MATH 2250 PRACTICE SHEET FOR FINAL EXAM

1. Use the definition of the derivative to find the derivative of the function
$$f(x) = 1 + \frac{1}{x}$$

2. Find an equation for the tangent line to the graph of the function
$$f(x) = 3x + \ln x$$
at $x = 1$.
Use this information to approximate $f(1.2)$.

3. Find the derivative of the function
$$f(x) = \frac{xe^x - 1}{\ln x}$$

4. Compute the following limit
$$\lim_{x \to 0} \frac{\tan^2 x}{x \cos(x^2)}$$

5. Compute the following limit
$$\lim_{x \to \infty} \left(1 - \frac{3}{x}\right)^x$$

6. Compute the following limit
$$\lim_{x \to 3} \frac{e^x - e^3}{x - 3}$$

7. Compute the following limit
$$\lim_{x \to 3} \frac{e^x - e^3}{x}$$

8. Find the absolute minimum and maximum values of the function $f(x) = x + \ln x$ on the interval $[1, e]$.

9. Consider the function $f(x) = \dfrac{3}{1 + x^3}$, and suppose that $F(x)$ is an antiderivative for $f(x)$ with $F(0) = 0$.
Explain why $F(x) = \displaystyle\int_0^x \frac{3}{1 + t^3} dt$

10. Suppose that we have functions $x(t), y(t), z(t)$ such that $x^2 + y^2 + z^2 = \ln t$.
Solve for $\frac{dz}{dt}$ by using implicit differentiation

11. Two people start walking from the same point, person A walking due north and person B walking due east. After some time, if person A is 40 feet from the starting point and walking at 3 feet per second, and if person B is 30 feet from the starting point and walking at 5 feet per second, how fast is the distance between the two people changing?

12. Compute
$$\int e^x \cos e^x dx$$

13. Compute
$$\int e^{2x} \cos e^x dx$$

14. Compute
$$\int (\sin x)^7 (\cos x) dx$$

15. Compute
$$\int x^3 \sqrt{1-x^2}\ dx$$

16. Compute
$$\int \cot x\ dx$$

17. Find two number a and b such that $3a + 4b = 9$ and such that ab is as large as possible.

18. Find all critical values of the following functions $x, x^{-1}, x^2, x^3, x^{2/3}, x^{-2/3}$.
Which of these critical values represent local minimums and which represent local maximums?

19. Use Riemann Sums with 3 rectangles and using right endpoints to approximate the value of the integral:
$$\int_0^1 \frac{1}{1+x^3}\ dx$$

20. A company would like to design a box (bottom, top and four sides), with square base with a volume of exactly 1000 cubic centimeters. How tall should the box be made so that it uses the least amount of material (surface area)?

21. Suppose that $f(x)$ is defined on $[-3, 3]$ which satisfies the following properties:
- $f(x)$ is increasing on the interval $[-3, 0]$,
- $f(x)$ is decreasing on $[0, 3]$,
- $f(x)$ is concave down on $[-3, 1]$, and
- $f(x)$ is concave up on $[1, 3]$.

Use this information to sketch the graph of $f(x)$.