Lecture 14: review for exam 2, part 2 (problems from practice sheet)

Monday, October 12, 2015 12:37 PM

2) $e^{xy} + \ln(x+y) = y$

$$e^{xy}\left(x\frac{dy}{dt} + y\boxed{\frac{dx}{dt}}\right) + \frac{1}{x+y}\left(\boxed{\frac{dx}{dt}} + \frac{dy}{dt}\right) = \frac{dy}{dt}$$

$$xe^{xy}\frac{dy}{dt} + \underbrace{ye^{xy}\frac{dx}{dt}} + \underbrace{\left(\frac{1}{x+y}\right)\frac{dx}{dt}} + \left(\frac{1}{x+y}\right)\frac{dy}{dt} = \frac{dy}{dt}$$

$$ye^{xy}\frac{dx}{dt} + \left(\frac{1}{x+y}\right)\frac{dx}{dt} = \frac{dy}{dt} - xe^{xy}\frac{dy}{dt} - \left(\frac{1}{x+y}\right)\frac{dy}{dt}$$

$$\frac{dx}{dt}\left(ye^{xy} + \frac{1}{x+y}\right) = \frac{dy}{dt} - xe^{xy}\frac{dy}{dt} - \left(\frac{1}{x+y}\right)\frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{\left(\frac{dy}{dt} - xe^{xy}\frac{dy}{dt} - \left(\frac{1}{x+y}\right)\frac{dy}{dt}\right)}{\left(ye^{xy} + \frac{1}{x+y}\right)}$$

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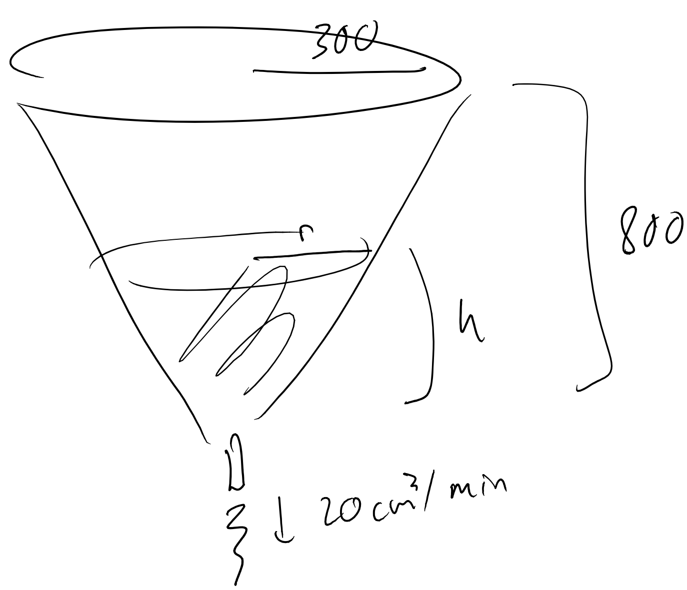
7)



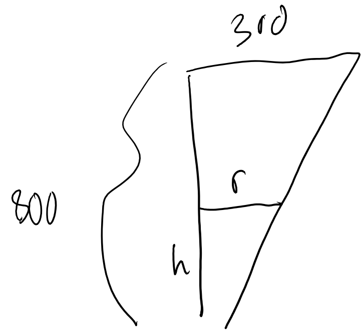
want $\frac{dh}{dt}$ when $h = 100$

$V = \frac{1}{3}\pi r^2 h$

dV

$V = \frac{}{3}$

$$\frac{dV}{dt} = -20$$



$$\frac{r}{h} = \frac{300}{800} = \frac{3}{8} \qquad \boxed{r = \frac{3}{8}h}$$

$$V = \frac{1}{3}\pi\left(\frac{3}{8}h\right)^2 h = \frac{1}{3}\pi\frac{3^2}{8^2}h^3 = \frac{3\pi}{64}h^3$$

$$\left(\frac{A}{B}C\right)^2 = \frac{A^2}{B^2}C^2$$

$$\frac{d}{dt}V = \frac{d}{dt}\left(\frac{3\pi}{64}h^3\right) = \frac{3\pi}{64}3h^2\cdot\frac{dh}{dt}$$

$$\overset{\shortparallel}{\frac{dV}{dt}}$$

$$\frac{dV}{dt} = \frac{9\pi}{64}h^2\boxed{\frac{dh}{dt}}$$

$$\frac{dh}{dt} = \left(\frac{dV}{dt}\right) \Big/ \left(\frac{9\pi}{64}h^2\right) = -20 \Big/ \left(\frac{9\pi}{64}100^2\right)$$

9)

$$f(g(x)) - 3g(x) + 2f(x+f(x)) = 9x$$

find $g'(x)$

$$\frac{d}{dx}\left[f(g(x)) - 3g(x) + 2f(x+f(x))\right] = \frac{d}{dx}(9x)$$

$$\underbrace{f'(g(x))\cdot \underbrace{g'(x)}}-\underbrace{3\underbrace{g'(x)}} + 2f'(x+f(x))\cdot(1+f'(x)) = 9$$

$f'(g(x))\cdot g'(x) \ldots = 6$

$$f'(g(x))g'(x) - 3g'(x) = 9 - 2f'(x+f(x))(1+f'(x))$$

$$g'(x)\left(f'(g(x)) - 3\right) = 9 - 2f'(x+f(x))(1+f'(x))$$

$$g'(x) = \frac{9-2f'(x+f(x))(1+f'(x))}{f'(g(x))-3}$$

15)

$$\lim_{x\to 0} (1-x)^{1/x} = L$$

$$\ln\left(\lim_{x\to 0}(1-x)^{1/x}\right) = \ln L$$

$$\lim_{x\to 0} \ln(1-x)^{1/x} = \lim_{x\to 0}\frac{1}{x}\ln(1-x) = \lim_{x\to 0}\frac{\ln(1-x)}{x}$$

l'hop

$\ln(1-0) = \ln 1 = 0$

$x \to 0$

$$\lim_{x\to 0}\frac{\left(\frac{1}{1-x}\right)\cdot(-1)}{1} = \lim_{x\to 0}\frac{-1}{1-x} = -1$$

$$\ln L = -1$$

$$\boxed{L = e^{-1} = \frac{1}{e}}$$

10)

$f(x) = x(6-2x)^2$ crit pts

$f(x) = x(36 - 24x + 4x^2)$

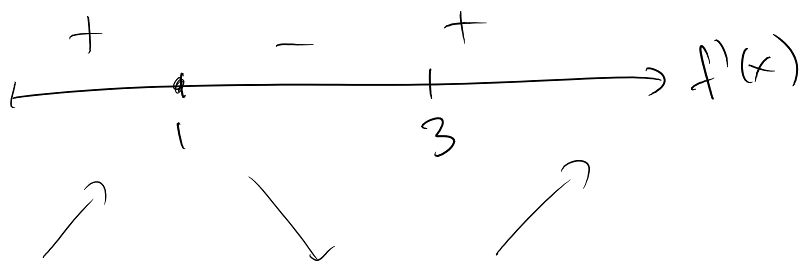
$f(x) = 36x - 24x^2 + 4x^3 \longrightarrow f'(x) = 36 - 48x + 12x^2$

$= 12(3 - 4x + x^2)$

$= 12(x^2 - 4x + 3)$

$= 12(x-3)(x-1)$

$\boxed{\text{crit pts } x = 1, 3}$



loc max: @ $x = 1$
value $f(1)$

loc min: @ $x = 3$
value $f(3)$
$= 3(6 - 2\cdot 3)^2$

either max @ $x=1$ or fcn is bigger somewhere to right of $x=3$

if bigger somewhere to right of $x=3$ then no max, since fcn keep increasing.

max only happens at endpts & crit pts.
no endpts. can check crit pts are not the max & min