Unit 1

1. Let X denote the number that shown up when an unfair die is tossed. Find the probability dist of X if face 1 to 5 of the die are equally likely while face 6 is twice as likely as any other.

Answer

~~Let~~ X denote the number that shown on unfair die

Let the probability function be

$$f(x) = \begin{cases} k & 1 \le x \le 5 \\ 2k & x = 6 \\ 0 & \text{otherwise} \end{cases}$$

We know that

$$\sum_{x=-\infty}^{\infty} f(x) = 1$$

$$\therefore 5k + 2k = 1$$

$$k = \frac{1}{7}$$

x =	1	2	3	4	5	6
$f(x)$ =	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{2}{7}$

2. X and Y are random variable with $Y = -2X + 3$. If we know that $E(Y) = 1$ and $E(Y^2) = 9$ find $E(X)$ and $V(X)$

Ans

$Y = -2X + 3$

$E(Y) = 1 \quad E(Y^2) = 9$

$E(Y) = E(-2X + 3) = 1$

$\Rightarrow -2E(X) + 3 = 1$

$\Rightarrow E(X) = 1$ —— (1)

$E(Y^2) = E((-2X + 3)^2) = 9$

$E(4X^2 + 9 - 12X) = 9$

$4E(X^2) - 12E(X) + 9 = 9$

$E(X^2) = \frac{12E(X)}{4} = 3E(X) = 3$ —— (2)

$V(X) = E(X^2) - [E(X)]^2$

$= 3 - (1)^2$

$= 3 - 1 = 2$

$V(X) = 2$

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3. Random variable X takes values 1, 2, 3 and 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$. Find PMF and CDF of X

We know that $\sum_{-\infty}^{\infty} f(x) = 1$.

$\therefore P(X=1) + P(X=2) \cdots P(X=4) = 1$

$P(X=1) + \frac{2}{3}P(X=1) + 2P(X=1) + \frac{2}{5}P(X=1) = 1$

$\left(1+\frac{2}{3}+2+\frac{2}{5}\right)(P(X=1)) = 1$

$\frac{61}{15}P(X=1) = 1$

$P(X=1) = \frac{15}{61}$

PMF

X	1	2	3	4
$f(x)$	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

CDF

$CDF = F(x) = \sum_{-\infty}^{x} f(x)$

$x =$	1	2	3	4
$F(x) =$	$\frac{15}{61}$	$\frac{25}{61}$	$\frac{55}{61}$	1

4) X is discrete random variable PMF

$$P_X(x) = \begin{cases} \frac{1}{21} & x \in (-10, 9 \ldots 10\} \\ 0 & \text{otherwise} \end{cases}$$

Define a new random variable Y as

$$Y = \begin{cases} 0 & \text{if } X \le 0 \\ X & \text{if } 0 < X < 5 \\ 5 & \text{if } X \ge 5 \end{cases}$$

Find PMF (Y), E(Y), V(Y)

Answer

$$Y = \begin{cases} 0 & \text{if } X \le 0 \\ X & \text{if } 0 < X < 5 \\ 5 & \text{if } X \ge 5 \end{cases}$$

Here X is $\{-10, 9, \ldots 7, 0, 1, \ldots 10\}$

Total number of elements 21

$P(Y=0)P(X \le 0) = 11/21$

$P(0 < X < 5) = P(X=1) + \cdots P(X=4)$

$P(Y=X) = \frac{1}{21} + \frac{1}{21} + \cdots \frac{1}{21}$

$= \frac{4}{21}$

$P(Y=5)\,P(X \ge 5) = 6P(X=5)$

$= \frac{6}{21}$

$$\therefore f(Y) = \begin{cases} \frac{11}{21} & X \le 0 \\ 4/21 & 0 < X < 5 \\ 6/21 & X \ge 5 \end{cases}$$

$E(Y) = \Sigma y f(y)$

$= 0 \cdot \frac{11}{21} + 1 \cdot \frac{4}{21} + 2 \cdot \frac{4}{21} + 3 \cdot \frac{}{2}$

$+ 4 \cdot \frac{4}{21} + 5 \cdot \frac{6}{21}$

$= 10 \cdot \frac{4}{21} + \frac{30}{21}$

$= \frac{70}{21} = \frac{10}{3} = \underline{3.333}$

$V(X) = E(X^2) - [E(X)]^2$

Here

$E(Y^2) = \Sigma y^2 f(y)$

$= 0 \cdot \frac{11}{21} +$

$(1 + 4 + 9 + 16) \frac{4}{21}$

$+ 25 \cdot \frac{6}{21}$

$= \frac{30 \cdot 4}{21} + \frac{25 \cdot 6}{21}$

$= \frac{120}{21} + \frac{150}{21}$

$= \frac{90}{7}$

$V(Y) = \frac{90}{7} - \left(\frac{10}{3}\right)^2$

$= \frac{110}{63} = \underline{1.7460}$

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1. If X is uniformly distributed over $(-\alpha, \alpha)$, $\alpha > 0$ find α so that

(i) $P(X>1) = \frac{1}{3}$

(ii) $P(|x|<1) = P(|x|>1)$

(i) $P(X>1) = 1 - P(X \le 1)$

$$= 1 - \int_{\alpha}^{x} \frac{1}{b-a}\,dx$$

$$= 1 - \int_{-\alpha}^{1} \frac{1}{2\alpha}\,dx \qquad \begin{array}{l} b=\alpha \\ a=-\alpha \end{array}$$

$$= 1 - \frac{[x]_{-\alpha}^{1}}{2\alpha}$$

$$= 1 - \frac{[1+\alpha]}{2\alpha}$$

$$= \frac{2\alpha - 1 - \alpha}{2\alpha}$$

$$= \frac{\alpha - 1}{2\alpha}$$

$$\frac{\alpha-1}{2\alpha} = \frac{1}{3}$$

$$3\alpha - 3 = 2\alpha$$

$$\alpha = 3$$

(ii) $P(|x|>1) = P(|x|<1)$

$\Rightarrow 2P(|x|<1) = 1$

$\Rightarrow P(|x|<1) = \frac{1}{2}$

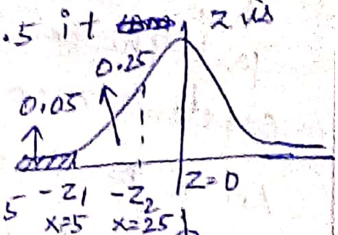
$\Rightarrow \int_{-1}^{1} \frac{1}{2\alpha}\,dx$

$\Rightarrow \frac{2}{2\alpha} = \frac{1}{2} \Rightarrow \frac{1}{\alpha} = \frac{1}{2}$

$\alpha = 2$

2. 5% of the observation in a normal distribution are below 5 and 25% of the observation are between 5 and 25. Find mean and SD

So the area for value below 5 and in b/w 5 & 25 is 0.05 and 0.25 respectively. And since both are < 0.5 it z is $-ve$.



$P(X<5) = 0.05$

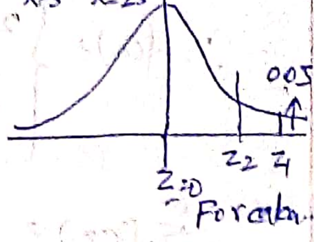
$\Rightarrow 0.5 - P(z_1 < z < 0) = 0.05$

$\Rightarrow P(0<z<z_1) = 0.45$

$F(z_1) = 0.45$

$\Rightarrow z_1 = 1.64$

Since its on left side.



$\frac{5-\mu}{\sigma} = -1.64 \Rightarrow \mu - 1.64\sigma = 5$ — (1)

$P(5<X<25) = P(-z_1 < z < -z_2) = 0.25$

For calculation

$P(z_2 < z < z_1) = 0.25$

$\Rightarrow P(0<z<z_1) - P(0<z<z_2) = 0.25$

$\Rightarrow F(z_1) - F(z_2) =$

$\Rightarrow 0.45 - F(z_2) = 0.25$

$F(z_2) = 0.45 - 0.25 = 0.20$

$\therefore z_2 = 0.52$

Since its on left side

$\frac{25-\mu}{\sigma} = -0.52$

$\Rightarrow \mu - 0.52\sigma = 25$ — (2)

From (1) & (2)

$\mu = 34.286 \quad \sigma = 17.857$

$\therefore$ Mean $= 34.29$ SD $= 17.857$

3. Find the value of k for the probability density f(x) given below and hence find its mean & variance

$$f(x) \begin{cases} kx^3 & 0<x<1 \\ 0 & \text{otherwise.} \end{cases}$$

$$\sum_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} f(x) = 1$$

$$\Rightarrow \int_0^1 kx^3 dx = 1 \Rightarrow k \cdot \frac{[x^4]_0^1}{4} = 1$$

$$\Rightarrow \frac{k}{4} = 1 \Rightarrow \underline{k=4}$$

$$\mu = \text{Mean} = \int_{-\infty}^{\infty} x f(x) dx = \int_0^1 x\, 4x^3 dx$$

$$= \frac{4[x^5]_0^1}{5} = \frac{4}{5}$$

$$\text{Variance} = E(X^2) - \mu^2$$

$$= \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$$

$$= \int_0^1 x^2\, 4x^3 dx - \frac{16}{25}$$

$$= \frac{4[x^6]_0^1}{6} - \frac{16}{25}$$

$$= \frac{4}{6} - \frac{16}{25}$$

$$= \frac{2}{75}$$

4. The amount of time that surveillance camera will run without having to be rest is a random variable having exponential distribution with the parameter 50 days. Find the prob that such a camera will

(i) have to be reset in less than 20 days.

(ii) not have to be reset in atleast 60 days.

~~(i) P(~~

Let X be the amount of time

$$f(x) = \frac{1}{50} e^{-x/50} \quad x \geq 0$$

(i) $P(X<20) = \frac{1}{50}\int_0^{20} e^{-x/50} dx$

$$= \frac{1}{50} \cdot \frac{-50}{1} \left[e^{-x/150}\right]_0^{20}$$

$$= -\left[e^{-x/50}\right]_0^{20}$$

$$= 1 - e^{-2/5}$$

(ii) P(not have to rest atleast 60 days)

$$= P(X > 60)$$

$$= \frac{1}{50}\int_{60}^{\infty} f(x)dx = \frac{1}{50}\left[\frac{-50}{1}\left[e^{-x/150}\right]_{60}^{\infty}\right]$$

$$= -1\left[e^{-x/50}\right]_{60}^{\infty}$$

$$= e^{-6/5}$$