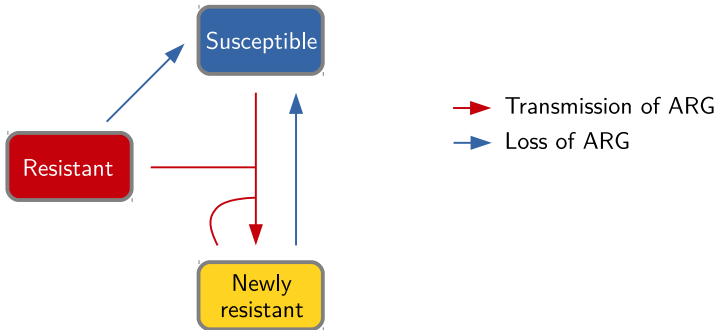
Contribution of  $\text{N}_2$ -fixation to Nitrogen balance?

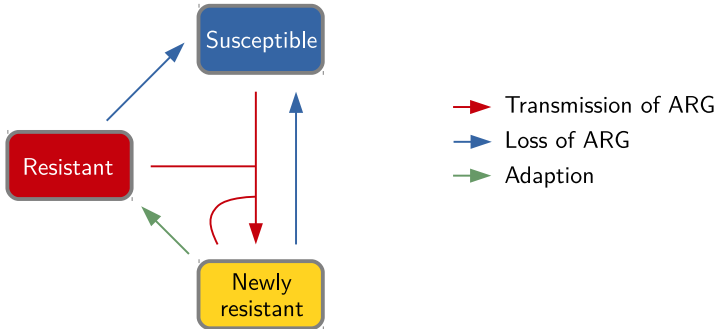


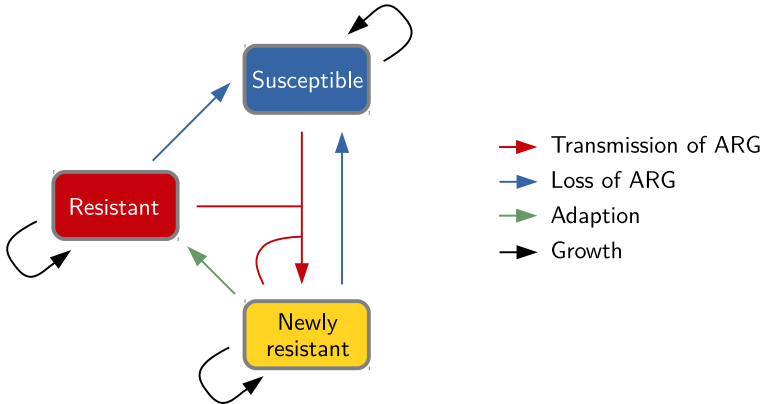


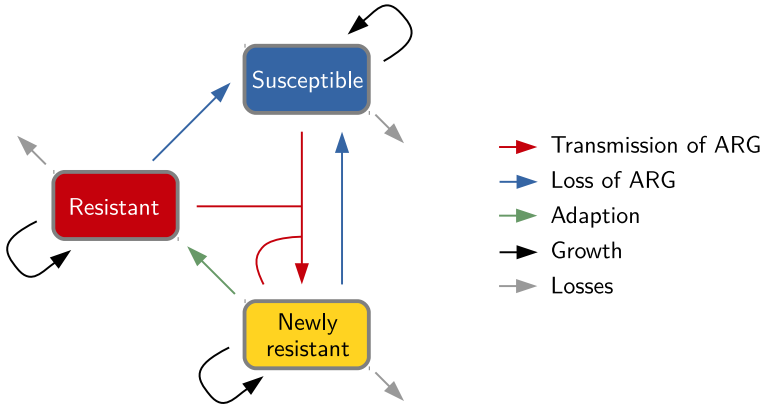


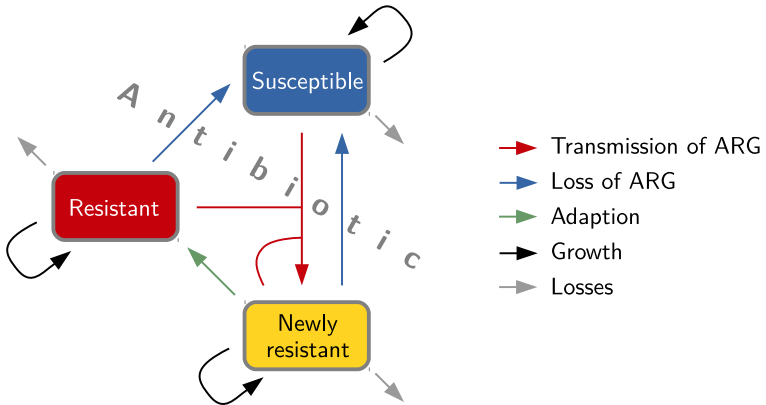
→ Transmission of ARG

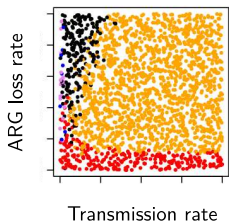
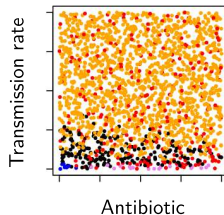
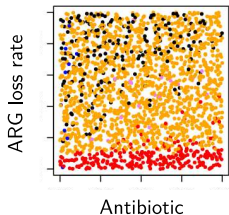




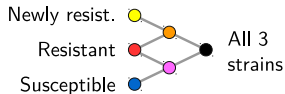


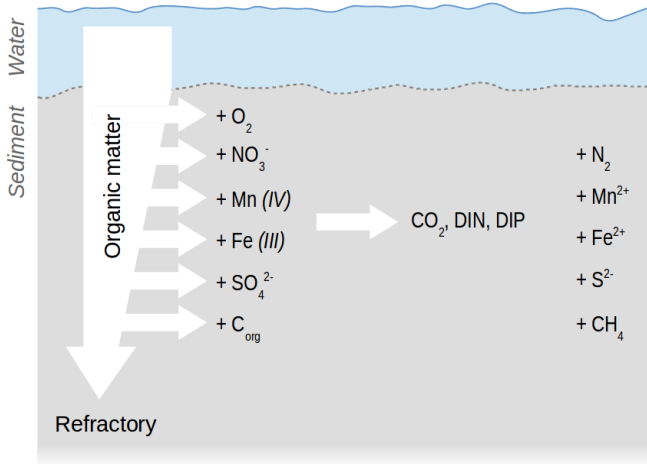




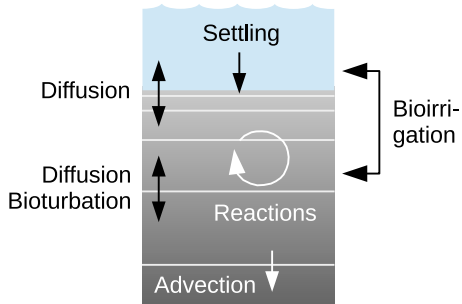


**Strains present after long time**









	C <sub>org</sub>	DIP	IMP	NH <sub>4</sub> <sup>+</sup>	sorb. NH <sub>4</sub> <sup>+</sup>	NO <sub>3</sub> <sup>-</sup>	O <sub>2</sub>	ODU
Settling	▲							
Mineral. (O <sub>2</sub> )	▼	▲		▲			▼	
Mineral. (NO <sub>3</sub> <sup>-</sup> )	▼	▲		▲		▼		
Mineral. (anox.)	▼	▲		▲				▲
Nitrification				▼		▲	▼	
Oxidation w. O <sub>2</sub>							▼	▼
Oxidation w. NO <sub>3</sub> <sup>-</sup>						▼		▼
P-Immobilization		▼	▲					
P-Remobilization		▲	▼					
NH <sub>4</sub> <sup>+</sup> -Sorption				▼	▲			
Advection ↓	●	●	●	●	●	●	●	●
Advection ↑		●		●		●	●	●
Diffus. transport	●	●	●	●	●	●	●	●
Diffus. at surface		●		●		●	●	●
Bioirrigation		●		●		●	●	●

Concentration

▲ increases

▼ decreases

● goes up or down

DIP: Dissolved inorg. P

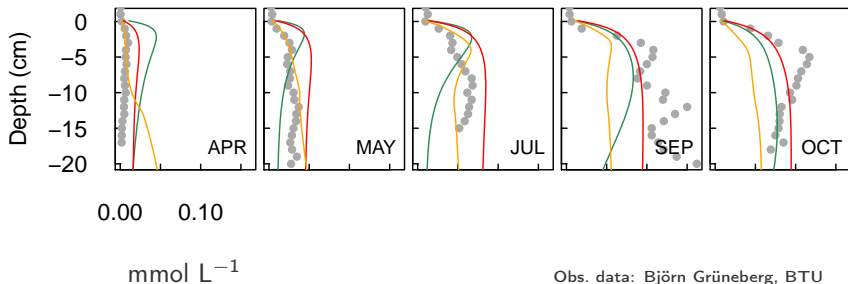
IMP: Immobile inorg. P

ODU: Mn<sup>2+</sup>, Fe<sup>2+</sup>, HS<sup>-</sup>, ...

Basic concepts borrowed  
from Soetaert et al. (1996)

● Phosphorus in pore water, observed

/// Simulated with different model structures



# Outline

Introduction

Objectives

Concepts

Minimalistic example

Specific features & limitations

Applications

**Summary**

Hands-on part

<b>Scope</b>	Implementation of ODE models (+ 1D PDE)
<b>Concepts</b>	Table-based notation & code generation
<b>Benefit</b>	Simplicity and performance
<b>Uses</b>	Project work & teaching
<b>Package</b>	<a href="https://cran.r-project.org/package=rodeo">https://cran.r-project.org/package=rodeo</a> <a href="https://github.com/dkneis/rodeo">https://github.com/dkneis/rodeo</a>
<b>Examples</b>	<a href="http://dkneis.github.io">http://dkneis.github.io</a> <a href="http://limno-live.hydro.tu-dresden.de/">http://limno-live.hydro.tu-dresden.de/</a>

Thanks !

Sponsored by the Federal Ministry of Education and Research,  
Germany, Grant no. 0033W015EN, 02WU1351A

# Outline

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**Hands-on part**

- ▶ Recent R version
- ▶ Developer tools (Rtools on Windows)
- ▶ R packages
  - ▶ `install.packages('deSolve')`
  - ▶ `install.packages('readxl')`
  - ▶ `install.packages('rodeo')`

Link to instructions on <http://dkneis.github.io>



<https://cran.r-project.org/package=rodeo>

- ▶ `rodeo` class is a 'reference class'
- ▶ Creation: `object <- new('rodeo', <data>)`
- ▶ Usage: `object$method()`

<https://cran.r-project.org/package=rodeo>

- ▶ `rodeo` class is a 'reference class'
- ▶ Creation: `object <- new('rodeo', <data>)`
- ▶ Usage: `object$method()`

```
install.packages('rodeo')    # done this already?  
library('rodeo')  
?rodeo  
vignette('rodeo')
```

<https://cran.r-project.org/package=deSolve>

- ▶ Switch between stiff and non-stiff methods
- ▶ Structure of Jacobian can be specified
- ▶ Works with compiled code in shared library

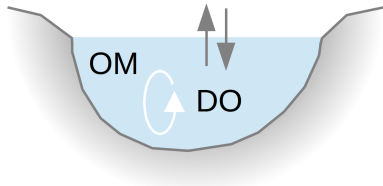
<https://cran.r-project.org/package=deSolve>

- ▶ Switch between stiff and non-stiff methods
- ▶ Structure of Jacobian can be specified
- ▶ Works with compiled code in shared library

```
install.packages('deSolve')    # done this already?  
library('deSolve')  
?lsoda  
?ode
```

- ▶ See links on <http://dkneis.github.io>
- ▶ Available in latest rodeo package (not on CRAN yet)

OM Organic matter (mg/L)  
DO Dissolved oxygen (mg/L)



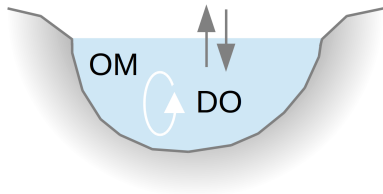
*Streeter, W. H. and Phelps, W. B. (1925): A study of the pollution and natural purification of the Ohio River. Public Health Bull. 146, US Public Health Service, Washington DC.*

→ Essential extensions developed in past 90 years

$$\frac{d}{dt} OM = -k_d \cdot OM$$

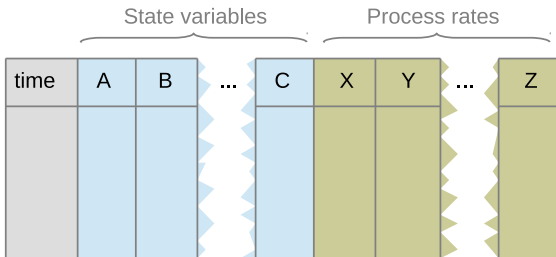
$$\frac{d}{dt} DO = -k_d \cdot OM \cdot s$$

$$+ k_a \cdot (DO_{sat} - DO)$$



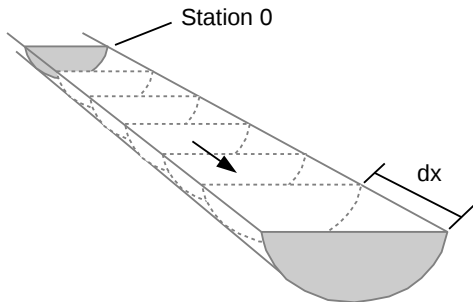
Symbol	Units	Descr.
$k_d$	$d^{-1}$	Decay rate
$k_a$	$d^{-1}$	Aeration rate
$s$	Mass ratio	DO consumed per degraded OM
$DO_{sat}$	mg/L	O <sub>2</sub> saturation level

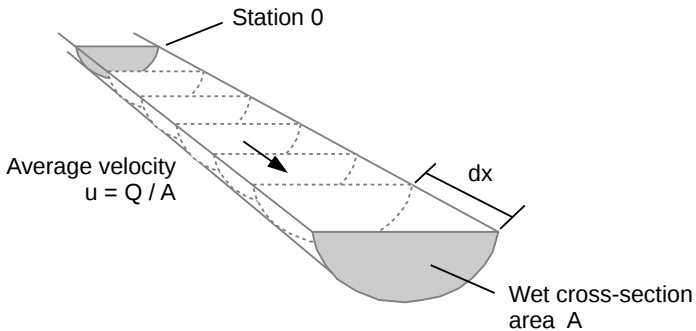
## deSolve output for 0D rodeo models

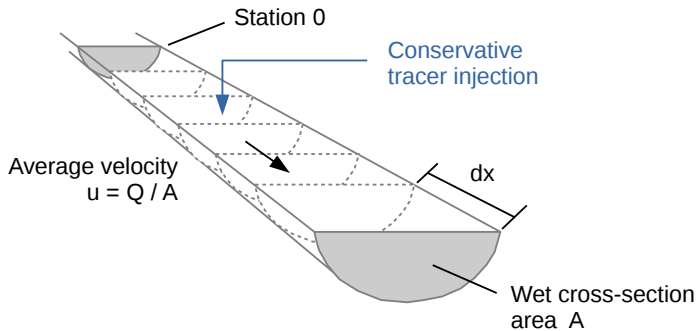


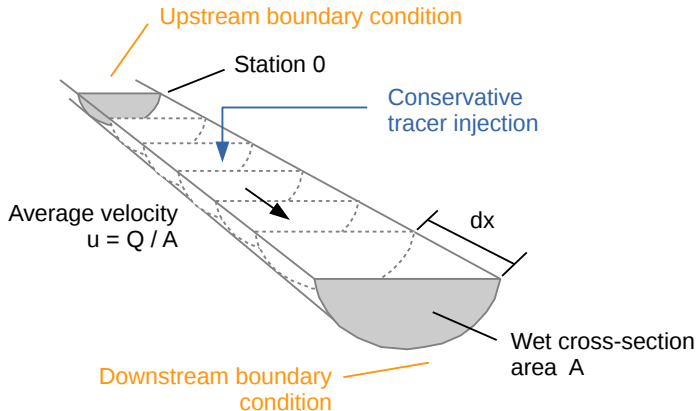
```
is.matrix(out)    # TRUE
ncol(out) == 1 + m$lenVars() + m$lenPros()    # m:  model
colnames(out) == c('time', m$namesVars(), m$namesPros())
```



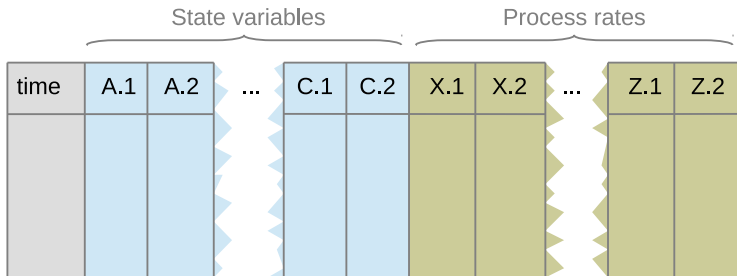






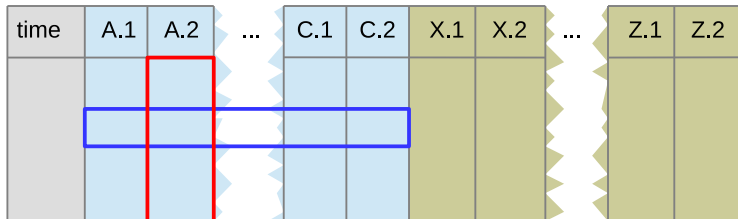


deSolve output for 1D rodeo models



```
ncol(out) == 1 + m$lenVars() * nBox + m$lenPros() * nBox
```

deSolve output for 1D rodeo models



- Snapshot of spatial distribution
- Breakthrough curve at particular station

**Delimited  
text**

- ▶ Powerful editors (regular exp., syntax highlight)
- ▶ Version control
- ▶ Many processing options ( $\text{\LaTeX}$ , data base, ...)
- ▶ Portable (but newline & encoding issues)

**Spreadsheet**

- ▶ Tabular view
- ▶ All tables kept in a single file
- ▶ Portable (different issues)

→ Best used in combination