# Gauss Seidal Iteration Method to solve Simultaneous Linear Equations

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## Gauss Seidal Iteration Method- Working Rule

- $a_1x + b_1y + c_1z = d_1$
  $a_2x + b_2y + c_2z = d_2$
  $a_3x + b_3y + c_3z = d_3$
- Rearrange theb system so that each leading diagonal coefficient is larger (in absolute value) than any other in its row.
- $x = \frac{1}{a_1}(d_1 - b_1y - c_1z)$
  $y = \frac{1}{b_2}(d_2 - a_2x - c_2z)$
  $z = \frac{1}{c_3}(d_3 - a_3x - b_3y)$
- Initial approximation is usually taken as $x_0 = 0, y_0 = 0, z_0 = 0$
- The most recent approximation is used while proceeding to next iteration.
- Stop the iteration when the values in the two successive iterations are equal.

## Problem

Apply Gauss seidal iteration method to solve the system

$5x - y = 9$

$-x + 5y - z = 4$

$-y + 5z = -6$

## Solution

Here the given system of equations satisfies the diagonal domination conditions. We approximate the values upto four places of decimal.

$x = \frac{1}{5}[9 + y]$

$y = \frac{1}{5}[4 + x + z]$

$z = \frac{1}{5}[-6 + y]$

Let $x_0 = 0, y_0 = 0, z_0 = 0$ be the initial approximation.

## Solution

First approximation

$$x_1 = \frac{1}{5}[9 + 0] = 1.8$$
$$y_1 = \frac{1}{5}[4 + 1.8 + 0] = 1.16$$
$$z_1 = \frac{1}{5}[-6 + 1.16] = -0.968$$

Second approximation

$$x_2 = \frac{1}{5}[9 + 1.16] = 2.032$$
$$y_2 = \frac{1}{5}[4 + 2.032 - 0.968] = 1.0128$$
$$z_2 = \frac{1}{5}[-6 + 1.0128] = -0.9974$$

## Solution

Third approximation

$$x_3 = \frac{1}{5}[9 + 1.0128] = 2.0026$$
$$y_3 = \frac{1}{5}[4 + 2.0026 - 0.9974] = 1.0010$$
$$z_3 = \frac{1}{5}[-6 + 1.0010] = -0.9998$$

Fourth approximation

$$x_4 = \frac{1}{5}[9 + 1.0010] = 2.0002$$
$$y_4 = \frac{1}{5}[4 + 2.0002 - 0.9998] = 1.0000$$
$$z_4 = \frac{1}{5}[-6 + 1.0000] = -1.0000$$

Fifth approximation

$$x_5 = \frac{1}{5}[9 + 1.0010] = 2.0000$$
$$y_5 = \frac{1}{5}[4 + 2.0000 - 0.9998] = 1.0000$$
$$z_5 = \frac{1}{5}[-6 + 1.0000] = -1.0000$$

The solution is $x = 2, y = 1, z = 1$

## AssignmentProblems

1. Apply Gauss seidal iteration method to solve the system
   $27x + 6y - z = 85$
   $x + y + 54z = 110$
   $6x + 15y + 2z = 72$
2. Apply Gauss seidal iteration method to solve the system
   $20x + y - 2z = 17$
   $3x + 20y - z = -18$
   $2x - 3y + 20z = 25$
3. Apply Gauss seidal iteration method to solve the system
   $x + 3y + 10z = 24$
   $28x + 4y - z = 32$
   $2x + 17y + 4z = 35$

# Thank You