

MATH 2250 PRACTICE SHEET FOR FINAL EXAM

1. Use the definition of the derivative to find the derivative of the function

$$f(x) = 1 + \frac{1}{x}$$

2. Find an equation for the tangent line to the graph of the function

$$f(x) = 3x + \ln x$$

at $x = 1$.

Use this information to approximate $f(1.2)$.

3. Find the derivative of the function

$$f(x) = \frac{xe^x - 1}{\ln x}$$

Compute the following limit

$$\lim_{x \rightarrow 0} \frac{\tan^2 x}{x \cos(x^2)}$$

4. Compute the following limit

$$\lim_{x \rightarrow 3} \left(1 - \frac{3}{n}\right)^n$$

5. Compute the following limit

$$\lim_{x \rightarrow 3} \frac{e^x - e^3}{x - 3}$$

6. Compute the following limit

$$\lim_{x \rightarrow 3} \frac{e^x - e^3}{x}$$

7. Find the absolute minimum and maximum values of the function $f(x) = x + \ln x$ on the interval $[1, e]$.

8. Consider the function $f(x) = \frac{3}{1+x^3}$, and suppose that $F(x)$ is an antiderivative for $f(x)$ with $F(0) = 0$.

Explain why $F(x) = \int_0^x \frac{3}{1+t^3} dt$

9. Suppose that we have functions $x(t), y(t), z(t)$ such that $x^2 + y^2 + z^2 = \ln t$.
Solve for $\frac{dz}{dt}$ by using implicit differentiation

10. Two people start walking from the same point, person A walking due north and person B walking due east. After some time, if person A is 40 feet from the starting point and walking at 3 feet per second, and if person B is 30 feet from the starting point and walking at 5 feet per second, how fast is the distance between the two people changing?

11. Compute

$$\int e^x \cos e^x dx$$

12. Compute

$$\int e^{2x} \cos e^x dx$$

13. Compute

$$\int (\sin x)^7 (\cos x) dx$$

14. Compute

$$\int x^3 \sqrt{1-x^2} dx$$

15. Compute

$$\int \cot x dx$$

16. Find two number a and b such that $3a + 4b = 9$ and such that ab is as large as possible.

17. Find all critical values of the following functions $x, x^{-1}, x^2, x^3, x^{2/3}, x^{-2/3}$.

Which of these critical values represent local minimums and which represent local maximums?

18. Use Riemann Sums with 3 rectangles and using right endpoints to approximate the value of the integral:

$$\int_0^1 \frac{1}{1+x^3} dx$$

19. A company would like to design a box (bottom, top and four sides), with square base with a volume of exactly 1000 cubic centimeters. How tall should the box be made so that it uses the least amount of material (surface area)?

20. Suppose that $f(x)$ is defined on $[-3, 3]$ which satisfies the following properties:

- $f(x)$ is increasing on the interval $[-3, 0]$,
- $f(x)$ is decreasing on $[0, 3]$,
- $f(x)$ is concave down on $[-3, 1]$, and
- $f(x)$ is concave up on $[1, 3]$.

Use this information to sketch the graph of $f(x)$.