1. Apply Gauss-seidal iteration to solve the system.

$$27x + 6y - z = 85$$
$$x + y + 54z = 110$$
$$6x + 15y + 2z = 72$$

Rearrange such that leading diagonal is large

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

$$x = \frac{1}{27}(85 - 6y + z)$$

$$y = \frac{1}{15}(72 - 6x - 2z)$$

$$z = \frac{1}{54}(110 - x - y)$$

Init $y_0 = z_0 = 0$

I

$$x_1 = \frac{85}{27} = 3.1481$$

$$y_1 = \frac{1}{15}(72 - 6 \cdot 3.1481)$$
$$= 8.54076$$

$$z_1 = \frac{1}{54}(110 - 3.1481 - 8.54076)$$
$$= 1.9132$$

II

$$x_2 = \frac{1}{27}(85 - 6 \times 3.54076 + 1.9132)$$
$$= 2.4322$$

$$y_2 = \frac{1}{15}(72 - 6 \times 2.4322 - 2 \times 1.9132)$$
$$= 3.5720$$

$$x_2 = \frac{1}{54}(110 - 2.4322 - 3.5720)$$
$$= 1.9258$$

III

$$x_3 = \frac{1}{27}(85 - 6 \times 2.4322 + 1.9258)$$
$$= 2.67899$$

$$y_3 = \frac{1}{15}(72 - 6 \times 2.67899 - 2 \times 1.9258)$$
$$= 3.465733$$

$$z_3 = \frac{1}{54}(110 - 2.67899 - 3.4673)$$
$$= 1.9282$$

IV

$$x_4 = \frac{1}{27}(85 - 6 \times 2.67899 + 1.9282)$$
$$= 2.6240$$

$$y_4 = \frac{1}{15}(72 - 6 \times 2.6240 - 2 \times 1.92)$$
$$= 3.49397$$

$$z_4 = \frac{1}{54}(110 - 2.6240 - 3.49397)$$
$$= 1.9237$$

$$\therefore x = 2.6 \quad y = 3.5 \quad z = 1.9$$

2.

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

$$x = (17 - y + 2z)\frac{1}{20}$$

$$y = (z - 18 - 3x)\frac{1}{20}$$

$$z = \frac{1}{20}(25 - 2x + 3y)$$

Let $x_0 = 0 \; y_0 = 0 \; z_0 = 0$

I

$$x_1 = \frac{17}{20} = 0.85$$

$$y_1 = \frac{1}{20}(-18 - 3 \cdot 0 \cdot 85)$$

$$= -1.0275$$

$$z_1 = \frac{1}{20}(25 - 2 \cdot 0.85 - 3 \cdot 1.0275)$$

$$= 1.00986$$

II

$$x_2 = \frac{1}{20}(17 + 1.0275 + 2 \times 1.00986)$$

$$= 1.002361$$

$$y_1 = \frac{1}{20}(-18 - 3 \times 1.023614 + 1.00986)$$

$$= -1.0030$$

$$z_1 = \frac{1}{20}(25 - 2 \times 1.002361 - 3 \times 1.0030)$$

$$= 0.999361$$

$$\therefore x = 1, \; y = 1, \; z = 1$$

3.

$$x + 3y + 10z = 24$$
$$28x + 4y - z = 32$$
$$2x + 17y + 4z = 35$$

Rearranging

$$28x + 4y - z = 32$$
$$2x + 17y + 4z = 35$$
$$x + 3y + 10z = 24$$

$$x = \frac{1}{28}(32 - 4y + z)$$

$$y = \frac{1}{17}(35 - 2x - 4z)$$

$$z = \frac{1}{10}(24 - x - 3y)$$

Let $x_0 = y_0 = z_0 = 0$

I

$$x_1 = \frac{32}{28} = 1.1429$$

$$y_1 = \frac{1}{17}(35 - 2 \cdot 1.1429)$$

$$= 1.92436$$

$$z_1 = \frac{1}{10}(24 - 1.1429 - 3 \times 1.92436)$$

$$= 1.7084$$

II

$$x_2 = \frac{1}{28}(32 - 4 \times 1.92436 + 1.7084)$$

$$y_2 = 0.9829$$

$$y_2 = \frac{1}{17}(35 - 2 \times 0.9829 - 4 \times 1.7084)$$

$$= 1.6118$$

$$z_2 = \frac{1}{10}(24 - 0.9829 - 3 \times 1.6118)$$

$$= 1.8182$$

III

$$x_3 = \frac{1}{28}(32 - 4 \times 1.6118 + 1.8182)$$

$$= 0.9775$$

$$y_3 = \frac{1}{17}(35 - 2 \times 0.9775 - 4(1.8182))$$

$$= 1.5160$$

$$z_3 = \frac{1}{10}(24 - 0.9775 - 3 \times 1.5160)$$

$$= 1.8475$$

IV

$$x_4 = \frac{1}{28}(32 - 4 \times 1.5160 + 1.8475)$$

$$= 0.9923$$

$$y_4 = \frac{1}{17}(35 - 2 \times 0.9923 - 4 \times 1.8475)$$

$$= 1.5074$$

$$z_4 = \frac{1}{10}(24 - 0.9923 - 3 \times 1.5074)$$

$$= 1.849$$

$$x = 1 \qquad y = 1.5 \qquad z = 1.8$$