Lecture 6: Chain rule and recap

Tuesday, September 8, 2015 9:35 AM

Notations for derivative

If $y = f(x)$ then these all mean the same thing:

$$f' = f'(x) = \frac{dy}{dx} = \frac{d}{dx}y = \frac{d}{dx}f(x) = \frac{df}{dx} = y'$$

$f(x) = 3x^2 + 2$ $\qquad$ $f'(x) = 6x$

$(3x^2+2)' = 6x$ $\qquad$ $\frac{d}{dx}(3x^2+2) = 6x$

Recap

Power Rule (3.2) $\qquad \frac{d}{dx}x^n = nx^{n-1}$ $\quad$ n a real #

Sum / Difference / Const mult rule (3.2)

$$\frac{d}{dx}(f(x)+g(x)) = f'(x)+g'(x)$$

$$\frac{d}{dx}(f(x)-g(x)) = f'(x)-g'(x)$$

$$\frac{d}{dx}(Cf(x)) = Cf'(x)$$

$\boxed{\text{SKIPPED 4 for now}}$

Product Rule (5.1) $\qquad \frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

Quotient Rule (5.2) $\qquad \frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

Quotient Rule (3.2) $\qquad \frac{d}{dx}\frac{\quad}{g(x)} = \frac{g\ \ldots}{(g(x))^2}$

$$\frac{d}{dx}\frac{HI}{HO} = \frac{HO\,dHI - HI\,dHO}{HO\,HO}$$

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examples

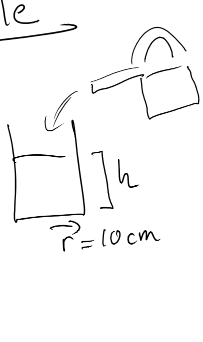
$$\left(\frac{3x^2e^x}{2x-3}\right)' = \frac{(2x-3)(3x^2e^x)' - (3x^2e^x)(2x-3)'}{(2x-3)^2}$$

$$(3x^2e^x)' = (3x^2)'e^x + (3x^2)(e^x)' = \boxed{6xe^x + 3x^2e^x}$$

$$(2x-3)' = \boxed{2}$$

$$= \frac{(2x-3)(6xe^x+3x^2e^x) - (3x^2e^x)(2)}{(2x-3)^2} \quad \text{(stop.)}$$

Chain Rule



$V = \pi r^2 h = 100\pi h$

pour at a rate of $10\,cm^3/sec$.

how fast is height changing? $\qquad \frac{dh}{dt}$

$h(t) \rightsquigarrow h'(t)$

we have $10\,cm^3/sec = \frac{dV}{dt} = 10$

$h = \frac{1}{\ldots} V \qquad \frac{dh}{\ldots} = \frac{1}{\ldots}$

$$h = \frac{1}{100\pi} \cdot V \qquad \frac{dh}{dV} = \frac{1}{100\pi}$$

$$\left(y = \frac{1}{100\pi} x\right)$$

each sec, get 10 units vol

each unit vol, get $\frac{1}{100\pi}$ units ht

$\Rightarrow$ each sec, get $\frac{10}{100\pi}$ units ht

$= \frac{1}{10\pi}$

$$\frac{dh}{dt} = \frac{1}{10\pi}$$

ie:

$$\boxed{\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}}$$

<u>Chain Rule:</u> f a fcn of u $f(u)$, u a fcn of x $u(x)$

$$\boxed{\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}}, \quad \boxed{\frac{d}{dx} f(u(x)) = f'(u(x)) \cdot u'(x)}$$

<u>ex:</u> $\boxed{u(x) = 3x^2 - 1 \qquad f(u) = \sqrt{u}}$ $\rightsquigarrow$ $f(x) = \sqrt{3x^2-1}$

$= f(u(x))$

$$f'(u) = \frac{1}{2} u^{\frac{1}{2}-1} = \frac{1}{2} u^{-1/2} = \frac{1}{2u^{1/2}} = \frac{1}{2\sqrt{u}}$$

$$f(u) = \sqrt{u} = u^{1/2}$$

$$\frac{df}{dx} = \frac{df}{du} \frac{du}{dx}$$

$$u'(x) = 6x$$

$$f'(x) = \frac{1}{2\sqrt{u}} \cdot 6x$$

$$\boxed{= \frac{1}{2\sqrt{3x^2-1}} \cdot 6x}$$

$\boxed{f(u) = \sin u \qquad u(x) = e^x}$

$f(x) = \sin(e^x)$

$f'(x) = f'(u(x)) \cdot u'(x)$

$= \cos(e^x)\, e^x$

$f'(u) = \cos u$

$u'(x) = e^x$

$f'(u(x)) = \cos(e^x)$

$f(x) = \sin(e^x)$

$u = e^x \qquad u'(x) = e^x$

$f'(x) = \cos(e^x) \cdot e^x$

dr of outer fcn $= \sin(u)$

plug in inner fcn $u(x)$

mult. by derivative of inner fcn.

$f(x) = \sin(2x^2)$

$\left(\begin{array}{c} u = 2x^2 \\ f(u) = \sin u \end{array} \right)$

$f'(x) = \cos(2x^2) \cdot (4x)$

$\frac{df}{du} \qquad \frac{du}{dx}$

$$e^{\ln x} = x \quad \leadsto \quad \frac{d}{dx}\left(e^{\ln x}\right) = \frac{d}{dx}(x)$$

$$e^{\ln x} \cdot (\ln x)' = 1$$

$$e^{\ln x} = x$$

$$\frac{d}{dx}\left(e^{\ln x}\right) = e^{\ln x} \cdot (\ln x)'$$

$u = \ln x$

e^u

$$x \cdot (\ln x)' = 1$$

$$\boxed{(\ln x)' = \frac{1}{x}}$$

Just for fun example

$$f(x) = \frac{2-x}{1-\frac{1}{1+\sin x}} \qquad f'(x) = \frac{\left(1-\frac{1}{1+\sin x}\right)\overbrace{(2-x)'}^{-1} - (2-x)\left(1-\frac{1}{1+\sin x}\right)'}{\left(1-\frac{1}{1+\sin x}\right)^2}$$

$$\left(1-\frac{1}{1+\sin x}\right)' = (1)' - \left(\frac{1}{1+\sin x}\right)' = -\left(\frac{(1+\sin x)\cdot \overbrace{1'}^{0} - 1\cdot(1+\sin x)'}{(1+\sin x)^2}\right)$$

$$= -\left(\frac{-(1+\sin x)'}{(1+\sin x)^2}\right) = \frac{\cos x}{(1+\sin x)^2}$$

$$f'(x) = \frac{\left(1-\frac{1}{1+\sin x}\right)(-1) - (2-x)\left(\frac{\cos x}{(1+\sin x)^2}\right)}{\left(1-\frac{1}{1+\sin x}\right)^2}$$