Physically-based Modelling

Kiematics

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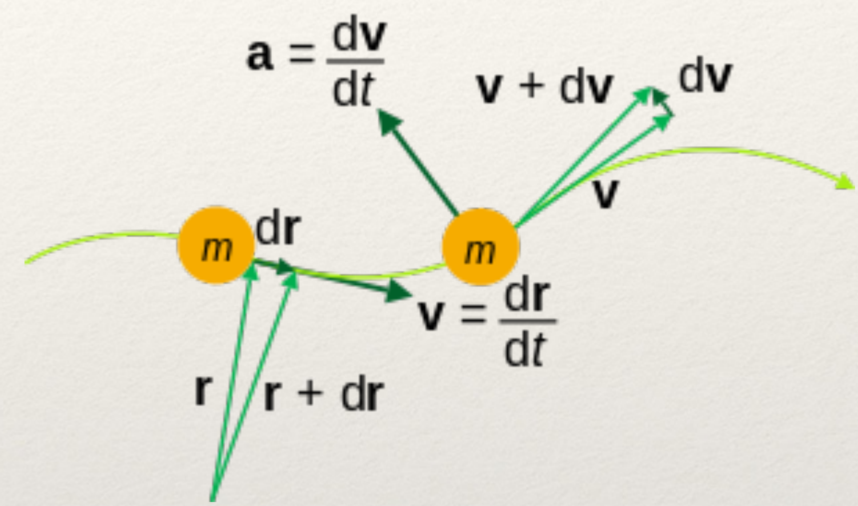
Kinematics

- Kinematics vs. Dynamics
  - kinematics
    - force is not considered
    - location, velocity, and acceleration are considered as
      - functions of time
      - x(t), v(t), a(t)
  - dynamics
    - force plays the most essential role
      - compute f(t)
      - a(t) = f(t)/m
      - v(t+dt) += a(t)dt
      - x(t+dt) += v(t+dt)dt



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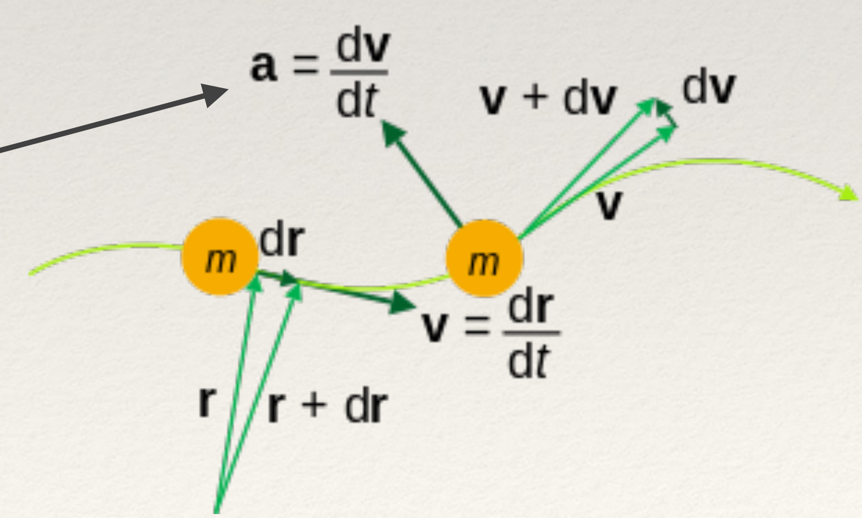


Statics, Kinetics, Kinematics and Dynamics

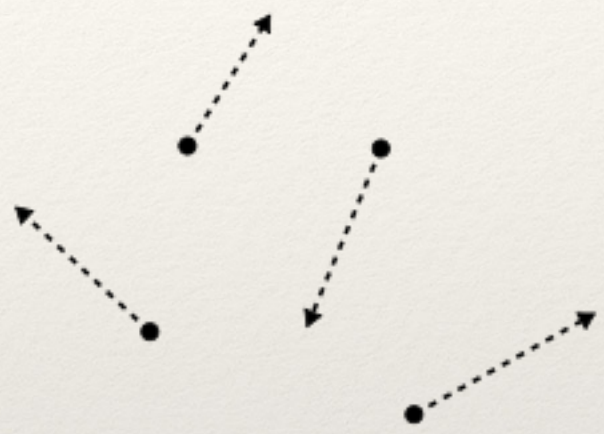
- Statics
  - the study of equilibrium and its relation to **forces**
- Kinetics
  - the study of motion and its relation to **forces**
- Kinematics
  - the study of observed motions without regard for circumstances causing them

classical mechanics

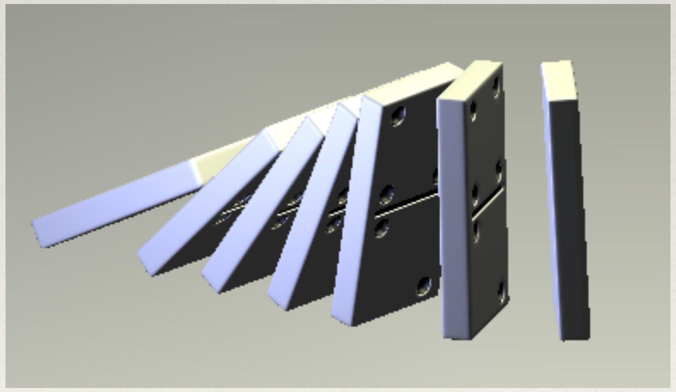
- Dynamics
  - Statics + Kinetics

Particle and Rigid Body

- Particle
  - very very tiny object with mass
    - volume can be ignored
  - rocket trajectory analysis
    - the volume of the rocket can be ignored
    - rocket can be regarded a particle



- Rigid body (= idealised solid body)
  - object with mass and volume
  - the shape does not change in any condition
  - valid animation of rigid body = rotation and translation



Velocity

- velocity
  - a vector
    - speed: the magnitude of velocity
    - velocity = (speed + direction)
  - direction of velocity
    - moving direction
  - magnitude of velocity
    - ratio of moving distance to time

$$\mathbf{v} = \frac{\Delta \mathbf{s}}{\Delta t}$$

ratio of displacement to time interval

Instantaneous Velocity

- Reduce the time interval
  - the more precise velocity at the moment
  - if the time interval approaches to 0
    - instantaneous velocity

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{s}}{\Delta t} = \frac{d\mathbf{s}}{dt}$$

displacement

- integration of velocity

$$\int \mathbf{v}dt = \int d\mathbf{s}$$

- integration from t1 to t2

$$\int_{t_1}^{t_2} \mathbf{v}dt = \int_{s(t_1)}^{s(t_2)} d\mathbf{s}$$

- equivalent to the displacement

$$\int_{t_1}^{t_2} \mathbf{v}dt = \mathbf{s}(t_2) - \mathbf{s}(t_1) = \Delta\mathbf{s}$$

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{s}}{\Delta t} = \frac{d\mathbf{s}}{dt}$$

$$\mathbf{v}dt = d\mathbf{s}$$

- If we know the velocity
  - we can predict where the particle will be located in the future

Acceleration

- Average acceleration
  - ratio of velocity change to the time interval

$$\mathbf{a} = \Delta\mathbf{v}/\Delta t$$

- Instantaneous acceleration

$$\mathbf{a} = \lim_{\Delta t \to 0} \Delta\mathbf{v}/\Delta t = d\mathbf{v}/dt$$

- Integration of acceleration $\quad \mathbf{a}dt = d\mathbf{v}$
  - velocity change

$$\int_{t_1}^{t_2} \mathbf{a}dt = \int_{\mathbf{v}(t_1)}^{\mathbf{v}(t_2)} d\mathbf{v} = \Delta\mathbf{v}$$

Constant Acceleration

- Simple problem for Kinematics
  - example of constant acceleration: gravity
  - gravitational acceleration
    - magnitude: 9.81 m/s^2
    - direction: downwards (0,-1,0)
- We can easily "integrate the acceleration" to compute
  - velocity change at every time
  - this makes it possible to know the velocity at every time: v(t)

Gravitational Acceleration

- gravitational acceleration: g
  - velocity change and the integration of acceleration

$$\int_{\mathbf{v}(t_1)}^{\mathbf{v}(t_2)} d\mathbf{v} = \int_{t_1}^{t_2} \mathbf{g} dt$$

$$\mathbf{v}_2 - \mathbf{v}_1 = \mathbf{g}(t_2 - t_1)$$

  - If we know every thing at time t_1
    - We can easily predict the velocity at time t_2

$$\mathbf{v}_2 = \mathbf{g}t_2 - \mathbf{g}t_1 + \mathbf{v}_1$$

    - if $t_1=0$ ($t_2=t$)

$$\mathbf{v}_2 = \mathbf{v}_1 + \mathbf{g}t$$

Velocity as f(Displacement)

- Another differential equation

$$\mathbf{v}\frac{d\mathbf{v}}{dt} = \frac{d\mathbf{s}}{dt}\mathbf{a}$$

$$\mathbf{v}d\mathbf{v} = \mathbf{a}d\mathbf{s}$$

- Integration $\displaystyle\int_{\mathbf{v}_1}^{\mathbf{v}_2} \mathbf{v}d\mathbf{v} = \int_{\mathbf{s}_1}^{\mathbf{s}_2} \mathbf{a}d\mathbf{s}$

$$\frac{1}{2}\mathbf{v}^2\Big|_{\mathbf{v}_1}^{\mathbf{v}_2} = \mathbf{as}\Big|_{\mathbf{s}_1}^{\mathbf{s}_2} = \mathbf{gs}\Big|_{\mathbf{s}_1}^{\mathbf{s}_2}$$

- Relation between velocity and displacement

$$\frac{1}{2}({\mathbf{v}_2}^2 - {\mathbf{v}_1}^2) = \mathbf{g}(\mathbf{s}_2 - \mathbf{s}_1)$$

$${\mathbf{v}_2}^2 = 2\mathbf{g}(\mathbf{s}_2 - \mathbf{s}_1) + {\mathbf{v}_1}^2$$

Location

- Velocity and displacement

$$\mathbf{v}dt = d\mathbf{s}$$
$$(\mathbf{v}_1 + \mathbf{g}t)dt = d\mathbf{s}$$

- Integration

$$\int_0^t (\mathbf{v}_1 + \mathbf{g}t)dt = \int_{\mathbf{s}_1}^{\mathbf{s}_2} d\mathbf{s}$$
$$v_1 t + \frac{1}{2}\mathbf{g}t^2 = \mathbf{s}_2 - \mathbf{s}_1$$

- Location at time t

$$\mathbf{s}_2 = v_1 t + \frac{1}{2}\mathbf{g}t^2 + \mathbf{s}_1$$