Unit 1

1. Let x denote the number that shown up when an unfair die is tossed. Find the probability dist of x if face 1 to 5 of the die are equally likely while face 6 is twice as likely as any other.

Answer

Let x denote the number that shown on unfair die

Let the probability function be

$$f(x) = \begin{cases} k & 1 \le x \le 5 \\ 2k & x = 6 \\ 0 & \text{otherwise} \end{cases}$$

We know that

$$\sum_{x=-\infty}^{\infty} f(x) = 1$$

$\therefore 5k + 2k = 1$

$k = \frac{1}{7}$

~~x 1 2 3 4 5 6~~

~~f(x) 1/3 1/3 1/3 1/3 1/3 2/3~~

$x =$	1	2	3	4	5	6
$f(x) =$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{2}{7}$

2. X and Y are random variable with $Y = -2X+3$. If we know that $E(Y) = 1$ and $E(Y^2) = 9$ find $E(X)$ and $V(X)$

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Ans

$Y = -2X + 3$

$E(Y) = 1 \quad E(Y^2) = 9$

$E(Y) = E(-2X+3) = 1$

$\Rightarrow -2E(X) + 3 = 1$

$\Rightarrow E(X) = 1$ —— (1)

$E(Y^2) = E((-2X+3)^2) = 9$

$E(4X^2 + 9 - 12X) = 9$

$4E(X^2) - 12E(X) + 9 = 9$

$E(X^2) = \frac{12E(X)}{4} = 3E(X) = 3$ —— (2)

$V(X) = E(X^2) - [E(X)]^2$

$= 3 - (1)^2$

$= 3 - 1 = 2$

$V(X) = 2$

3. Random variable X takes values 1, 2, 3 and 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$. Find PMF and CDF of X

We know that $\sum_{-\infty}^{\infty} f(x) = 1$.

$\therefore P(X=1) + P(X=2) \cdots P(X=4) = 1$

$P(X=1) + \frac{2}{3}P(X=1) + 2P(X=1) + \frac{2}{5}P(X=1) = 1$

$\left(1 + \frac{2}{3} + 2 + \frac{2}{5}\right)(P(X=1)) = 1$

$\frac{61}{15} P(X=1) = 1$

$P(X=1) = \frac{15}{61}$

PMF

X	1	2	3	4
f(x)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

CDF

$CDF = F(x) = \sum_{-\infty}^{x} f(x)$

X =	1	2	3	4
F(x) =	$\frac{15}{61}$	$\frac{25}{61}$	$\frac{55}{61}$	1

4) X is discrete random variable PMF

$P_X(x) = \begin{cases} \frac{1}{21} & x \in (-10, 9 \dots 10\} \\ 0 & \text{otherwise} \end{cases}$

Define a new random variable Y as

$Y = \begin{cases} 0 & \text{if } X \le 0 \\ X & \text{if } 0 < X < 5 \\ 5 & \text{if } X \ge 5 \end{cases}$

Find PMF (Y), E(Y), V(Y)

Answer

$Y = \begin{cases} 0 & \text{if } X \le 0 \\ X & \text{if } X \le 5 \\ 5 & \text{if } X \ge 5 \end{cases}$

Here X is $\{-10, 9, \dots 7, 0, 1, \dots 10\}$

Total number of elements 21

$P(Y=0)P(X \le 0) = \frac{11}{21}$

$P(X < 5) = P(X=1) + \cdots P(X=4)$

$P(Y=X) = \frac{1}{21} + \frac{1}{21} + \cdots \frac{1}{21}$

$= \frac{4}{21}$

$P(Y=5) = P(X \ge 5) =$

$= P(X=5)$

$= \frac{6}{21}$

$\therefore f(Y) = \begin{cases} \frac{11}{21} & X \le 0 \\ 4/21 & 0 < X < 5 \\ 6/21 & X \ge 5 \end{cases}$

$E(Y) = \Sigma y f(y)$

$= 0 \cdot \frac{11}{21} + 1 \cdot \frac{4}{21} + 2 \cdot \frac{4}{21} + 3 \cdot \frac{}{2}$

$+ 4 \cdot \frac{4}{21} + 5 \cdot \frac{6}{21}$

$= 10 \cdot \frac{4}{21} + \frac{30}{21}$

$= \frac{70}{21} = \frac{10}{3} = \underline{3.333}$

$V(X) = E(X^2) - [E(X)]^2$

Here

$E(Y^2) = \Sigma y^2 f(y)$

$= 0 \cdot \frac{11}{21} +$

$(1 + 4 + 9 + 16) \frac{4}{21}$

$+ 25 \cdot \frac{6}{21}$

$= \frac{30 \cdot 4}{21} + \frac{25 \cdot 6}{21}$

$= \frac{120}{21} + \frac{150}{21}$

$= \frac{90}{7}$

$V(Y) = \frac{90}{7} - \left(\frac{10}{3}\right)^2$

$= \frac{110}{63} = \underline{1.7460}$

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