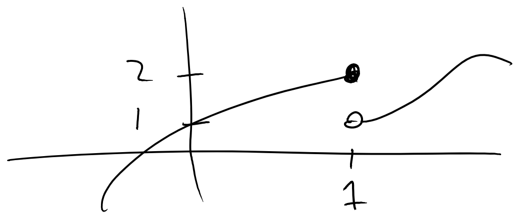
Last time: one-sided limits (1.1)

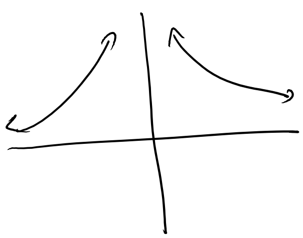


$$\lim_{x\to 1^+} f(x) = 1 \qquad f(x) = 2$$

$$\lim_{x\to 1^-} f(x) = 2$$

Def $\lim\limits_{x\to a^+} f(x) = L$ means $f(x)$ gets close to L whenever x gets close to & is bigger than a.

Vertical Asymptotes (2.1)



$\frac{1}{x^2}$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1}{x^2} = \infty$$

is a new kind of sentence

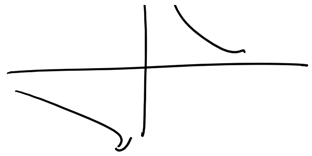
what does this mean?

Defn $\lim\limits_{x\to a} f(x) = \infty$ means that we can make $f(x)$ as as big as we want, if we make x sufficiently close to a.

$$\lim_{x\to 0} \frac{1}{x} = \text{DNE}$$

$$\lim_{x\to 0^+} \frac{1}{x} = \infty$$

$$\lim_{x\to 0^-} \frac{1}{x} = -\infty$$



$$\lim_{x\to 0^-} \frac{1}{x} = -\infty$$

<u>Def</u> $\lim_{x\to a^+} f(x) = -\infty$ means we can make $f(x)$ as largely negative as we want, by making x sufficently close to, but larger than a.

<u>ex:</u> find and describe vertical asymptotes of $f(x) = \frac{x^2-9x+14}{x^2-5x+6}$

- If $f(x)$ has a vertical asymptote at $x=a$, then $f(x)$ is <u>not</u> continuous at $x=a$
- Rational functions are only discontinous when denominator $=0$.

$$f(x) = \frac{(x-7)(x-2)}{(x-3)(x-2)} \left(\neq \frac{x-7}{x-3} \right)$$

$$\lim_{x\to 2} \frac{(x-7)(x-2)}{(x-3)(x-2)} = \lim_{x\to 2} \frac{x-7}{x-3} = \frac{-5}{-1} = 5$$ no asymp at $x=2$.

$$\lim_{x\to 3^+} \frac{(x-7)(x-2)}{(x-3)(x-2)} = \lim_{x\to 3^+} \frac{x-7}{x-3} = \lim_{x\to 3^+} \frac{1}{x-3}(x-7)$$

" ?

$$\left(\lim_{x\to 3^+} \frac{1}{x-3}\right)\left(\underbrace{\lim_{x\to 3^+} x-7}\right)$$

$$\left(\frac{1}{\text{small}+}\right)(-4)$$

$$= (\text{BIG}+)(-4)$$

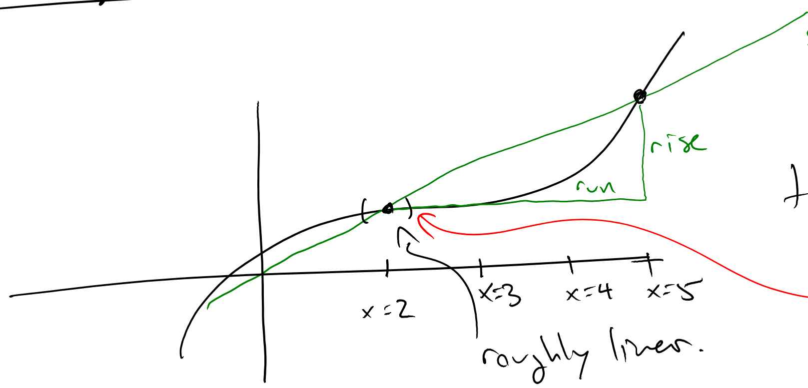
$$= -\text{BIG}$$

$$\lim_{x\to 3^-} \frac{(x-7)(x-2)}{(x-3)(x-2)} = \lim_{x\to 3^-} \frac{x-7}{x-3}$$

$$= \infty$$

$= -\infty$

Tangent lines (and secant lines)

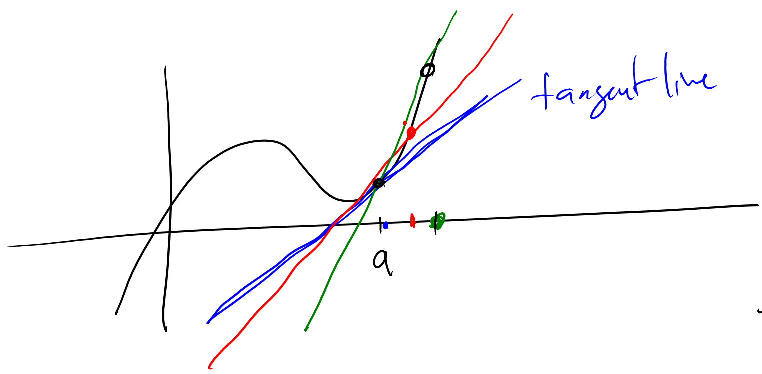


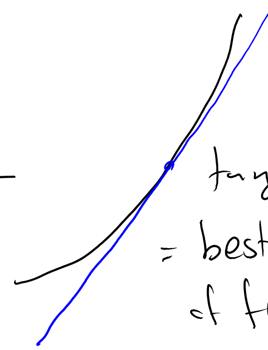
secant line : line through 2 pts in graph.

to get a line, need a point, and a slope.

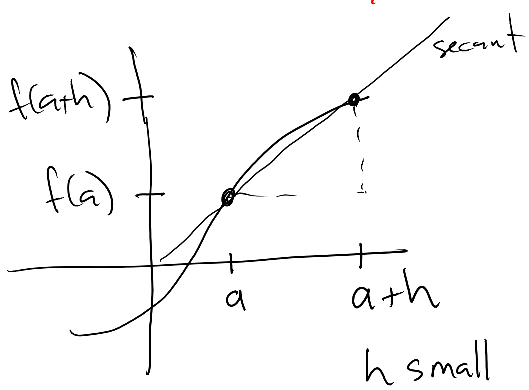
slope : need a $\frac{\text{rise}}{\text{run}}$ (another point)

to get a better view of what's happening at $x=2$, want to bring second pt *much* closer.





tangent line = best approximation of $f(x)$ near $x=a$ using a line.



h small

$$\text{slope of secant line} = \frac{\text{rise}}{\text{run}} = \frac{f(a+h)-f(a)}{(a+h)-a}$$

$$\boxed{= \frac{f(a+h)-f(a)}{h}}$$

difference quotient

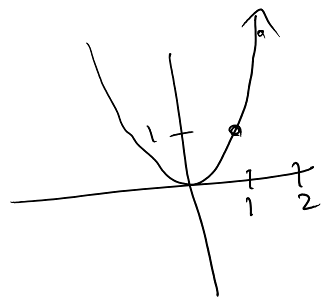
$$\boxed{\text{slope of tangent line} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}}$$

$$\boxed{\text{slope of tangent line} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

<u>Practice</u>: • Find slope of secant line between $x=1$ & $x=2$

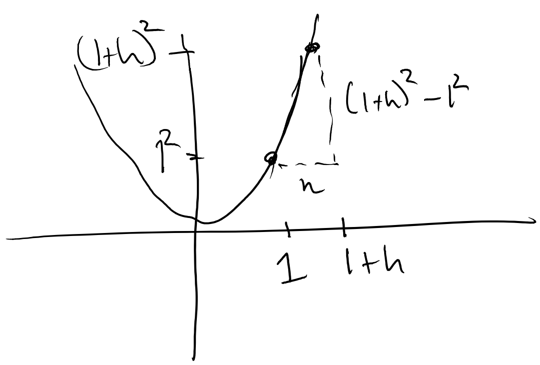
• Find slope of tang. line at $x=1$

for $f(x) = x^2$



• $\dfrac{f(2)-f(1)}{2-1} = \dfrac{2^2 - 1^2}{1} = \dfrac{4-1}{1} = 3.$

• $\displaystyle \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{(1+h)^2 - 1^2}{h} = \lim_{h\to 0} \frac{1+2h+h^2-1}{h}$



$\displaystyle = \lim_{h\to 0} \frac{2h+h^2}{h} = \lim_{h\to 0} \frac{h}{h}(2+h)$

$\displaystyle = \lim_{h\to 0} 2+h = 2$