INVERSE LAPLACE TRANSFORM

KOWSIK ND
ROLL NO 31

1. $\frac{s^2-3s+8}{s^3}$

$F(s) = \frac{s^2-3s+8}{s^3}$

$L^{-1}[F(s)] = L^{-1}\left[\frac{s^2-3s+8}{s^3}\right]$

$= L^{-1}\left[\frac{1}{s} - \frac{3}{s^2} + \frac{8}{s^3}\right]$

$= L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{3}{s^2}\right] + L^{-1}\left[\frac{8}{s^3}\right]$

$= 1 - 3\cdot t + 8\cdot\frac{t^2}{2!}$

$= 4t^2 - 3t + 1$

2. $\frac{5}{s^2+3s+7}$

$L^{-1}\left[\frac{5}{s^2+3s+7}\right] = L^{-1}\left[\frac{5}{s^2+3s+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+7}\right]$

$= L^{-1}\left[\frac{5}{\left(s+\frac{3}{2}\right)^2+\frac{19}{4}}\right]$

$= 5\sqrt{\frac{4}{19}}\, L^{-1}\left[\frac{\sqrt{\frac{19}{4}}}{\left(s+\frac{3}{2}\right)^2+\frac{19}{4}}\right]$

$= \sqrt{\frac{100}{19}}\cdot \sin\left(\sqrt{\frac{19}{4}}\,t\right)\cdot e^{-3/2\,t}$

3. $\frac{s}{s^2+6s+25}$

$L^{-1}\left[\frac{s}{s^2+6s+25}\right] = L^{-1}\left[\frac{s}{s^2+6s+9-9+25}\right]$

$= L^{-1}\left[\frac{(s+3)-3}{(s+3)^2+4^2}\right]$

$= \cos(4t)\cdot e^{-3t} - \frac{3\sin 4t\cdot e^{-3t}}{4}$

4. $\frac{3s-2}{s^2+2s+6}$

$L^{-1}\left[\frac{3s-2}{s^2+2s+6}\right] = L^{-1}\left[\frac{3(s+1)+1}{(s+1)^2+5}\right]$

$= 3\,L^{-1}\left[\frac{(s+1)}{(s+1)^2+\sqrt{5}^2}\right] + \frac{1}{\sqrt{5}}L^{-1}\left[\frac{\sqrt{5}}{(s+1)^2+\sqrt{5}^2}\right]$

$= e^{-t}\left[3\cos(\sqrt{5}\,t) + \frac{1}{\sqrt{5}}\sin(\sqrt{5}\,t)\right]$

5. $\frac{7s+4}{4s^2+4s+9}$

$L^{-1}\left[\frac{7s+4}{4s^2+4s+9}\right] = \frac{1}{4}L^{-1}\left[\frac{7s+4}{s^2+s+\frac{9}{4}}\right]$

$= \frac{1}{4}\left[7\,L^{-1}\left[\frac{s}{s^2+s+\frac{9}{4}}\right] + L^{-1}\left[\frac{4}{s^2+s+\frac{9}{4}}\right]\right]$

$= \frac{1}{4}\left[7\cdot L^{-1}\left[\frac{\left(s+\frac{1}{2}\right)-\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2-\frac{1}{4}+\frac{9}{4}}\right] + L^{-1}\left[\frac{4}{\left(s+\frac{1}{2}\right)^2-\frac{1}{4}+\frac{9}{4}}\right]\right]$

$= \frac{1}{4}\left[7\cdot L^{-1}\left(\frac{\left(s+\frac{1}{2}\right)-\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+2}\right) + L^{-1}\left[\frac{\frac{4}{\sqrt{2}}\sqrt{2}}{\left(s+\frac{1}{2}\right)^2+2}\right]\right]$

$= \frac{7}{4}\cos(\sqrt{2}\,t)\,e^{-1/2\,t} - \frac{7}{4}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\sin(\sqrt{2}\,t)e^{-1/2\,t}$

$+ \frac{1\cdot 4}{4\sqrt{2}}\sin\sqrt{2}\,t\cdot e^{-1/2\,t}$

$= e^{-1/2\,t}\left[\frac{7}{4}\cos\sqrt{2}\,t + \frac{\sqrt{2}}{16}\sin\sqrt{2}\,t\right]$

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