Physically-based Modelling

Forces

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Force

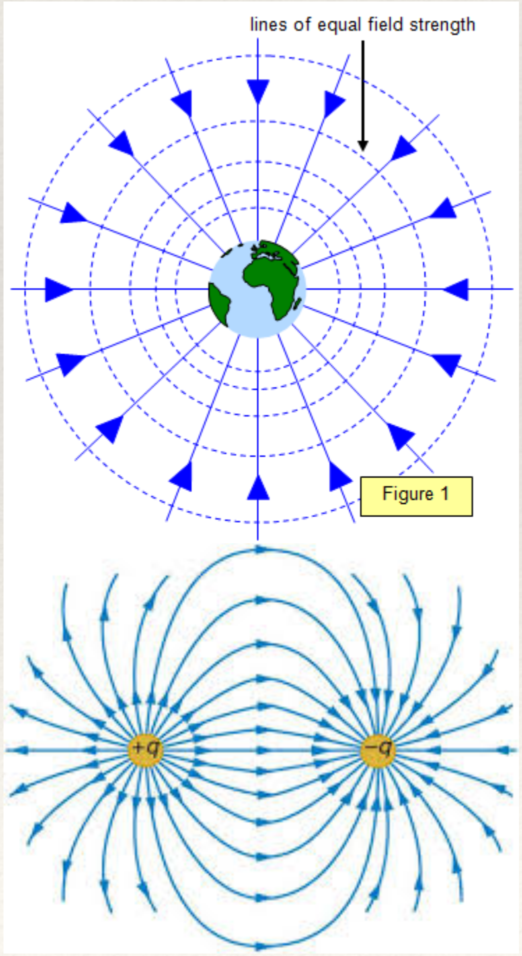
- Limitation of kinematics
  - difficult to handle “various changing forces”
- Dynamics
  - force-based understanding of motions
- What is force?
  - something that causes motion

Concepts

- Force field : example - gravitational force
- Friction: a contact force that resists motion
- Fluid dynamic drag: resistance to objects moving in fluid
- Pressure: force per area
- Buoyancy: force pushing objects “upwards” when immersed in a fluid
- Springs and dampers: elastically ties objects
- Force applies torque to an object, and make it “rotate.”

Force field

- Force field
  - a vector field indicating the force exerted by on object on another
- Very good example
  - gravitational force field
  - electromagnetic force field



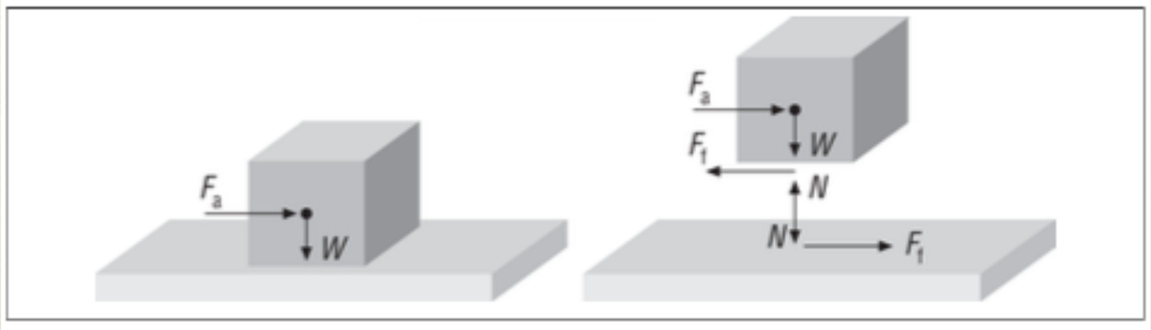
Gravitational force field

- universal gravitational force $|\mathbf{f}_u| = Gm_1m_2/r^2$
  - G: gravitation constance $6.673 \times 10^{-11}(N \cdot m^2)/kg^2$
  - r: distance between masses
  - m_{1,2}: mass of each objects
- gravity on Earth
  - mass of Earth: $5.98 \times 10^{24}kg$
  - radius of Earth: 6.38×10^6m
  - gravitational acceleration

$$\frac{Gm_{earth}}{r^2} \simeq (\frac{6.673 \times 5.98}{6.38^2}) \times 10m/s^2 \simeq 9.8034m/s^2$$

friction

- resistance force due to the contacting surfaces
    - contact force
    - normal force: N is important



- two friction
    - static friction: maximum friction

$$|\mathbf{f}_{max}| = \mu_s \mathbf{N}$$

    - kinetic friction

$$|\mathbf{f}_k| = \mu_k \mathbf{N}$$

Coefficients of friction

- Well known surfaces

Surface condition	M_s	M_u	% difference
Dry glass on glass	0.94	0.4	54%
Dry iron on iron	1.1	0.15	86%
Dry rubber on pavement	0.55	0.4	27%
Dry steel on steel	0.78	0.42	46%
Dry Teflon on Teflon	0.04	0.04	—
Dry wood on wood	0.38	0.2	47%
Ice on ice	0.1	0.03	70%
Oiled steel on steel	0.10	0.08	20%

Fluid dynamic drag

- similar to friction
    - friction is also important component of drag
    - but, it's not the only one
- viscous drag for “slow-moving” objects (laminar)
    - $f = -C\, v$
- for “fast-moving” objects (turbulence)
    - $f = -C\, v^2$

Pressure

- Pressure is not force
  - pressure = force per unit area
  - $F = PA$ (force = pressure x area)
  - $P = F / A$
- Pressure is important in simulating
  - boats, hovercrafts,…

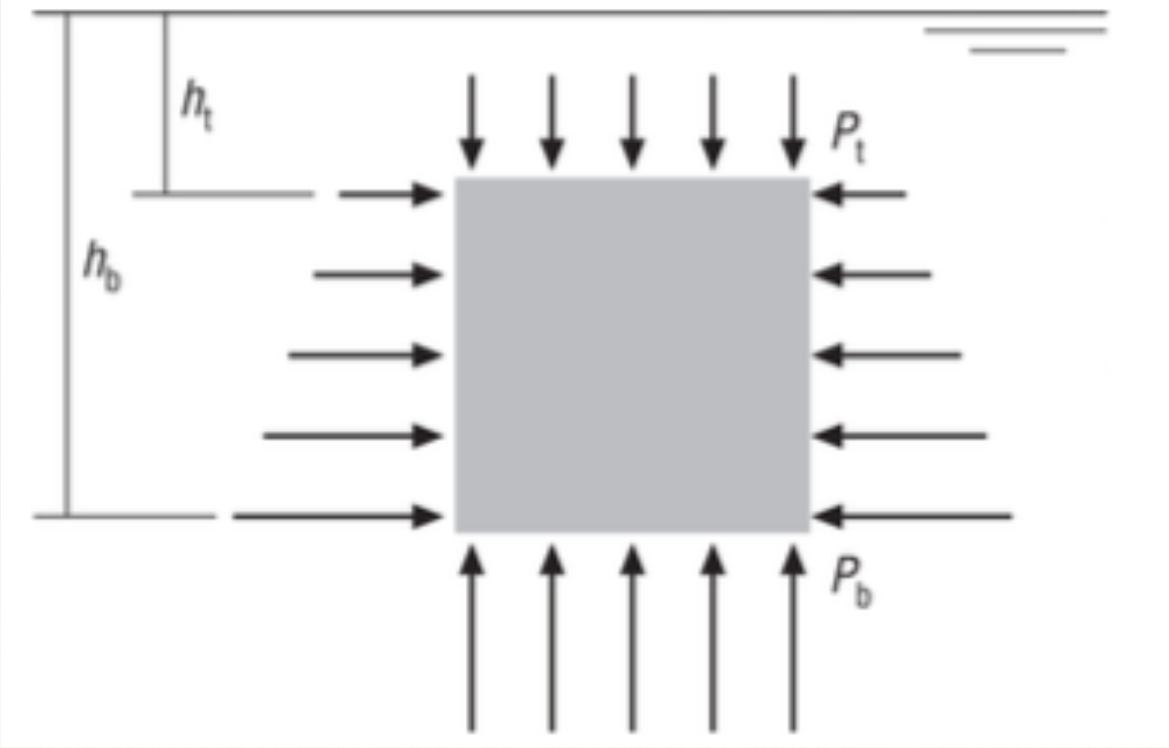
Buoyancy

- Different pressure in fluid
- Horizontal net force = 0
- Vertical net force = bottom force - top force
- F = PA
- Pressures: function(density, gravity)
  - on top surface

$$\mathbf{P}_t = \rho \mathbf{g} h_t$$

  - on bottom surface

$$\mathbf{P}_b = \rho \mathbf{g} h_b$$



Buoyancy

- forces $$\mathbf{f}_t = \mathbf{P}_t A_t = \rho \mathbf{g} h_t s^2$$
$$\mathbf{f}_b = \mathbf{P}_b A_b = \rho \mathbf{g} h_b s^2$$

- difference

$$\begin{aligned}
\mathbf{f}_b - \mathbf{f}_t &= \rho \mathbf{g} h_b s^2 - \rho \mathbf{g} h_t s^2 \\
&= \rho \mathbf{g} (h_b - h_t) s^2 \\
&= -\rho \mathbf{g} s^3 \\
&= -\rho \mathbf{g} V \quad (V : volume)
\end{aligned}$$

Spring force

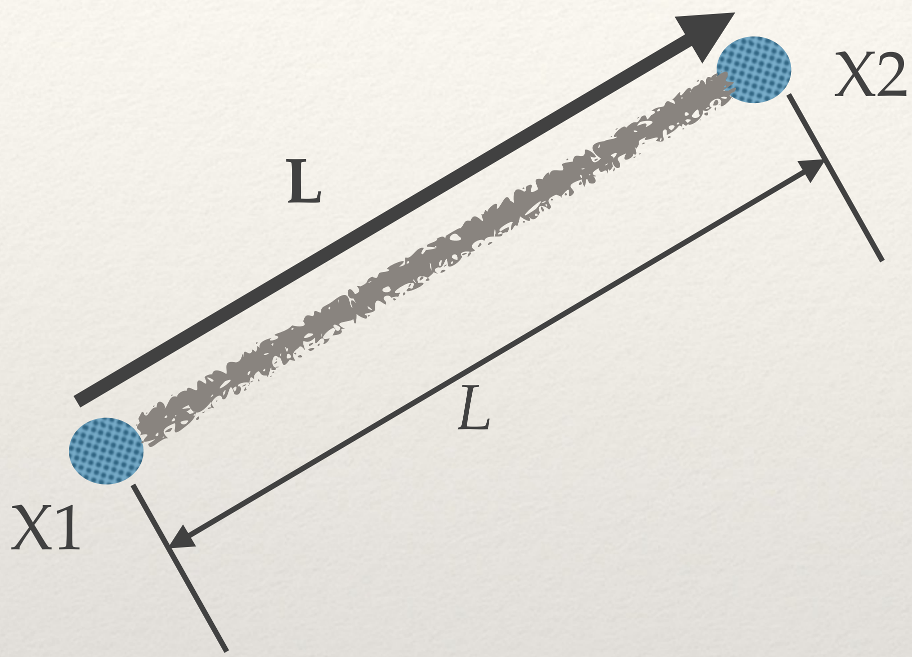
- Hooke's law
  - force needed to extend or compress a spring by a distance x is proportional to that distance
    - f = -k x
    - k: spring constant
- rest length of spring:
- current length of spring:
  - force magnitude: $|\mathbf{f}| = k_s(L - r)$
  - force direction: obj1 and obj2 are linked and located at x1 and x2.
    - $\dfrac{\mathbf{x}_1 - \mathbf{x}_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$ and $-\dfrac{\mathbf{x}_1 - \mathbf{x}_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$

Damper

- Springs do not oscillate forever
  - energy dissipation
  - simple model
    - damping force

$$\mathbf{f}_d = k_d(\mathbf{v}_1 - \mathbf{v}_2)$$

Spring and Damper



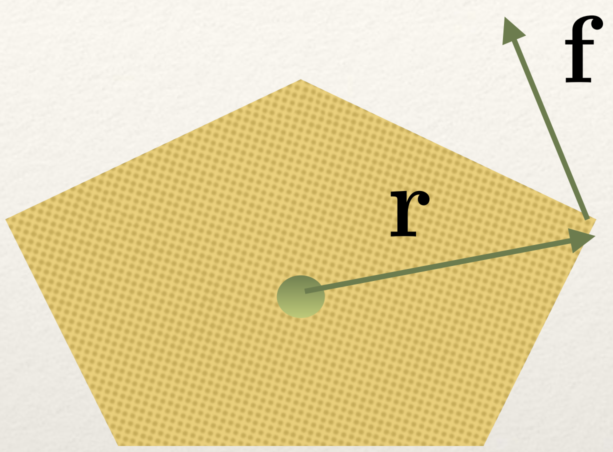
$$\mathbf{f}_1 = -(k_s(L - r) + k_d(\mathbf{v}_1 - \mathbf{v}_2) \cdot \frac{\mathbf{L}}{L})\frac{\mathbf{L}}{L}$$

$$\mathbf{f}_2 = -\mathbf{f}_1$$

Force and Torque

Force and Torque

- Force
  - causes linear acceleration
- Torque
  - causes angular acceleration
- Torque: τ
  - a vector
    - magnitude
      - how quickly the angular velocity is changed
        - $|\mathbf{r} \times \mathbf{f}|$
    - direction
      - rotational axis = $(\mathbf{r} \times \mathbf{f}) / |\mathbf{r} \times \mathbf{f}|$



$$\tau = \mathbf{r} \times \mathbf{f}$$