Newton's method

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May 21, 2015

1 Introduce

1.1 Formulating optimization problem

Problem, which needs to be solved, is to find minimum point in set non-linear, multidimensional function. If function has more than one minimum, algorithm is looking for the nearest local minimum from initial point. In this project method to find these special points is Newton's method. This algorithm allows to find local minimum points, but this particular method can be optimized by using other algorithm to find optimal step, which is used in the main optimization program. Thanks to this, whole algorithm can find the solution in a faster way.

1.2 Newton's method

Newton's method is a local optimization type algorithm. To implement the method, stop criteria should be also known. In this case we have three main stop criteria for Newton's method and one additional that ensures that program stops counting after exceeding maximum compute time.

a. $< gradf(x) \cdot gradf(x) > \leqslant \epsilon_1$

b. $||x_i - x_{i-1}|| \leqslant \epsilon_2$

c. $|f(x_i) - f(x_{i-1})| \leqslant \epsilon_3$

d. max number of iterations

Newton's method in optimization is a numeric method, which is used to find local extrema in a defined, differentiable function f. In this method we need to construct a sequence x_n from initial point x_0 to x_* such that $f'(x_*) = 0$. Last point is a local extremum point which we are looking for.

1.2.1 The Newton's method iteration

Let x_0 be a point

2 Examples

2.1 Function with four local minima

Function equation:

$$y = x_1^4 + x_2^4 - 0.62x_1^2 - 0.62x_2^2 \tag{1}$$

Function figures:

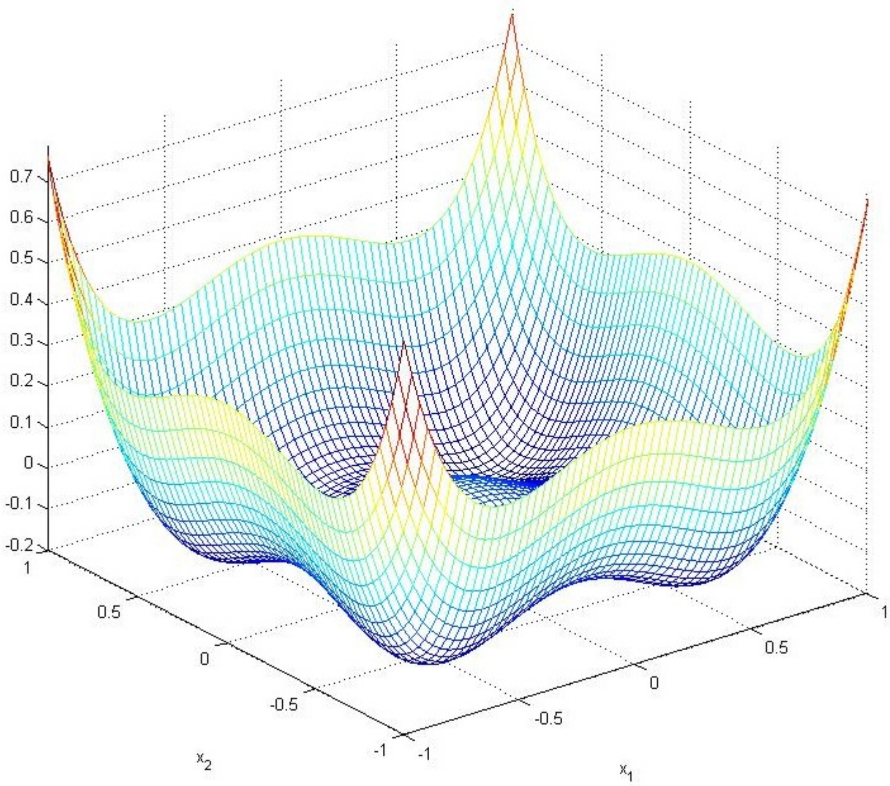

Figure 1: Analyzed function 3D view.

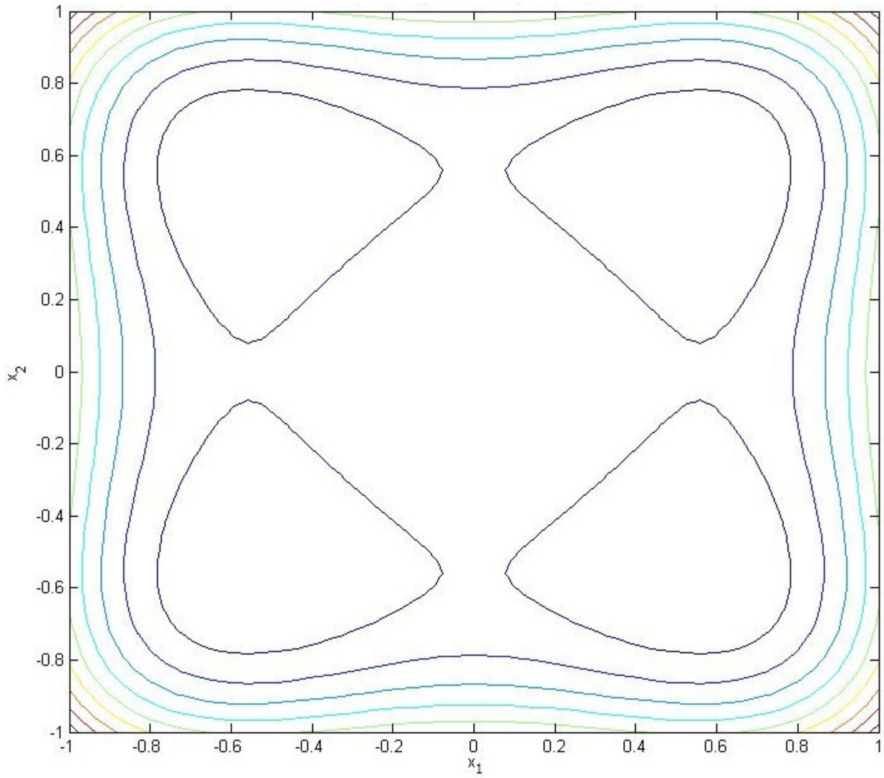

Figure 2: Analyzed function contour view.

Results:

iteration	point coordinates	function value	C_1 value	C_2 value	C_3 value
0	$1, 1$	0.76	0.132	-	-
1	$0.743, 0.743$	-0.0743	0.132	0.363	0.834
2	$0.61, 0.61$	-0.185	0.0358	0.189	0.11
3	$5.63 \cdot 10^{-1}, 5.63 \cdot 10^{-1}$	$-1.92 \cdot 10^{-1}$	$4.36 \cdot 10^{-3}$	$6.6 \cdot 10^{-2}$	$7.5 \cdot 10^{-3}$
4	$5.63 \cdot 10^{-1}, 5.63 \cdot 10^{-1}$	$-1.92 \cdot 10^{-1}$	$7.35 \cdot 10^{-5}$	$6.6 \cdot 10^{-2}$	$7.5 \cdot 10^{-3}$

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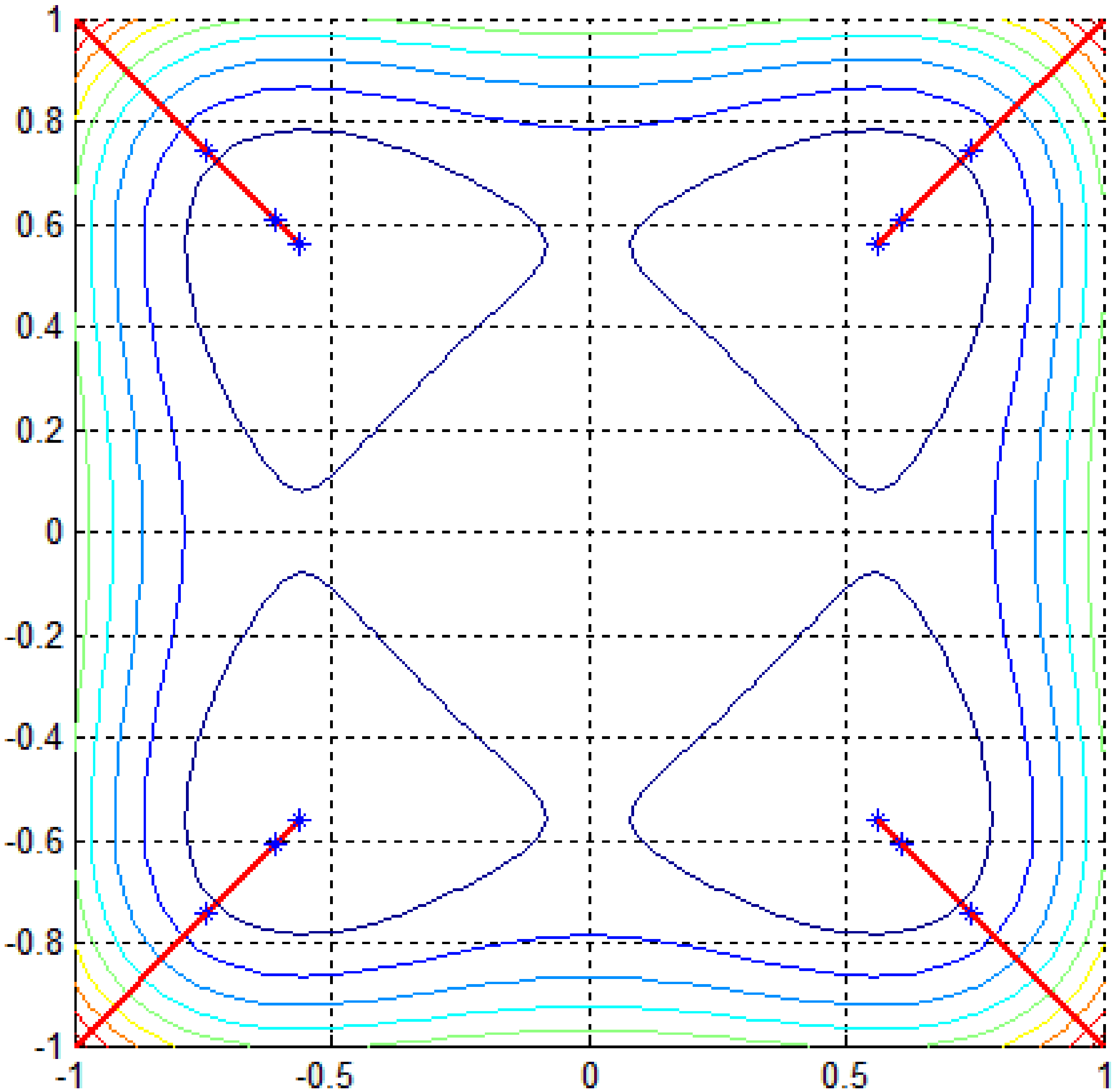

Figure 3: Location of four minima.