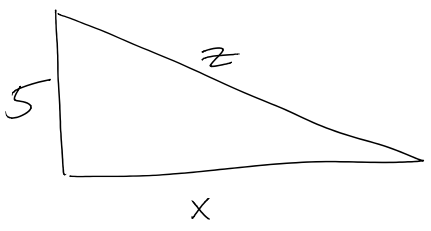
Lecture 9: More related rates, etcetera

Thursday, September 24, 2015 12:36 PM

A boat is pulled in to a dock by a rope with one end attached to the front of the boat and the other end passing through a ring attached to the dock at a point 5ft higher than the front of the boat. The rope is being pulled through the ring at a rate of 0.6 ft/sec. How fast is the boat approaching the dock when 13 ft of rope are out?

Strategy

1. Read, read, read
2. Draw a picture and/or diagram
   Diagram should have variables
3. Find an equation relating variables
   - whose rates of change we know
   - whose rates of change we want
4. Implicit diff, solve
5. Plug in values.



$\frac{dz}{dt} = -0.6$

$\frac{dx}{dt}$ = want.

$x^2 + 5^2 = z^2$

d/dt

$$\frac{d}{dt}(x^2+5^2)=\frac{d}{dt}(z^2)$$

$$2x\frac{dx}{dt}=2z\frac{dz}{dt}$$

$$\frac{dx}{dt}=\frac{z}{x}\frac{dz}{dt}$$

13

$= -0.6$

$x^2+5^2=13^2$

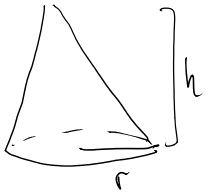
$x^2+25=169$

$x^2=144$

$x=12$

$$\frac{dx}{dt}=\frac{13}{12}(-.6)$$

Sand is poured onto a surface at $15\,cm^3/sec$, forming a conical pile whose base diameter is always equal to its altitude. How fast is the altitude of the pile increasing when the pile is 3cm high?



$V=\frac{1}{3}\pi r^2 h$

$2r=h$

$\frac{dV}{dt}=15$

want $\frac{dh}{dt}$

easy way: $r=\frac{h}{2}$ $\longrightarrow$ $V=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h=\frac{\pi}{3}\frac{h^2}{4}h$

$V = \frac{\pi}{3} \cdot \frac{1}{4} h^3$

$\frac{d}{dt}$ both sides

$\frac{dV}{dt} = \frac{\pi}{3} \cdot \frac{1}{4} 3h^2 \frac{dh}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt} \implies \frac{dh}{dt} = \frac{4}{\pi h^2} \frac{dV}{dt} = \frac{4}{\pi (3)^2} (15)$

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hard way:

$V = \frac{\pi r^2 h}{3}$ ~~$2r = h$~~

$r + \ln(r) = h$

$\frac{dV}{dt} = 15$

want $\frac{dh}{dt}$ when $h = 3$.

$\frac{d}{dt}$

$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{\pi r^2 h}{3}\right)$

$= \frac{\pi}{3}\left(\frac{d}{dt}(r^2 h)\right)$

$\frac{dV}{dt} = \frac{\pi}{3}\left(2r\frac{dr}{dt}h + r^2 \frac{dh}{dt}\right)$

need $\frac{dr}{dt}$: look for an eqn

$r + \ln r = h$

$\frac{d}{dt}(r + \ln r) = \frac{d}{dt} h$

$\frac{dr}{dt} + \frac{1}{r}\frac{dr}{dt} = \frac{dh}{dt}$

$\frac{dr}{dt} = \frac{\left(\frac{dh}{dt}\right)}{(1 + 1/r)}$

solve

$\frac{dV}{dt} + \frac{1}{r}\frac{dV}{dt} = \frac{dV}{dt}$

$\frac{dr}{dt}\left(1+\frac{1}{r}\right) = \frac{dh}{dt}$

solve

$\frac{dh}{dt} = \text{Something}\left(r, h, \frac{dV}{dt}\right)$

no prob!

uh oh.

solve

$r + \ln r = 3$

guess: $r=1 \rightsquigarrow 1+\ln(1) = 1+0 = 1$ too small

$r=3 \rightsquigarrow 3+\ln(3) =$ bigger than 3.

IVT: between 1 & 3

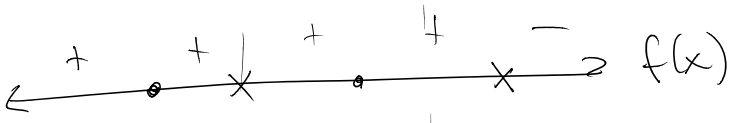
guess $r=2 \rightsquigarrow 2+\ln(2)$ less than 3

IVT: between 2 & 3

⋮

Next time: Curve sketching

Use sign charts for a fcn & its derivative(s) to sketch graphs or find info. about them.



mark pts where $f(x)=0$ •

$f(x)$ is discont X