Name:

Homework: Math Preliminaries and Notation

CSCI-561

Fall 2021

Instructions: Answer the following questions. You are encouraged to use the provided LaTeX source to edit and submit your answers.

1. Set Builder Notion: Find the elements of the following sets

   (a) $a = \{\, x \in \mathbb{Z} \mid x \text{ is even and between 0 and 5, exclusive} \,\}$

   (b) $a = \{\, (x \in \mathbb{Z}, y \in \mathbb{Z}) \mid (0 \leq x < 2) \wedge (0 \leq y < 2) \,\}$

2. Set Operations

   Given

   - $a = \{1, 2, 3, 5, 7\}$
   - $b = \{1, 3, 5, 7, 9\}$
   - $c = \{2, 4\}$

   Find

   (a) $a \cup b$

   (b) $b \cup a$

   (c) $a \cap b$

   (d) $b \cap a$

   (e) $a \setminus b$

   (f) $b \setminus a$

   (g) $a \times c$

   (h) $c \times a$

   (i) $\mathcal{P}(c) =$

3. Boolean Algebra: Simplify and write in terms of a, b, AND, OR, NOT, 0, and 1.

   (a) $a \vee \neg a$

   (b) $a \wedge \neg a$

   (c) $\neg(a \iff b)$

4. Use a truth table to prove the distributivity or $\vee$ over $\wedge$:
   $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$.

5. Set Algebra: Simplify

   (a) $a \cup a$

   (b) $a \cap \overline{a}$

   (c) $a \cup (a \cap b)$

6. Set Algebra: Always true, always false, or unknown (i.e., depends on the values of a and b).

   (a) $a \setminus b \subseteq a$

(b) $a \cap b \subseteq a$

(c) $a \cup b \subseteq a$

(d) $\{1, 2\} = \{2, 1\}$

(e) $(1, 2) = (2, 1)$

7. Function Notion: Translate the following C function prototypes to mathematical function notation. Assume that the C `int` type corresponds to mathematical integers and the C `float` type corresponds to mathematical reals. *(Hint: You may find the* `cdecl` *program useful to explain the C syntax. See* `man cdecl` *for usage details.)*

   (a) `int f(int, int)`

   (b) `float g(_Bool, float)`

   (c) `int h(int(*)(int,int),int)`

8. Prove by induction that merge-sort is correct, i.e., it returns the sorted sequence.