

② If the sum of the mean and var of binomial distribution for 5 trials is 1.8. Find the probability distribution function.

$$\text{Mean } \mu = np$$

$$\text{Variance } \sigma^2 = npq$$

$$np + npq = 1.8$$

$$np(1+q) = 1.8$$

$$np(1+1-p) = 1.8$$

$$= np(2-p) = 1.8$$

$$\Rightarrow 5p(2-p) = 1.8$$

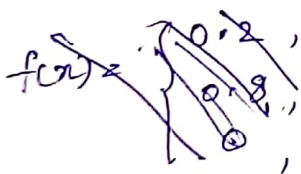
$$\Rightarrow -5p^2 + 10p - 1.8 = 0$$

$$\Rightarrow 5p^2 - 10p + 1.8 = 0$$

$$\Rightarrow p = \frac{9}{5}, \frac{1}{5} \quad p \neq \frac{9}{5}$$

$$\therefore p = \frac{1}{5} = 0.2$$

$$q = 0.8$$



$$p = 0.2 \quad q = 0.8$$

~~f(x)~~ x

$$f(x) = \begin{cases} 0.2 & \text{if success} \\ 0.8 & \text{otherwise} \end{cases}$$

$$\text{PDF} = \begin{cases} 5C_x \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{5-x} & x=0, \dots, 5 \\ 0 & \text{otherwise} \end{cases}$$

③ It is known that 2% of the accounts in company are delinquent. If 5 accounts selected at random.

compute probabilities

① at most 2 accounts will be deliq.

② At most 4 will be \quad

$$p = 2\% = 0.02 \quad q = 1-p = 0.98$$

$$n = 5$$

① Let X denote the delinquent account then $X \sim B(x, n, p)$ $P(X=x) = {}^nC_x p^x q^{n-x}$

$$\begin{aligned} \text{① } P[X \leq 2] &= P[X=0] + P[X=1] + P[X=2] \\ &= {}^5C_0 p^0 q^5 + {}^5C_1 p^1 q^4 + {}^5C_2 p^2 q^3 \\ &= 1 + 5 \times 0.02 \times 0.98^4 + 10 \times 0.02^2 \times 0.98^3 \\ &= 0.9999 \end{aligned}$$

② At most 4

$$\begin{aligned} P[X \leq 4] &= 1 - P[X > 4] \\ &= 1 - P[X = 5] \\ &= 1 - {}^5C_5 p^5 q^0 \\ &= 1 - 0.02^5 = 0.9999 \end{aligned}$$

4. A random var X takes the val $-3, -2, -1, 0, 1, 2, 3$ such that

$$P(X=0) = P(X>0) = P(X<0)$$

and

$$P(X=-3) = P(X=-2) = P(X=-1) \\ = P(X=1) = P(X=2) = P(X=3)$$

obtain probability dist of fun of X

~~Let the prob of~~

$$f(x) = \begin{cases} k, & x = -3, -2, -1, 1, 2, 3 \\ 3k, & x = 0 \end{cases}$$

$$\therefore \sum_{x=-\infty}^{\infty} f(x) = 1 \Rightarrow \cancel{4k} \frac{1}{4}$$

$$\Rightarrow 6k + 3k = 1$$

$$\Rightarrow 9k = 1 \Rightarrow k = \frac{1}{9}$$

| | | | | | | | |
|--------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $x =$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| $f(x)$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{3}$ | $\frac{1}{9}$ | $\frac{1}{9}$ | $\frac{1}{9}$ |

$$\therefore f(x) = \begin{cases} \frac{1}{9}, & x = -3, -2, -1, 1, 2, 3 \\ \frac{1}{3}, & x = 0 \\ 0, & \text{otherwise} \end{cases}$$

Thursday 1

A coin is biased so that head
twice as likely to appear as the
tail. The coin is tossed twice. find
expectance also variance of head.
Let X be the number of head.

$$\therefore f(x) = \begin{cases} 2k & x=0, 2 \\ k & x=1 \end{cases}$$

$$\sum_{-\infty}^{\infty} f(x) = 1 \Rightarrow 3k = 1 \Rightarrow k = \frac{1}{3}$$

$$p = P(\text{Head}) = \frac{2}{3} \quad q = \frac{1}{3}$$

$$n = \text{No of trials} = 2$$

$$\therefore \text{Expectance of binomial disto} \\ \text{is } = np$$

$$= 2 \times \frac{2}{3} = \underline{\underline{\frac{4}{3}}}$$

$$\text{Variance of binomial distribution} = npq$$

$$= 2 \cdot \frac{2}{3} \cdot \frac{1}{3}$$

$$= \underline{\underline{\frac{4}{9}}}$$

1. A die is tossed thrice.
A success is getting 1 or 6. Find
mean & variance of the success.

$$n = 3$$

$$1) P(\text{Getting 1 or 6}) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = p$$

$$q = \frac{4}{6}$$

$$\text{Mean} = np = 3 \times \frac{2}{6} = 1$$

$$2) \text{ Variance} = npq = 3 \times \frac{2}{6} \times \frac{4}{6} = \frac{4}{3}$$

2. A random var. X has the following
prob func

| | | | | | | |
|--------------|---|-----|------|------|------|-------------|
| Value of X | 0 | 1 | 2 | 3 | 4 | 5-6 |
| $X:$ | 0 | 1 | 2 | 3 | 4 | 5-6 |
| $f(x)P(X):$ | 0 | k | $2k$ | $2k$ | $3k$ | $k^2 + k^2$ |
| | | | | | | $7k^2$ |

$$\frac{1}{9} \text{ We know that } \sum_{x=-\infty}^{\infty} f(x) = 1$$

$$\therefore k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 = 1$$

$$\Rightarrow 10k^2 + 9k - 1 = 0$$

$$\Rightarrow k = \frac{1}{10} \text{ or } -1$$

$$k \neq -1 \text{ since } P(X=1) \neq 0$$

$$\therefore k = \frac{1}{10}$$

$$P(X \geq 6) = 2k^2 + 7k^2 + k = 9k^2 + k$$

$$= 9 \times \frac{1}{100} + \frac{1}{10} = \frac{9+10}{100} = \frac{19}{100}$$

$$P(X < 6) = 0 + k + 2k + 2k + 3k + k^2$$

$$= 8k + k^2$$

$$= \frac{8}{10} + \frac{1}{100} = \frac{81}{100}$$

$$\begin{aligned}
 P(3 < X \leq 6) &= 3k + k^2 + 2k^2 \\
 &= 3k + 3k^2 \\
 &= \frac{3}{10} + \frac{3}{100} \\
 &= \frac{33}{100}
 \end{aligned}$$