Government Engineering College, Thrissur

MA202 - PROBABILITY DISTRIBUTIONS,TRANSFORMS AND NUMERICAL METHODS

Assignment -

Lagrange's Interpolation Formula

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1.] Interpolate the value of function corresponding to $x = 4$ using Legrange's interpolation formula from the following set of data.

$x = 4$

X	$x_0 = 2$	$x_1 = 3$	$x_2 = 5$	$x_3 = 8$	$x_4 = 12$
$f(x)$	10	15	25	40	60

$$P_0 = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} f(x_0)$$

$$= \frac{(4-3)(4-5)(4-8)(4-12)}{(2-3)(2-5)(2-8)(2-12)} 10$$

$$= \frac{-16}{9}$$

$$P_1 = \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} f(x_1)$$

$$= \frac{(4-2)(4-5)(4-8)(4-12)}{(3-2)(3-5)(3-8)(3-12)} 15$$

$$= \frac{32}{3}$$

$$P_2 = \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} f(x_2)$$

$$= \frac{(4-2)(4-3)(4-8)(4-12)}{(5-2)(5-3)(5-8)(5-12)} 25$$

$$= \frac{800}{63}$$

$$P_3 = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} f(x_3)$$

$$= \frac{(4-2)(4-3)(4-5)(4-12)}{(8-2)(8-3)(8-5)(8-12)} 40$$

$$= \frac{-16}{9}$$

$$P_4 = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} f(x_4)$$

$$= \frac{(4-2)(4-3)(4-5)(4-8)}{(12-2)(12-3)(12-5)(12-8)} 60 = \frac{4}{21}$$

$$f(4) = P_0 + P_1 + P_2 + P_3 + P_4 = 20$$

2] The population of Mississippi durin three census period was as follo Interpolate the population during 1

Year	1951	1961	1971
Population	2.8	3.2	4.5

Solution

X	x_0 0	x_1 1	x_2 2
$f(x)$	2.8	3.2	4.5

Taking scale as $\frac{1}{10}$ ∴ 1966 will be $x = 1.5$

$$P_0 = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0)$$

$$= \frac{(1.5-1)(1.5-2)}{(0-1)(0-2)} 2.8 = -\frac{7}{20}$$

$$P_1 = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1)$$

$$= \frac{(1.5-0)(1.5-2)}{(1-0)(1-2)} 3.2 = \frac{12}{5}$$

$$P_2 = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$$= \frac{(1.5-0)(1.5-1)}{(2-0)(2-1)} 4.5 = \frac{27}{16}$$

$$f(x=1.5) = 3.7375$$

Population in 1966 is 3.7375

3] Estimate $f(5)$ using Lagrange interpolation formula

	x_0	x_1	x_2	x_3	x_4	x_5	x_6
x	1	2	3	4	6	8	10
$f(x)$	2	2.5	7	10.5	12.75	13	13

$x = 5$

$$P_0 = \frac{(x-x_1)(x-x_2)(x-x_3)\cdots(x-x_6)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_6)} f(x_0)$$

$$= \frac{(5-2)(5-3)\cdots(5-10)}{(1-2)(1-3)\cdots(1-10)}\, 2$$

$$= \frac{-2}{21}$$

$$P_1 = \frac{(x-x_0)(x-x_2)\cdots(x-x_6)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_6)} f(x_1)$$

$$= \frac{(5-1)(5-3)\cdots(5-10)}{(2-1)(2-3)\cdots(2-10)}\, 2.5$$

$$= \frac{25}{32}$$

$$P_2 = \frac{(x-x_0)(x-x_1)\cdots(x-x_6)}{(x_2-x_0)(x_2-x_0)\cdots(x_2-x_0)} f(x_2)$$

$$= \frac{(5-1)(5-2)\cdots(5-10)}{(3-1)(3-2)\cdots(3-10)}\, 7 = -6$$

$$P_3 = \frac{(x-x_0)(x-x_1)\cdots(x-x_6)}{(x_3-x_0)(x_3-x_1)\cdots(x_3-x_6)} f(x_3)$$

$$= \frac{(5-1)(5-2)\cdots(5-10)}{(4-1)(4-2)\cdots(4-10)}\, 10.5 = \frac{10.5}{8}$$

$$P_4 = \frac{(x-x_0)(x-x_1)\cdots(x-x_6)}{(x_4-x_0)(x_4-x_1)\cdots(x_4-x_6)} f(x_4)$$

$$= \frac{(5-1)(5-2)\cdots(5-10)}{(6-1)(6-2)\cdots(6-10)}\, 12.75 = \frac{153}{32}$$

$$P_5 = \frac{(x-x_0)(x-x_1)\cdots(x-x_6)}{(x_5-x_0)(x_5-x_1)\cdots(x_5-x_6)} f(x_5)$$

$$= \frac{(5-1)(5-2)\cdots(5-10)}{(8-1)(8-2)\cdots(8-10)}\, 13 = \frac{-13}{28}$$

$$P_6 = \frac{(x-x_0)(x-x_1)\cdots(x-x_5)}{(x_6-x_0)(x_6-x_1)\cdots(x_6-x_5)} f(x_6)$$

$$= \frac{(5-1)(5-2)\cdots(5-8)}{(10-1)(10-2)\cdots(10-8)}\, 13 = \frac{13}{336}$$

$$f(5) = P_0 + P_1 + P_2 + \cdots + P_6$$

$$= 12.1667$$

4] Refer to pressure-volume data to use Lagrange interpolation to estimate the volume V at $P = 1.2$ atm

	x_0	x_1	x_2	x_3
P(atm)	0.969	1.090	1.341	1.60
V(L)	25	22.2	18	15

$x = 1.2$

$$P_0 = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0)$$

$$= \frac{(1.2-1.090)(1.2-1.341)(1.2-1.605)}{(0.969-1.090)(0.969-1.341)(0.969-1.605)}\, 25$$

$$= -5.485565$$

$$P_1 = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$

$$= \frac{(1.2-0.969)(1.2-1.341)(1.2-1.605)}{(1.090-0.969)(1.090-0.969)(1.090-1.605)}\, 22.2$$

$$= 18.722884$$

$$P_2 = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2)$$

$$= \frac{(1.2-0.969)(1.2-1.090)(1.2-1.605)}{(1.341-0.969)(1.341-1.090)(1.341-1.605)}\, 18$$

$$= 7.514699$$

$$P_3 = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_4)} f(x_3)$$

$$= \frac{(1.2-0.969)(1.2-1.090)(1.2-1.341)}{(1.605-0.969)(1.605-1.090)(1.605-1.341)} 1.5$$

$$= -0.621608$$

$f(1.2) = 20.1305107$

Volume at $P = 1.2$ atm is 20.1305

5] Consider the vapor - liquid equilibrium mole fraction data below for the binary system of methanol and water at 1atm

	x_0	x_1	x_2	x_3	x_4
x	1	0.880	0.765	0.653	0.545
y	1	0.929	0.849	0.764	0.673

	x_5	x_6	x_7	x_8
x	0.449	0.344	0.25	0.159
y	0.575	0.471	0.359	0.241

	x_9	x_{10}
x	0.072	0
y	0.114	0

Determine the vapor mole fraction of methanol (y) considering corresponding to the liquid mole fraction of methanol of $x = 0.15$ by lagranges interpolation

$x = 0.15$

$$P_0 = \frac{(x-x_1)(x-x_2)\ \ldots\ (x-x_{10})}{(x_0-x_1)\ \ldots\ (x_0-x_{10})} f(x_0)$$

$= 5.606 \times 10^{-5}$

$P_1 = -0.000727$

$P_2 = 0.0041946$

$P_3 = -0.0147607$

$P_4 = 0.0354 7577$

$P_5 = -0.0615777$

$P_6 = 0.0799213$

$P_7 = -0.0780866$

$P_8 = 0.2625275$

$P_9 = 0.66409761$

$P_{10} = 0$

$f(0.15) = 0.343404$

Vapour pressure at liquid mole fraction 0.15 is 0.343404 atm