Should have read Ch. 0 in Mooculus

This lecture: Ch. 1

- nuts & bolts of limits
- laws of limits, limits of some nice fcns
- trig functions

We say that the limit of $f(x)$ as x approaches a is L if: $f(x)$ gets close to L whenever x gets close to a. $\neq a$

What does close mean?

replace close by "as close as we want"?

Actual definition:

<u>Def</u>: $\lim_{x \to a} f(x) = L$ means for any positive number $\varepsilon > 0$

we can find a positive number $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x-a| < \delta$

Example

$$\lim_{x \to 3} 2x = 6 \quad \text{(guess)}$$

How about $\varepsilon = 1$? $\quad \delta = \ldots$

$$|2x-6| < 1 \quad \text{whenever } 0 < |x-3| < \delta$$

$\delta = 3$? $\quad 0 < |x-3| < 3$ does this mean $|2x-6| < 1$?

ex: $x = 5 \rightsquigarrow |2x-6| = |2\cdot 5 - 6| = 4 <$

no!

$\delta = 1$? $\quad 0 < |x-3| < 1$?

$x = 3 + 2/3 \rightsquigarrow |2(3+2/3) - 6| = |6 + 4/3 - 6| = |4/3| < 1$

no!!

$$|2x-6| < 1$$

$$2|x-3| < 1 \longleftrightarrow |x-3| < \frac{1}{2}$$

$\delta = \frac{1}{2}$ then if $0 < |x-3| < \frac{1}{2}$ then $2|x-3| < 1$

$$|2x-6| < 1$$

$\varepsilon = \frac{1}{2} \qquad |2x-6| < \frac{1}{2}$ if $0 < |x-3| < \delta = \frac{1}{4}$

$\delta = \cancel{8}\ \frac{1}{4}$

$$\lim_{x\to 3} 2x = 6 \quad \text{(we hope)}$$

$\varepsilon = \frac{1}{10}$ $\delta = \frac{1}{20}$ | $\varepsilon = \frac{1}{100}$ $\delta = \frac{1}{200}$

let $\varepsilon > 0$ be any positive number.

set $\delta = \frac{1}{2}\varepsilon$ then whenever $0 < |x-3| < \delta = \frac{1}{2}\varepsilon$

we have $\underset{\underset{|2x-6|}{\shortparallel}}{2|x-3|} < \varepsilon$. And so by the definition

$$\lim_{x\to 3} 2x = 6.$$

Limit laws

Suppose $f(x), g(x)$ are functions and

$\lim_{x\to a} f(x) = L$, $\lim_{x\to a} g(x) = M$

then

1. $\lim_{x\to a} f(x) + g(x) = L + M$

2. $\lim_{x\to a} f(x)g(x) = LM$

3. $\lim_{x\to a} C f(x) = CL$

4. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{M}$ if $M \neq 0$

Limit Facts

1. $\lim_{x\to a} C = C$

2. $\lim_{x\to a} x = a$

ex: $\lim_{x\to 3} 2x = CL = 2\cdot 3 = 6$

law 3

$C = 2$

$x = f(x)$

$\lim_{x\to 3} f(x) = \lim_{x\to 3} x = 3 = L$

Fact 2

Practice

1. $\lim_{x\to 2} 3x^2-1$

$$\lim_{x\to 2} 3x^2+(-1)$$

$$= \lim_{x\to 2} 3x^2 + \lim_{x\to 2}(-1) = 3\lim_{x\to 2} x^2 + \lim_{x\to 2}(-1)$$

$$= 3\lim_{x\to 2} x\cdot x + \lim_{x\to 2}(-1) = 3\left(\left(\lim_{x\to 2} x\right)\left(\lim_{x\to 2} x\right)\right) + \lim_{x\to 2}(-1)$$

$$= 3(2)(2) + (-1) = 11$$

2. $\lim_{x\to 3} \frac{(x^2-9)2x}{x-3}$

2. $$\lim_{x\to 3} \frac{(x-3)(x+3)2x}{x-3} = \lim_{x\to 3}\left(\frac{x-3}{x-3}\right)\lim_{x\to 3}(x+3)(2x$$

$$\lim_{x\to 3}\frac{x-3}{x-3} = \lim_{x\to 3} 1 = 1$$

$$\frac{x-3}{x-3} = 1 \quad \text{if } x\neq 3$$

$$\lim_{x\to 2} (x^2-4) = \lim_{x\to 2} (x+2)(x-2)$$

$$= \left(\lim_{x\to 2} (x-2)\right)\left(\lim_{x\to 2} (x+2)\right) = 0$$

0! don't care 4

$$\lim_{x\to 2} x-2 = \lim_{x\to 2} x + \lim_{x\to 2} (-2) = 2+(-2) = 0.$$

maybe I care...

$$\lim_{x\to 2} x+2 = \lim_{x\to 2} x + \lim_{x\to 2} 2 = 2+2 = 4$$

$$\lim_{x\to 0} \frac{x}{x} = \lim_{x\to 0} x \cdot \frac{1}{x} \overset{\times}{=} \left(\lim_{x\to 0} x\right)\left(\lim_{x\to 0} \frac{1}{x}\right) = 0!$$

0 ~~don't care~~ D.N.E.

$$\|$$

$$\lim_{x\to 0} 1 = 1$$