Inverse Laplace Transform

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Definition

- If $L[f(t)] = F(s)$, then $f(t)$ is the inverse Laplace transform of $F(s)$ and is denoted by $L^{-1}[F(s)] = f(t)$.
- Linearity Property
  $L^{-1}[aF(s) + bG(s)] = aL^{-1}[F(s) + bL^{-1}[G(s)]$
- Shifting Property
  If $L[f(t)] = F(s)$, then $L^{-1}[F(s-a)] = e^{at}f(t)$

Table of Inverse Laplace Transforms

- $L^{-1}[\frac{1}{s}] = 1$
- $L^{-1}[\frac{1}{s^2}] = t$
- $L^{-1}[\frac{1}{s^{n+1}}] = \frac{t^n}{n!}, n = 1, 2, 3....$
- $L^{-1}[\frac{1}{s-a}] = e^{at}$
- $L^{-1}[\frac{a}{s^2+a^2}] = \sin at$
- $L^{-1}[\frac{s}{s^2+a^2}] = \cos at$
- $L^{-1}[\frac{a}{s^2-a^2}] = \sinh at$
- $L^{-1}[\frac{s}{s^2-a^2}] = \cosh at$

Methods of finding Inverse Transforms

There are six methods to inverse Laplace Transforms.

- Using Shifting Property
- Using Partial Fractions
- Using Derivaties
- Using Integration
- Using unit step function
- Using Convolution theorem

Inverse Using Shifting property

$L^{-1}[\frac{1}{s^2+6s+18}]$

$= L^{-1}[\frac{1}{s^2+6s+3^2-3^2+18}]$

$= L^{-1}[\frac{1}{(s+3)^2+9}]$

$= L^{-1}[\frac{3}{3\{(s+3)^2+9\}}]$

$= \frac{1}{3}e^{-3t}L^{-1}[\frac{3}{s^2+9}]$

$= \frac{e^{-3t}\sin 3t}{3}$

AssignmentProblems

1. $\frac{s^2-3s+8}{s^3}$
2. $\frac{5}{s^2+3s+7}$
3. $\frac{s}{s^2+6s+25}$
4. $\frac{3s-2}{s^2+2s+6}$
5. $\frac{7s+4}{4s^2+4s+9}$

Thank You