Math 2250, Fall 2012, Practice Sheet for Exam 2

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(1) Solve for $\frac{dy}{dx}$ given that

$$x^2 - 4y^2 = \sin(xy)$$

(2) Solve for $\frac{dx}{dt}$ given that

$$e^{xy} + \ln(x+y) = y$$

(3) Given that

$$x + 3y - \sin(y) = 17, \text{ and } \frac{dy}{dt} = 3$$

solve for $\frac{dx}{dt}$ when $y = 0$.

(4) Suppose that two ships leave from the same port, the first travelling due north, and the second due east. If we let y represent the distance between the first ship and the port, x represent the distance between the second ship and the port, and z represent the distance between the ships, find an equation which relates the three quantities x, y and z together.

(5) A 30 foot ladder is sliding down a wall so that the top is falling at a constant speed of 4 ft/sec. How fast is the angle between the ladder and the floor decreasing (in terms of degrees/second) when the bottom of the ladder is 20 feet from the wall?

(6) Water is draining from a cylindrical tank with a radius of 100cm at a constant rate of $20\text{cm}^3/\text{min}$. Find the rate of change of the height of the water in the tank when the water's height is 500cm.
Recall that the formula for the volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius and h is the height

(7) Water is in a conical tank with a height of 800cm and radius at the top of 300cm. If the water is draining out at a constant rate of $20\text{cm}^3/\text{min}$, find the rate of change of the height of the water in the tank when the water's height is 100cm.
Recall that the formula for the volume of a circular cone is given by $V = \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height

(8) Water is draining from a cylindrical tank with a radius of 100cm at a rate inversely proportional to the amount of water in the tank given by the formula:

$$\frac{dV}{dt} = -1/10V$$

where V is the volume of the tank in cc's (cubic centimeters), and where the speed is given in cc/sec. Find the rate of change of the height of the water in the tank when the water's height is 500cm.

(9) Suppose that we have functions $f(x), g(x)$ such that

$$f(g(x)) - 3g(x) + 2f(x + f(x)) = 9x.$$

Solve for $g'(x)$.

(10) Consider the function $f(x) = x(6 - 2x)^2$.
(a) Find all the critical points of $f(x)$
(b) Find the intervals on which $f(x)$ is increasing or decreasing
(c) Find all local minima and maxima
(d) Find all absolute minima and maxima

(11) Find the minimum and maximum values of the function $f(x) = -3(x+1)^{2/3}$ for $-2 \leq x \leq 0$.

(12) Find the minimum and maximum values of the function $f(x) = 2x^2 - 3x + 1$ on the interval $[-2, 3]$.

(13) Suppose that $f(x), g(x)$ are functions such that $f(g(x)) = -x^2$. If $g(2) = 5$, and $g'(2) = 7$, find $f'(5)$.

(14) Compute the limit: $\lim_{x \to \infty} \frac{(\ln x)^2}{x}$

(15) Compute the limit: $\lim_{x \to 0} (1 - x)^{1/x}$