Lecture 19: integration practice, parts

Monday, November 2, 2015 10:10 AM

Practice:

2. $\int \tan x \, dx$

1. $\int \sin x \cos x \, dx$

3. $\int \sin^3 x \, dx$

1. $\int \sin x \cos x \, dx = \int u \, du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2 x + C$

$u = \sin x$

$du = \cos x \, dx$

$\int \sin x \cos x \, dx = -\int u \, du = -u^2 + C = -\frac{1}{2}\cos^2 x + C$

$u = \cos x$

$du = -\sin x \, dx$

2. $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{1}{u} du$

$= -\ln|u| + C$

$u = \cos x$

$du = -\sin x \, dx$

$= -\ln|\cos x| + C$

3. $\int \sin^3 x \, dx = \int \sin x (1 - \cos^2 x) \, dx$

$$= \int \sin x \, dx - \int \sin x \cos^2 x \, dx$$

$$= -\cos x - \int \sin x \cos^2 x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$$= -\cos x + \int u^2 \, du$$

$$= -\cos x + \frac{1}{3} u^3 + C$$

$$= -\cos x + \frac{1}{3} \cos^3 x + C$$

$$\int \cos^5 x \, dx = \int (\cos^2 x)(\cos^2 x) \cos x \, dx$$

$$= \int (1 - \sin^2 x)(1 - \sin^2 x) \cos x \, dx$$

$u = \sin x \quad du = \cos x \, dx$

$$= \int (1 - u^2)^2 \, du \ldots$$

$\int \cos^2 x \, dx$?

Anti Product Rule

Product Rule: Given $u = u(x)$ $v = v(x)$ then

$$(uv)' = u'v + uv'$$

$$\int (uv)'\,dx = \int u'v\,dx + \int uv'\,dx$$

$$uv = \int u'v\,dx + \int uv'\,dx$$

$$\Rightarrow \int uv'\,dx = uv - \int u'v\,dx$$

$$\int u(x)\,v'(x)\,dx = u(x)\,v(x) - \int u'(x)\,v(x)\,dx$$

ex: $\int xe^x\,dx$

$u(x) = x$ - would be better if we differentiate

$v'(x) = e^x$ - not afraid of its anti-derivative.

$\Rightarrow v(x) = e^x$ (anti-der of $v'(x)$)

$$\int xe^x\,dx = \int u(x)\,v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$$

$$= xe^x - \int 1\cdot e^x\,dx = xe^x - e^x + C.$$

$$\int x\sin x\,dx = -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + C$$

$u(x) = x \qquad u'(x) = 1$

$v'(x) = \sin x \qquad v(x) = -\cos x$

Notation: $du = u'(x)dx \qquad dv = v'(x)dx$

$$\int u \underbrace{v' dx}_{dv} = uv - \int v \underbrace{u' dx}_{du}$$

$$\int u\, dv = uv - \int v\, du$$

$$\int \cos^2 x\, dx = \int \cos x \cos x\, dx = \cos x \sin x + \int \sin^2 x\, dx$$

$u = \cos x \qquad du = -\sin x\, dx$

$dv = \cos x\, dx \qquad v = \sin x$

$$= \cos x \sin x + \int (1 - \cos^2 x)\, dx$$

$$\int \cos^2 x\, dx = \cos x \sin x + \int 1\, dx - \int \cos^2 x\, dx$$

$$\int \cos^2 x\, dx = \cos x \sin x + x - \int \cos^2 x\, dx$$

$$+ \int \cos^2 x\, dx \qquad\qquad + \int \cos^2 x\, dx$$

$$2 \int \cos^2 x\, dx = \cos x \sin x + x + C$$

$2\int \cos \cancel{x} \ldots$

$$\boxed{\int \cos^2 x\,dx = \frac{1}{2}(\cos x \sin x + x) + C}$$

$$\int \ln x\,dx = \int u e^u\,du = ue^u - e^u + C$$

$$= (\ln x)e^{\ln x} - e^{\ln x} + C$$

$$= \boxed{x\ln x - x + C}$$

$u = \ln x$

$e^u = x$

$e^u u'(x) = 1$

$e^u \underbrace{u'(x)dx}_{du} = dx$

$e^u\,du = dx$