Numerical Integration

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Numerical Integration

- The process of evaluating a definite integral from a set of tabulated values of the integrand $f(x)$ is called numerical integration.
- This process when applied to a function of a single variable, is known as quadrature.
- We learn two methods of numerical integration
  1. Trapezoidal Rule
  2. Simpson's one-third rule.

Trapezoidal Rule

- $\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 \ldots y_{n-1})]$.
- Here $x_0, x_1, \ldots x_n$ are the x values and $y_0, y_1, \ldots y_n$ are the corresponding function values.
- h is the interval width.

Simposon's One-third Rule

- $\int_{x_0}^{x_0+nh} f(x)dx =$
  $\frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 \ldots y_{n-1}) + 2(y_2 + y_4 \ldots y_{n-2})]$
- Here $x_0, x_1, \ldots x_n$ are the x values and $y_0, y_1, \ldots y_n$ are the corresponding function values
- h is the interval width.

Problem

- Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using
  (i) Trapezoidal Rule
  (ii) Simpson's $\frac{1}{3}$ Rule
  and compare the results with the actual value.
- Solution:
  Divide the interval $(0, 6)$ into six parts each of width $h = 1$.
  The values of $f(x) = \frac{dx}{1+x^2}$ are given below:

x	0	1	2	3	4	5	6
$f(x) = y$	1	0.5	0.2	0.1	0.0588	0.0385	0.027

Using Trapezoidal Rule

- $\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 \ldots y_{n-1})]$
  $= \frac{1}{2}[(1 + 0.027) + 2(0.5 + 0.2 + 0.1 + 0.0588 + 0.0385)]$
  $= 1.1408.$

Using Simpson's One-Third Rule

- $\int_{x_0}^{x_0+nh} f(x)dx =$
  $\frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 \ldots y_{n-1}) + 2(y_2 + y_4 \ldots y_{n-2})]$
  $= \frac{1}{3}[(1 + 0.027) + 4(0.5 + 0.1 + 0.0385) + 2(0.2 + 0.0588)]$
  $= 1.3662.$

By Actual Integration

- $\int_0^6 \frac{dx}{1+x^2}$
  $\tan^{-1} x_0^6 = \tan^{-1} 6 = 1.4056$

Problems

- Evaluate $\int_0^{\frac{\pi}{2}} \cos x dx$ using trapezoidal rule choosing $n = 6$.
- Evaluate $\int_0^2 xe^x dx$ using Simpson's one third rule, choosing $n = 8$.

Thank You