Lecture 22: integrating secant cubed

Monday, November 9, 2015 10:19 AM

Last time:

$$\int_0^1 \sqrt{1+(2t)^2}\,dt = \frac{1}{2}\int_0^{\arctan 2} \sqrt{1+\tan^2 y}\,\sec^2 y\,dy = \frac{1}{2}\int_0^{\arctan 2} \sec^3 y\,dy$$

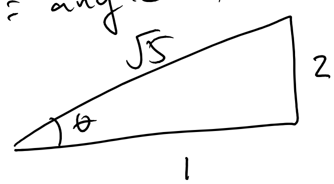
$\tan y = 2t$ $\qquad y = \arctan 2t$

$\sec^2 y = 2dt$

$\frac{1}{2}\sec^2 y\,dy = dt$

limits: $t=0 \quad y = \arctan(2\cdot 0) = 0$

$t=1 \quad y = \arctan(2\cdot 1) =$ angle θ in



Need $\displaystyle\int \sec^3 y\,dy = \int (1+\tan^2 y)\sec y\,dy$

$$= \int (\sec y + \tan^2 y \sec y)\,dy$$

$$= \int \sec y\,dy + \int \tan^2 y \sec y\,dy$$

Let's look at

$$\int \sec x\,dx = \int \sec x \,\frac{\sec x + \tan x}{\sec x + \tan x}\,dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\,dx$$

$u = \sec x + \tan x$

$du = (\sec^2 x + \sec x \tan x)\,dx$

$$\sec x(\sec x + \tan x) = \sec^2 x + \sec x \tan x = (\sec x + \tan x)'$$

$$\int \frac{1}{u}\,du = \ln|u| + C = \ln|\sec x + \tan x| + C.$$

let's look at:

$$\int \tan^2 x \sec x \, dx = \int \underbrace{\tan x}_{\substack{\text{diff} \\ \text{this}}} \underbrace{\tan x \sec x \, dx}_{\text{can integrate}} = \tan x \sec x - \int \sec^3 x \, dx$$

$u = \tan x$ $\quad du = \sec^2 x \, dx$

$dv = \tan x \sec x \, dx$ $\quad v = \sec x$

Look at this all together:

$$\int \sec^3 y \, dy = \int \sec y (1 + \tan^2 y) \, dy = \int \sec y \, dy + \int \sec y \tan^2 y \, dy$$

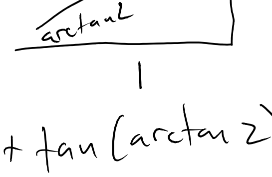
$$= \ln|\sec y + \tan y| + \tan y \sec y - \int \sec^3 y \, dy$$

$$+ \int \sec^3 y \, dy \qquad\qquad + \int \sec^3 y \, dy$$

$$2 \int \sec^3 y \, dy = \ln|\sec y + \tan y| + \tan y \sec y$$

$$\int \sec^3 y \, dy = \frac{1}{2} \left[\ln|\sec y + \tan y| + \tan y \sec y \right] + C$$

want: $$\frac{1}{2} \int_0^{\arctan 2} \sec^3 y \, dy = \frac{1}{4} \left[\ln|\sec y + \tan y| + \tan y \sec y \right]_0^{\arctan 2}$$



$$\frac{1}{4} \Big[\left(\ln|\sec(\arctan 2) + \tan(\arctan 2)| + \tan(\arctan 2)\sec(\arctan 2) \right)$$
$$- \left(\ln|\sec 0 + \tan 0| + \tan(0)\sec(0) \right) \Big]$$

$$\left. - \left(\ln|\sec 0 + \tan 0| + \tan(0)\sec(0)\right) \right]$$

$$= \frac{1}{4}\left[\left(\ln|\sqrt{5}+2| + 2\cdot\sqrt{5}\right) - \left(\underbrace{\ln|1|}_{0} + 0\cdot 1\right)\right]$$

$$\boxed{\frac{1}{4}\left(\ln|2+\sqrt{5}| + 2\sqrt{5}\right)} \approx 1.4789$$

Comment: Parts w/ definite integrals

$$\int u\,dv = uv - \int v\,du$$

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int v(x)u'(x)\,dx$$

$$\int_0^1 xe^x\,dx = xe^x - e^x\Big]_0^1 = (1e - e) - (0e^0 - e^0)$$
$$= -(-e^0) = 1$$

$$\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + C$$

$u = x$ $\quad du = dx$

$dv = e^x dx$ $\quad v = e^x$

Observe: $\left[xe^x - e^x\right]_0^1 = \left[xe^x\right]_0^1 + \left[-e^x\right]_0^1$

$$\int_0^1 xe^x\,dx = \left[xe^x\right]_0^1 - \int_0^1 e^x\,dx$$

$u = x$ $\quad du = dx$

$dv = e^x\,dx$ $\quad v = e^x$