1. Apply Runge Kutta method to find approx value for y for $x = 0.2$ & 0.4 in step 0.2 if $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ given $(y(0)) = 1$

KOWSIK ND
ROLL NO 31

Given $h = 0.2$ $f(x_0, y_0) = \frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ $x_0 = 0$ $y_0 = 1$

$k_1 = hf(x_0, y_0)$

$= 0.2 f(0, 1)$

$= 0.2 \left(\frac{+1}{1}\right)$

$= 0.2000$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$= 0.2 f(0 + 0.1, 1 + 0.1)$

$= 0.2 f(0.1, 1.1)$

$= 0.2 \times \frac{60}{61}$

$= 0.19672$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$

$= 0.2 f(0.1, 1 + 0983)$

$= 0.19671$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.2 f(0.2, 1.19671)$

$= 0.18913$

$k = \frac{1}{6}[k_1 + k_4 + 2(k_2 + k_3)]$

$= 0.195998 \approx 0.196$

$\therefore y$ when $x = 0.2$

$y(0.2) = k + y_0 = 1.196$

$x_1 = 0.2$ $y_1 = 1.196$ $h = 0.2$

$k_1 = hf(x_1, y_1)$

$= 0.2 f(0.2, 1.196)$

$= 0.18911$

$k_2 = hf(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2})$

$= 0.2 f(0.3, 1.196 + \frac{0.18911}{2})$

$= \cancel{0.89448}$ 0.1795

$k_3 = 0.2 f(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2})$

$= 0.2 f(0.3,$

$= 0.17934$

$k_4 = 0.2 f(x_1 + h, y + k_3)$

$= 0.16880$

$k = \frac{h}{6}(k_1 + k + 2(k_2 + k_3))$

$= 0.1793$

$y(0.4) = y_1 + k = 1.3752$

Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial condition $y=1$ at $x=0$ find y for $x=0.1$ by Euler's method

Given $f(x,y) = \frac{y-x}{y+x}$ $y_0 = 1$ $x_0 = 0$ $y(0.1) = ?$

Let division be 10 division b/w 0 and 0.1

$\therefore h = 0.01$

x	y	$\frac{y-x}{y+x} = f(x,y)$	$0.01 f(x,y) + old\,y = new\,y$
0	1	1	1.01
0.01	1.01	0.98039	1.0198
0.02	1.0198	0.96153	1.0294
0.03	1.0294	0.94336	1.0388
0.04	1.0388	0.92584	1.0480
0.05	1.0480	0.9089	1.0570
0.06	1.0570	0.89256	1.0659
0.07	1.0659	0.8767	1.07467
0.08	1.07467	0.8614	1.08328
0.09	1.08328	0.84658	1.091745
0.1	1.091745		

$\therefore y(0.1) = 1.091745$