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1) Apply Gauss Elimination method to solve the system.

$$10x - 7y + 3z + 5u = 6$$
$$-6x + 8y - z - 4u = 5$$
$$3x + y + 4z + 11u = 2$$
$$5x - 9y - 2z + 4u = 7$$

In matrix form:

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ -6 & 8 & -1 & -4 \\ 3 & 1 & 4 & 11 \\ 5 & -9 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 2 \\ 7 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 10 & -7 & 3 & 5 & 6 \\ -6 & 8 & -1 & -4 & 5 \\ 3 & 1 & 4 & 11 & 2 \\ 5 & -9 & -2 & 4 & 7 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & -0.7 & 0.3 & 5 & 0.6 \\ -6 & 8 & -1 & -4 & 5 \\ 3 & 1 & 4 & 11 & 2 \\ 5 & -9 & -2 & 4 & 7 \end{array}\right]$$

$$R_3 = \left[\begin{array}{cccc|c} 1 & -0.7 & 0.3 & 5 & 0.6 \\ 0 & 3.8 & 0.8 & -1 & 8.6 \\ 0 & 3.1 & 3.1 & 9.5 & 0.8 \\ 0 & -5.5 & -3.5 & 1.5 & 4 \end{array}\right]$$

$$R_3' = \left[\begin{array}{cccc|c} 1 & -0.7 & 0.3 & 5 & 0.6 \\ 0 & 1 & 0.21 & 9.86 & 2.26 \\ 0 & 0 & 2.49 & 10.31 & 6.804 \\ 0 & 0 & 0.5 & 0.05 & 16.43 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & -0.7 & 0.3 & 5 & 0.6 \\ 0 & 1 & 0.21 & 9.86 & 2.26 \\ 0 & 0 & 1 & 4.81 & 2.779 \\ 0 & 0 & 0.5 & 0.05 & 16.43 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & -0.7 & 0.3 & 5 & 0.6 \\ 0 & 1 & 0.21 & 9.86 & 2.26 \\ 0 & 0 & 1 & 4.81 & 2.779 \\ 0 & 0 & 0 & 17.81 & 17.81 \end{array}\right]$$

$\therefore u = 1, x = -7, y = 4, z = 5$

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2)

$x - 2y + 3z - u = 10$

$2x + 3y - 3z - u = 5$

$3x + 2y - 4 + 3 = 2$

$2x - y + 2z + 3u = 7$

$$\therefore \left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 10 \\ 2 & 3 & -3 & -1 & 5 \\ 3 & 2 & -4 & 3 & 2 \\ 2 & -1 & 2 & 3 & 7 \end{array}\right]$$

-2

$$= \left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 10 \\ 0 & 7 & -9 & 1 & -15 \\ 0 & 8 & -13 & 6 & -28 \\ 0 & 3 & -4 & 5 & -13 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 10 \\ 0 & 1 & -9/7 & 1/7 & -15/7 \\ 0 & 8 & -13 & 6 & -28 \\ 0 & 3 & -4 & 5 & -13 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 10 \\ 0 & 1 & -9/7 & 1/7 & -15/7 \\ 0 & 0 & -19/7 & 34/7 & -76/7 \\ 0 & 0 & -1/7 & 82/7 & -164/7 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 10 \\ 0 & 1 & -9/7 & 1/7 & -15/7 \\ 0 & 0 & 1 & -34/19 & 4 \\ 0 & 0 & 0 & 82/19 & -6 \end{array}\right]$$

$\therefore \frac{82}{19}u = -6$

$\rightarrow u = \frac{-57}{41}$

$x = 4 + \frac{34}{19}u$

$= \frac{62}{41}$

$y = \frac{-15}{7} + \frac{9}{7}x - \frac{1}{7}u$

$= \frac{-15 + 9x - u}{7}$

$= 0$

$x = 10 + u - 3z + 2y$

$= \frac{167}{41}$

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3) $x - 2y + 3z = -2$

$-x + y - 2z = 3$

$2x - y + 3z = 1$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & -2 \\ -1 & 1 & -2 & 3 \\ 2 & -1 & 3 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & -2 \\ 0 & -1 & 1 & 1 \\ 0 & 3 & -3 & 5 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 3 & -3 & 5 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 8 \end{array}\right]$$

System has no solution since $0 \neq 2$

Rank $(A) \neq$ Rank (AK)

and

Rank (A) < Rank (AK)

$\therefore$ Inconsistent

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