Inverse Laplace Transform using Partial Fraction

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Definition

- If $L[f(t)] = F(s)$, then $f(t)$ is the inverse Laplace transform of $F(s)$ and is denoted by $L^{-1}[F(s)] = f(t)$.

- Linearity Property
  $L^{-1}[aF(s) + bG(s)] = aL^{-1}[F(s) + bL^{-1}[G(s)]$

- Shifting Property

  If $L[f(t)] = F(s)$, then $L^{-1}[F(s-a)] = e^{at}f(t)$

Table of Inverse Laplace Transforms

- $L^{-1}[\frac{1}{s}] = 1$
- $L^{-1}[\frac{1}{s^2}] = t$
- $L^{-1}[\frac{1}{s^{n+1}}] = \frac{t^n}{n!}, n = 1, 2, 3....$
- $L^{-1}[\frac{1}{s-a}] = e^{at}$
- $L^{-1}[\frac{a}{s^2+a^2}] = \sin at$
- $L^{-1}[\frac{s}{s^2+a^2}] = \cos at$
- $L^{-1}[\frac{a}{s^2-a^2}] = \sinh at$
- $L^{-1}[\frac{s}{s^2-a^2}] = \cosh at$

Methods of finding Inverse Transforms

There are six methods to inverse Laplace Transforms.

- Using Shifting Property
- Using Partial Fractions
- Using Derivaties
- Using Integration
- Using unit step function
- Using Convolution theorem

Inverse Using partial fraction

Find the inverse Laplace Transform of $\frac{s^2-10s+13}{s^3-4s^2+s+6}$

Solution

$\frac{s^2-10s+13}{s^3-4s^2+s+6} = \frac{s^2-10s+13}{(s+1)(s-2)(s-3)} = \frac{A}{s+1} + \frac{B}{s-2} + \frac{C}{s-3}$

$\Longrightarrow s^2-10s+13 = A(s-2)(s-3)+B(s+1)(s-3)+C(s+1)(s-2)$

Put $s=-1 \Longrightarrow A=2$

Put $s=2 \Longrightarrow B=1$

Put $s=3 \Longrightarrow C=-2$

$\frac{s^2-10s+13}{s^3-4s^2+s+6} = \frac{2}{s+1} + \frac{1}{s-2} - \frac{2}{s-3}$

$L^{-1}[\frac{s^2-10s+13}{s^3-4s^2+s+6}] = 2e^{-t} + e^{2t} - 2e^{3t}$

AssignmentProblems

1. $\frac{s+2}{(s+1)^2(s-2)}$

2. $\frac{3s+2}{(s-1)(s^2+1)}$

3. $\frac{5s+3}{(s-1)(s^2+2s+5)}$

4. $\frac{s}{s^4+s^2+1}$

5. $\frac{s^3}{s^4-a^4}$

Thank You