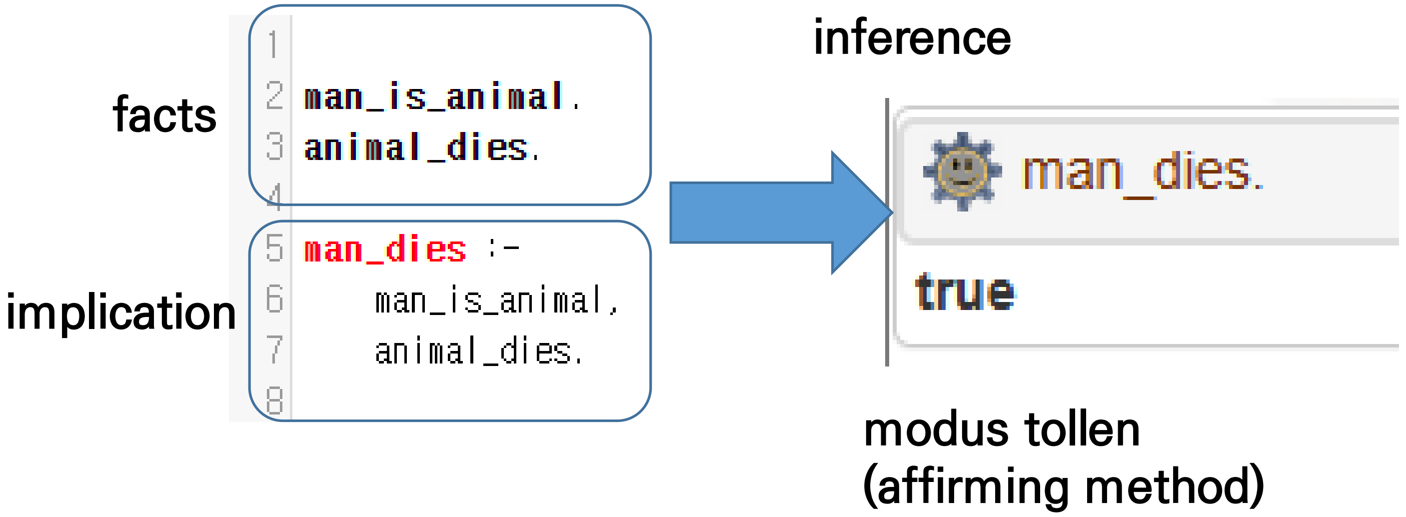
이산수학 (1~4강)
배운 거 활용해 보기

동명대학교 게임공학과

2024. 1학기

연역법

- 연역: 주어진 사실과 고리에 입각하여 추론을 통해 새로운 사실을 도출
- Prolog
  - Modus pones (긍정 법칙) 예시



p: man is animal

q: animal dies

r: man dies

$(p \wedge q) \rightarrow r$

$s = (p \wedge q)$

$s \rightarrow r$

s is true, therefore r is true (modus pones)

연역법

- Modus tollen (부정 법칙)

```
bird(sparrow).
bird(macpie).
bird(seagull).

insect(beatle).

mammal(human).
mammal(elephant).
mammal(bat).

havewings(beatle).
havewings(bat).


flyingAnimal(X) :-
    (   (insect(X); mammal(X)), havewings(X) );
    bird(X).

flies(X) :-
    bird(X);
    flyingAnimal(X).
```

modus tollen
(denying method)

```
not_bird(X):-
    \+ flies(X).
```

연역법

- Modus tollen (부정 법칙)

```
flyingAnimal(X).
X = beatle
X = bat

flyingAnimal(X).
X = beatle
X = bat
X = sparrow
X = macpie
X = seagull

bird(X).
X = sparrow
X = macpie
X = seagull

not_bird(human).
true
```

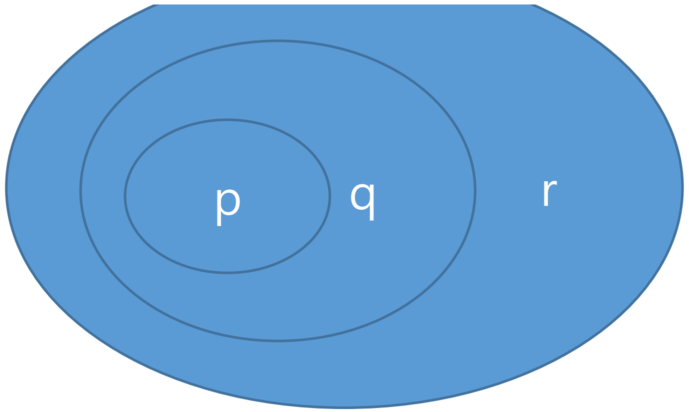
Modus tollen에 의해 확정적으로 새가 아니라고 하는 것

```
not_bird(bat).
false
```

새라는 의미가 아니고 확정적으로 새가 아니라 할 수 없다는 것

연역법

$$\begin{pmatrix} p \to q \\ q \to r \end{pmatrix} \Longrightarrow p \to r$$

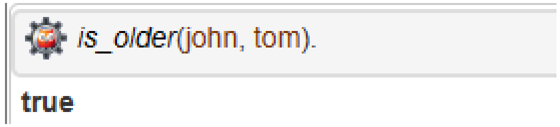


- 삼단 논법(Hypothetical syllogism – 조건 추론)

```
% Facts
is_parent(john, mary).
is_parent(mary, tom).
```

John is older than mary
Mary is older than tom

```
% If X is a parent of Y, then X is older than Y.
is_older(X, Y) :- is_parent(X, Y).

% If X is older than Y and Y is older than Z, then X is older than Z.
is_older(X, Z) :- is_older(X, Y), is_older(Y, Z).
```



연역법

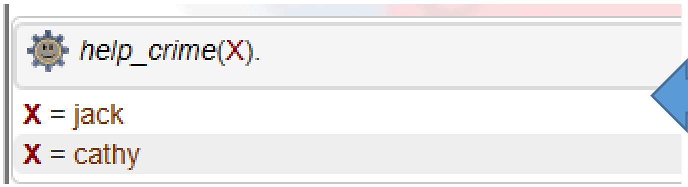
OR　　　　양자택일

- 선언 삼단 논법(disjunctive dilemma)

$$\begin{pmatrix} p \vee q \\ \neg p \end{pmatrix} \Longrightarrow q$$

유죄는 범죄자(p) 혹은 방조자(q)

누군가가 유죄라면 p v q
범죄자는 아니다 ~p
→ 방조자일 수밖에 없다.

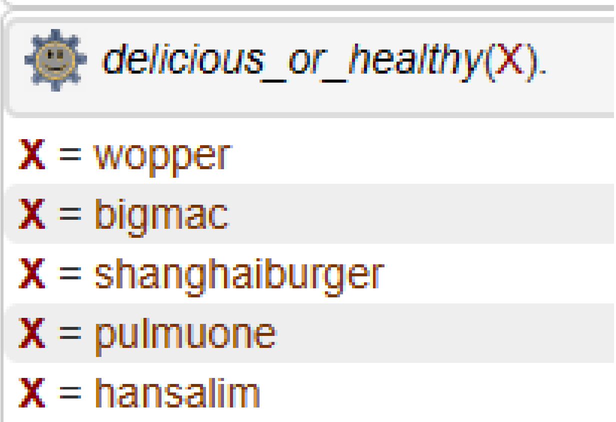



```
commit_crime(john).
commit_crime(bob).
commit_crime(susan).
commit_crime(sue).

sentenced_guilty(john).
sentenced_guilty(susan).
sentenced_guilty(jack).
sentenced_guilty(cathy).

criminal(X) :-
    commit_crime(X).

guilty(X) :-
    % 유죄의 조건
    criminal(X);  sentenced_guilty(X).

help_crime(X) :-
    guilty(X), \+ criminal(X).
```

연역법

- 양도 법칙 (constructive dilemma)

$$\begin{pmatrix} (p \to q) \land (r \to s) \\ p \lor r \end{pmatrix} \implies q \lor s$$




```
kind_of_hamburger(wopper).
kind_of_hamburger(bigmac).
kind_of_hamburger(shanghaiburger).

kind_of_pizza(mrpizza).
kind_of_pizza(domino).
kind_of_pizza(pizzanara).

organic(pulmuone).
organic(hansalim).

nonorganic(shinramyon).

food(X) :-
   kind_of_hamburger(X); kind_of_pizza(X); organic(X); nonorganic(X).


delicious(X) :-
   kind_of_hamburger(X).

healthy(X) :-
   organic(X).

delicious_or_healthy(X) :-
    kind_of_hamburger(X); organic(X) .
```

연역법

- 파괴적 법칙 (destructive dilemma)

$$\begin{pmatrix} (p \to q) \land (r \to s) \\ \neg q \lor \neg s \end{pmatrix} \implies \neg p \lor \neg s$$

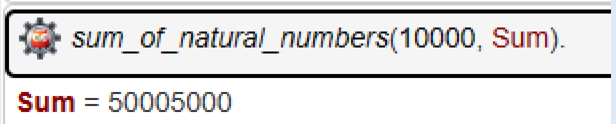
```
kind_of_hamburger(wopper).
kind_of_hamburger(bigmac).
kind_of_hamburger(organicburger).
kind_of_hamburger(shanghaiburger).

kind_of_pizza(mrpizza).
kind_of_pizza(domino).
kind_of_pizza(pizzanara).

organic(pulmuone).
organic(hansalim).
organic(organicburger).

nonorganic(shinramyon).

food(X) :-
   kind_of_hamburger(X); kind_of_pizza(X); organic(X).

delicious(X) :-
   kind_of_hamburger(X).

healthy(X) :-
   organic(X).

not_delicious_or_not_healthy(X) :-
   food(X), (   \+ kind_of_hamburger(X) ; \+ organic(X)).
```

```
not_delicious_or_not_healthy(X).

X = wopper
X = bigmac
X = shanghaiburger
X = mrpizza
X = mrpizza
X = domino
X = domino
X = pizzanara
X = pizzanara
X = pulmuone
X = hansalim
```

수학적 귀납법

- 수학적 귀납법으로 증명되는 명제는 점화식으로 표현 가능
  - 점화식
    - 1에서 n까지 자연수의 합 $A(n) = A(n-1) + n$
    - $n(n+1)/2$
      - $n(n+1)/2 = (n-1)n/2 + n = (n2+n) / 2 = n(n+1)/2$

```
% Base case
sum_of_natural_numbers(1, 1).

% Inductive step
sum_of_natural_numbers(N, Sum) :-
    N > 1,
    Prev is N - 1,
    sum_of_natural_numbers(Prev, PrevSum),
    Sum is PrevSum + N.
```



수학적 귀납법

- 피보나치 수열
  - 점화식
    - F(0), F(1) = 0, 1
    - F(n) = F(n-1) + F(n-2)

- 팩토리얼
  - 점화식
    - 0! = 1
    - n! = (n-1)! * n

```
sum_of_natural_numbers(10000, Sum).
Sum = 50005000
```

대우 증명법 (contrapositive proof)

- parent(x,y) → ischild(y, x)

Contrapositive rule

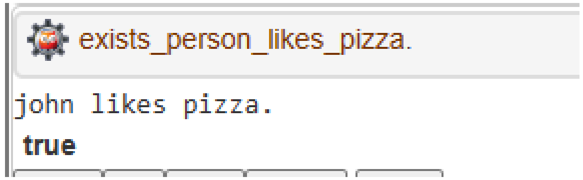
~ ischild(y,x) → ~parent(x,y)

```
% Facts
parent(john, alice).
parent(john, bob).
parent(jack, mary).

% Rule
% If Y is a child of X, then X is a parent of Y.
is_child(Y, X) :- parent(X, Y).

% Contrapositive Rule
% If Y is not a child of X, then X is not a parent of Y.
not_parent_of(X, Y) :- \+ is_child(Y, X).

% Query
% Is it true that John is not a parent of Mary?
```

존재 증명법 (existence proof)

- Prolog에서는 조건을 만족하는 값을 찾는 일

```
% Facts
likes(john, pizza).
likes(mary, sushi).
likes(alex, pizza).

% Rule
% To prove there exists a person who likes pizza.
exists_person_likes_pizza :-
    likes(Person, pizza),
    format('~w likes pizza.~n', [Person]).

% Query
% Is there a person who likes pizza?
```



집합을 다루는 prolog 코드

```prolog
% Define sets
set(a, [1, 2, 3, 4, 5]).
set(b, [4, 5, 6, 7, 8]).

% Union of two sets
union_set(Set1, Set2, Union) :-
    findall(X, (member(X, Set1); member(X, Set2)), Union).

% Intersection of two sets
intersection_set([], _, []).
intersection_set([X|Set1], Set2, [X|Intersection]) :-
    member(X, Set2),
    !,
    intersection_set(Set1, Set2, Intersection).
intersection_set([_|Set1], Set2, Intersection) :-
    intersection_set(Set1, Set2, Intersection).

% Difference of two sets
difference_set([], _, []).
difference_set([X|Set1], Set2, [X|Difference]) :-
    \+ member(X, Set2),
    !,
    difference_set(Set1, Set2, Difference).
difference_set([_|Set1], Set2, Difference) :-
    difference_set(Set1, Set2, Difference).
```

```
?- set(a, SetA), set(b, SetB),
   union_set(SetA, SetB, Union),
   intersection_set(SetA, SetB, Intersection),
   difference_set(SetA, SetB, Difference).
```

```
set(a, SetA), set(b, SetB), union_set(SetA, SetB, Union), intersection_set(SetA, SetB, Intersection), difference_set(SetA, SetB, Difference).
Difference = [1, 2, 3],
Intersection = [4, 5],
SetA = [1, 2, 3, 4, 5],
SetB = [4, 5, 6, 7, 8],
Union = [1, 2, 3, 4, 5, 4, 5, 6, 7, 8]
```