Lecture 11: curve sketching

Tuesday, September 29, 2015 9:33 AM

<u>examples</u>

$y = x^2 - x$ use sign charts for $f(x)$ & $f'(x)$

<u>sign chart for $f(x)$</u>

<u>Method</u>: if $f(x)$ changes sign from pos to neg or vise-versa, it must do so either by

- passing through 0 or
- by having a discontinuity. } IVT

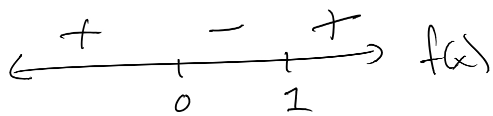
$y = f(x) = x^2 - x \longrightarrow = 0$? $\quad x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ or 1

$\searrow$ discontinuous?
always cont.

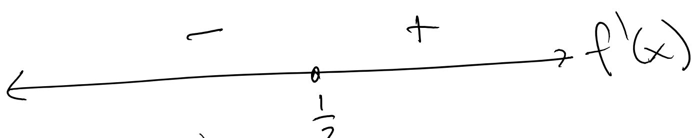
mark the points on # line

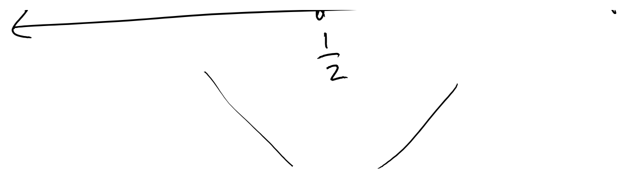


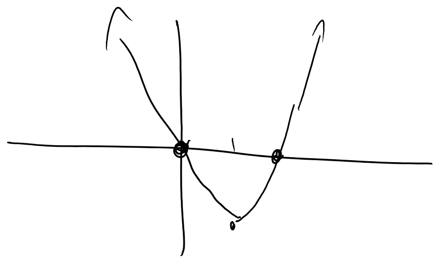
test values: $f(x) = x^2 - x$
$= x(x-1)$

$f'(x) = (x^2 - x)' = 2x - 1$ $\quad f'(x) = 0 \longrightarrow x = \frac{1}{2}$

$f'(x) = \text{d.n.e} \longrightarrow$ nothing.







Practice

1. $f(x) = \dfrac{x+1}{x-1}$

sign chart for $f(x)$

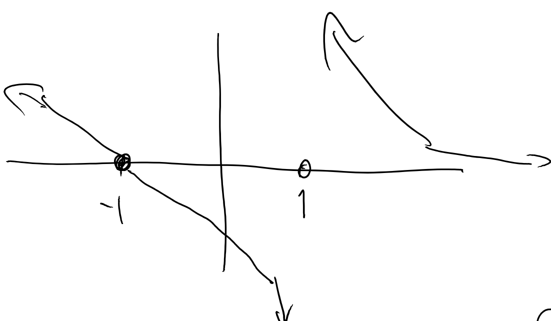
$+ \quad - \quad +$

$-1 \quad 1$

sign chart for $f'(x) = \dfrac{(x-1)(1)-(x+1)(1)}{(x-1)^2} = \dfrac{-2}{(x-1)^2}$

$- \quad -$

1



2. $f(x) = \dfrac{x^4}{4} + \dfrac{x^3}{3} - x^2$

$f'(x) = x^3 + x^2 - 2x$

leave off sign chart for $f(x)$.

$f'(x) = 0$ gives

$x^3 + x^2 - 2x = 0$

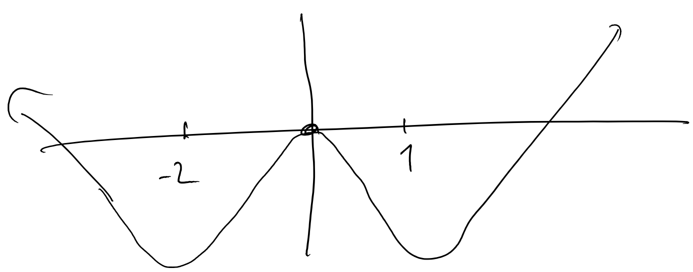
$x(x^2 + x - 2) = 0$

$x(x+2)(x-1) = 0$

$f'(x)$

$- \quad + \quad - \quad +$

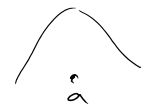
$-2 \quad 0 \quad 1$



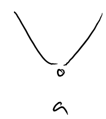
Fact

If $f(x)$ is increasing just to left of $x=a$ & decreasing just to

If $f(x)$ is increasing just to left of $x=a$ & decreasing just to right of $x=a$, then $(a, f(x))$ is a local maximum



If $f(x)$ is decreasing just left, increasing just to right then local min.



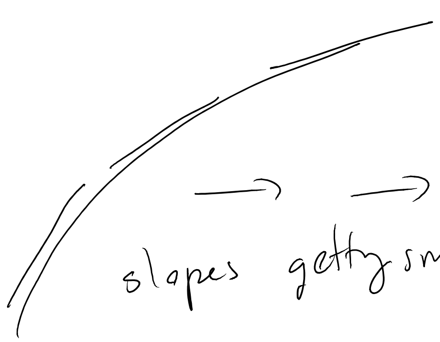
First derivative test

If $f'(x) > 0$ to left of a & $f'(x) < 0$ to right of a then a is a local max

If $f'(x) < 0$ to left of a & $f'(x) > 0$ to right of a then a local min.

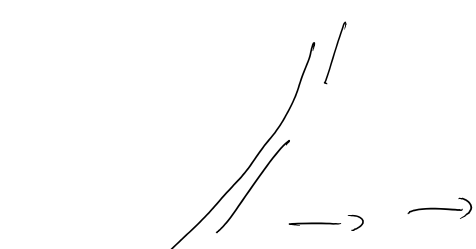
Second derivatives

$f''(x)$ = second derivative of $f(x)$ = rate of change of slopes.



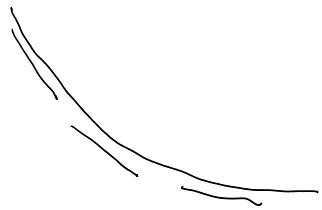
slopes getting smaller

$f''(x) < 0$
slopes decreasing



slopes getting bigger

$f''(x) > 0$
slopes increasing.



$f''(x) > 0$

slopes going from neg $\to$ close to 0

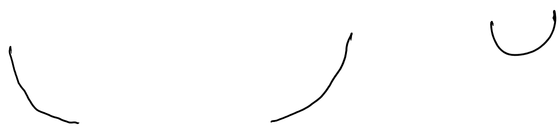
(increasing)

$f''(x) < 0$

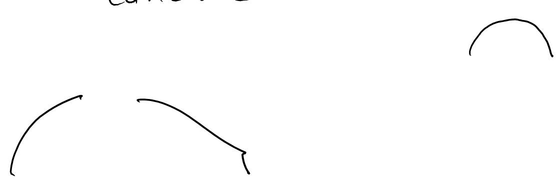
slopes getting more negative

$f'' \longleftrightarrow$ concavity

$f'' > 0$ "concave up" c.up

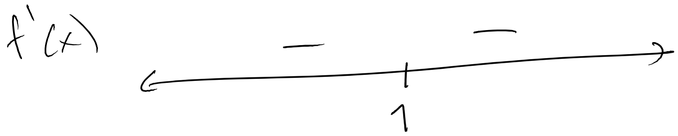


$f'' < 0$ "concave down"



$f(x) = \dfrac{x+1}{x-1}$

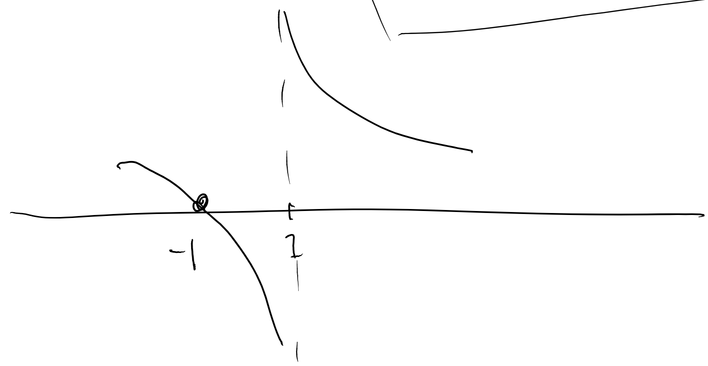
$f'(x) = \dfrac{-2}{(x-1)^2}$



$-2)'$ -3 4

$$f''(x) = \left(-2(x-1)^{-2}\right)' = 4(x-1)^{-3}(1) = \frac{4}{(x-1)^3}$$

$f''(x)$: $-$ | 1 | $+$



$$f(x) = x^3 - x$$

$$f(x) = 0 = x^3 - x$$

$$0 = x(x^2-1) = x(x-1)(x+1)$$

$-$ | -1 | $+$ | 0 | $-$ | 1 | $+$

$$f'(x) = 3x^2 - 1 = 0$$

$$x^2 = \frac{1}{3} \quad x = \pm\sqrt{\frac{1}{3}}$$

$+$ | $-\sqrt{\frac{1}{3}}$ | $-$ | $\sqrt{\frac{1}{3}}$ | $+$

↗ ↘ ↗

$$f''(x) = 6x$$

$-$ | 0 | $+$

c. down | 0 | c. up

inc	dec	dec	inc
c. down	c. down	c. up	c. up

$-\sqrt{\frac{1}{3}}$ | 0 | $\sqrt{\frac{1}{3}}$