Stochastic survival of the densest in neurons

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1 Glossary of model parameters

μ = death rate or degradation rate
γ = transport rate between two units
δ = mutant deficiency ratio
C_b = adaptive birth rate control strength parameter
C_t = adaptive transport rate control strength parameter
NSS_A = carrying capacity of axon
NSS_S = carrying capacity of soma
wt_{soma} = number of wildtype mtDNA molecules in the soma
mt_{soma} = number of mutant mtDNA molecules in the soma
wt_{axon} = number of wildtype mtDNA molecules in the axon
mt_{axon} = number of mutant mtDNA molecules in the axon

2 Equations governing per capita reaction rates in the model

All per capita reaction rates are identical for wildtype (wt) and mutant (mt) mtDNA molecules, and defined as follows:

2.0.1 Birth rates:

mtDNA is produced in the soma only. The rate of production is sufficient to supply both the soma and the axon.

$$Birth_{soma} = 2\mu + C_b \times (Nss_{soma} - wt_{soma} - \delta_m mt_{soma}) \tag{1}$$

$$Birth_{axon} = 0 \tag{2}$$

2.0.2 Death rates:

mtDNA is degraded with equal rates in the soma and axon.

$$Death_{soma} = \mu \tag{3}$$

$$Death_{axon} = \mu \tag{4}$$

2.0.3 Transport rates:

mtDNA is transported between the soma and the axon. At steady state, anterograde (soma to axon) transport is 2x the rate of retrograde transport.

$$Move_{soma-axon} = 2\gamma + C_t \times (Nss_{axon} - wt_{axon} - \delta_m mt_{axon}) \tag{5}$$

$$Move_{axon-soma} = \gamma \tag{6}$$

3 Conditions for stability

A crucial steady state for the system occurs when:

$$wt_{soma} = wt_{axon} \wedge mt_{soma} = mt_{axon} \wedge Nss_{soma} - wt_{soma} - \delta_m mt_{soma} = 0 \wedge Nss_{axon} - wt_{axon} - \delta_m mt_{axon} = 0 \quad (7)$$

In such cases, the adaptive birth rate and transport rate sections are both 0:

$$C_b \times (Nss_{soma} - wt_{soma} - \delta_m mt_{soma}) = 0 \wedge C_t \times (Nss_{axon} - wt_{axon} - \delta_m mt_{axon}) = 0 \quad (8)$$

Thus, the system is reduced such that:

$$Birth_{soma} = 2\mu \quad (9)$$

$$Birth_{axon} = 0 \quad (10)$$

$$Death_{soma} = \mu \quad (11)$$

$$Death_{axon} = \mu \quad (12)$$

$$Move_{soma-axon} = 2\gamma \quad (13)$$

$$Move_{axon-soma} = \gamma \quad (14)$$

Thus the absolute rates of change for the four model variables are:

$$\frac{\partial wt_{soma}}{\partial t} = wt_{soma} \times [2\mu + \gamma - 2\gamma - \mu] \quad (15)$$

$$\frac{\partial mt_{soma}}{\partial t} = mt_{soma} \times [2\mu + \gamma - 2\gamma - \mu] \quad (16)$$

$$\frac{\partial wt_{axon}}{\partial t} = wt_{axon} \times [2\gamma - \gamma - \mu] \quad (17)$$

$$\frac{\partial mt_{axon}}{\partial t} = mt_{axon} \times [2\gamma - \gamma - \mu] \quad (18)$$

These rates of change $= 0$ when $\mu = \gamma$.