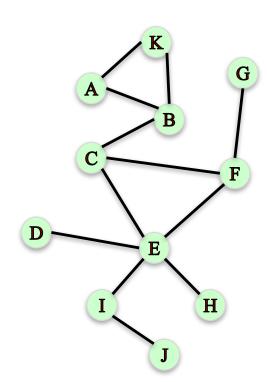


How "far" is node A from node H?

Are nodes far away or close to each other in this network?

Which nodes are "closest" and "farthest" to other nodes?

We need a sense of distance between nodes to answer these questions





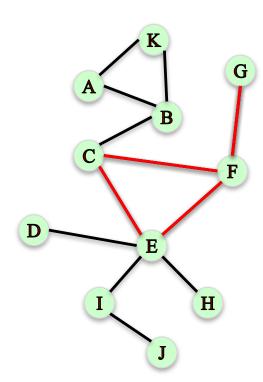
# **Paths**

**Path:** A sequence of nodes connected by an edge.

Find two paths from node G to node C:

$$G-F-C$$

$$G-F-E-C$$





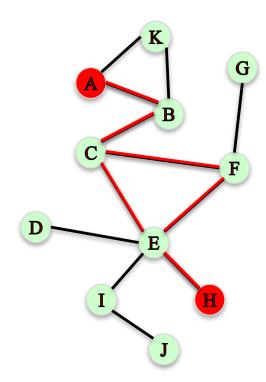
How far is node A from node H?

Path I: 
$$A - B - C - E - H$$
 (4"hops")

Path 2: 
$$A - B - C - F - E - H$$
 (5 "hops")

**Path length:** Number of steps it contains from beginning to end.

Path I has length 4, Path 2 has length 5





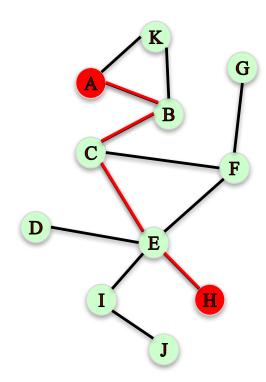
**Distance between two nodes**: the length of the shortest path between them.

The distance between node A and H is 4

In: nx.shortest\_path(G,'A', 'H')
Out: ['A', 'B', 'C', 'E', 'H']

In: nx.shortest\_path\_length(G,'A', 'H')

Out: 4

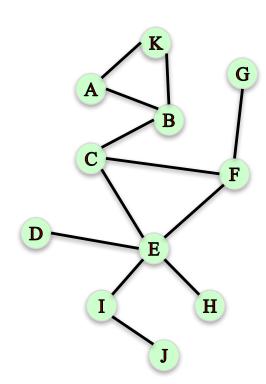




Finding the distance from node A to every other node.

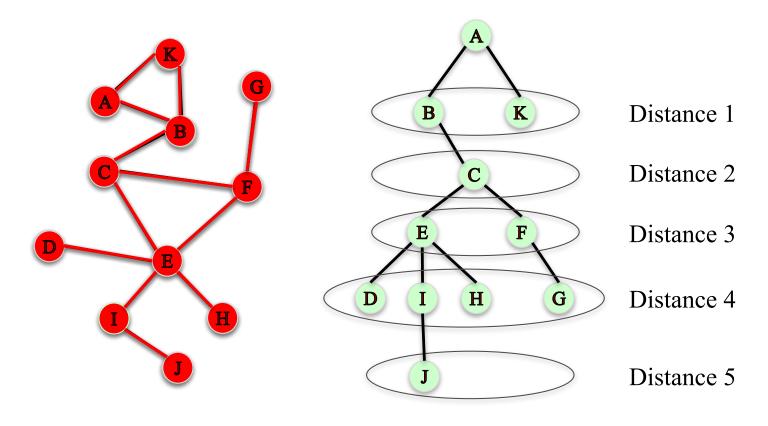
Easy to do manually in small networks but tedious in large (real) networks.

**Breadth-first search:** a systematic and efficient procedure for computing distances from a node to all other nodes in a large network by "discovering" nodes in layers.





# **Breadth-First Search**





# **Breadth-First Search**

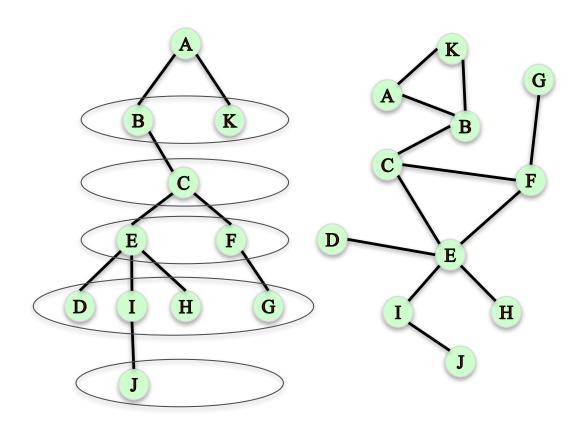
In:  $T = nx.bfs\_tree(G, 'A')$ 

In: T.edges()

Out: [('A', 'K'), ('A', 'B'), ('B', 'C'), ('C', 'E'), ('C', 'F'), ('E', 'I'), ('E', 'H'), ('E', 'D'), ('F', 'G'), ('I', 'J')]

In: nx.shortest\_path\_length(G,'A')

Out: {'A': 0, 'B': 1, 'C': 2, 'D': 4, 'E': 3, 'F': 3, 'G': 4, 'H': 4, 'I': 4, 'J': 5, 'K': 1}



### Distance Measures

How to characterize the distance between all pairs of nodes in a graph?

Average distance between every pair of nodes.

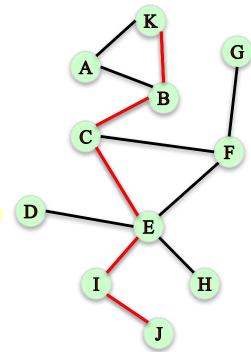
In: nx.average\_shortest\_path\_length(G)

Out: 2.52727272727

**Diameter:** maximum distance between any pair of nodes.

In: nx.diameter(G)

Out: 5





### Distance Measures

How to summarize the distances between all pairs of nodes in a graph?

The **Eccentricity** of a node n is the largest distance between n and all other nodes.

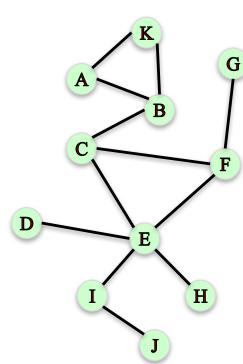
In: nx.eccentricity(G)

Out: {'A': 5, 'B': 4, 'C': 3, 'D': 4, 'E': 3, 'F': 3, 'G': 4, 'H': 4, 'I': 4, 'J': 5, 'K': 5}

The **radius** of a graph is the minimum eccentricity.

In: nx.radius(G)

Out: 3





# Distance Measures

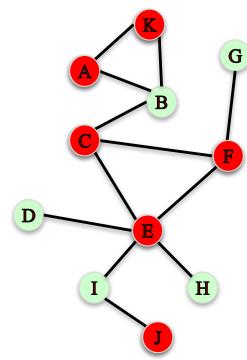
How to summarize the distances between all pairs of nodes in a graph?

The **Periphery** of a graph is the set of nodes that have eccentricity equal to the diameter.

In: nx.periphery(G)
Out: ['A', 'K', 'J']

The **center** of a graph is the set of nodes that have eccentricity equal to the radius.

In: nx.center(G)
Out: ['C', 'E', 'F']



# Karate Club Network

G = nx.karate\_club\_graph()

G = nx.convert\_node\_labels\_to\_integers(G,first\_label=1)

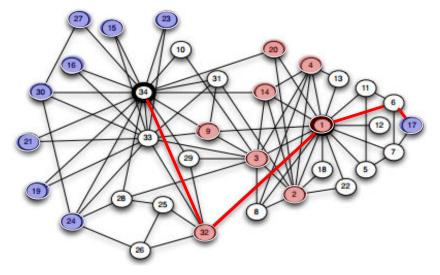
Average shortest path = 2.41

Radius = 3

Diameter = 5

Center = [1, 2, 3, 4, 9, 14, 20, 32]

Periphery: [15, 16, 17, 19, 21, 23, 24, 27, 30]



Friendship network in a 34-person karate club

Node 34 looks pretty "central". However, it has distance 4 to node 17

# Summary

**Distance between two nodes:** length of the shortest path between them. **Eccentricity** of a node n is the largest distance between n and all other nodes.

#### Characterizing distances in a network:

Average distance between every pair of nodes.

**Diameter:** maximum distance between any pair of nodes.

**Radius:** the minimum eccentricity in the graph.

#### Identifying central and peripheral nodes:

The **Periphery** is the set of nodes with eccentricity = diameter.

The **center** is the set of nodes with eccentricity = radius.