Newton's method

Dominik Koszkul
Michał Oleszczyk

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1 Introduce

1.1 Newton's method

Newton's method in optimization is a numeric method and is used to find local extrema/roots in a defined, differentiable function *f*. In this method we need to construct a sequence x_n from initial point x_0 to x_* such that $f'(x_*) = 0$. Last point is a local extremum point which we are looking for.

1.1.1 The Newton's method iteration

Let x_0 be a point

2 Examples

2.1 Function with four local minima

Function equation:

$$y = x_1^4 + x_2^4 - 0.62x_1^2 - 0.62x_2^2 \tag{1}$$

Function figures:

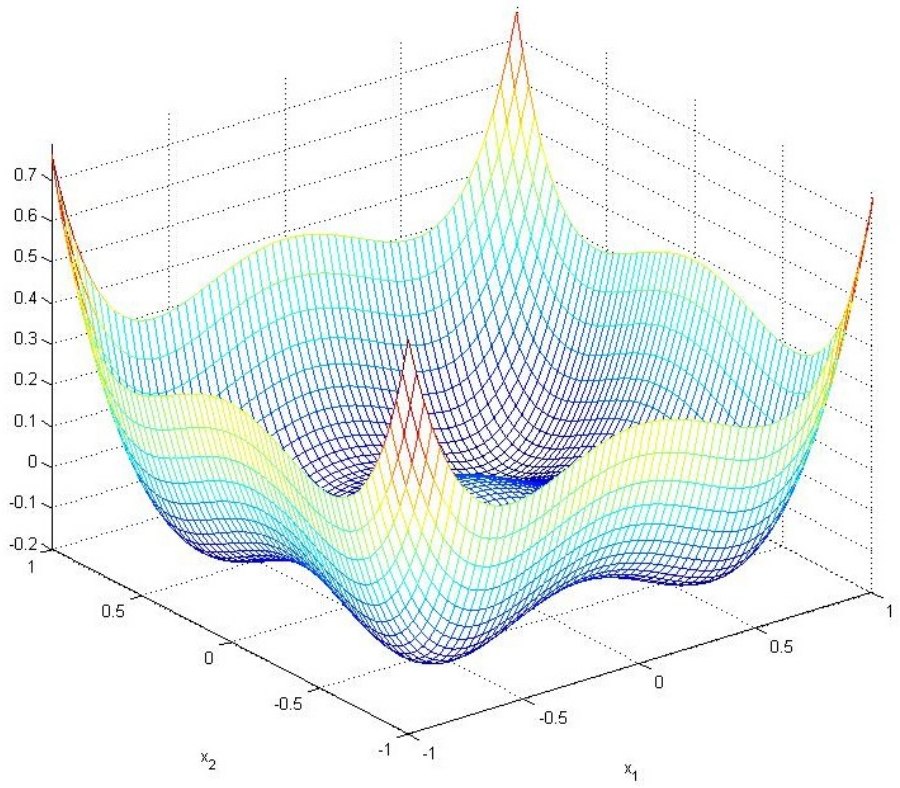

Figure 1: Analyzed function 3D view.

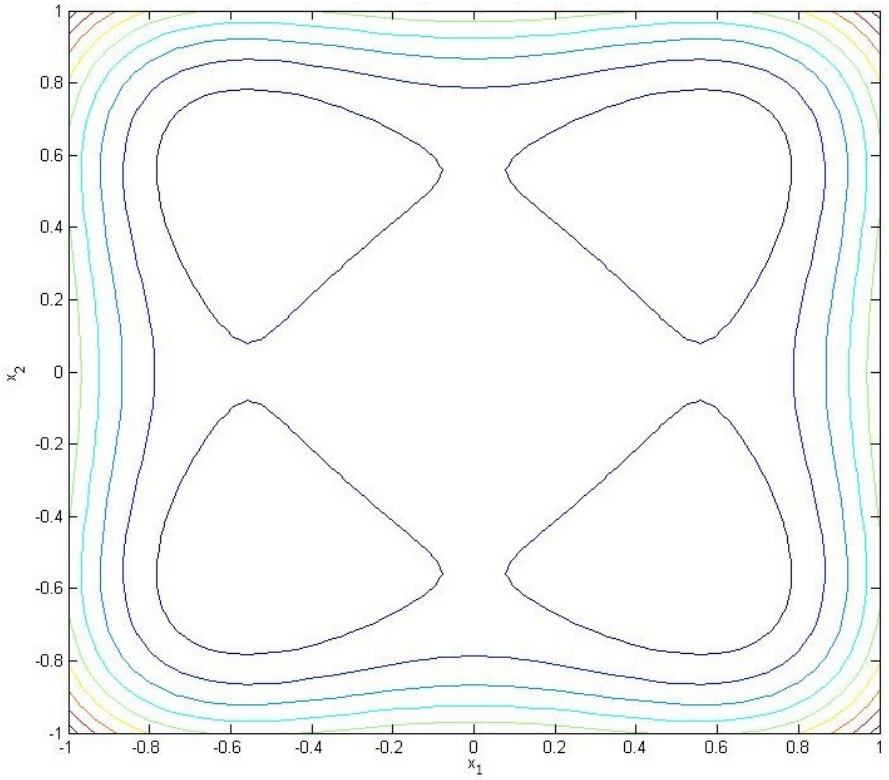

Figure 2: Analyzed function contour view.

Results:

iteration	point coordinates	function value	C_1 value	C_2 value	C_3 value
0	$1, 1$	0.76	0.132	-	-
1	$0.743, 0.743$	-0.0743	0.132	0.363	0.834
2	$0.61, 0.61$	-0.185	0.0358	0.189	0.11
3	$5.63 \cdot 10^{-1}, 5.63 \cdot 10^{-1}$	$-1.92 \cdot 10^{-1}$	$4.36 \cdot 10^{-3}$	$6.6 \cdot 10^{-2}$	$7.5 \cdot 10^{-3}$
4	$5.63 \cdot 10^{-1}, 5.63 \cdot 10^{-1}$	$-1.92 \cdot 10^{-1}$	$7.35 \cdot 10^{-5}$	$6.6 \cdot 10^{-2}$	$7.5 \cdot 10^{-3}$

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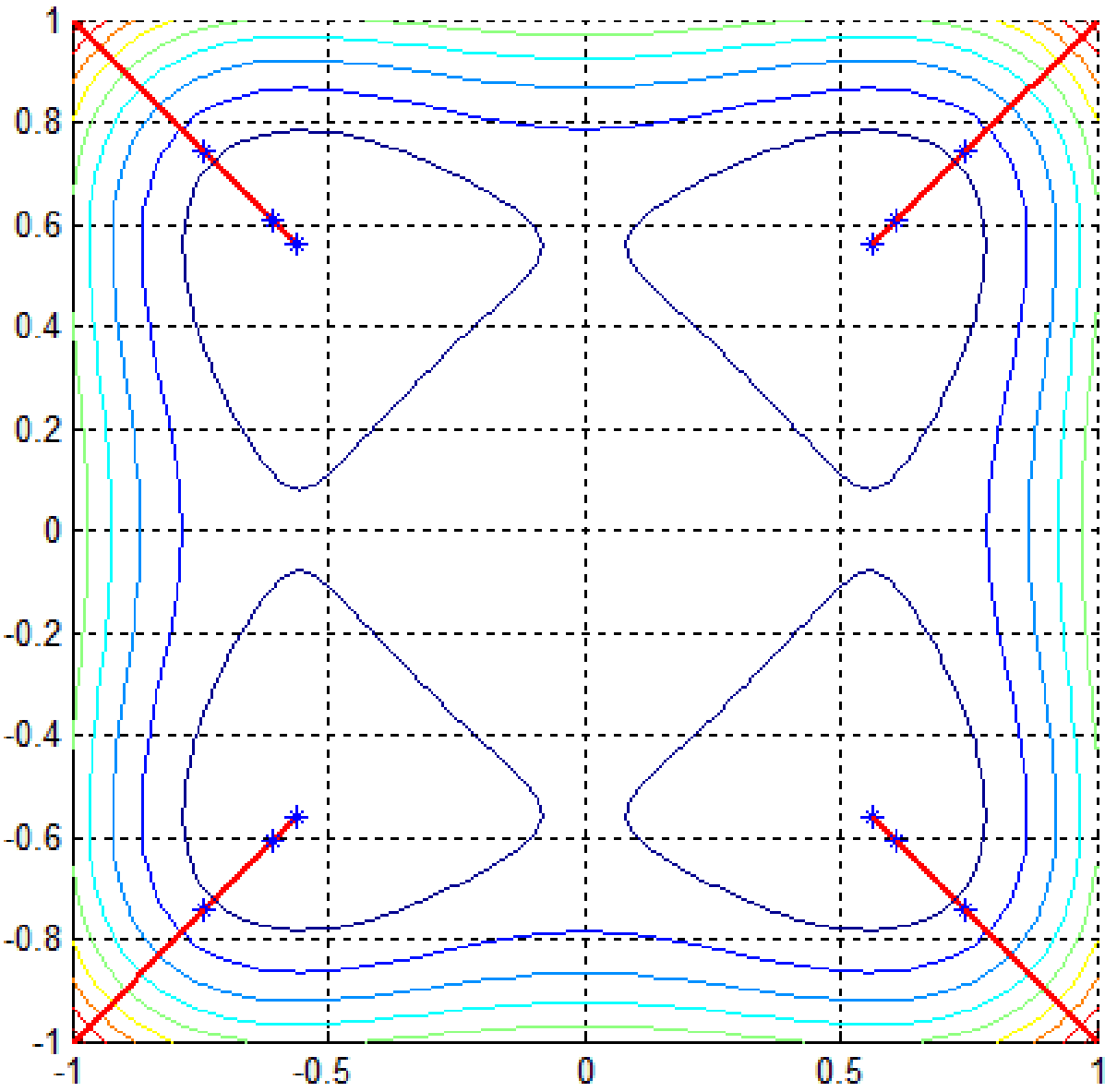

Figure 3: Location of four minima.