Lecture 15: Anti-derivatives

Thursday, October 15, 2015 9:35 AM

<u>Definition</u>: An anti-derivative for a function $f(x)$ is a function $F(x)$ such that $F'(x) = f(x)$.

<u>example</u> x^2 is an antiderivative for $2x$

what are some antiderivatives for $3x+1$?

$\frac{3}{2}x^2 + x \xrightarrow{\frac{d}{dx}}$ | $\frac{3}{2}x^2 + x + 1 \xrightarrow{d/dx}$

$\frac{3}{2}\cdot 2x + 1 = 3x+1$! | $3x+1$

$\frac{3}{2}x^2 + x + \pi$ $\quad$ $\frac{3}{2}x^2 + x + C$, C any constant.

These are all the anti-derivatives:

<u>main observation</u>: if a function $f(x)$ is differentiable, and $f'(x)$ is <u>zero</u>, then $f(x)$ is constant.

Assume this, suppose $F(x)$ & $G(x)$ are both anti-derivatives of the same function $f(x)$

$F'(x) = G'(x) = f(x)$ but therefore, $\big(F(x) - G(x)\big)' = f(x) - f(x)$
$= 0$

but this means the difference $F(x) - G(x)$ is a function w/ 0 derivative $\Rightarrow F(x) - G(x) = C$ constant

$\Rightarrow F(x) = G(x) + C$

what are all the anti-derivatives of $f(x) = \cos x$?

$\sin x + C$, C any constant.

Practice Find all anti-derivatives for the following functions:

1. $3x^4 - 2x + \sqrt{x}$
2. $\sin 2x$
3. $\sec^2(3x+1)$
4. $\dfrac{x^3}{\sqrt{x^4-7}}$

1. $\frac{3}{5}x^5 - x^2 + \frac{2}{3}x^{3/2} + C$
2. $-\frac{1}{2}\cos 2x + C$
3. $\frac{1}{3}\tan(3x+1) + C$
4. $\frac{1}{2}(x^4-7)^{1/2} + C$

C any constant

Notation: The indefinite integral of $f(x)$ means the general form of an anti-derivative of $f(x)$.

$\int f(x)\,dx \leftarrow$ "indefinite integral of $f(x)$ with respect to the variable x"

idea of notation: $\int dy$ "=" y

$$\int \frac{dy}{dx}\,dx \;=\; \overset{\text{imagination}}{\int \frac{dy}{\cancel{dx}}\,\cancel{dx}} = \int dy = y$$

$$\int 2x\,dx = x^2 + C \qquad \int \cos x\,dx = \sin x + C$$

Integration Rules

$$\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C$$

k constant

anti-power rule $\quad \int x^n\,dx = \frac{1}{n+1}x^{n+1} + C \qquad n \neq -1$

anti-log rule $\quad \int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$

$\frac{d}{dx}\ln x = \frac{1}{x} + C$ only for $x > 0$

$\frac{d}{dx}\ln(-x) = \frac{1}{x} + C$ only for $x < 0$

$\frac{d}{dx} \ln(-x) = \frac{1}{x} + C$ only for $x<0$

$$\ln|x| = \begin{cases} \ln x & x > 0 \\ \ln(-x) & x < 0 \end{cases}$$

$$\boxed{\int k f(x)\,dx = k \int f(x)\,dx}$$

Look in mooculus, p.179 thm 11.a.1.1

$$\int \frac{1}{x^2+1}\,dx = \arctan x + C$$