Lecture 3: practice with limits, one-sided limits

Monday, August 24, 2015 10:12 AM

Practice (warmup)

1. $\lim_{x\to 0} x\cot x = \lim_{x\to 0} x\frac{\cos x}{\sin x} \overset{?}{=} \left(\lim_{x\to 0}\frac{x}{\sin x}\right)\left(\lim_{x\to 0}\cos x\right)$

$$= \lim_{x\to 0}\frac{1}{\left(\frac{\sin x}{x}\right)} \underbrace{\lim_{x\to 0}\cos x}_{\text{1 since } \cos x \text{ is continuous}} = \frac{\lim_{x\to 0} 1}{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)}(1) = \frac{1}{1}\cdot 1$$

$$= 1$$

2. $\lim_{x\to 0^+}\frac{x}{\sqrt{\sin x}} = \lim_{x\to 0^+}\frac{x}{\sqrt{\sin x}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}} = \left(\lim_{x\to 0^+}\frac{x}{\sin x}\right)\left(\lim_{x\to 0^+}\sqrt{\sin x}\right)$

$$= \lim_{x\to 0^+}\frac{1}{\left(\frac{\sin x}{x}\right)}\sqrt{\lim_{x\to 0^+}\sin x} = 1\cdot 0 = 0 \qquad \boxed{!}$$

3. $\lim_{x\to 0}\frac{\sin x}{3x} = \frac{1}{3}\lim_{x\to 0}\frac{\sin x}{x} = \frac{1}{3}\cdot 1 = \frac{1}{3}$

4. $\lim_{x\to 0}\frac{\sin 3x}{x} = \lim_{x\to 0}\frac{\sin 2x\cos x + \sin x\cos 2x}{x}$

$$= \lim_{x\to 0}\frac{2(\sin x\cos x)\cos x + \sin x(\cos^2 x - \sin^2 x)}{x}$$

$$= \lim_{x\to 0} 2\cos^2 x\frac{\sin x}{x} + \lim_{x\to 0}\frac{\sin x}{x}\cos^2 x - \lim_{x\to 0}\frac{\sin^3 x}{x}$$

$$2 + 1 + 0 = 3.$$

easy way:

$$\lim_{x\to 0} \frac{\sin 3x}{x} = \lim_{x\to 0} \frac{3\sin 3x}{3x} = 3\lim_{x\to 0} \frac{\sin 3x}{3x}$$

$$\underbrace{\lim_{x\to 0} 3x = 0 \qquad \lim_{x\to 0} \frac{\sin x}{x} = 1}$$

composition Law

$$\lim_{x\to 0} \frac{\sin 3x}{3x} = 1$$

another way to think of this $\quad u = 3x \qquad \lim_{x\to 0} u = \lim_{x\to 0} 3x = 0$

$$\lim_{x\to 0} \frac{\sin 3x}{3x} = \lim_{x\to 0} \frac{\sin u}{u} = \lim_{u\to 0} \frac{\sin u}{u} = 1$$

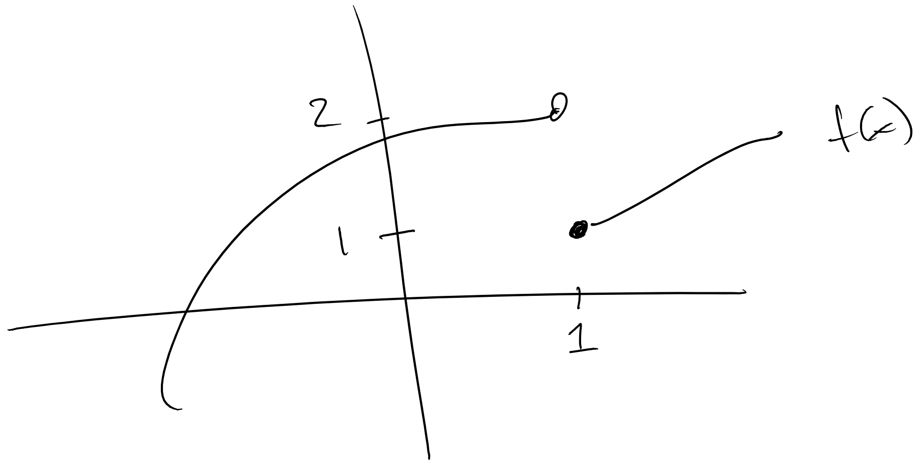
<u>One sided limits:</u>

$\lim_{x\to a^+} f(x) = L$ means $f(x)$ gets close to L whenever x is close to, <u>but larger</u> than a

$f(x)$ approaches L as x approaches a from the right

$\lim_{x\to a^-} f(x) = L$ means $f(x)$ gets close to L whenever x is close to, <u>but less than</u> a.

All laws hold for one-sided limits.


2
1
1
f(x)