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Abstract

1 Ornstein-Uhlenbeck approximation

1.1 Introduction

The changes in copy number of the system over time can be approximated as an Ornstein-Uhlenbeck process occurring in a given potential energy landscape. The change in copy number (P) is given by:

$$dP(t) = -\alpha \nabla U(P(t))dt + \sqrt{2D}dW(t) \tag{1}$$

The gradient of the potential energy is defined for a given P according to the expected change in P due to the combined effects of actively controlled births, and deaths occurring with a constant rate:

$$\nabla U(P(t))dt = -P(max[0, Birth + C_b(NSS - P)] - Death)dt \tag{2}$$

Thus the full equation takes the form:

$$dP(t) = \alpha \times P(max[0, Birth + C_b(NSS - P)] - Death)dt + \sqrt{2D}dW(t) \tag{3}$$

1.2 Parameter estimation

As the $Birth$, $Death$, NSS, and C_b parameters are predetermined, simulating with the model requires the inference of the D and α parameters.

1.2.1 Estimating D

The estimation of D is relatively straightforward. The Gillespie algorithm can be use to simulate a high number of replicates of the system with $C_b = 0$. With $C_b = 0$, copy numbers are no longer stabilized, and P reverts to a simpler Brownian motion. For such 1-dimensional Brownian motion, the variance at time t is given by:

$$Var(P(t)) = 2Dt \tag{4}$$

Therefore D can be estimated from the data observed during the Gillespie simulations as the best linear fit (with intercept 0) to the observed variance over time.

$$2D = \frac{\sum_i t_i \times Var(\hat{P}(t))_i}{\sum_i t_i^2} \tag{5}$$

1.2.2 Estimating α

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