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Roll No 31

2. If X be a uniformly distributed with mean 1 and var $\frac{4}{3}$ find $P(|X-2|<2)$

Let the uniform dist be defined on $[a, b]$

$$\text{Mean} = \frac{a+b}{2} = 1$$

$$a+b = 2 \quad \text{—(1)}$$

$$\text{Variance} = \frac{(b-a)^2}{12} = \frac{4}{3}$$

$$(b-a)^2 = \frac{4 \times \cancel{12}^{4}}{\cancel{3}} = 16$$

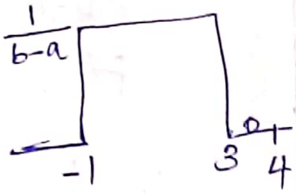
$$b-a = \pm 4 \qquad \begin{aligned} a+b &= 2 \\ -a+b &= 4 \end{aligned}$$

$$b = 3 \qquad a = -1$$

$$P(|X-2|<2) = P(-2<X-2<2)$$

$$= P(-2+2<X<2+2)$$

$$= P(0<X<4)$$



$$= \int_0^3 \frac{1}{b-a}\,dx + \int_3^4 0\,dx$$

$$= \int_0^3 \frac{1}{3+1}\,dx$$

$$= \frac{1}{4}\int_0^3 dx =$$

$$= \frac{3}{4}$$

3. The time in hours req to repair a machine is exponentially distributed with mean 20. What is prob that the required time
(i) Exceeds 30 h (ii) B/w 16 hrs & 24 hrs

In exponential distribution

$$\text{Mean} = \beta = 20$$

(i)
$$P(X \geq 30) = \int_{30}^{\infty} \frac{1}{\beta} e^{-\frac{1}{\beta}x}\,dx$$

$$= \frac{1}{20}\int_{30}^{\infty} e^{-\frac{1}{20}x}\,dx.$$

$$= \cancel{20 \cdot \frac{-1}{20}\left[e^{-20x}\right]_{30}^{\infty}}$$

$$= \frac{1}{20} \cdot \frac{-20}{1}\left[e^{-x/20}\right]_{30}^{\infty}$$

$$= e^{-3/2}$$

(ii)
$$P(16<X<24) = \int_{16}^{24} \frac{1}{\beta} e^{-1/\beta\, x}\,dx$$

$$= \frac{1}{20}\int_{16}^{24} e^{-x/20}\,dx$$

$$= \frac{1}{20} \cdot \frac{-20}{1}\left[e^{-x/20}\right]_{16}^{24}$$

$$= e^{-16/20} - e^{-24/20}$$

$$= e^{-4/5} - e^{-6/5}$$

1. ① If the distribution of a random variable is given by

$$F(x) = \begin{cases} 1 - \frac{1}{x^2} & \text{for } x \geq 1 \\ 0 & \text{for } x < 1 \end{cases}$$

Find probability that RV
① < 3 ② $4 < X < 5$

$$f(x) = F'(x) = \begin{cases} \frac{2}{x^3} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

$$P(X \leq 3) = \int_1^3 f(x)\,dx = 2\left[\cancel{\log}\ \frac{1}{x^3}\right]_1^3 = \frac{8}{9}$$

$$P(4 < X < 5) = \int_4^5 \frac{2}{x^3}\,dx = \frac{9}{400}$$

4. Prove that binomial dist in parameters n & p can be approximated to poisson distribution when n is larger than p is small with $np = \lambda$ a constant

Given $n \to \infty$ $p \to 0$ $np = \lambda$

Binomial distribution is ${}^nC_x p^x (1-p)^{n-x}$ $p = \frac{\lambda}{n}$

$$= {}^nC_x \left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{n!}{(n-x)!\,x!} \frac{\lambda^x}{n^x} \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{n(n-1)(n-2)\cdots(n-x+1)}{x!} \frac{\lambda^x}{n^x}$$

$$= \frac{\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)\cdots\left(1 - \frac{x+1}{n}\right)\lambda^x \left(1 - \frac{\lambda}{n}\right)^{n-x}}{x!}$$

Now $\lim_{n \to \infty}$

$$= \frac{1}{x!} \lambda^x \lim_{n \to \infty} \left(1 - \frac{\lambda}{x}\right)^{n-x}$$

$$= (e^{-\lambda} \cdot \lambda^x)/x!$$

5. Derive mean & variance of uniform distribution.

Uniform distribution

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{Otherwise} \end{cases}$$

Mean $= \mu = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$$= \int_{-\infty}^{a} 0\,dx + \int_{b}^{\infty} 0\,dx + \int_{a}^{b} \frac{1}{b-a}\,x\,dx$$

$$= \frac{1}{b-a} \cdot \left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{b^2-a^2}{2(b-a)} = \frac{b+a}{2}$$

Variance $= E(x^2) - \mu^2$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_{-\infty}^{a} 0\,dx + \int_{b}^{\infty} 0\,dx + \int_{a}^{b} \frac{x^2}{b-a}\,dx$$

$$= \frac{1}{b-a} \int_a^b x^2\,dx$$

$$= \frac{1}{b-a} \left(\frac{x^3}{3}\right)_a^b$$

$$= \frac{b^3-a^3}{(b-a)3} = \frac{(b-a)(b^2+a^2+ab)}{(b-a)3}$$

variance $= \frac{b^2+ab+a^2}{3} - \frac{b^2+a^2+2ab}{4}$

$$= \frac{4b^2+4ab+4a^2-3b^2-3a^2-6ab}{12}$$

$$= \frac{b^2+a^2-2ab}{12}$$

$$= \frac{(b-a)^2}{12}$$