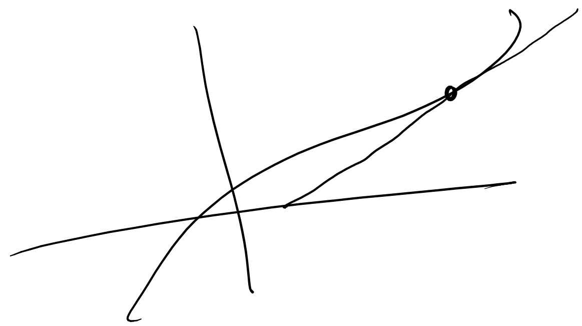
Lecture 23: Approximation

Tuesday, November 17, 2015 9:32 AM

Approximation

Approximation:

A (reasonable) function is approximated by its tangent line.



simple example

$\sqrt{9.1}$

$f(x) = \sqrt{x} = x^{1/2}$ $\quad f(9) = 3$

$f(x)$ near 3 $\approx$ tang line of $f(x)$ at 3.

tangent line: $y - 3 = f'(9)(x-9) = \frac{1}{6}(x-9)$

at pt $(9,3)$

$(9, \sqrt{9})$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$ $\quad f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

tang line

$y = \frac{1}{6}(x-9) + 3$

$$\sqrt{x} \approx \frac{1}{6}(x-9)+3 \quad \text{if } x \text{ close to } 9.$$

$$\sqrt{9.1} \approx \frac{1}{6}(9.1-9)+3 = \frac{1}{6}\left(\frac{1}{10}\right)+3 = 3+\frac{1}{60}$$

$\frac{1}{60}$ a little less than $\frac{1}{50}$

$\sqrt{9.1} \approx 3.018$? .018??

actual: 3.0166...

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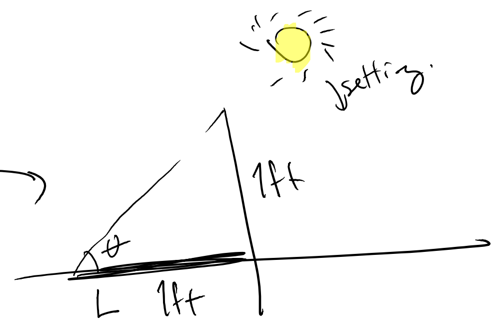
put stick in ground

$\boxed{\pi \approx 3}$

sun makes a shadow:
(sun@45°)



Q: how much does shadow grow (approx) in one minute?

$$\tan\theta = \frac{1}{L} \qquad \theta \approx \frac{\pi}{4} - \frac{1}{240} \quad (\text{using } 2\pi \approx 6)$$

$$\tan\left(\frac{\pi}{4} - \frac{1}{240}\right) = \frac{1}{L} \longrightarrow \text{approx } \frac{1}{L} = \tan\left(\frac{\pi}{4} - \frac{1}{240}\right)$$

then invert.

$$\cot\left(\frac{\pi}{4} - \frac{1}{240}\right) = L$$

approx L directly.

cot. near $\frac{\pi}{4}$ $\approx$ eqn for tang line to $\cot(x)$ at $x=\frac{\pi}{4}$

$$y - \cot\left(\tfrac{\pi}{4}\right) = -\csc^2\left(\tfrac{\pi}{4}\right)\left(x - \tfrac{\pi}{4}\right)$$

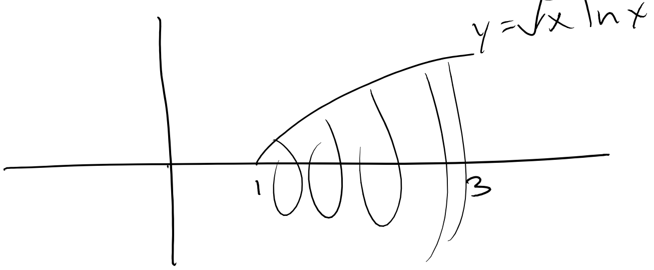
$$y - 1 = -(\sqrt{2})^2\left(x - \tfrac{\pi}{4}\right)$$

$$y = -2(x - \pi/4) + 1$$

$$\cot\left(\tfrac{\pi}{4} - \tfrac{1}{240}\right) \approx -2\left(\left(\tfrac{\pi}{4} - \tfrac{1}{240}\right) - \tfrac{\pi}{4}\right) + 1$$

$$\approx -2\left(-\tfrac{1}{240}\right) + 1 = 1 + \tfrac{1}{120}$$

increases by about 1/10 inch.



change in volume if we end at 3.1?

$$V(x) = \int_1^x A(t)\,dt = \int_1^x \pi r^2\,dt$$

$$= \int_1^x \pi\, t(\ln t)^2\,dt$$

$$V'(x) = \pi\, x(\ln x)^2 \quad (\text{FTC I})$$

$V(3.1) \approx V'(3)(x-3) + V(3)$ $\to$ change in Volume $\approx V'(3)(x-3)$

$= \pi \cdot 3(\ln 3)^2(3.1-3)$

$\approx 3 \cdot 3(1)^2(.1)$

$\approx V(3) - V'(3)(x-3)$

tangent line $y - V(3) = V'(3)(x-3)$

$y = V'(3)(x-3) + V(3)$

$\approx 3 \cdot 3(1)^2(.1)$

$\approx 9 \cdot (\frac{1}{10}) \approx \boxed{1}$