ST 518 Project

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Executive Summary

A one or two paragraph summary that includes a description of the experiment, significant results (including any interesting numerical results), and any conclusions you draw. The reader should be able to glean all the important aspects of your work from the executive summary. Effectively and succinctly convey objectives, summary of experimental design, and results and conclusions drawn from experiment.

Introduction

The purpose of this experiment is to investigate specific factors and their effect on the amount of time it takes to dissolve a cold medicine tablet in water. The data is from an "Effervescent Experiment" designed to compare dissolving times of two different brands of tablets (name brand and store brand) at three different equally spaced water temperatures (6°C, 23°C, and 40°C). The run order number, as well as whether or not the sample was stirred, were also recorded and are investigated in the analysis portions of this report.

Experimental Design

The experiment carried out was a complete block design where b=2 blocks (by stirred status) were selected with n=4 observations on each of the treatment combinations in each block. In Block I, the water was stirred using a magnetic stirring plate at 350 revolutions per minute, whereas in Block II, the water was not stirred. The time for the tablet to dissolve was measured from the moment the tablet was dropped into the water to the time the tablet was completely dissolved, and each tablet was dropped from a fixed height into 60mL of water. The observation was taken as an average of the times as measured by four experimenters and was recorded, along with the run order for each observation. The primary effects being investigated are Brand and Temp, though Stirred and Order are also considered in the analysis portion below.

Exploratory Analysis

Summary statistics for each variable can be seen below. For the Brand, Temp, Stirred, and Order variables, we can see counts for each level. For the Time variable, we can see a five-number summary for the variable.

_	Brand	Temp	Stirred	Order	Time
Ī	name :24	6:16	no :24	1:1	Min. :58.24
	store:24	23:16	yes:24	2:1	1st Qu.:66.09
	NA	40:16	NA	3:1	Median $:70.92$
	NA	NA	NA	4:1	Mean : 70.77
	NA	NA	NA	5:1	3rd Qu.:76.93
	NA	NA	NA	6:1	Max. :80.92
	NA	NA	NA	(Other):42	NA

Table 1: Summary Stats for Variables

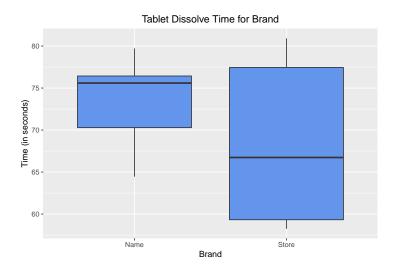
The summary statistics for the Time by Group Table below shows a decrease in mean dissolving time as temperature increases regardless of Brand or Stirred status. "Name Brand" generally had a higher mean dissolving time, but occasionally, "Store Brand" had a higher mean dissolve time. The same was true for when the water was not stirred, both instances at the 6 degrees Celsius. There were also some unusually high standard deviations for "Name Brand" at 23 and 40 degrees with their standard deviations at 2.64 and 1.6, respectively. In both of these instances, the water was stirred. "Store Brand" no stirred at 6 degrees also had a higher standard deviation of 1.3 whereas, overall, the standard deviations tended to range between 0.2 and 0.77.

Table 2.	Summary	State	for	Time	bw	Croun
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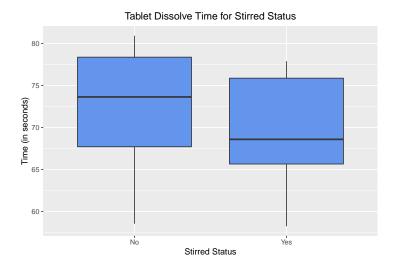
Brand	Stirred	Temp	Mean	SD	Min	Q1	Median	Q3	Max
name	no	6	78.99	0.64	78.15	78.80	79.04	79.24	79.72
name	no	23	76.36	0.33	76.07	76.20	76.28	76.43	76.83
name	no	40	70.56	0.50	70.00	70.29	70.51	70.78	71.20
name	yes	6	76.20	0.68	75.81	75.83	75.89	76.26	77.22
name	yes	23	72.69	2.64	69.09	71.82	73.15	74.02	75.38
name	yes	40	65.85	1.60	64.45	64.87	65.44	66.42	68.08
store	no	6	79.49	1.30	77.78	79.02	79.63	80.10	80.92
store	no	23	67.52	0.45	67.08	67.14	67.52	67.90	67.93
store	no	40	58.96	0.35	58.54	58.77	58.99	59.19	59.33
store	yes	6	77.34	0.77	76.24	77.07	77.61	77.88	77.89
store	yes	23	66.19	0.20	65.93	66.09	66.23	66.33	66.38

Brand	Stirred	Temp	Mean	SD	Min	Q1	Median	Q3	Max
store	yes	40	59.13	0.66	58.24	58.91	59.22	59.43	59.82

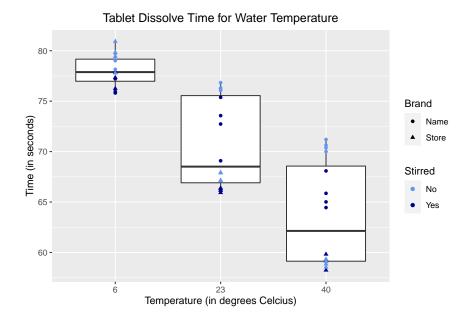
The box plot below displays a five-number summary of dissolving time for each brand of tablet. The plot displays Time as a function of Brand and indicates that there is an effect of the brand on time.



The box plot below displays a five-number summary of dissolving time for each stirred status. The box plot displays Time as a function of Stirred and indicates that there is an effect of the stirred status on time.



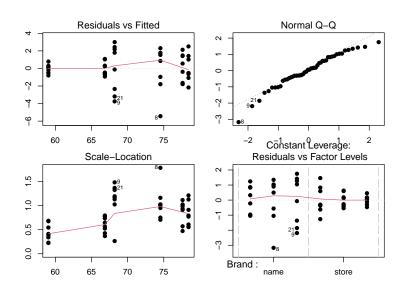
The box plot below displays the dissolving time for different water temperatures. Time is displayed as a function of Temperature, however, we can also see how the Brand and Stirred variables affect the dissolving time by observing the color and shape of the points. It is clear that a warmer temperature reduces the dissolving time. It also appears that, as mentioned above, stirring the water reduces the dissolving time and that at the higher temperatures, the store brand dissolves more quickly than the name brand tablets.



Analysis and Results

Two Factor Crossed Mixed Effects Model

The diagnostic plots look pretty good. The residuals plot shows residuals randomly scattered around zero, and the Q-Q plot suggests they are normally distributed (with the possible exception of one observation, which may be an outlier). The scale-location plot shows the spread is constant over the range.

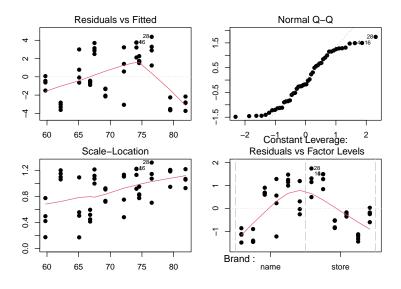


```
##
               Df Sum Sq Mean Sq F value
                                           Pr(>F)
                           342.0
                                 101.75 8.65e-13 ***
## Brand
                   342.0
## Temp
                  1654.7
                           827.4
                                  246.16 < 2e-16 ***
                           115.9
## Brand:Temp
                2
                   231.9
                                   34.49 1.37e-09 ***
## Residuals
                             3.4
                   141.2
                   0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Signif. codes:
```

```
Our model is Y_{ijk} = \mu + \alpha_i + B_j + (\alpha B)_{ij} + \epsilon_{ijk} for i = 1, 2; j = 1, 2, 3; k = 1, ..., 8 where B_j \sim iid \ N(0, \sigma_B^2); \ (\alpha B)_{ij} \sim iid \ N(0, \sigma_{\alpha B}^2); \ \epsilon_{ijk} \sim iid \ N(0, \sigma^2) We also require that each of the above are independent of each other. Y_{ijk} is the dissolving time \mu is the overall mean \alpha_i is the fixed effect due to the ith brand B_j is the random effect due to the jth temperature \alpha B_{ij} is the interaction effect between the ith brand and jth temp \epsilon_{ijk} is the error term
```

Three Factor Analysis of Variance Model

The diagnostic plots look pretty good. The residuals plot shows residuals randomly scattered around zero, and the Q-Q plot suggests they are normally distributed in general. However, this model is more affected by outliers compared to the previous one. The scale-location plot shows the spread is constant over the range.



```
##
               Df Sum Sq Mean Sq F value
                                            Pr(>F)
                           342.0
## Brand
                   342.0
                                  48.514 1.44e-08
                  1654.7
## Temp
                           827.4 117.364
                                           < 2e-16 ***
                    69.9
                             69.9
                                    9.914
                                           0.00298 **
## Stirred
                1
                   303.1
## Residuals
               43
                             7.0
                   0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Signif. codes:
```

Our model is $Y_{ijk} = \mu + \alpha_i + B_j + \gamma_k + \epsilon_{ijkl}$

for
$$i = 1, 2$$
; $j = 1, 2, 3$; $k = 1, 2$; $l = 1, 2, 3, 4$

where $B_j \sim iid \ N(0, \sigma_B^2)$; $\epsilon_{ijkl} \sim iid \ N(0, \sigma^2)$

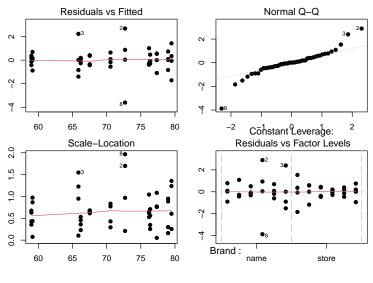
We also require that each of the above are independent of each other.

 Y_{ijkl} is the dissolving time μ is the overall mean α_i is the fixed effect due to the ith brand

 B_j is the random effect due to the jth temperature γ_k is the fixed effect due to the kth stirred status ϵ_{ijkl} is the error term

Three Factor Crossed Mixed Effects Model

The diagnostic plots look pretty good. The residuals plot shows residuals randomly scattered around zero, and the Q-Q plot suggests they are normally distributed in general. However, this model is more affected by outliers compared to the first model. The scale-location plot shows the spread is constant over the range.



```
##
                       Df Sum Sq Mean Sq F value
                                                    Pr(>F)
## Brand
                           342.0
                                   342.0 296.041
                                                   < 2e-16 ***
## Temp
                        2
                          1654.7
                                   827.4 716.169
                                                   < 2e-16 ***
## Stirred
                            69.9
                                           60.495 3.22e-09 ***
## Brand:Temp
                        2
                           231.9
                                   115.9 100.345 1.90e-15 ***
## Brand:Stirred
                            20.5
                                     20.5
                                           17.753 0.000161 ***
  Temp:Stirred
                        2
                             0.1
                                      0.1
                                            0.054 0.947535
## Brand:Temp:Stirred
                        2
                             9.1
                                      4.5
                                            3.919 0.028838 *
##
  Residuals
                       36
                                      1.2
                            41.6
                   0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
## Signif. codes:
```

Our model is $Y_{ijkl} = \mu + \alpha_i + B_j + \gamma_k + (\alpha B)_{ij} + (\alpha \gamma)_{ik} + (B\gamma)_{jk} + (\alpha B\gamma)_{ijk} + \epsilon_{ijkl}$

for i = 1, 2; j = 1, 2, 3; k = 1, 2; l = 1, 2, 3, 4

where $B_j \sim iid\ N(0, \sigma_B^2)$; $(\alpha B)_{ij} \sim iid\ N(0, \sigma_{\alpha B}^2)$; $B\gamma_{jk} \sim iid\ N(0, \sigma_{B\gamma}^2)$; $\alpha B\gamma_{ijk} \sim iid\ N(0, \sigma_{\alpha B\gamma}^2)$; $\epsilon_{ijk} \sim iid\ N(0, \sigma^2)$

We also require that each of the above are independent of each other.

```
Y_{ijkl} is the dissolving time
```

 μ is the overall mean

 α_i is the fixed effect due to the ith brand

 B_j is the random effect due to the jth temperature

 γ_k is the fixed effect due to the kth stirred status

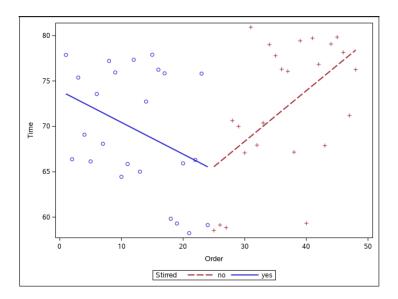
 αB_{ij} is the interaction effect between the ith brand and jth temp

 $\alpha \gamma_{ik}$ is the interaction effect between the ith brand and kth stirred status

 $B\gamma_{jk}$ is the interaction effect between the jth temp and kth stirred status $\alpha B\gamma_{ijk}$ is the interaction effect between the ith brand jth temperature and kth stirred status ϵ_{ijkl} is the error term

Using Order as a Covariate with Time

When we graph the Order variable with respect to Time and group it by Stirred, there appears to be a linear association between the two blocks, albeit weak, but when we look at the two blocks separately, we can see the potential of using Order as a covariate to assist us in determining if there is a difference in dissolving times between the two blocks.



For Block 1, where the liquid was stirred, there appears to be a downward linear trend in the dissolving time of the cold medicine tablets. For Block 2, where the liquid was not stirred, there appears to be an upward linear trend in the dissolving time of the cold medicine tablets. Since the Order variable directly matches the Stirred variable, meaning the first 24 values of Order coincide with Block 1 and the second 24 values of Order coincide with Block 2 and the slopes for the two Blocks are not moving in the same direction as we can observe looking at our initial graph, we can write our model as follows:

$$Time = Mean + Brand + Temp + Brand * Temp + Error$$

When we look at our model for Block 1 and Block 2, we see that our regression slopes are not all equal to 0 given the p-values (<0.001) for both models are less than our significance level of 0.05 and that there is a significant interaction effect. Since we have different slopes we can look at the differences between Brand and Temp at the different levels. For both Blocks, we can see that there was not a significant difference between the Brands at 6 degrees Celsius.

Conclusion

The model which describes this experiment best is the Three Factor Crossed Mixed Effects Model. When comparing MSE between the models, this model has the lowest value at 1.2. MSE is the average of the squared errors, so a lower value indicates a lower error and a better model fit. This model includes three explanatory variables, Brand, Temp, and Stirred. While Brand and Stirred are fixed effects because we are only interested in comparing "Name Brand" vs "Store Brand" and "Stirred" vs "Not Stirred", Temp is

a random effect since the three temperatures have presumably been selected as a sample from the total population of temperatures at which one might dissolve a tablet.

Previously, we looked at the following image displaying the Type I SS analyses:

```
##
                      Df Sum Sq Mean Sq F value
## Brand
                          342.0
                                  342.0 296.041
                                                 < 2e-16 ***
## Temp
                       2 1654.7
                                  827.4 716.169
                                                 < 2e-16 ***
## Stirred
                           69.9
                                   69.9 60.495 3.22e-09 ***
                       1
                          231.9
## Brand:Temp
                       2
                                  115.9 100.345 1.90e-15 ***
## Brand:Stirred
                           20.5
                                   20.5
                                        17.753 0.000161 ***
                       1
## Temp:Stirred
                       2
                            0.1
                                    0.1
                                          0.054 0.947535
                                    4.5
## Brand:Temp:Stirred
                      2
                            9.1
                                          3.919 0.028838 *
## Residuals
                      36
                           41.6
                                    1.2
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

However, because this is a mixed model, we must consider the Type III SS analyses, which are calculated below...

The following results display the F and p values for the main effect Brand. We can see that this effect is *not* significant at the 5% level.

```
## [1] 2.950221
```

[1] 0.2280041

The following results display the F and p values for the main effect Stirred. We can see that this effect is significant at the 5% level.

```
## [1] 1120.842
```

[1] 0.0008909945

The following results display the F and p values for the interaction effect Brand:Temp. We can see that this effect is significant at the 5% level.

```
## [1] 25.60166
```

[1] 0.03759163

The following results display the F and p values for the interaction effect Brand:Stirred. We can see that this effect is *not* significant at the 5% level.

```
## [1] 4.529539
```

[1] 0.1671136

The following results display the F and p values for the interaction effect Temp:Stirred. We can see that this effect is *not* significant at the 5% level.

```
## [1] 0.01377035
```

[1] 0.9864167

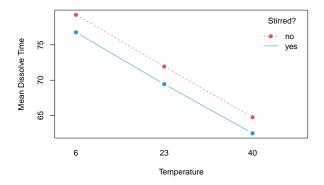
The following results display the F and p values for the interaction effect Brand:Temp:Stirred. We can see that this effect is significant at the 5% level.

```
## [1] 3.919484
```

[1] 0.02883762

In reviewing the analysis of variance table for this model, we can see that all model terms are significant at the 5% level aside from one, the interaction effect between Temperature and Stirred. This is confirmed in the interaction plot below, where we observe the two lines to be parallel. We can see that the temperature reduces the dissolve time and whether or not it is stirred also affects dissolve time, but that temperature of the water and stirred status do not depend on the value of one another.

Interaction Plot for Temp and Stirred Status



Additionally, we had to consider what to do with the blocking variable, Stirred. In order to investigate the effect of this on the response, we considered a two factor crossed mixed effects model without it and a three factor crossed mixed effects model with it. It is clear that the model which includes this variable is a much better model and the Stirred term was highly significant at the 1% level.

Finally, we should address the limitations on our analysis...

Appendix

```
##
     Tukey multiple comparisons of means
##
       95% family-wise confidence level
##
##
  Fit: aov(formula = Time ~ Brand * Temp * Stirred, data = data)
##
##
  $Brand
##
                                          upr p adj
                    diff
                               lwr
  store-name -5.338595 -5.967868 -4.709322
##
##
##
  $Temp
##
               diff
                            lwr
                                        upr p adj
                                 -6.386316
## 23-6
          -7.315177
                      -8.244038
```

```
## 40-6 -14.381286 -15.310147 -13.452425
## 40-23 -7.066109 -7.994971 -6.137248
                                              0
##
## $Stirred
##
               diff
                          lwr
                                    upr p adj
  yes-no -2.413294 -3.042567 -1.784021
##
  $'Brand:Temp'
##
                            diff
                                         lwr
                                                    upr
                                                            p adj
## store:6-name:6
                       0.8181992
                                  -0.7986616
                                               2.435060 0.6525123
## name:23-name:6
                      -3.0690321 -4.6858929
                                              -1.452171 0.0000238
## store:23-name:6
                     -10.7431221 -12.3599829
                                              -9.126261 0.0000000
## name:40-name:6
                      -9.3922395 -11.0091002
                                              -7.775379 0.0000000
                     -18.5521336 -20.1689943 -16.935273 0.0000000
## store:40-name:6
## name:23-store:6
                                              -2.270371 0.0000002
                      -3.8872313 -5.5040921
## store:23-store:6
                    -11.5613213 -13.1781821
                                              -9.944461 0.0000000
                                              -8.593578 0.0000000
## name:40-store:6
                     -10.2104387 -11.8272994
## store:40-store:6
                    -19.3703328 -20.9871935 -17.753472 0.0000000
## store:23-name:23
                                              -6.057229 0.0000000
                      -7.6740900 -9.2909507
## name:40-name:23
                      -6.3232073 -7.9400681
                                              -4.706347 0.0000000
## store:40-name:23
                    -15.4831014 -17.0999622 -13.866241 0.0000000
## name:40-store:23
                                 -0.2659781
                                               2.967743 0.1469933
                      1.3508826
## store:40-store:23 -7.8090115 -9.4258722
                                             -6.192151 0.0000000
## store:40-name:40
                      -9.1598941 -10.7767548 -7.543033 0.0000000
##
  $'Brand:Stirred'
##
                           diff
                                      lwr
                                                  upr
                                                          p adj
## store:no-name:no
                      -6.645947 -7.827734 -5.46416037 0.0000000
## name:yes-name:no
                      -3.720646 -4.902433 -2.53885959 0.0000000
                      -7.751889 -8.933676 -6.57010224 0.0000000
## store:yes-name:no
## name:yes-store:no
                       2.925301 1.743514 4.10708769 0.0000005
## store:yes-store:no -1.105942 -2.287729 0.07584503 0.0735994
   store:yes-name:yes -4.031243 -5.213030 -2.84945575 0.0000000
##
##
   $'Temp:Stirred'
##
                       diff
                                   lwr
                                               upr
                                                        p adj
## 23:no-6:no
                  -7.301989
                             -8.918849
                                        -5.6851278 0.0000000
## 40:no-6:no
                 -14.482213 -16.099074 -12.8653522 0.0000000
## 6:yes-6:no
                             -4.088647
                                        -0.8549259 0.0006750
                  -2.471787
## 23:yes-6:no
                 -9.800152 -11.417012 -8.1832908 0.0000000
## 40:yes-6:no
                 -16.752146 -18.369007 -15.1352852 0.0000000
## 40:no-23:no
                                       -5.5633637 0.0000000
                  -7.180224
                            -8.797085
## 6:yes-23:no
                  4.830202
                              3.213341
                                        6.4470627 0.0000000
                 -2.498163
                                       -0.8813022 0.0005841
## 23:yes-23:no
                            -4.115024
## 40:yes-23:no
                  -9.450157 -11.067018
                                        -7.8332966 0.0000000
## 6:yes-40:no
                            10.393566
                                       13.6272871 0.0000000
                  12.010426
                  4.682061
## 23:yes-40:no
                              3.065201
                                         6.2989222 0.0000000
## 40:yes-40:no
                  -2.269933
                             -3.886794
                                        -0.6530722 0.0020112
## 23:yes-6:yes
                  -7.328365
                             -8.945226
                                        -5.7115041 0.0000000
## 40:yes-6:yes
                 -14.280359 -15.897220 -12.6634986 0.0000000
## 40:yes-23:yes
                 -6.951994 -8.568855 -5.3351337 0.0000000
## $'Brand:Temp:Stirred'
##
                                    diff
                                                 lwr
                                                             upr
                                                                     p adj
```

```
## store:6:no-name:6:no
                               0.5017854
                                          -2.1509398
                                                        3.1545105 0.9999308
## name:23:no-name:6:no
                                                        0.0256226 0.0542189
                              -2.6271025
                                          -5.2798277
                             -11.4750892 -14.1278144
                                                       -8.8223641 0.0000000
## store:23:no-name:6:no
## name:40:no-name:6:no
                              -8.4355000 -11.0882252
                                                       -5.7827749 0.0000000
## store:40:no-name:6:no
                             -20.0271405 -22.6798657 -17.3744154 0.0000000
## name:6:yes-name:6:no
                                          -5.4409256
                                                       -0.1354753 0.0322182
                              -2.7882004
## store:6:yes-name:6:no
                                          -4.3063126
                                                        0.9991377 0.5760360
                              -1.6535874
## name:23:yes-name:6:no
                                                       -3.6464370 0.0000000
                              -6.2991622
                                          -8.9518873
## store:23:yes-name:6:no
                             -12.7993555 -15.4520806 -10.1466303 0.0000000
## name:40:yes-name:6:no
                             -13.1371794 -15.7899045 -10.4844543 0.0000000
## store:40:yes-name:6:no
                             -19.8653271 -22.5180522 -17.2126019 0.0000000
## name:23:no-store:6:no
                                          -5.7816130
                                                       -0.4761628 0.0099431
                              -3.1288879
## store:23:no-store:6:no
                             -11.9768746 -14.6295997
                                                       -9.3241494 0.0000000
## name:40:no-store:6:no
                                                       -6.2845602 0.0000000
                              -8.9372854 -11.5900105
                             -20.5289259 -23.1816510 -17.8762008 0.0000000
## store:40:no-store:6:no
## name:6:yes-store:6:no
                              -3.2899858
                                          -5.9427109
                                                       -0.6372607 0.0055453
## store:6:yes-store:6:no
                                          -4.8080979
                                                        0.4973523 0.2078431
                              -2.1553728
## name:23:yes-store:6:no
                              -6.8009476
                                          -9.4536727
                                                       -4.1482224 0.0000000
## store:23:yes-store:6:no
                             -13.3011408 -15.9538660 -10.6484157 0.0000000
## name:40:yes-store:6:no
                             -13.6389648 -16.2916899 -10.9862396 0.0000000
## store:40:yes-store:6:no
                             -20.3671124 -23.0198376 -17.7143873 0.0000000
## store:23:no-name:23:no
                              -8.8479867 -11.5007118
                                                       -6.1952615 0.0000000
## name:40:no-name:23:no
                                                       -3.1556723 0.0000003
                                          -8.4611226
                              -5.8083975
## store:40:no-name:23:no
                             -17.4000380 -20.0527631 -14.7473128 0.0000000
                              -0.1610979
                                                        2.4916272 1.0000000
## name:6:yes-name:23:no
                                           -2.8138230
## store:6:yes-name:23:no
                               0.9735151
                                          -1.6792100
                                                        3.6262402 0.9764962
## name:23:yes-name:23:no
                              -3.6720597
                                          -6.3247848
                                                       -1.0193345 0.0013190
## store:23:yes-name:23:no
                             -10.1722529 -12.8249781
                                                       -7.5195278 0.0000000
## name:40:yes-name:23:no
                             -10.5100769 -13.1628020
                                                       -7.8573517 0.0000000
## store:40:yes-name:23:no
                             -17.2382245 -19.8909497 -14.5854994 0.0000000
## name:40:no-store:23:no
                               3.0395892
                                            0.3868641
                                                        5.6923143 0.0136471
## store:40:no-store:23:no
                              -8.5520513 -11.2047765
                                                       -5.8993262 0.0000000
## name:6:yes-store:23:no
                               8.6868888
                                            6.0341636
                                                       11.3396139 0.0000000
## store:6:yes-store:23:no
                                                       12.4742269 0.0000000
                               9.8215018
                                            7.1687766
## name:23:yes-store:23:no
                               5.1759270
                                            2.5232019
                                                        7.8286522 0.0000035
## store:23:yes-store:23:no
                                                        1.3284589 0.8366681
                              -1.3242663
                                           -3.9769914
## name:40:yes-store:23:no
                              -1.6620902
                                           -4.3148153
                                                        0.9906349 0.5686505
## store:40:yes-store:23:no
                              -8.3902379 -11.0429630
                                                       -5.7375127 0.0000000
## store:40:no-name:40:no
                             -11.5916405 -14.2443657
                                                       -8.9389154 0.0000000
## name:6:yes-name:40:no
                                            2.9945744
                                                        8.3000247 0.0000006
                               5.6472996
## store:6:yes-name:40:no
                                                        9.4346377 0.0000000
                               6.7819126
                                            4.1291874
## name:23:yes-name:40:no
                               2.1363378
                                          -0.5163873
                                                        4.7890630 0.2179241
## store:23:yes-name:40:no
                              -4.3638555
                                           -7.0165806
                                                       -1.7111303 0.0000881
## name:40:yes-name:40:no
                                                       -2.0489542 0.0000230
                              -4.7016794
                                          -7.3544045
## store:40:yes-name:40:no
                             -11.4298271 -14.0825522
                                                       -8.7771019 0.0000000
## name:6:yes-store:40:no
                              17.2389401
                                           14.5862150
                                                       19.8916652 0.0000000
## store:6:yes-store:40:no
                              18.3735531
                                           15.7208279
                                                       21.0262782 0.0000000
## name:23:yes-store:40:no
                                                       16.3807035 0.0000000
                              13.7279783
                                           11.0752532
## store:23:yes-store:40:no
                               7.2277851
                                            4.5750599
                                                        9.8805102 0.0000000
## name:40:yes-store:40:no
                               6.8899611
                                            4.2372360
                                                        9.5426863 0.0000000
## store:40:yes-store:40:no
                                                        2.8145386 1.0000000
                               0.1618135
                                           -2.4909117
## store:6:yes-name:6:yes
                               1.1346130
                                          -1.5181121
                                                        3.7873381 0.9330499
## name:23:yes-name:6:yes
                              -3.5109618
                                          -6.1636869
                                                       -0.8582366 0.0024347
## store:23:yes-name:6:yes
                                                       -7.3584299 0.0000000
                             -10.0111550 -12.6638802
```

```
## name:40:yes-name:6:yes
                           -10.3489790 -13.0017041 -7.6962538 0.0000000
## store:40:yes-name:6:yes
                           -17.0771266 -19.7298518 -14.4244015 0.0000000
## name:23:yes-store:6:yes
                           -4.6455747 -7.2982999 -1.9928496 0.0000288
## store:23:yes-store:6:yes
                           -11.1457680 -13.7984932 -8.4930429 0.0000000
## name:40:yes-store:6:yes
                            -11.4835920 -14.1363171 -8.8308668 0.0000000
## store:40:yes-store:6:yes
                           -18.2117396 -20.8644648 -15.5590145 0.0000000
## store:23:yes-name:23:yes
                            -6.5001933 -9.1529184 -3.8474681 0.0000000
## name:40:yes-name:23:yes
                            -6.8380172 -9.4907424 -4.1852921 0.0000000
## store:40:yes-name:23:yes -13.5661649 -16.2188900 -10.9134397 0.0000000
## name:40:yes-store:23:yes
                            -0.3378239 -2.9905491
                                                    2.3149012 0.9999988
## store:40:yes-store:23:yes -7.0659716 -9.7186967 -4.4132465 0.0000000
## store:40:yes-name:40:yes
                             -6.7281477 -9.3808728 -4.0754225 0.0000000
```