

# ST 518 Project

Rachel Hencher, Dane Korver, Mohammed Mira

2022-12-04

## Contents

<b>Executive Summary</b>	<b>1</b>
<b>Introduction</b>	<b>2</b>
<b>Experimental Design</b>	<b>2</b>
<b>Exploratory Analysis</b>	<b>2</b>
<b>Analysis and Results</b>	<b>4</b>
Two Factor Crossed Mixed Effects Model . . . . .	4
Three Factor Analysis of Variance Model . . . . .	5
Three Factor Crossed Mixed Effects Model . . . . .	6
Using Order as a Covariate with Time . . . . .	7
<b>Conclusion</b>	<b>8</b>
<b>Appendix</b>	<b>9</b>

## Executive Summary

In this “Effervescent Experiment”, we looked at four models: Two Factor Crossed Mixed Effects Model, Three Factor Analysis of Variance Model, the Three Factor Crossed Mixed Effects Model, and whether using Order as a Covariate with Time would be appropriate in assisting us in whether there is a difference in the dissolving time of cold medicine tablets at varying degrees of temperature, whether agitation of the water by stirring it or not stirring it and whether type of Brand helped decrease the dissolve time. We found that our “Three Factor Crossed Mixed Effects Model” better fit the data when comparing the MSE values. When we investigated using Order as a Covariate with Time to assist us in differentiating between the treatments, it further supported the evidence of using the “Three Factor Crossed Mixed Effects Model” because of the different slopes between when the water was stirred and not stirred on dissolving time.

## Introduction

The purpose of this experiment is to investigate specific factors and their effect on the amount of time it takes to dissolve a cold medicine tablet in water. The data is from an “Effervescent Experiment” designed to compare dissolving times of two different brands of tablets (name brand and store brand) at three different equally spaced water temperatures (6°C, 23°C, and 40°C). The run order number, as well as whether or not the sample was stirred, were also recorded and are investigated in the analysis portions of this report.

## Experimental Design

The experiment carried out was a complete block design where  $b = 2$  blocks (by stirred status) were selected with  $n = 4$  observations on each of the treatment combinations in each block. In Block I, the water was stirred using a magnetic stirring plate at 350 revolutions per minute, whereas in Block II, the water was not stirred. The time for the tablet to dissolve was measured from the moment the tablet was dropped into the water to the time the tablet was completely dissolved, and each tablet was dropped from a fixed height into 60mL of water. The observation was taken as an average of the times as measured by four experimenters and was recorded, along with the run order for each observation. The primary effects being investigated are **Brand** and **Temp**, though **Stirred** and **Order** are also considered in the analysis portion below.

## Exploratory Analysis

Summary statistics for each variable can be seen below. For the **Brand**, **Temp**, **Stirred**, and **Order** variables, we can see counts for each level. For the **Time** variable, we can see a five-number summary for the variable.

Table 1: Summary Stats for Variables

Brand	Temp	Stirred	Order	Time
name :24	6 :16	no :24	1 : 1	Min. :58.24
store:24	23:16	yes:24	2 : 1	1st Qu.:66.09
NA	40:16	NA	3 : 1	Median :70.92
NA	NA	NA	4 : 1	Mean :70.77
NA	NA	NA	5 : 1	3rd Qu.:76.93
NA	NA	NA	6 : 1	Max. :80.92
NA	NA	NA	(Other):42	NA

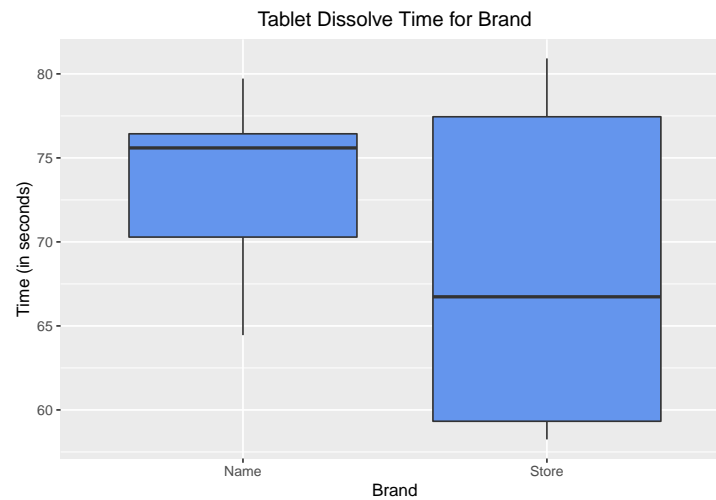
The summary statistics for the **Time by Group Table** below shows a decrease in mean dissolving time as temperature increases regardless of **Brand** or **Stirred** status. “Name Brand” generally had a higher mean dissolving time, but occasionally, “Store Brand” had a higher mean dissolve time. The same was true for when the water was not stirred, both instances at the 6 degrees Celsius. There were also some unusually high standard deviations for “Name Brand” at 23 and 40 degrees with their standard deviations at 2.64 and 1.6, respectively. In both of these instances, the water was stirred. There was also one instance of “Store Brand” in the “not stirred” block at 6 degrees that had a standard deviation of 1.3 whereas, overall, the standard deviations tended to range between 0.2 and 0.77.

Table 2: Summary Stats for Time by Group

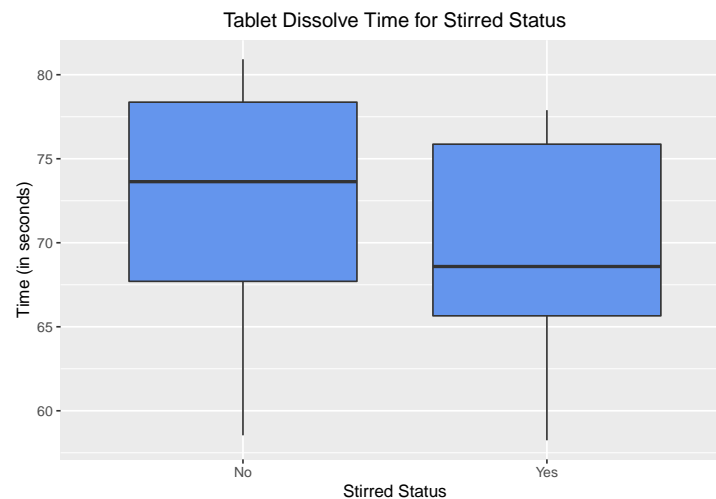
Brand	Stirred	Temp	Mean	SD	Min	Q1	Median	Q3	Max
name	no	6	78.99	0.64	78.15	78.80	79.04	79.24	79.72

Brand	Stirred	Temp	Mean	SD	Min	Q1	Median	Q3	Max
name	no	23	76.36	0.33	76.07	76.20	76.28	76.43	76.83
name	no	40	70.56	0.50	70.00	70.29	70.51	70.78	71.20
name	yes	6	76.20	0.68	75.81	75.83	75.89	76.26	77.22
name	yes	23	72.69	2.64	69.09	71.82	73.15	74.02	75.38
name	yes	40	65.85	1.60	64.45	64.87	65.44	66.42	68.08
store	no	6	79.49	1.30	77.78	79.02	79.63	80.10	80.92
store	no	23	67.52	0.45	67.08	67.14	67.52	67.90	67.93
store	no	40	58.96	0.35	58.54	58.77	58.99	59.19	59.33
store	yes	6	77.34	0.77	76.24	77.07	77.61	77.88	77.89
store	yes	23	66.19	0.20	65.93	66.09	66.23	66.33	66.38
store	yes	40	59.13	0.66	58.24	58.91	59.22	59.43	59.82

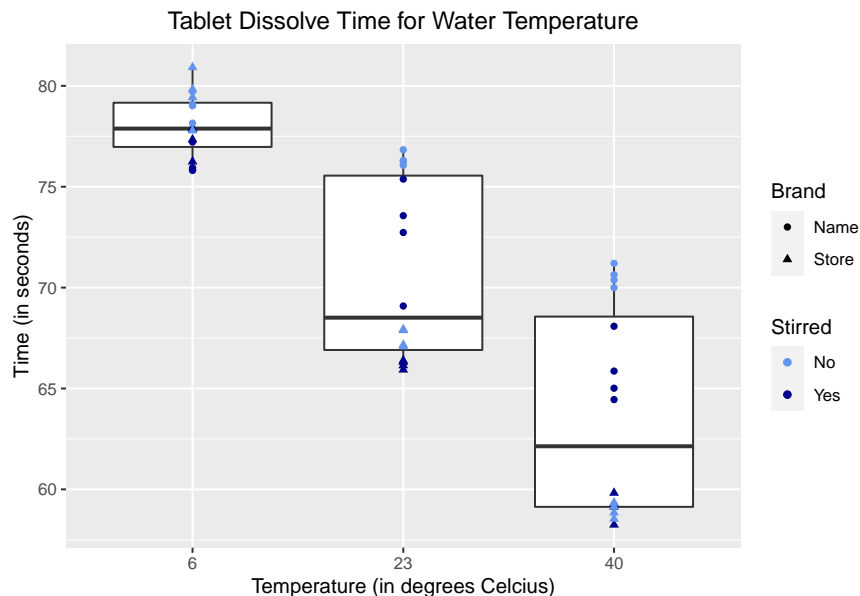
The box plot below displays a five-number summary of dissolving time for each brand of tablet. The plot displays **Time** as a function of **Brand** and indicates that there is an effect of the brand on time.



The box plot below displays a five-number summary of dissolving time for each stirred status. The box plot displays **Time** as a function of **Stirred** and indicates that there is an effect of the stirred status on time.



The box plot below displays the dissolving time for different water temperatures. Time is displayed as a function of **Temperature**, however, we can also see how the **Brand** and **Stirred** variables affect the dissolving time by observing the color and shape of the points. It is clear that a warmer temperature reduces the dissolving time. It also appears that, as mentioned above, stirring the water reduces the dissolving time and that at the higher temperatures, the store brand dissolves more quickly than the name brand tablets.



## Analysis and Results

### Two Factor Crossed Mixed Effects Model

Our first model that we tried is a two factor crossed mixed effects model.

$$Y_{ijk} = \mu + \alpha_i + B_j + (\alpha B)_{ij} + \epsilon_{ijk}$$

for  $i = 1, 2; j = 1, 2, 3; k = 1, \dots, 8$

where  $B_j \sim iid N(0, \sigma_B^2); (\alpha B)_{ij} \sim iid N(0, \sigma_{\alpha B}^2); \epsilon_{ijk} \sim iid N(0, \sigma^2)$

We also require that each of the above are independent of each other.

$Y_{ijk}$  is the dissolving time

$\mu$  is the overall mean

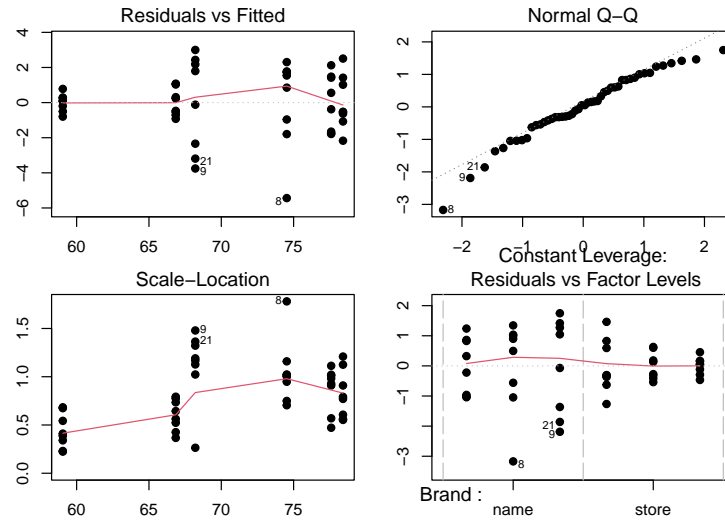
$\alpha_i$  is the fixed effect due to the  $i$ th brand

$B_j$  is the random effect due to the  $j$ th temperature

$\alpha B_{ij}$  is the interaction effect between the  $i$ th brand and  $j$ th temp

$\epsilon_{ijk}$  is the error term

The diagnostic plots look pretty good. The residuals plot shows residuals randomly scattered around zero, and the Q-Q plot suggests they are normally distributed (with the possible exception of one observation, which may be an outlier). The scale-location plot shows the spread is constant over the range.



```
##           Df Sum Sq Mean Sq F value    Pr(>F)
## Brand         1   342.0    342.0   101.75 8.65e-13 ***
## Temp          2  1654.7    827.4   246.16 < 2e-16 ***
## Brand:Temp     2   231.9    115.9    34.49 1.37e-09 ***
## Residuals    42   141.2      3.4
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## Three Factor Analysis of Variance Model

The next model that we tried is a three factor analysis of variance model.

$$Y_{ijk} = \mu + \alpha_i + B_j + \gamma_k + \epsilon_{ijkl}$$

for  $i = 1, 2$ ;  $j = 1, 2, 3$ ;  $k = 1, 2$ ;  $l = 1, 2, 3, 4$

where  $B_j \sim iid N(0, \sigma_B^2)$ ;  $\epsilon_{ijkl} \sim iid N(0, \sigma^2)$

We also require that each of the above are independent of each other.

$Y_{ijkl}$  is the dissolving time

$\mu$  is the overall mean

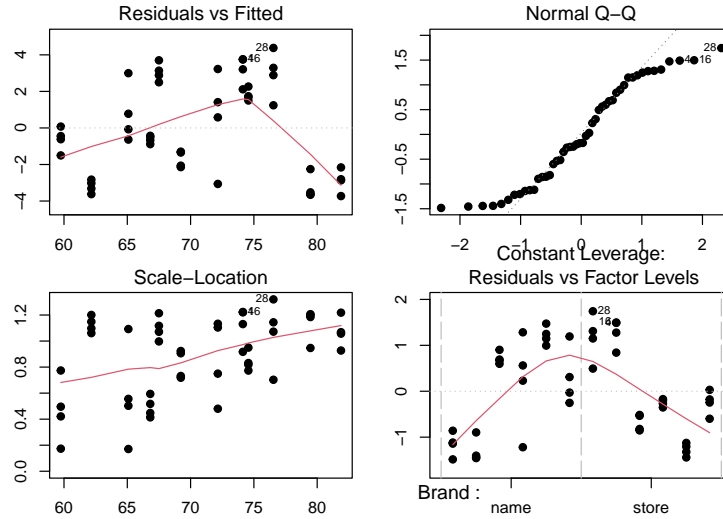
$\alpha_i$  is the fixed effect due to the  $i$ th brand

$B_j$  is the random effect due to the  $j$ th temperature

$\gamma_k$  is the fixed effect due to the  $k$ th stirred status

$\epsilon_{ijkl}$  is the error term

The diagnostic plots look pretty good. The residuals plot shows residuals randomly scattered around zero, and the Q-Q plot suggests they are normally distributed in general. However, this model is more affected by outliers compared to the previous one. The scale-location plot shows the spread is constant over the range.



```
##           Df Sum Sq Mean Sq F value    Pr(>F)
## Brand      1  342.0   342.0  48.514 1.44e-08 ***
## Temp       2 1654.7   827.4 117.364 < 2e-16 ***
## Stirred    1   69.9    69.9   9.914 0.00298 **
## Residuals 43  303.1     7.0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## Three Factor Crossed Mixed Effects Model

The next model that we tried is a three factor crossed mixed effects model.

$$Y_{ijkl} = \mu + \alpha_i + B_j + \gamma_k + (\alpha B)_{ij} + (\alpha \gamma)_{ik} + (B \gamma)_{jk} + (\alpha B \gamma)_{ijk} + \epsilon_{ijkl}$$

for  $i = 1, 2$ ;  $j = 1, 2, 3$ ;  $k = 1, 2$ ;  $l = 1, 2, 3, 4$

where  $B_j \sim iid N(0, \sigma_B^2)$ ;  $(\alpha B)_{ij} \sim iid N(0, \sigma_{\alpha B}^2)$ ;  $B \gamma_{jk} \sim iid N(0, \sigma_{B \gamma}^2)$ ;  $\alpha B \gamma_{ijk} \sim iid N(0, \sigma_{\alpha B \gamma}^2)$ ;  $\epsilon_{ijk} \sim iid N(0, \sigma^2)$

We also require that each of the above are independent of each other.

$Y_{ijkl}$  is the dissolving time

$\mu$  is the overall mean

$\alpha_i$  is the fixed effect due to the  $i$ th brand

$B_j$  is the random effect due to the  $j$ th temperature

$\gamma_k$  is the fixed effect due to the  $k$ th stirred status

$\alpha B_{ij}$  is the interaction effect between the  $i$ th brand and  $j$ th temp

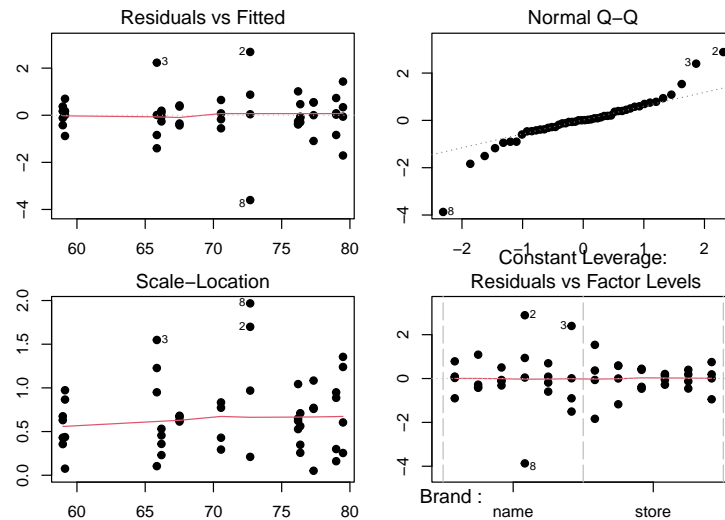
$\alpha \gamma_{ik}$  is the interaction effect between the  $i$ th brand and  $k$ th stirred status

$B \gamma_{jk}$  is the interaction effect between the  $j$ th temp and  $k$ th stirred status

$\alpha B \gamma_{ijk}$  is the interaction effect between the  $i$ th brand  $j$ th temperature and  $k$ th stirred status

$\epsilon_{ijkl}$  is the error term

The diagnostic plots look pretty good. The residuals plot shows residuals randomly scattered around zero and the Q-Q plot suggests they are normally distributed in general. However, this model is more affected by outliers compared to the first model. The scale-location plot shows the spread is constant over the range.

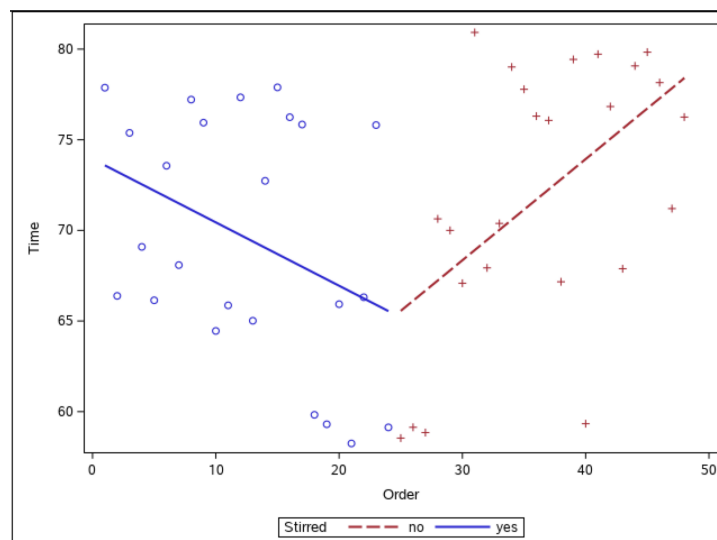


```
##               Df Sum Sq Mean Sq F value    Pr(>F)
## Brand           1  342.0    342.0 296.041 < 2e-16 ***
## Temp            2 1654.7    827.4 716.169 < 2e-16 ***
## Stirred         1   69.9     69.9  60.495 3.22e-09 ***
## Brand:Temp      2  231.9    115.9 100.345 1.90e-15 ***
## Brand:Stirred   1   20.5     20.5  17.753 0.000161 ***
## Temp:Stirred    2    0.1      0.1   0.054 0.947535
## Brand:Temp:Stirred 2    9.1      4.5   3.919 0.028838 *
## Residuals      36   41.6      1.2

## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## Using Order as a Covariate with Time

When we graph the **Order** variable with respect to **Time** and group it by **Stirred**, there appears to be a linear association between the two blocks, albeit weak, but when we look at the two blocks separately, we can see the potential of using **Order** as a covariate to assist us in determining if there is a difference in dissolving times between the two blocks.



For Block I, where the liquid was stirred, there appears to be a downward linear trend in the dissolving time of the cold medicine tablets. For Block II, where the liquid was not stirred, there appears to be an upward linear trend in the dissolving time of the cold medicine tablets. Since the **Order** variable directly matches the **Stirred** variable, meaning the first 24 values of **Order** coincide with Block I and the second 24 values of **Order** coincide with Block II and the slopes for the two blocks are not moving in the same direction as we can observe looking at our initial graph, we can write our model similar to our “Three Factor Crossed Mixed Effects Model”, but with a condition on the **Order** variable to match the slope of our initial graph.

The first model will match the **Stirred**=“yes” block:

$$Y_{ijkl} = \mu + \alpha_i + B_j + \gamma_k + (\alpha B)_{ij} + \epsilon_{ijkl}$$

for  $i = 1, 2$ ;  $j = 1, 2, 3$ ;  $k = 1, 2, \dots, 24$ ;  $l = 1, 2, 3, 4$

The second model will match the **Stirred**=“no” block:

$$Y_{ijkl} = \mu + \alpha_i + B_j + \gamma_k + (\alpha B)_{ij} + \epsilon_{ijkl}$$

for  $i = 1, 2$ ;  $j = 1, 2, 3$ ;  $k = 25, 26, \dots, 48$ ;  $l = 1, 2, 3, 4$

where  $Y_{ijkl}$  is the dissolving time;  $\mu$  is the overall mean;  $\alpha_i$  represents **Brand**;  $B_j$  represents **Temp**;  $\gamma_k$  represents **Order**;  $(\alpha B)_{ij}$  represents our interaction between **Brand** and **Temp**; and  $\epsilon_{ijk} \sim iid N(0, \sigma^2)$

When we look at our model for Block I and Block II, we see that our regression slopes are not all equal to 0 given the p-values ( $<0.001$ ) for both models are less than our significance level of 0.05 and that there is a significant interaction effect. Since we have different slopes we can look at the differences between **Brand** and **Temp** at the different levels. For both blocks, we can see that there was not a significant difference between the brands at 6 degrees Celsius.

## Conclusion

The model which describes this experiment best is the Three Factor Crossed Mixed Effects Model. When comparing the MSE between the models, this model has the lowest value at 1.2. MSE is the average of the squared errors, so a lower value indicates a lower error and a better model fit. This model includes three explanatory variables, **Brand**, **Temp**, and **Stirred**. While **Brand** and **Stirred** are fixed effects because we are only interested in comparing “Name Brand” vs “Store Brand” and “Stirred” vs “Not Stirred”, **Temp** is a random effect since the three temperatures have presumably been selected as a sample from the total population of temperatures at which one might dissolve a tablet. One of the things we had to consider was what to do with the blocking variable, **Stirred**. So, in order to investigate the effect of this on the response, we considered a two factor crossed mixed effects model without it and a three factor crossed mixed effects model with it. It is clear that the model which includes this variable is a much better model and we can see below that the main effect **Stirred** was highly significant at the 1% level.

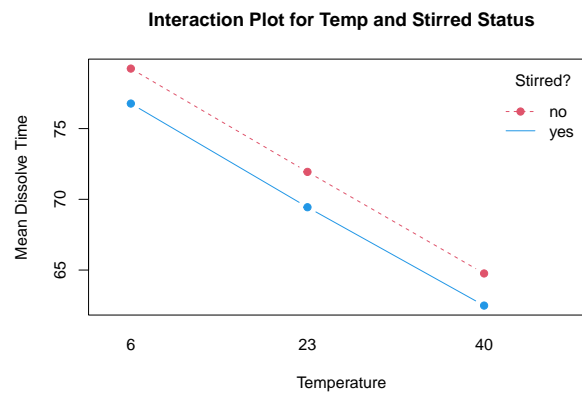
Previously, we looked at the following image displaying the Type I SS analyses:

```
##              Df Sum Sq Mean Sq F value    Pr(>F)
## Brand          1  342.0    342.0 296.041 < 2e-16 ***
## Temp           2 1654.7    827.4 716.169 < 2e-16 ***
## Stirred         1   69.9     69.9  60.495 3.22e-09 ***
## Brand:Temp      2  231.9    115.9 100.345 1.90e-15 ***
## Brand:Stirred   1   20.5     20.5  17.753 0.000161 ***
## Temp:Stirred    2    0.1      0.1   0.054 0.947535
## Brand:Temp:Stirred 2    9.1      4.5   3.919 0.028838 *
## Residuals      36   41.6      1.2
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



However, because this is a mixed model, we must consider the Type III SS analyses. For the main effect **Brand**, the F statistic is 2.9502207 and the P-value is 0.2280041 which is *not* significant at the 5% level. For the main effect **Stirred**, the F statistic is 1120.8416565 and the P-value is  $8.9099446 \times 10^{-4}$  which is significant at the 5% level. For the interaction effect **Brand:Temp**, the F statistic is 25.6016645 and the P-value is 0.0375916 which is significant at the 5% level. For the interaction effect **Brand:Stirred**, the F statistic is 4.5295394 and the P-value is 0.1671136 which is *not* significant at the 5% level. For the interaction effect **Temp:Stirred**, the F statistic is 0.0137704 and the P-value is 0.9864167 which is *not* significant at the 5% level. For the interaction effect **Brand:Temp:Stirred**, the F statistic is 3.9194837 and the P-value is 0.0288376 which is significant at the 5% level.

In reviewing the above, we can see that while not all model terms are significant at the 5% level, the interaction effect between **Temperature** and **Stirred** stands out as being the least significant. This is confirmed in the interaction plot below, where we observe the two lines to be parallel. We can see that the temperature reduces the dissolve time and whether or not it is stirred also affects dissolve time, but that temperature of the water and stirred status do not depend on the value of one another.



Finally, we should address the limitations on our analysis... The sample size was on the smaller side, so the outliers did have an effect on the normality, as seen in the Q-Q plots. Additionally, we don't know whether the researcher was more concerned about these three temperatures as fixed effects, or whether they were interested in the effect of temperature in general. We assumed the latter throughout our analysis.

## Appendix

```
## Tukey multiple comparisons of means
## 95% family-wise confidence level
##
## Fit: aov(formula = Time ~ Brand * Temp * Stirred, data = data)
##
## $Brand
##              diff              lwr              upr p adj
## store-name -5.338595 -5.967868 -4.709322      0
##
## $Temp
##              diff              lwr              upr p adj
## 23-6    -7.315177    -8.244038    -6.386316      0
## 40-6   -14.381286   -15.310147   -13.452425      0
## 40-23   -7.066109   -7.994971   -6.137248      0
##
## $Stirred
```

```

##          diff          lwr          upr p adj
## yes-no -2.413294 -3.042567 -1.784021    0
##
## $'Brand:Temp'
##          diff          lwr          upr          p adj
## store:6-name:6      0.8181992 -0.7986616  2.435060 0.6525123
## name:23-name:6     -3.0690321 -4.6858929 -1.452171 0.0000238
## store:23-name:6    -10.7431221 -12.3599829 -9.126261 0.0000000
## name:40-name:6     -9.3922395 -11.0091002 -7.775379 0.0000000
## store:40-name:6    -18.5521336 -20.1689943 -16.935273 0.0000000
## name:23-store:6    -3.8872313 -5.5040921 -2.270371 0.0000002
## store:23-store:6   -11.5613213 -13.1781821 -9.944461 0.0000000
## name:40-store:6    -10.2104387 -11.8272994 -8.593578 0.0000000
## store:40-store:6   -19.3703328 -20.9871935 -17.753472 0.0000000
## store:23-name:23   -7.6740900 -9.2909507 -6.057229 0.0000000
## name:40-name:23    -6.3232073 -7.9400681 -4.706347 0.0000000
## store:40-name:23   -15.4831014 -17.0999622 -13.866241 0.0000000
## name:40-store:23    1.3508826 -0.2659781  2.967743 0.1469933
## store:40-store:23  -7.8090115 -9.4258722 -6.192151 0.0000000
## store:40-name:40   -9.1598941 -10.7767548 -7.543033 0.0000000
##
## $'Brand:Stirred'
##          diff          lwr          upr          p adj
## store:no-name:no   -6.645947 -7.827734 -5.46416037 0.0000000
## name:yes-name:no   -3.720646 -4.902433 -2.53885959 0.0000000
## store:yes-name:no  -7.751889 -8.933676 -6.57010224 0.0000000
## name:yes-store:no  2.925301  1.743514  4.10708769 0.0000005
## store:yes-store:no -1.105942 -2.287729  0.07584503 0.0735994
## store:yes-name:yes -4.031243 -5.213030 -2.84945575 0.0000000
##
## $'Temp:Stirred'
##          diff          lwr          upr          p adj
## 23:no-6:no        -7.301989 -8.918849 -5.6851278 0.0000000
## 40:no-6:no        -14.482213 -16.099074 -12.8653522 0.0000000
## 6:yes-6:no        -2.471787 -4.088647 -0.8549259 0.0006750
## 23:yes-6:no       -9.800152 -11.417012 -8.1832908 0.0000000
## 40:yes-6:no       -16.752146 -18.369007 -15.1352852 0.0000000
## 40:no-23:no       -7.180224 -8.797085 -5.5633637 0.0000000
## 6:yes-23:no       4.830202  3.213341  6.4470627 0.0000000
## 23:yes-23:no      -2.498163 -4.115024 -0.8813022 0.0005841
## 40:yes-23:no      -9.450157 -11.067018 -7.8332966 0.0000000
## 6:yes-40:no       12.010426 10.393566 13.6272871 0.0000000
## 23:yes-40:no       4.682061  3.065201  6.2989222 0.0000000
## 40:yes-40:no      -2.269933 -3.886794 -0.6530722 0.0020112
## 23:yes-6:yes      -7.328365 -8.945226 -5.7115041 0.0000000
## 40:yes-6:yes     -14.280359 -15.897220 -12.6634986 0.0000000
## 40:yes-23:yes     -6.951994 -8.568855 -5.3351337 0.0000000
##
## $'Brand:Temp:Stirred'
##          diff          lwr          upr          p adj
## store:6:no-name:6:no      0.5017854 -2.1509398  3.1545105 0.9999308
## name:23:no-name:6:no     -2.6271025 -5.2798277  0.0256226 0.0542189
## store:23:no-name:6:no    -11.4750892 -14.1278144 -8.8223641 0.0000000
## name:40:no-name:6:no     -8.4355000 -11.0882252 -5.7827749 0.0000000

```

## store:40:no-name:6:no	-20.0271405	-22.6798657	-17.3744154	0.0000000
## name:6:yes-name:6:no	-2.7882004	-5.4409256	-0.1354753	0.0322182
## store:6:yes-name:6:no	-1.6535874	-4.3063126	0.9991377	0.5760360
## name:23:yes-name:6:no	-6.2991622	-8.9518873	-3.6464370	0.0000000
## store:23:yes-name:6:no	-12.7993555	-15.4520806	-10.1466303	0.0000000
## name:40:yes-name:6:no	-13.1371794	-15.7899045	-10.4844543	0.0000000
## store:40:yes-name:6:no	-19.8653271	-22.5180522	-17.2126019	0.0000000
## name:23:no-store:6:no	-3.1288879	-5.7816130	-0.4761628	0.0099431
## store:23:no-store:6:no	-11.9768746	-14.6295997	-9.3241494	0.0000000
## name:40:no-store:6:no	-8.9372854	-11.5900105	-6.2845602	0.0000000
## store:40:no-store:6:no	-20.5289259	-23.1816510	-17.8762008	0.0000000
## name:6:yes-store:6:no	-3.2899858	-5.9427109	-0.6372607	0.0055453
## store:6:yes-store:6:no	-2.1553728	-4.8080979	0.4973523	0.2078431
## name:23:yes-store:6:no	-6.8009476	-9.4536727	-4.1482224	0.0000000
## store:23:yes-store:6:no	-13.3011408	-15.9538660	-10.6484157	0.0000000
## name:40:yes-store:6:no	-13.6389648	-16.2916899	-10.9862396	0.0000000
## store:40:yes-store:6:no	-20.3671124	-23.0198376	-17.7143873	0.0000000
## store:23:no-name:23:no	-8.8479867	-11.5007118	-6.1952615	0.0000000
## name:40:no-name:23:no	-5.8083975	-8.4611226	-3.1556723	0.0000003
## store:40:no-name:23:no	-17.4000380	-20.0527631	-14.7473128	0.0000000
## name:6:yes-name:23:no	-0.1610979	-2.8138230	2.4916272	1.0000000
## store:6:yes-name:23:no	0.9735151	-1.6792100	3.6262402	0.9764962
## name:23:yes-name:23:no	-3.6720597	-6.3247848	-1.0193345	0.0013190
## store:23:yes-name:23:no	-10.1722529	-12.8249781	-7.5195278	0.0000000
## name:40:yes-name:23:no	-10.5100769	-13.1628020	-7.8573517	0.0000000
## store:40:yes-name:23:no	-17.2382245	-19.8909497	-14.5854994	0.0000000
## name:40:no-store:23:no	3.0395892	0.3868641	5.6923143	0.0136471
## store:40:no-store:23:no	-8.5520513	-11.2047765	-5.8993262	0.0000000
## name:6:yes-store:23:no	8.6868888	6.0341636	11.3396139	0.0000000
## store:6:yes-store:23:no	9.8215018	7.1687766	12.4742269	0.0000000
## name:23:yes-store:23:no	5.1759270	2.5232019	7.8286522	0.0000035
## store:23:yes-store:23:no	-1.3242663	-3.9769914	1.3284589	0.8366681
## name:40:yes-store:23:no	-1.6620902	-4.3148153	0.9906349	0.5686505
## store:40:yes-store:23:no	-8.3902379	-11.0429630	-5.7375127	0.0000000
## store:40:no-name:40:no	-11.5916405	-14.2443657	-8.9389154	0.0000000
## name:6:yes-name:40:no	5.6472996	2.9945744	8.3000247	0.0000006
## store:6:yes-name:40:no	6.7819126	4.1291874	9.4346377	0.0000000
## name:23:yes-name:40:no	2.1363378	-0.5163873	4.7890630	0.2179241
## store:23:yes-name:40:no	-4.3638555	-7.0165806	-1.7111303	0.0000881
## name:40:yes-name:40:no	-4.7016794	-7.3544045	-2.0489542	0.0000230
## store:40:yes-name:40:no	-11.4298271	-14.0825522	-8.7771019	0.0000000
## name:6:yes-store:40:no	17.2389401	14.5862150	19.8916652	0.0000000
## store:6:yes-store:40:no	18.3735531	15.7208279	21.0262782	0.0000000
## name:23:yes-store:40:no	13.7279783	11.0752532	16.3807035	0.0000000
## store:23:yes-store:40:no	7.2277851	4.5750599	9.8805102	0.0000000
## name:40:yes-store:40:no	6.8899611	4.2372360	9.5426863	0.0000000
## store:40:yes-store:40:no	0.1618135	-2.4909117	2.8145386	1.0000000
## store:6:yes-name:6:yes	1.1346130	-1.5181121	3.7873381	0.9330499
## name:23:yes-name:6:yes	-3.5109618	-6.1636869	-0.8582366	0.0024347
## store:23:yes-name:6:yes	-10.0111550	-12.6638802	-7.3584299	0.0000000
## name:40:yes-name:6:yes	-10.3489790	-13.0017041	-7.6962538	0.0000000
## store:40:yes-name:6:yes	-17.0771266	-19.7298518	-14.4244015	0.0000000
## name:23:yes-store:6:yes	-4.6455747	-7.2982999	-1.9928496	0.0000288
## store:23:yes-store:6:yes	-11.1457680	-13.7984932	-8.4930429	0.0000000

## name:40:yes-store:6:yes	-11.4835920	-14.1363171	-8.8308668	0.0000000
## store:40:yes-store:6:yes	-18.2117396	-20.8644648	-15.5590145	0.0000000
## store:23:yes-name:23:yes	-6.5001933	-9.1529184	-3.8474681	0.0000000
## name:40:yes-name:23:yes	-6.8380172	-9.4907424	-4.1852921	0.0000000
## store:40:yes-name:23:yes	-13.5661649	-16.2188900	-10.9134397	0.0000000
## name:40:yes-store:23:yes	-0.3378239	-2.9905491	2.3149012	0.9999988
## store:40:yes-store:23:yes	-7.0659716	-9.7186967	-4.4132465	0.0000000
## store:40:yes-name:40:yes	-6.7281477	-9.3808728	-4.0754225	0.0000000