# ST 518 Project

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## Executive Summary

In this "Effervescent Experiment", we looked at four models: a Two Factor Crossed Mixed Effects Model, a Three Factor Analysis of Variance Model, a Three Factor Crossed Mixed Effects Model, and a model using Order as a covariate with Time. We aimed to determine which would be most appropriate in determining whether there is a difference in the dissolving time of cold medicine tablets when varying degrees of temperature, whether agitation of the water by stirring it or not stirring it, and whether type of brand helped decrease the dissolve time. We found that our "Three Factor Crossed Mixed Effects Model" best fit the data when comparing the MSE values. When we investigated using Order as a covariate with Time to assist us in differentiating between the treatments, it further supported the evidence of using the "Three Factor Crossed Mixed Effects Model" because of the different slopes between when the water was stirred and not stirred on dissolving time.

### Introduction

The purpose of this experiment is to investigate specific factors and their effect on the amount of time it takes to dissolve a cold medicine tablet in water. The data is from an "Effervescent Experiment" designed to compare dissolving times of two different brands of tablets (name brand and store brand) at three different equally spaced water temperatures (6°C, 23°C, and 40°C). The run order number, as well as whether or not the sample was stirred, were also recorded and are investigated in the analysis portions of this report.

### Experimental Design

The experiment carried out was a complete block design where b=2 blocks (by stirred status) were selected with n=4 observations on each of the treatment combinations in each block. In Block I, the water was stirred using a magnetic stirring plate at 350 revolutions per minute, whereas in Block II, the water was not stirred. The time for the tablet to dissolve was measured from the moment the tablet was dropped into the water to the time the tablet was completely dissolved, and each tablet was dropped from a fixed height into 60mL of water. The observation was taken as an average of the times as measured by four experimenters and was recorded, along with the run order for each observation. The primary effects being investigated are Brand and Temp, though Stirred and Order are also considered in the analysis portion below.

## **Exploratory Analysis**

To begin our exploratory analysis, we view the summary statistics for each variable as seen below. For the Brand, Temp, Stirred, and Order variables, we can see counts for each level. For the Time variable, we can see a five-number summary for the variable.

| _ |          |       |         |            |                |
|---|----------|-------|---------|------------|----------------|
|   | Brand    | Temp  | Stirred | Order      | Time           |
|   | name :24 | 6:16  | no :24  | 1:1        | Min. :58.24    |
|   | store:24 | 23:16 | yes:24  | 2:1        | 1st Qu.:66.09  |
|   | NA       | 40:16 | NA      | 3:1        | Median :70.92  |
|   | NA       | NA    | NA      | 4:1        | Mean : $70.77$ |
|   | NA       | NA    | NA      | 5:1        | 3rd Qu.:76.93  |
|   | NA       | NA    | NA      | 6:1        | Max. $:80.92$  |
|   | NA       | NA    | NA      | (Other):42 | NA             |
|   |          |       |         |            |                |

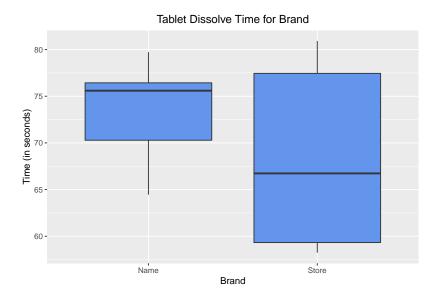
Table 1: Summary Stats for Variables

The summary statistics for the Time by Group Table below shows a decrease in mean dissolving time as temperature increases regardless of Brand or Stirred status. "Name Brand" generally had a higher mean dissolving time, but occasionally, "Store Brand" had a higher mean dissolve time. The same was true for when the water was not stirred, both instances at the 6 degrees Celsius. There were also some unusually high standard deviations for "Name Brand" at 23 and 40 degrees with their standard deviations at 2.64 and 1.6, respectively. In both of these instances, the water was stirred. There was also one instance of "Store Brand" in the "not stirred" block at 6 degrees that had a standard deviation of 1.3 whereas, overall, the standard deviations tended to range between 0.2 and 0.77.

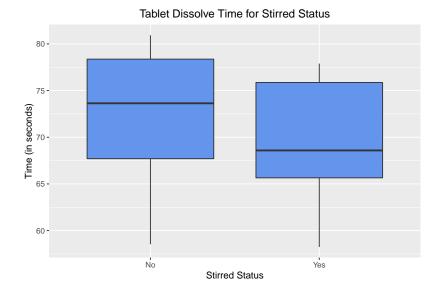
Table 2: Summary Stats for Time by Group

| Brand | Stirred | Temp | Mean  | SD   | Min   | Q1    | Median | Q3    | Max   |
|-------|---------|------|-------|------|-------|-------|--------|-------|-------|
| name  | no      | 6    | 78.99 | 0.64 | 78.15 | 78.80 | 79.04  | 79.24 | 79.72 |
| name  | no      | 23   | 76.36 | 0.33 | 76.07 | 76.20 | 76.28  | 76.43 | 76.83 |
| name  | no      | 40   | 70.56 | 0.50 | 70.00 | 70.29 | 70.51  | 70.78 | 71.20 |
| name  | yes     | 6    | 76.20 | 0.68 | 75.81 | 75.83 | 75.89  | 76.26 | 77.22 |
| name  | yes     | 23   | 72.69 | 2.64 | 69.09 | 71.82 | 73.15  | 74.02 | 75.38 |
| name  | yes     | 40   | 65.85 | 1.60 | 64.45 | 64.87 | 65.44  | 66.42 | 68.08 |
| store | no      | 6    | 79.49 | 1.30 | 77.78 | 79.02 | 79.63  | 80.10 | 80.92 |
| store | no      | 23   | 67.52 | 0.45 | 67.08 | 67.14 | 67.52  | 67.90 | 67.93 |
| store | no      | 40   | 58.96 | 0.35 | 58.54 | 58.77 | 58.99  | 59.19 | 59.33 |
| store | yes     | 6    | 77.34 | 0.77 | 76.24 | 77.07 | 77.61  | 77.88 | 77.89 |
| store | yes     | 23   | 66.19 | 0.20 | 65.93 | 66.09 | 66.23  | 66.33 | 66.38 |
| store | yes     | 40   | 59.13 | 0.66 | 58.24 | 58.91 | 59.22  | 59.43 | 59.82 |

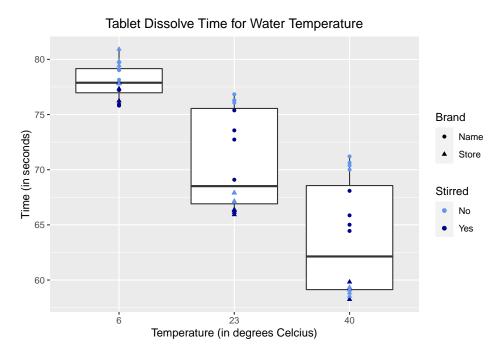
The box plot below displays a five-number summary of dissolving time for each brand of tablet. The plot displays Time as a function of Brand and indicates that there is an effect of the brand on time.



The box plot below displays a five-number summary of dissolving time for each stirred status. The box plot displays Time as a function of Stirred and indicates that there is an effect of the stirred status on time.



The box plot below displays the dissolving time for different water temperatures. Time is displayed as a function of Temperature, however, we can also see how the Brand and Stirred variables affect the dissolving time by observing the color and shape of the points. It is clear that a warmer temperature reduces the dissolving time. It also appears that, as mentioned above, stirring the water reduces the dissolving time and that at the higher temperatures, the store brand dissolves more quickly than the name brand tablets.



## Analysis and Results

### Two Factor Crossed Mixed Effects Model

Our first model that we tried is a two factor crossed mixed effects model.

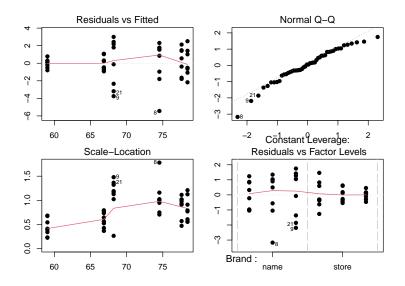
$$Y_{ijk} = \mu + \alpha_i + B_j + (\alpha B)_{ij} + \epsilon_{ijk}$$

```
for i = 1, 2; j = 1, 2, 3; k = 1, ..., 8
```

where  $B_j \sim iid\ N(0, \sigma_B^2)$ ;  $(\alpha B)_{ij} \sim iid\ N(0, \sigma_{\alpha B}^2)$ ;  $\epsilon_{ijk} \sim iid\ N(0, \sigma^2)$ We also require that each of the above are independent of each other.

 $Y_{ijk}$  is the dissolving time  $\mu$  is the overall mean  $\alpha_i$  is the fixed effect due to the ith brand  $B_j$  is the random effect due to the jth temperature  $\alpha B_{ij}$  is the interaction effect between the ith brand and jth temp  $\epsilon_{ijk}$  is the error term

The diagnostic plots look pretty good. The residuals plot shows residuals randomly scattered around zero, and the Q-Q plot suggests they are normally distributed (with the possible exception of one observation, which may be an outlier). The scale-location plot shows the spread is constant over the range.



```
Df Sum Sq Mean Sq F value
##
                                            Pr(>F)
## Brand
                   342.0
                           342.0
                                  101.75 8.65e-13 ***
## Temp
                2
                  1654.7
                           827.4
                                  246.16
                                          < 2e-16 ***
## Brand:Temp
                2
                   231.9
                           115.9
                                    34.49 1.37e-09 ***
## Residuals
                   141.2
                              3.4
                   0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Signif. codes:
```

### Three Factor Analysis of Variance Model

The next model that we tried is a three factor analysis of variance model.

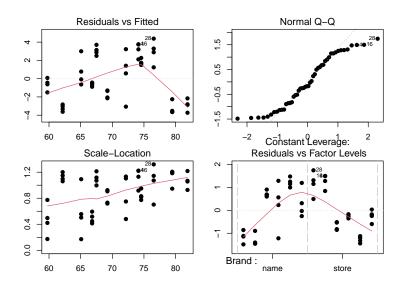
$$Y_{ijk} = \mu + \alpha_i + B_j + \gamma_k + \epsilon_{ijkl}$$
  
for  $i = 1, 2; j = 1, 2, 3; k = 1, 2; l = 1, 2, 3, 4$   
where  $B_j \sim iid \ N(0, \sigma_B^2); \epsilon_{ijkl} \sim iid \ N(0, \sigma^2)$ 

We also require that each of the above are independent of each other.

 $Y_{ijkl}$  is the dissolving time  $\mu$  is the overall mean  $\alpha_i$  is the fixed effect due to the ith brand  $B_j$  is the random effect due to the jth temperature

 $\gamma_k$  is the fixed effect due to the kth stirred status  $\epsilon_{ijkl}$  is the error term

The diagnostic plots look pretty good. The residuals plot shows residuals randomly scattered around zero, and the Q-Q plot suggests they are normally distributed in general. However, this model is more affected by outliers compared to the previous one. The scale-location plot shows the spread is constant over the range.



```
Df Sum Sq Mean Sq F value
##
                                            Pr(>F)
                            342.0 48.514 1.44e-08 ***
## Brand
                   342.0
## Temp
                  1654.7
                            827.4 117.364
                                           < 2e-16 ***
                             69.9
## Stirred
                1
                     69.9
                                    9.914
                                           0.00298 **
                   303.1
                              7.0
  Residuals
               43
##
                   0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
## Signif. codes:
```

### Three Factor Crossed Mixed Effects Model

The next model that we tried is a three factor crossed mixed effects model.

$$Y_{ijkl} = \mu + \alpha_i + B_j + \gamma_k + (\alpha B)_{ij} + (\alpha \gamma)_{ik} + (B\gamma)_{jk} + (\alpha B\gamma)_{ijk} + \epsilon_{ijkl}$$
  
for  $i = 1, 2; j = 1, 2, 3; k = 1, 2; l = 1, 2, 3, 4$ 

where  $B_j \sim iid\ N(0, \sigma_B^2)$ ;  $(\alpha B)_{ij} \sim iid\ N(0, \sigma_{\alpha B}^2)$ ;  $B\gamma_{jk} \sim iid\ N(0, \sigma_{B\gamma}^2)$ ;  $\alpha B\gamma_{ijk} \sim iid\ N(0, \sigma_{\alpha B\gamma}^2)$ ;  $\epsilon_{ijk} \sim iid\ N(0, \sigma^2)$ 

We also require that each of the above are independent of each other.

```
Y_{ijkl} is the dissolving time
```

 $\mu$  is the overall mean

 $\alpha_i$  is the fixed effect due to the ith brand

 $B_j$  is the random effect due to the jth temperature

 $\gamma_k$  is the fixed effect due to the kth stirred status

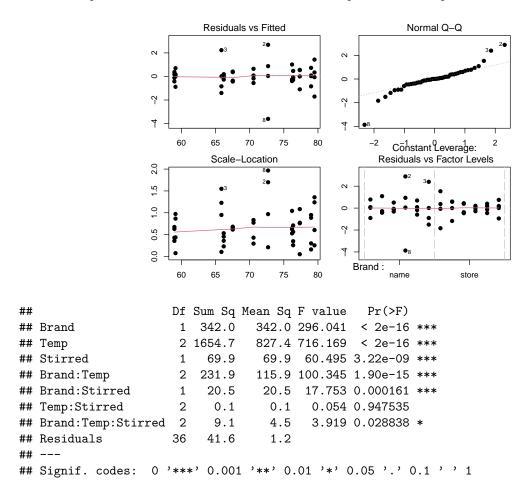
 $\alpha B_{ij}$  is the interaction effect between the ith brand and jth temp

 $\alpha \gamma_{ik}$  is the interaction effect between the ith brand and kth stirred status

 $B\gamma_{jk}$  is the interaction effect between the jth temp and kth stirred status

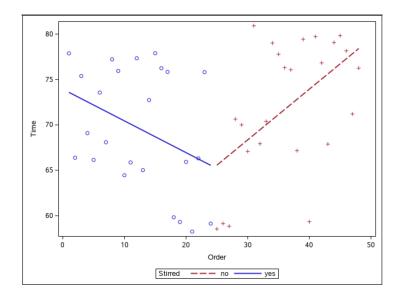
 $\alpha B \gamma_{ijk}$  is the interaction effect between the ith brand jth temperature and kth stirred status  $\epsilon_{ijkl}$  is the error term

The diagnostic plots look pretty good. The residuals plot shows residuals randomly scattered around zero and the Q-Q plot suggests they are normally distributed in general. However, this model is more affected by outliers compared to the first model. The scale-location plot shows the spread is constant over the range.



### Using Order as a Covariate with Time

When we graph the Order variable with respect to Time and group it by Stirred, there appears to be a linear association between the two blocks, albeit weak, but when we look at the two blocks separately, we can see the potential of using Order as a covariate to assist us in determining if there is a difference in dissolving times between the two blocks.



For Block I, where the liquid was stirred, there appears to be a downward linear trend in the dissolving time of the cold medicine tablets. For Block II, where the liquid was not stirred, there appears to be an upward linear trend in the dissolving time of the cold medicine tablets. Since the Order variable directly matches the Stirred variable, meaning the first 24 values of Order coincide with Block I and the second 24 values of Order coincide with Block II and the slopes for the two blocks are not moving in the same direction as we can observe looking at our initial graph, we can write our model similar to our "Three Factor Crossed Mixed Effects Model", but with a condition on the Order variable to match the slope of our initial graph.

The first model will match the Stirred="yes" block:

$$Y_{ijkl} = \mu + \alpha_i + B_j + \gamma_k + (\alpha B)_{ij} + \epsilon_{ijkl}$$

for 
$$i = 1, 2$$
;  $j = 1, 2, 3$ ;  $k = 1, 2, ..., 24$ ;  $l = 1, 2, 3, 4$ 

The second model will match the Stirred="no" block:

$$Y_{ijkl} = \mu + \alpha_i + B_j + \gamma_k + (\alpha B)_{ij} + \epsilon_{ijkl}$$

for 
$$i = 1, 2$$
;  $j = 1, 2, 3$ ;  $k = 25, 26, ..., 48$ ;  $l = 1, 2, 3, 4$ 

where  $Y_{ijkl}$  is the dissolving time;  $\mu$  is the overall mean;  $\alpha_i$  represents Brand;  $B_j$  represents Temp;  $\gamma_k$  represents Order;  $(\alpha B)_{ij}$  represents our interaction between Brand and Temp; and  $\epsilon_{ijk} \sim iid \ N(0, \sigma^2)$ 

When we look at our model for Block I and Block II, we see that our regression slopes are not all equal to 0 given the p-values (<0.001) for both models are less than our significance level of 0.05 and that there is a significant interaction effect. Since we have different slopes we can look at the differences between Brand and Temp at the different levels. For both blocks, we can see that there was not a significant difference between the brands at 6 degrees Celsius.

### Conclusion

The model which describes this experiment best is the Three Factor Crossed Mixed Effects Model. When comparing the MSE between the models, this model has the lowest value at 1.2. MSE is the average of the squared errors, so a lower value indicates a lower error and a better model fit. This model includes three explanatory variables, Brand, Temp, and Stirred. While Brand and Stirred are fixed effects because we are only interested in comparing "Name Brand" vs "Store Brand" and "Stirred" vs "Not Stirred", Temp is a random effect since the three temperatures have presumably been selected as a sample from the total population of temperatures at which one might dissolve a tablet. One of the things we had to consider was what to do with the blocking variable, Stirred. So, in order to investigate the effect of this on the response,

we considered a two factor crossed mixed effects model without it and a three factor crossed mixed effects model with it. It is clear that the model which includes this variable is a much better model and we can see below that the main effect Stirred was highly significant at the 1% level.

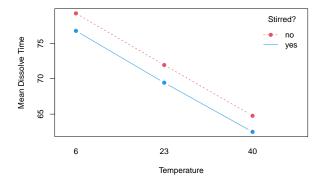
Previously, we looked at the following image displaying the Type I SS analyses:

```
##
                       Df Sum Sq Mean Sq F value
                                                     Pr(>F)
## Brand
                           342.0
                                    342.0 296.041
## Temp
                        2
                          1654.7
                                    827.4 716.169
                                                    < 2e-16
## Stirred
                        1
                            69.9
                                     69.9
                                           60.495 3.22e-09 ***
## Brand:Temp
                        2
                           231.9
                                    115.9 100.345 1.90e-15 ***
## Brand:Stirred
                        1
                            20.5
                                     20.5
                                           17.753 0.000161 ***
  Temp:Stirred
                        2
                             0.1
                                      0.1
                                            0.054 0.947535
                        2
                                            3.919 0.028838 *
## Brand:Temp:Stirred
                             9.1
                                      4.5
## Residuals
                       36
                                      1.2
                            41.6
                    0 '*** 0.001 '** 0.01 '* 0.05 '. ' 0.1 ' 1
## Signif. codes:
```

However, because this is a mixed model, we must consider the Type III SS analyses. For the main effect Brand, the F statistic is 2.9502207 and the P-value is 0.2280041 which is not significant at the 5% level. For the main effect Stirred, the F statistic is 1120.8416565 and the P-value is  $8.9099446 \times 10^{-4}$  which is significant at the 5% level. For the interaction effect Brand:Temp, the F statistic is 25.6016645 and the P-value is 0.0375916 which is significant at the 5% level. For the interaction effect Brand:Stirred, the F statistic is 4.5295394 and the P-value is 0.1671136 which is not significant at the 5% level. For the interaction effect Temp:Stirred, the F statistic is 0.0137704 and the P-value is 0.9864167 which is not significant at the 5% level. For the interaction effect Brand:Temp:Stirred, the F statistic is 3.9194837 and the P-value is 0.0288376 which is significant at the 5% level.

In reviewing the above, we can see that while not all model terms are significant at the 5% level, the interaction effect between Temperature and Stirred stands out as being the least significant. This is confirmed in the interaction plot below, where we observe the two lines to be parallel. We can see that the temperature reduces the dissolve time and whether or not it is stirred also affects dissolve time, but that temperature of the water and stirred status do not depend on the value of one another.

#### Interaction Plot for Temp and Stirred Status



Finally, we should address the limitations on our analysis... The sample size was on the smaller side, so the outliers did have an effect on the normality, as seen in the Q-Q plots. Additionally, we don't know whether the researcher was more concerned about these three temperatures as fixed effects, or whether they were interested in the effect of temperature in general. We assumed the latter throughout our analysis.

### **Appendix**

```
##
     Tukey multiple comparisons of means
##
       95% family-wise confidence level
##
## Fit: aov(formula = Time ~ Brand * Temp * Stirred, data = data)
##
##
  $Brand
##
                   diff
                              lwr
                                        upr p adj
##
  store-name -5.338595 -5.967868 -4.709322
##
## $Temp
##
               diff
                                      upr p adj
         -7.315177
                    -8.244038
                                -6.386316
## 23-6
  40-6 -14.381286 -15.310147 -13.452425
                                              0
  40-23 -7.066109 -7.994971 -6.137248
                                              0
##
## $Stirred
               diff
                          lwr
                                    upr p adj
  yes-no -2.413294 -3.042567 -1.784021
##
## $'Brand:Temp'
##
                            diff
                                         lwr
                                                    upr
                                                             p adj
## store:6-name:6
                                  -0.7986616
                       0.8181992
                                               2.435060 0.6525123
## name:23-name:6
                      -3.0690321 -4.6858929
                                              -1.452171 0.0000238
## store:23-name:6
                    -10.7431221 -12.3599829
                                              -9.126261 0.0000000
## name:40-name:6
                      -9.3922395 -11.0091002
                                              -7.775379 0.0000000
                     -18.5521336 -20.1689943 -16.935273 0.0000000
## store:40-name:6
## name:23-store:6
                      -3.8872313 -5.5040921
                                              -2.270371 0.0000002
## store:23-store:6
                    -11.5613213 -13.1781821
                                             -9.944461 0.0000000
## name:40-store:6
                     -10.2104387 -11.8272994
                                             -8.593578 0.0000000
## store:40-store:6
                    -19.3703328 -20.9871935 -17.753472 0.0000000
                      -7.6740900 -9.2909507
## store:23-name:23
                                              -6.057229 0.0000000
## name:40-name:23
                      -6.3232073 -7.9400681
                                              -4.706347 0.0000000
## store:40-name:23
                    -15.4831014 -17.0999622 -13.866241 0.0000000
## name:40-store:23
                       1.3508826
                                  -0.2659781
                                               2.967743 0.1469933
## store:40-store:23
                      -7.8090115 -9.4258722
                                              -6.192151 0.0000000
  store:40-name:40
                      -9.1598941 -10.7767548
                                             -7.543033 0.0000000
##
## $'Brand:Stirred'
##
                           diff
                                      lwr
## store:no-name:no
                      -6.645947 -7.827734 -5.46416037 0.0000000
                      -3.720646 -4.902433 -2.53885959 0.0000000
## name:yes-name:no
## store:yes-name:no
                      -7.751889 -8.933676 -6.57010224 0.0000000
## name:yes-store:no
                       2.925301 1.743514 4.10708769 0.0000005
## store:yes-store:no -1.105942 -2.287729 0.07584503 0.0735994
  store:yes-name:yes -4.031243 -5.213030 -2.84945575 0.0000000
  $'Temp:Stirred'
##
##
                       diff
                                   lwr
                                               upr
## 23:no-6:no
                  -7.301989
                             -8.918849
                                        -5.6851278 0.0000000
## 40:no-6:no
                 -14.482213 -16.099074 -12.8653522 0.0000000
## 6:yes-6:no
                             -4.088647
                                        -0.8549259 0.0006750
                  -2.471787
## 23:yes-6:no
                  -9.800152 -11.417012 -8.1832908 0.0000000
```

```
## 40:yes-6:no
                 -16.752146 -18.369007 -15.1352852 0.0000000
## 40:no-23:no
                                        -5.5633637 0.0000000
                  -7.180224
                             -8.797085
                                          6.4470627 0.0000000
## 6:yes-23:no
                   4.830202
                              3.213341
## 23:yes-23:no
                  -2.498163
                             -4.115024
                                        -0.8813022 0.0005841
## 40:yes-23:no
                  -9.450157 -11.067018
                                        -7.8332966 0.0000000
## 6:yes-40:no
                  12.010426
                             10.393566
                                        13.6272871 0.0000000
## 23:yes-40:no
                   4.682061
                              3.065201
                                          6.2989222 0.0000000
## 40:yes-40:no
                  -2.269933
                             -3.886794
                                         -0.6530722 0.0020112
## 23:yes-6:yes
                  -7.328365
                             -8.945226
                                         -5.7115041 0.0000000
  40:yes-6:yes
                 -14.280359 -15.897220 -12.6634986 0.0000000
   40:yes-23:yes
                 -6.951994
                             -8.568855
                                        -5.3351337 0.0000000
##
  $'Brand:Temp:Stirred'
##
                                     diff
                                                  lwr
                                                              upr
                                                                       p adj
## store:6:no-name:6:no
                               0.5017854
                                           -2.1509398
                                                        3.1545105 0.9999308
## name:23:no-name:6:no
                              -2.6271025
                                           -5.2798277
                                                        0.0256226 0.0542189
## store:23:no-name:6:no
                             -11.4750892 -14.1278144
                                                       -8.8223641 0.0000000
## name:40:no-name:6:no
                              -8.4355000 -11.0882252
                                                       -5.7827749 0.0000000
## store:40:no-name:6:no
                             -20.0271405 -22.6798657 -17.3744154 0.0000000
                              -2.7882004
## name:6:yes-name:6:no
                                          -5.4409256
                                                       -0.1354753 0.0322182
## store:6:yes-name:6:no
                              -1.6535874
                                          -4.3063126
                                                        0.9991377 0.5760360
## name:23:yes-name:6:no
                                                       -3.6464370 0.0000000
                              -6.2991622
                                          -8.9518873
                             -12.7993555 -15.4520806 -10.1466303 0.0000000
## store:23:yes-name:6:no
## name:40:yes-name:6:no
                             -13.1371794 -15.7899045 -10.4844543 0.0000000
## store:40:yes-name:6:no
                             -19.8653271 -22.5180522 -17.2126019 0.0000000
## name:23:no-store:6:no
                              -3.1288879
                                          -5.7816130
                                                       -0.4761628 0.0099431
## store:23:no-store:6:no
                             -11.9768746 -14.6295997
                                                       -9.3241494 0.0000000
## name:40:no-store:6:no
                              -8.9372854 -11.5900105
                                                       -6.2845602 0.0000000
## store:40:no-store:6:no
                             -20.5289259 -23.1816510 -17.8762008 0.0000000
## name:6:yes-store:6:no
                                          -5.9427109
                                                       -0.6372607 0.0055453
                              -3.2899858
## store:6:yes-store:6:no
                              -2.1553728
                                           -4.8080979
                                                        0.4973523 0.2078431
  name:23:yes-store:6:no
                              -6.8009476
                                          -9.4536727
                                                       -4.1482224 0.0000000
## store:23:yes-store:6:no
                             -13.3011408 -15.9538660 -10.6484157 0.0000000
## name:40:yes-store:6:no
                             -13.6389648 -16.2916899 -10.9862396 0.0000000
## store:40:yes-store:6:no
                             -20.3671124 -23.0198376
                                                      -17.7143873 0.0000000
## store:23:no-name:23:no
                                                       -6.1952615 0.0000000
                              -8.8479867 -11.5007118
## name:40:no-name:23:no
                              -5.8083975
                                          -8.4611226
                                                       -3.1556723 0.0000003
## store:40:no-name:23:no
                             -17.4000380 -20.0527631 -14.7473128 0.0000000
## name:6:yes-name:23:no
                                                        2.4916272 1.0000000
                              -0.1610979
                                           -2.8138230
## store:6:yes-name:23:no
                                                        3.6262402 0.9764962
                               0.9735151
                                          -1.6792100
## name:23:yes-name:23:no
                              -3.6720597
                                           -6.3247848
                                                       -1.0193345 0.0013190
## store:23:yes-name:23:no
                                                       -7.5195278 0.0000000
                             -10.1722529 -12.8249781
## name:40:yes-name:23:no
                             -10.5100769 -13.1628020
                                                       -7.8573517 0.0000000
## store:40:yes-name:23:no
                             -17.2382245 -19.8909497 -14.5854994 0.0000000
## name:40:no-store:23:no
                               3.0395892
                                            0.3868641
                                                        5.6923143 0.0136471
## store:40:no-store:23:no
                                                       -5.8993262 0.0000000
                              -8.5520513 -11.2047765
## name:6:yes-store:23:no
                               8.6868888
                                            6.0341636
                                                       11.3396139 0.0000000
## store:6:yes-store:23:no
                               9.8215018
                                            7.1687766
                                                       12.4742269 0.0000000
## name:23:yes-store:23:no
                               5.1759270
                                            2.5232019
                                                        7.8286522 0.0000035
## store:23:yes-store:23:no
                              -1.3242663
                                           -3.9769914
                                                        1.3284589 0.8366681
## name:40:yes-store:23:no
                                          -4.3148153
                                                        0.9906349 0.5686505
                              -1.6620902
## store:40:yes-store:23:no
                              -8.3902379 -11.0429630
                                                       -5.7375127 0.0000000
## store:40:no-name:40:no
                             -11.5916405 -14.2443657
                                                       -8.9389154 0.0000000
## name:6:yes-name:40:no
                               5.6472996
                                            2.9945744
                                                        8.3000247 0.0000006
```

```
9.4346377 0.0000000
## store:6:yes-name:40:no
                             6.7819126
                                          4.1291874
## name:23:yes-name:40:no
                              2.1363378 -0.5163873
                                                      4.7890630 0.2179241
## store:23:yes-name:40:no
                             -4.3638555
                                        -7.0165806
                                                     -1.7111303 0.0000881
## name:40:yes-name:40:no
                                         -7.3544045
                                                     -2.0489542 0.0000230
                             -4.7016794
## store:40:yes-name:40:no
                            -11.4298271 -14.0825522
                                                     -8.7771019 0.0000000
## name:6:yes-store:40:no
                             17.2389401
                                        14.5862150
                                                     19.8916652 0.0000000
## store:6:yes-store:40:no
                             18.3735531 15.7208279
                                                     21.0262782 0.0000000
## name:23:yes-store:40:no
                             13.7279783 11.0752532 16.3807035 0.0000000
## store:23:yes-store:40:no
                              7.2277851
                                          4.5750599
                                                      9.8805102 0.0000000
## name:40:yes-store:40:no
                                                      9.5426863 0.0000000
                              6.8899611
                                          4.2372360
## store:40:yes-store:40:no
                              0.1618135
                                        -2.4909117
                                                      2.8145386 1.0000000
## store:6:yes-name:6:yes
                                         -1.5181121
                                                      3.7873381 0.9330499
                              1.1346130
## name:23:yes-name:6:yes
                              -3.5109618 -6.1636869
                                                     -0.8582366 0.0024347
## store:23:yes-name:6:yes
                            -10.0111550 -12.6638802
                                                     -7.3584299 0.0000000
## name:40:yes-name:6:yes
                             -10.3489790 -13.0017041
                                                     -7.6962538 0.0000000
## store:40:yes-name:6:yes
                             -17.0771266 -19.7298518 -14.4244015 0.0000000
## name:23:yes-store:6:yes
                             -4.6455747 -7.2982999
                                                     -1.9928496 0.0000288
## store:23:yes-store:6:yes
                            -11.1457680 -13.7984932
                                                     -8.4930429 0.0000000
## name:40:yes-store:6:yes
                             -11.4835920 -14.1363171
                                                     -8.8308668 0.0000000
## store:40:yes-store:6:yes
                            -18.2117396 -20.8644648 -15.5590145 0.0000000
                                                     -3.8474681 0.0000000
## store:23:yes-name:23:yes
                             -6.5001933 -9.1529184
## name:40:yes-name:23:yes
                              -6.8380172 -9.4907424
                                                    -4.1852921 0.0000000
## store:40:yes-name:23:yes
                            -13.5661649 -16.2188900 -10.9134397 0.0000000
## name:40:yes-store:23:yes
                                         -2.9905491
                                                       2.3149012 0.9999988
                              -0.3378239
## store:40:yes-store:23:yes
                                                     -4.4132465 0.0000000
                             -7.0659716 -9.7186967
## store:40:yes-name:40:yes
                              -6.7281477 -9.3808728 -4.0754225 0.0000000
```