Lecture 13: Review for Exam 2, part 1

Tuesday, October 6, 2015 9:33 AM

Practice w/ L'Hopital's Rule

1. $\lim\limits_{x\to 0} \frac{\sin 5x}{x}$

2. $\lim\limits_{t\to 0} \frac{\cos t - 1}{e^t - t - 1}$

3. $\lim\limits_{x\to\infty} (\ln x)^{1/x}$

4. $\lim\limits_{x\to 0^+} (\sin x)\ln x$

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1. $\lim\limits_{x\to 0} \frac{\sin 5x}{x} = \lim\limits_{x\to 0} \frac{(\cos 5x)\cdot 5}{1} = \frac{(\cos 0)\cdot 5}{1} = 5$

"$\frac{0}{0}$"

2. $\lim\limits_{t\to 0} \frac{\cos t - 1}{e^t - t - 1} = \lim\limits_{t\to 0} \frac{-\sin t}{e^t - 1} = \lim\limits_{t\to 0} \frac{-\cos t}{e^t} = \frac{-\cos 0}{e^0}$

$= -1$

"$\frac{1-1}{1-0-1}$" $=$ "$\frac{0}{0}$"

"$\frac{0}{0}$" again

3. $\lim\limits_{x\to\infty} (\ln x)^{1/x} = L = 1.$

$\ln\left(\lim\limits_{x\to\infty} (\ln x)^{1/x}\right) = \ln L$

$\lim\limits_{x\to\infty}$ "$\ln\left((\ln x)^{1/x}\right)$" $= \lim\limits_{x\to\infty} \frac{1}{x}(\ln\ln x) = \lim\limits_{x\to\infty} \frac{\ln(\ln x)}{x}$

$$\lim_{x\to\infty} \ln\left((\ln x)^{\cdots}\right) = \lim_{x\to\infty} \frac{1}{x}(\ln \ln x) = \lim_{x\to\infty} \qquad x$$

$$\text{“}\frac{\infty}{\infty}\text{”}$$

L'Hop.

$$\lim_{x\to\infty} \frac{\left(\frac{1}{\ln x}\right)\cdot \frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x \ln x} = 0 \Rightarrow \ln L = 0$$

$$\frac{1}{\text{Bigger \& bigger}}$$

$$L = e^0 = 1.$$

$$e^{\ln L} = e^0$$

4. $\lim_{x\to 0^+} (\sin x)\ln x$

"$0\cdot(-\infty)$"

$$= \lim_{x\to 0^+} \frac{\ln x}{(1/\sin x)}$$

$$= \lim_{x\to 0^+} \frac{\sin x}{(1/\ln x)}$$

$$= \lim_{x\to 0^+} \frac{\ln x}{(\sin x)^{-1}} = \lim_{x\to 0^+} \frac{\left(\frac{1}{x}\right)}{-(\sin x)^{-2}\cos x} = \lim_{x\to 0^+} \frac{-\sin^2 x}{x\cos x}$$

"$\frac{-\infty}{\infty}$" indet. form.

$$\frac{1}{\text{small}} \longrightarrow \infty$$

"$\frac{0}{0}$"

$$= \lim_{x\to 0^+} \frac{-2\sin x\cos x}{\cos x + x(-\sin x)}$$

I don't like this. there is probably an easier way.

$$\left(\lim_{x\to 0^+} \frac{\sin x}{x}\right)\left(\lim_{x\to 0} \frac{-\sin x}{\cos x}\right)$$

$$1 \cdot \frac{-0}{1} = 0.$$

examples

$$\lim_{x\to\infty} \sqrt{x^2+x} - x = \lim_{x\to\infty} \left(\frac{\sqrt{x^2+x} - x}{1}\right)\left(\frac{\sqrt{x^2+x} + x}{\sqrt{x^2+x} + x}\right)$$

$\infty - \infty$

$$= \lim_{x\to\infty} \frac{(x^2+x) - x^2}{\sqrt{x^2+x} + x} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+x} + x}$$

$= \dots$ L'hop "$\frac{\infty}{\infty}$"

$$\lim_{x\to 0^+} x^2(\ln x) = \lim_{x\to 0^+} \frac{\ln x}{x^{-2}} \dots \text{L'hop}$$

$$\lim_{x\to 0^+} x^x = L$$

$$\ln\left(\lim_{x\to 0^+} x^x\right) = \ln L$$

"

$$\lim_{x\to 0^+} \ln(x^x) = \lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{x^{-1}} = \dots$$

"$0\cdot(-\infty)$" "$\frac{-\infty}{\infty}$" l'Hop ...

Optimization

Local extrema can only happen at critical points:

crit pt: $f'(x)$ is either 0 or not defined.

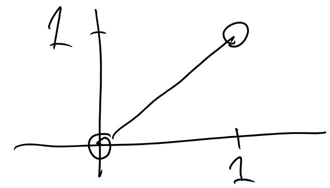
Extrema are therefore either

- loc extrema (crit pts) or
- happen on boundary. (end pt)

Closed interval theorem:

If $f(x)$ is a continuous function on a closed interval $[a,b]$ then $f(x)$ must have a minimum & maximum value.

non-example: $f(x) = x$ on $(0,1)$



So, to find min & max values for fcns on closed intervals.

Choices

1. make a sign chart for $f'(x)$ (might tell us everything)

2. check all crit pts & endpts, and see what the biggest & smallest values are.

3. Some combination of 1 & 2.

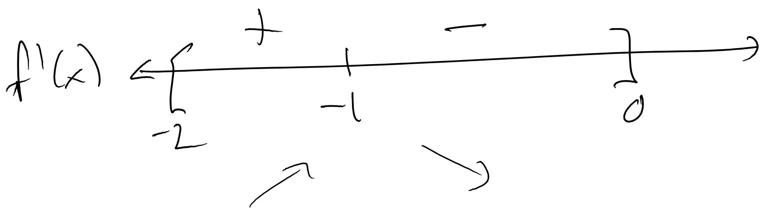
(prob 11)

ex: $f(x) = -3(x+1)^{2/3}$, $-2 \leq x \leq 0$ (closed interval)

find min & max

$$f'(x) = -3(2/3)(x+1)^{-1/3} = -2\frac{1}{\sqrt[3]{x+1}} = -\frac{2}{\sqrt[3]{x+1}}$$

$$f'(x) = -3\left(\tfrac{2}{3}\right)(x+1)^{-1/3} = -2\,\frac{1}{\sqrt[3]{x+1}} = -\frac{2}{\sqrt[3]{x+1}}$$



$f(-1) = \text{max!}$

$f(-1) = -3(-1+1)^{2/3} = 0$

min either $f(-2)$ or $f(0)$

$f(-2) = -3(-2+1)^{2/3} = -3$

$f(0) = -3(1)^{2/3} = -3$