Math 6030, Graduate Algebra, Spring 2025, Homework 8

Instructor: Danny Krashen

Discussing the problems with other people is encouraged,
but you must write up your own work independently!

1. Let R be the ring of differentiable (C^∞) functions on the unit interval and let I be the ideal of functions vanishing at 0. Show that $\bigcap I^n \neq 0$.

2. Let $S \subset R$ be a multiplicative subset in a commutative ring R. Show that for a proper ideal $I \triangleleft R$, we have $I = IR_S \cap R$ if and only if I is S-saturated.

3. Suppose I is an ideal in a commutative Noetherian ring R and P is a minimal prime over I. If P is maximal in R show that R/I is Artinian.

4. Let $R = \mathbb{C}[x, y]$, and $S = \{y^n \mid n \in \mathbb{N}\}$. Find an example of an ideal $I \triangleleft R$ such that I is not primary, but the S-saturation I^S of I is primary.

5. Suppose R is a PID. Give a characterization of the primary ideals of R.

6. (Nakayama variant via Krull intersection). Suppose that R is a Noetherian domain and $I, J \triangleleft R$ are proper ideals such that $IJ = J$. Show that we must have $J = 0$.