23 Jun 2020

MA202

Series 2

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1. $f(x) = \begin{cases} cx^2 & -1<x<1 \\ 0 & \text{otherwise} \end{cases}$

(a) We know that $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\therefore \int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{-1} 0\,dx + \int_{-1}^{1} cx^2\,dx + \int_{1}^{\infty} 0\,dx = 1$

$\Rightarrow c\left[\frac{x^3}{3}\right]_{-1}^{1} = 1$

$\therefore c[1+1] = 3$

$\Rightarrow c = \frac{3}{2}$

$\therefore f(x) = \begin{cases} \frac{3}{2}x^2. & |x| \le 1 \\ 0, & \text{otherwise} \end{cases}$

(b) $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_{-\infty}^{-1} 0\,dx + \int_{-1}^{1} x \cdot \frac{3}{2}x^2\,dx + \int_{1}^{\infty} 0\,dx$

$= \frac{3}{2}\int_{-1}^{1} x^3\,dx$

$= \frac{3}{2}\left[\frac{x^4}{4}\right]_{-1}^{1}$

$= \underline{\underline{0}}$

$V(x) = E(x^2) - E(x)$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$= \int_{-\infty}^{-1} 0\,dx + \int_{-1}^{1} x^2 \cdot \frac{3}{2}x^2\,dx + \int_{1}^{\infty} 0\,dx$

~~[illegible]~~

$\mu + 1.a1 -$

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$$= \int_{-1}^{1} \frac{3}{2} x^4 \, dx$$

$$= \frac{3}{2} \int_{-1}^{1} x^4 \, dx = \frac{3}{2} \left[\frac{x^5}{5}\right]_{-1}^{1}$$

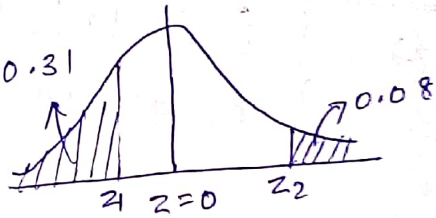
$$= \frac{3}{5}$$

$$\therefore Var(x) = E(x^2) - E(x)$$

$$= \frac{3}{5} - 0$$

$$= \frac{3}{5}$$

2. 0.31 % item under 45 & 8% ⇒ 0.08 over 64 ~~find~~
$\mu = ?$ $\sigma = ?$



For 31% under 45

$$P(x \leq 45) = 0.31$$

$$\Rightarrow P\left(z < \frac{45-\mu}{\sigma}\right) = 0.31$$

$$\Rightarrow 0.5 - P\left(0 < z < \frac{\mu-45}{\sigma}\right) = 0.31$$

$$\Rightarrow P\left(0 < z < \frac{\mu-45}{\sigma}\right) = 0.19$$

$$\Rightarrow F\left(\frac{\mu-45}{\sigma}\right) = 0.19$$

$$\Rightarrow \frac{\mu-45}{\sigma} = 0.5$$

$$\Rightarrow \mu - 0.5\sigma = 45 \quad \text{--- (1)}$$

For 8% over 64

$$P(x > 64) = 0.08$$

$$\Rightarrow P\left(z > \frac{64-\mu}{\sigma}\right) = 0.08$$

$$\Rightarrow 0.5 - P\left(0 < z < \frac{64-\mu}{\sigma}\right) = 0.08$$

$$\Rightarrow P\left(0 < z < \frac{64-\mu}{\sigma}\right) = 0.42$$

$\Rightarrow$

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$\therefore F\left(\frac{64-\mu}{\sigma}\right) = 0.42$

$\frac{64-\mu}{\sigma} = 1.41$

$\therefore$ $\mu + 1.41\sigma = 64$ —— (2)

Solving (1) & (2)

$\mu = 49.9738 \quad \sigma = 9.9476$

Mean $= 49.9738$ St. Deviation $= 9.9476$

3. Gauss elimination

$2x + 2y + z = 12$

$3x + 2y + 2z = 8$

$5x + 10y - 8z = 10$

In matrix form $A : P = B$

$$\begin{pmatrix} 2 & 2 & 1 \\ 3 & 2 & 2 \\ 5 & 10 & -8 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 8 \\ 10 \end{pmatrix}$$

Rank of $(A : B)$

$$\begin{array}{c} 5 \\ 2 \\ 2 \end{array}\left[\begin{array}{ccc|c} 2 & 2 & 1 & 12 \\ 3 & 2 & 2 & 8 \\ 5 & 10 & -8 & 10 \end{array}\right] \quad \begin{array}{l} R_2 \to 2R_2 - 3R_1 \\ R_3 \to 2R_3 - 5R_1 \end{array}$$

$$\begin{array}{cccc} 6 & 6 & 3 & 36 \\ -6 & -4 & -4 & -16 \\ \hline 0 & 2 & -1 & 20 \end{array}$$

$\frac{51}{4}$ $\frac{45}{5}$ $\frac{35}{4}$

$$\begin{array}{cccc} 6 & 4 & 4 & 16 \\ -6 & -6 & -3 & -36 \\ \hline 0 & -2 & 1 & -20 \end{array}$$

$$\begin{array}{cccc} 2 & 2 & 1 & 12 \\ 0 & 2 & -1 & 22 \end{array}$$

$$\left[\begin{array}{cccc} 2 & 2 & 1 & 12 \\ 0 & -2 & 1 & -20 \\ 0 & 10 & 21 & -40 \end{array}\right] \quad ~~R_3 \to R_3 + 5R_2~~$$

$$\begin{array}{cccc} 10 & 20 & -16 & 20 \\ -10 & -10 & -5 & -60 \\ \hline 0 & 10 & 21 & -40 \end{array}$$

$$\left[\begin{array}{cccc} 2 & 2 & 1 & 12 \\ 0 & -2 & 0.5 & -10 \\ 0 & 5 & -20.5 & -100 \end{array}\right]$$

$$\begin{array}{cccc} 0 & 10 & 21 & -40 \\ 0 & -10 & 5 & -60 \\ \hline & 0 & 26 & -100 \end{array}$$

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$$\left(\begin{array}{ccc|c} 1 & 1 & 0.5 & 6 \\ 0 & 1 & -0.5 & 10 \\ 0 & 0 & -8 & -70 \end{array}\right)$$

$$\therefore z = \frac{70}{8} = \frac{35}{4}$$

$$y = 10 + \frac{1}{2}z = \frac{115}{8}$$

$$x = 12 - z - 2y = \frac{-51}{4}$$

$$= -12.75$$

4 $\displaystyle\int_1^7 f(t)\,dt$ $n = 6$ division $t_a = 1$ $t_b = 7$

Here $\frac{t_b - t_a}{n} = h = \frac{7-1}{6} = 1$

$\therefore h = 1$

Simpson's rule is $\int_a^b f(x)dx = \frac{1}{3} h\left[f_a + f_b + 4(f_{even}) + 2(f_{odd})\right]$

$$\int_1^7 f(t)dt = \frac{1}{3}\left[81 + 60 + 4(80+78) + 2(75+83+70)\right]$$

$$= 409.6667$$