Physically-based Modelling

Collision

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Impulse

- Impulse
  - force that acts over very short time
  - examples
    - force exerted on a bullet when fired from a gun
    - force between colliding objects
- Physical meaning
  - impulse: vector quantity equal to the change in momentum
    - linear impulse
      - $= m(\mathbf{v}_+ - \mathbf{v}_-)$
    - angular impulse
      - $= \mathbf{I}(\omega_+ - \omega_-)$

Impulse Example

- bullet fired from a gun
  - mass of the bullet: 0.15 kg
  - bullet speed at the muzzle: 756 m/s
  - the length of barrel: 0.610 m
  - time for bullet to go through the barrel: 0.0008 s
  - impulse = change of momentum
    - mv = 0.15*756 kg m/s = 113.4 kgm/s
  - average impulse force = impulse / time
    - 113.4 / 0.00008 N = 141,750 N

Conservation of Momentum

- two objects (with masses m1 and m2) are colliding
  - velocities before the collision v-
  - velocities after the collision v+

$$m_1\mathbf{v}_1^+ + m_2\mathbf{v}_2^+ = m_1\mathbf{v}_1^- + m_2\mathbf{v}_2^-$$

Conservation of Kinetic Energy

- Linear Kinetic Energy

$$K_l = \frac{1}{2} m |\mathbf{v}|^2$$

- Angular Kinetic Energy

$$K_a = \frac{1}{2} I |\omega|^2$$

- If the energy is conserved…

$$m_1 \mathbf{v}_1^{+\,2} + m_2 \mathbf{v}_2^{+\,2} = m_1 \mathbf{v}_1^{-\,2} + m_2 \mathbf{v}_2^{-\,2}$$

Colliding Objects

- Conservation of Momentum

$$m_1 \mathbf{v}_1^+ + m_2 \mathbf{v}_2^+ = m_1 \mathbf{v}_1^- + m_2 \mathbf{v}_2^-$$

$$\boxed{m_1(\mathbf{v}_1^+ - \mathbf{v}_1^-) = -m_2(\mathbf{v}_2^+ - \mathbf{v}_2^-)}$$

- Conservation of Energy

$$m_1 {\mathbf{v}_1^+}^2 + m_2 {\mathbf{v}_2^+}^2 = m_1 {\mathbf{v}_1^-}^2 + m_2 {\mathbf{v}_2^-}^2$$

$$m_1({\mathbf{v}_1^+}^2 - {\mathbf{v}_1^-}^2) = -m_2({\mathbf{v}_2^+}^2 - {\mathbf{v}_2^-}^2)$$

$$m_1(\mathbf{v}_1^+ - \mathbf{v}_1^-)(\mathbf{v}_1^+ + \mathbf{v}_1^-) = -m_2(\mathbf{v}_2^+ - \mathbf{v}_2^-)(\mathbf{v}_2^+ + \mathbf{v}_2^-)$$

$$\boxed{\mathbf{v}_1^+ + \mathbf{v}_1^- = \mathbf{v}_2^+ + \mathbf{v}_2^-}$$

Find new velocities

$$\boxed{\begin{aligned} m_1(\mathbf{v}_1^+ - \mathbf{v}_1^-) &= -m_2(\mathbf{v}_2^+ - \mathbf{v}_2^-) \\ \mathbf{v}_1^+ + \mathbf{v}_1^- &= \mathbf{v}_2^+ + \mathbf{v}_2^- \end{aligned}}$$

$$\boxed{\begin{aligned} m_1\mathbf{v}_1^+ + m_2\mathbf{v}_2^+ &= m_1\mathbf{v}_1^- + m_2\mathbf{v}_1^- \\ \mathbf{v}_1^+ - \mathbf{v}_2^+ &= \mathbf{v}_2^- - \mathbf{v}_1^- \end{aligned}}$$

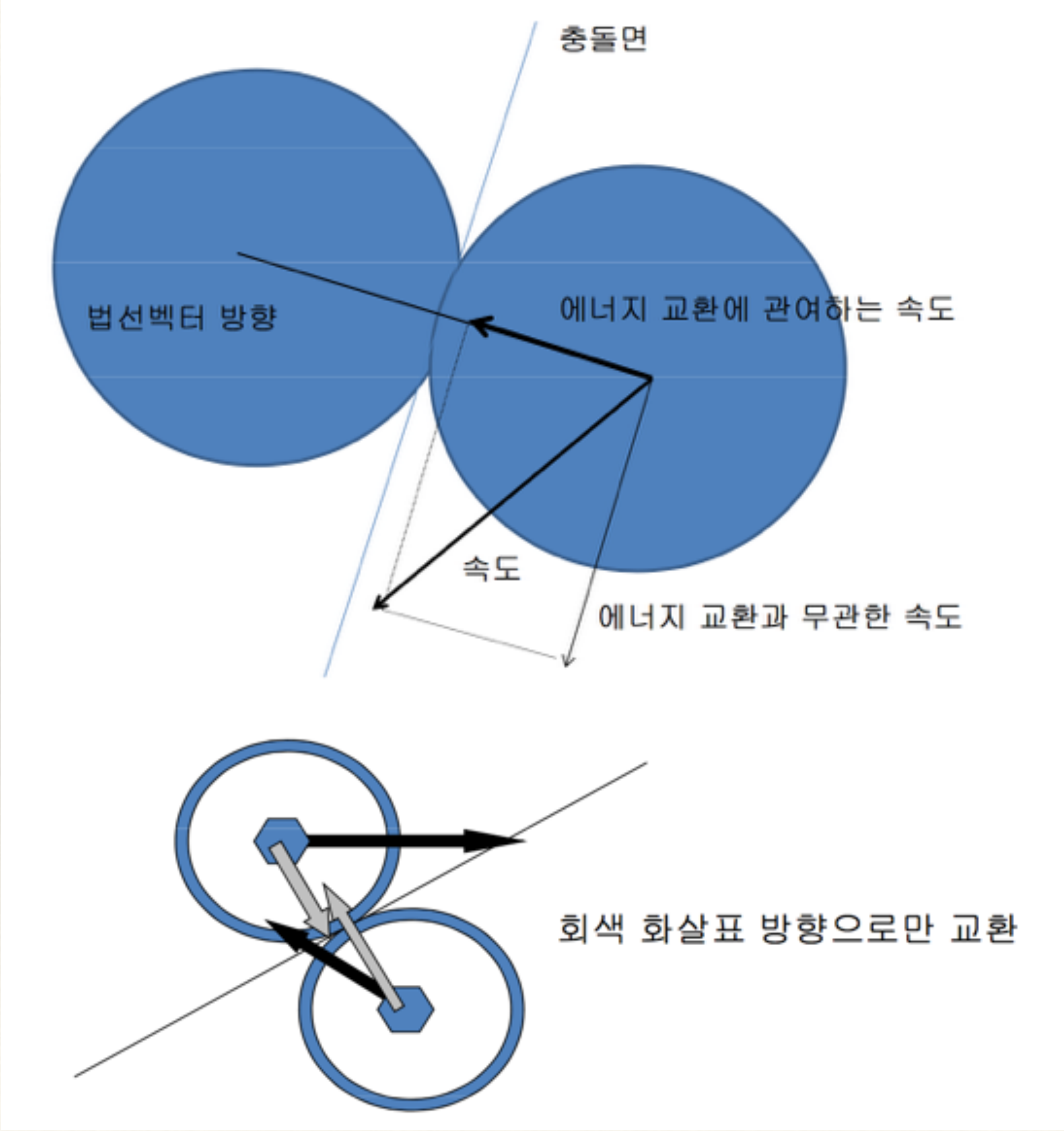
$$\mathbf{v}_1^+ = \frac{(m_1 - m_2)\mathbf{v}_1^- + 2m_2\mathbf{v}_2^-}{m_1 + m_2}$$

$$\mathbf{v}_2^+ = \frac{2m_1\mathbf{v}_1^- + (m_2 - m1)\mathbf{v}_2^-}{m_1 + m_2}$$

Velocity Change

- Direct impact vs. Oblique impact
  - direct impact
    - use the previous equation
  - oblique impact
    - line of action of impact
    - velocity along this line should be changed

Velocity Change



Line of Action of Impact

- Centres of particles
  - p1, p2
- Direction of the “line of action of impact”
  - $N = (p1-p2) / |p1-p2|$
- Collision Detection
  - radius of the particles
    - r1, r2
    - $|p1-p2| < r1+r2$

Velocity along the line of action

- The velocity along the line before the collision

$$v_1^- = \mathbf{v}_1 \cdot \mathbf{N}$$

$$v_2^- = \mathbf{v}_2 \cdot \mathbf{N}$$

- Collision handing
  - only when the objects are approaching to each other

$$v_2^- - v_1^- > 0$$

Velocity Update

$$v_1^+ = \frac{(m_1 - m_2)v_1^- + 2m_2v_2^-}{m_1 + m_2}$$

$$v_2^+ = \frac{(m_2 - m_1)v_2^- + 2m_1v_1^-}{m_1 + m_2}$$

$$\boxed{\begin{aligned} \mathbf{v}_1 &= \mathbf{v}_1 - v_1^-\mathbf{N} + v_1^+\mathbf{N} \\ \mathbf{v}_2 &= \mathbf{v}_2 - v_2^-\mathbf{N} + v_2^+\mathbf{N} \end{aligned}}$$