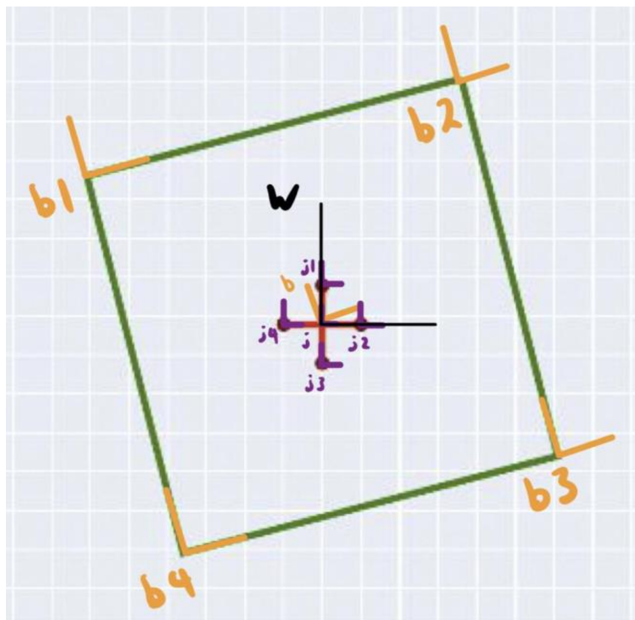
ME314 Final Project Writeup:

1.

For my ME314 Final Project, I chose to do it on the 'default project'.

2.

In the diagram below, you will be able to see the different frames used in the simulation as well as their labels (b1, b2, b3, b4, j1, j2, j3, j4). Additionally, you will be able to see the main intermediary frames/rigid body transformations to get to those frames, namely frames b and j.



These frames are clearly shown in the code via comments.

3.

To find the Euler-Lagrange equations, the first step that needed to be made was to find the body velocities of both objects, this is required since it is less straightforward to determine the kinetic energy of an object with both translational and rotational transformations. This step involves the use of the rigid body transformations mentioned above. Once the body velocities are found, Lagrangian can be calculated with the kinetic and potential energies of the system.

With the Lagrangian calculated, we can use it to set up the left-hand side of our main simultaneous equation so that the entire equation looks like this:

$$\frac{d}{dt}\frac{dL}{d\dot{q}} - \frac{dL}{dq} = \sum Fext$$

You will be see that the right-hand side of the simultaneous equation contains some arbitrary external forces that are exerted on the two objects, in the case of my final it is Fy and Ftheta which keeps the box from accelerating downwards forever due to gravity and gives the box a perpetual rotational motion respectively.

Once both the left-hand and right-hand sides of the simultaneous equations are set up, the forced Euler-Lagrange equations are calculated with the different qddot variables as the subject, q being the configuration variables (x_box, y_box, theta_box, x_jack, y_jack, theta_jack). For further use in the final project, these equations are then lambdified.

After these are solved, I solve for the 16 different impact constraint equations – one for each combination corner/wall of the jack and box that come into contact with each other – by implementing the below equations in the my code and utilizing dummy variables:

$$\boxed{\begin{aligned} \frac{\partial L}{\partial \dot{q}}\bigg|_{\tau^-}^{\tau^+} &= \lambda \frac{\partial \phi}{\partial q} \\ \left[\frac{\partial L}{\partial \dot{q}} \cdot \dot{q} - L(q, \dot{q})\right]_{\tau^-}^{\tau^+} &= 0. \end{aligned}}$$

This is solved by finding the Generalized Momentum and Hamiltonian as seen below:

$$\boxed{\begin{aligned} p\bigg|_{\tau^-}^{\tau^+} &= \lambda \frac{\partial \phi}{\partial q} \\ H\bigg|_{\tau^-}^{\tau^+} &= 0. \end{aligned}}$$

The dummy variables help calculate how impacts affect the value of the configuration variables the instant after impact is made. With all of that these calculations made, I am then able to accurately simulate the jack in a box system.

4.
The success or accuracy of this simulation largely hinges on the interaction between the jack and the walls, and every time the jack collided with a wall the resulting motions aligned with my expectations or basic intuitions. Additionally, other simulated parameters like the jack's downward acceleration due to gravity, appeared to be fairly realistic. The rotations of both the dice and the box can be attributed to rotational inertia and external forces, as anticipated. Given the how much smaller the mass of the dice was compared to the box, the lack of effect on the box’s motion caused by the jack’s impacts were not unexpected.

Simulation Plots: