Kousik Nandagopan D
Roll No 81
CSE S4

1. If X is uniformly distributed over $(-\alpha, \alpha)$, $\alpha > 0$ find α so that

(i) $P(X>1) = \frac{1}{3}$

(ii) $P(|x|<1) = P(|x|>1)$

(i) $P(X>1) = 1 - P(X \le 1)$

$= 1 - \int_{\alpha}^{x} \frac{1}{b-a}\,dx$

$= 1 - \int_{-\alpha}^{1} \frac{1}{2\alpha}\,dx \qquad \begin{array}{l} b=\alpha \\ a=-\alpha \end{array}$

$= 1 - \frac{[x]_{-\alpha}^{1}}{2\alpha}$

$= 1 - \frac{[1+\alpha]}{2\alpha}$

$= \frac{2\alpha - 1 - \alpha}{2\alpha}$

$= \frac{\alpha - 1}{2\alpha}$

$\frac{\alpha-1}{2\alpha} = \frac{1}{3}$

$3\alpha - 3 = 2\alpha$

$\alpha = 3$

(ii) $P(|x|>1) = P(|x|<1)$

$\Rightarrow 2P(|x|<1) = 1$

$\Rightarrow P(|x|<1) = \frac{1}{2}$

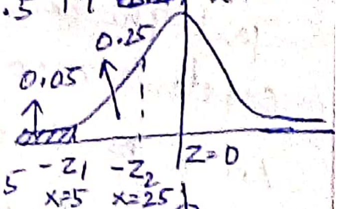
$\Rightarrow \int_{-1}^{1} \frac{1}{2\alpha}\,dx$

$\Rightarrow \frac{2}{2\alpha} = \frac{1}{2} \Rightarrow \frac{1}{\alpha} = \frac{1}{2}$

$\alpha = 2$

2. 5% of the observation in a normal distribution are below 5 and 25% of the observation are between 5 and 25 find mean and SD

So the area for value below 5 and in b/w 5 & 25 is 0.05 and 0.25 respectively. And since both are <0.5 it z is −ve.



$P(X<5) = 0.05$

$\Rightarrow 0.5 - P(z_1 < z < 0) = 0.05$

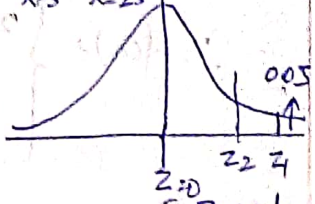
$\Rightarrow P(0<z<z_1) = 0.45$

$F(z_1) = 0.45$

$\Rightarrow z_1 = 1.64$

Since its on left side

$\frac{5-\mu}{\sigma} = -1.64 \Rightarrow \mu - 1.64\sigma = 5$ — (1)



$P(5<X<25) = P(-z_1 < z < -z_2) = 0.25$

For calculation

$P(z_2 < z < z_1) = 0.25$

$\Rightarrow P(0<z<z_1) - P(0<z<z_2) = 0.25$

$\Rightarrow F(z_1) - F(z_2) =$

$\Rightarrow 0.45 - F(z_2) = 0.25$

$F(z_2) = 0.45 - 0.25 = 0.20$

$\therefore z_2 = 0.52$

Since its on left side

$\frac{25-\mu}{\sigma} = -0.52$

$\Rightarrow \mu - 0.52\sigma = 25$ — (2)

From (1) & (2)

$\mu = 34.286 \quad \sigma = 17.857$

$\therefore$ Mean = 34.29 SD = 17.857

3. Find the value of K for the probabi density $f(x)$ given below and hence find its mean & variance

$$f(x) = \begin{cases} kx^3 & 0<x<1 \\ 0 & \text{otherwise.} \end{cases}$$

$$\sum_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} f(x) = 1$$

$$\Rightarrow \int_0^1 kx^3 dx = 1 \Rightarrow k \cdot \frac{[x^4]_0^1}{4} = 1$$

$$\Rightarrow \frac{k}{4} = 1 \Rightarrow \underline{k=4}$$

$$\mu = \text{Mean} = \int_{-\infty}^{\infty} x f(x) dx = \int_0^1 x\, 4x^3 dx$$

$$= \frac{4[x^5]_0^1}{5} = \underline{\frac{4}{5}}$$

$$\text{Variance} = E(X^2) - \mu^2$$

$$= \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$$

$$= \int_0^1 x^2\, 4x^3 dx - \frac{16}{25}$$

$$= \frac{4[x^6]_0^1}{6} - \frac{16}{25}$$

$$= \frac{4}{6} - \frac{16}{25}$$

$$= \underline{\frac{2}{75}}$$

4. The amount of time that surveillance camera will run without having to be rest is a random variable having exponent distribution with the parameter 50 days. Find the prob that such a camera will

(i) have to be reset in less than 20 days.

(ii) not have to be reset in atleast 60 days.

~~(i) P(~~

Let X be the amount of time

$$f(x) = \frac{1}{50} e^{-x/50} \quad x \geq 0$$

(i) $P(X<20) = \frac{1}{50}\int_0^{20} e^{-x/50} dx$

$$= \frac{1}{50} \cdot \frac{-50}{1} \left[e^{-x/150}\right]_0^{20}$$

$$= -\left[e^{-x/50}\right]_0^{20}$$

$$= \underline{1 - e^{-2/5}}$$

(ii) P(not have to rest atleast 60 days)

$= P(X > 60)$

$$= \frac{1}{50} \int_{60}^{\infty} f(x) dx = \frac{1}{50}\left[\frac{-50}{1}\left[e^{-x/150}\right]_{60}^{\infty}\right]$$

$$= -1\left[e^{-x/50}\right]_{60}^{\infty}$$

$$= \underline{e^{-6/5}}$$