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S4 CSE Roll No 31

PROBABILITY DISTRO, TRANSFORMS & NUMERICAL METHODS

MA 202

1.

(i)

We know that $\sum_{x=-\infty}^{\infty} f(x) = 1$

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(a) $\therefore a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$

$\Rightarrow 81a = 1$

$\Rightarrow a = \frac{1}{81}$

$P(X<3) = \sum_{x=0}^{2} P(X=x) = a + 3a + 2a = 6a = \frac{6}{81} = 0.07407$

(b)

$P(X \geq 3) = 1 - P(X<3)$

$= 1 - \frac{6}{81} = \frac{25}{27} = 0.9259$

$P(2 \leq X < 5) = \sum_{x=2}^{4} P(X=x) = 5a + 7a + 9a = 21a$

$= \frac{21}{81} = 0.25925$

(ii) P(Ship destroyed in voyage is) $= 0.02 = p$

Number of trials/ships $n = 6$

$p = 0.02 \quad q = 1 - p = 0.98 \quad n = 6$ Let x denote the number of ships destroied.

So it is binomial distribution

$X \sim B(x, n, p)$

(a) P(losing one ship) $P(X=1)$

$= {}^nC_x p^x q^{n-x}$

$= {}^6C_1 \, 0.02^1 \, 0.98^5$

$= 0.10847$

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(b) P(losing atmost two ships)

$= P(X \le 2)$

$= \sum_{x=0}^{2} {}^6C_x \, 0.02^x \, 0.98^{6-x}$

$= \underline{\underline{0.999847}}$

2. $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 5y = 4e^{3t}$ $\quad y = 2 \quad y' = 7$ When $t=0$

$y'' - 4y' + 5y = 4e^{3t}$ —— (1) $\quad$ Let $L(y) = Y$

$L(y'') = s^2Y - s\,y(0) - y'(0)$

$= s^2Y - 2s - 7$ —— (2)

$L(y') = sY - y(0)$

$= sY - 2$ —— (3)

$L(y) = Y$ —— (4)

$L(4e^{3t}) = 4L(e^{3t}) = 4 \cdot \frac{1}{s-3}$ —— (5)

∴ Taking Laplace transform on both side of ODE

$(s^2Y - 2s - 7) - 4(sY - 2) + 5Y = \frac{4}{s-3}$

$\Rightarrow (s^2 - 4s + 5)Y - 2s - 7 + 8 = \frac{4}{s-3}$

$\Rightarrow (s^2 - 4s + 5)Y = 2s - 1 + \frac{4}{s-3}$

$= \frac{2s(s-3) - s + 3 + 4}{s-3}$

$= \frac{2s^2 - 6s - s + 7}{s-3}$

$= \frac{2s^2 - 7s + 7}{s-3}$

$\Rightarrow Y = \frac{2s^2 - 7s + 7}{(s-3)(s^2 - 4s + 5)}$ —— (6)

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Partial fraction.

$$\frac{2s^2-7s+7}{(s-3)(s^2-4s+5)} = \frac{A}{s-3} + \frac{Bs+C}{s^2-4s+5}$$

$$2s^2-7s+7 = A(s^2-4s+5)+(Bs+C)(s-3)$$

$$= As^2 - 4As + 5A + Bs^2 - 3Bs + Cs - 3C$$

$\therefore A+B=2$ // Solved using matrix in calculator 991ES

$-4A-3B+C=-7$

$5A-3C=7$

$A=2 \quad B=0 \quad C=1$

From (6)

$$\therefore Y = \frac{2}{s-3} + \frac{1}{s^2-4s+5}$$

$$= \frac{2}{s-3} + \frac{1}{(s-2)^2-4+5}$$

$$= \frac{2}{s-3} + \frac{1}{(s-2)^2+1^2}$$

$$\therefore y = L^{-1}(Y)$$

$$= L^{-1}\left(\frac{2}{s-3}\right) + L^{-1}\left(\frac{1}{(s-2)^2+1}\right)$$

$$= 2 \cdot L^{-1}\left(\frac{1}{s-3}\right) + L^{-1}\left(\frac{1}{(s-2)^2+1}\right)$$

$$= 2 \cdot e^{+3t} + e^{2t}\sin t$$

$$\underline{\underline{y = 2e^{3t} + e^{2t}\sin t}}$$

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