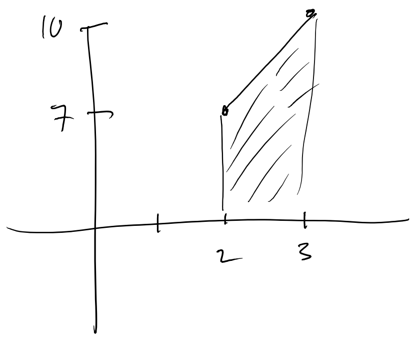
ex:



$f(x) = 1 + 3x$

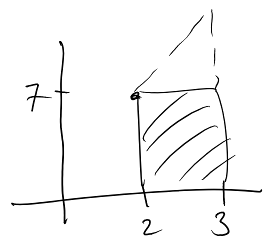
x between 2 & 3

subdivide $[2,3]$ into pieces & draw rectgles, add up areas.

left hand endpts for heights:

ex: 1 rectangle. $[2,3]$ $ht = f(2) = 1 + 3(2) = 7$

$b \cdot h = 1 \cdot 7 = 7$ ← approx area.



ex: 2 rectangles: $[2, 2+1/2]$ ← $ht = f(2) = 7$

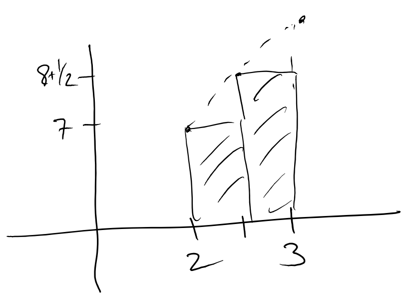
$[2+1/2, 3]$ ← $ht = f(2+1/2) = 1 + 3(2+1/2)$

$= 1 + 6 + 3/2 = 8 + 1/2$

area $= \underbrace{b \cdot h}_{1st} + \underbrace{b \cdot h}_{2nd} = (1/2)(7) + (1/2)(8+1/2)$

$= 7/2 + 4 + 1/4 = 14/4 + 16/4 + 1/4$

$= 31/4 = 7 + 3/4$



ex: n rectangles:

$[2, 2+1/n]$ ← $ht = f(2) = 7$

$[2+1/n, 2+2/n]$ ← $ht = f(2+1/n) = 1 + 3(2+1/n)$

$= 7 + 3/n$

$[2+2/n, 2+3/n]$ ← $ht = f(2+2/n) = 1 + 3(2+2/n)$

$\vdots \qquad\qquad = 7 + 6/n$

$\left[2+i/n,\ 2+\frac{i+1}{n}\right] \leftarrow ht = f(2+i/n) = 1+3(2+i/n)$

$= 7 + 3i/n$

$\vdots$

$\left[2+\frac{n-1}{n},\ 3\right] \leftarrow f(2+\frac{n-1}{n}) = 1+3\left(2+\frac{n-1}{n}\right)$

$= 7 + 3\left(\frac{n-1}{n}\right)$

<u>Area:</u> $\frac{1}{n}\cdot 7 + \frac{1}{n}(7+3/n) + \frac{1}{n}(7+6/n) + \cdots + \frac{1}{n}(7+3i/n) + \cdots + \frac{1}{n}\left(7+\frac{3(n-1)}{n}\right)$

$= \frac{1}{n}\left(7 + (7+3/n) + (7+6/n) + \cdots + (7+3i/n) + \cdots + \left(7+\frac{3(n-1)}{n}\right)\right)$

$= \frac{1}{n}\left(7n + \left(3/n + 6/n + \cdots + 3i/n + \cdots + 3(n-1)/n\right)\right)$

$= \frac{1}{n}\left[7n + \frac{1}{n}\left(3+6+9+12+\cdots+3i+\cdots+3(n-1)\right)\right]$

$= \frac{1}{n}\left[7n + \frac{3}{n}\underbrace{\left(1+2+3+4+\cdots+(n-1)\right)}_{\frac{(n-1)}{2}\cdot n}\right]$

$= \frac{1}{n}\left[7n + \frac{3}{\cancel{n}}\left(\frac{(n-1)\cancel{n}}{2}\right)\right] = \frac{1}{n}\left(7n + \frac{3(n-1)}{2}\right)$

$= \frac{7n + \left(\frac{3}{2}\right)(n-1)}{n}$

as $n \to \infty$, get area is $\lim_{n\to\infty} \frac{7n + \left(\frac{3}{2}\right)(n-1)}{n}$

$= \lim_{x\to\infty} \frac{7x + (3/2)x - (3/2)}{x}$

$\Big\downarrow$ L'hopital!

$= \quad 7 + (3/2) \quad =$

$$= \lim_{x\to\infty} \frac{7+(3/2)}{1} = 7 + 3/2$$

We have just shown:

$$\int_2^3 (1+3x)\,dx = 7 + 3/2$$

Important fact: If $f(x)$ is a continuous function in a bounded region, then $f(x)$ is Riemann Integrable.

this means: as long as spacing goes to 0, always get same answer.

$$\frac{1}{n}\Big(7 + (7+3/n) + (7+6/n) + \cdots + (7+3i/n) + \cdots (7 + \tfrac{3(n-1)}{n})\Big)$$

$7 + 3(0)/n$

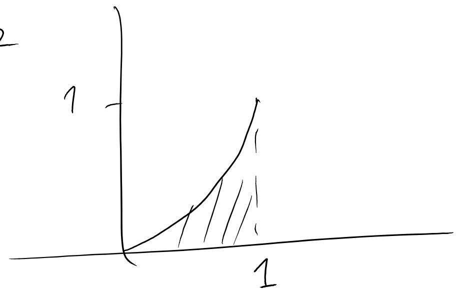
Notation: $\displaystyle\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$

$$\sum_{i=5}^{32} a_i = a_5 + a_6 + a_7 + \cdots + a_{32}$$

$$\frac{1}{n}\sum_{i=0}^{n-1}(7 + 3i/n) = \frac{1}{n}\left[\left(\sum_{i=0}^{n-1} 7\right) + \left(\sum_{i=0}^{n-1}(3i/n)\right)\right]$$

$$\frac{1}{n}\left(\sum_{i=0}^{n-1} 7 + \frac{3}{n} \sum_{i=0}^{n-1} i \right)$$

$f(x) = x^2$



- 2 rectangles

$[0, 1/2] \leftarrow$ ht $1/4$

$[1/2, 1] \leftarrow$ ht 1

$$(1/2)(1/4) + (1/2)(1) = \boxed{5/8}$$

calculate area

- n rectangles

$[0, 1/n] \leftarrow ht = (1/n)^2$

$[1/n, 2/n] \leftarrow ht = (2/n)^2$

$\vdots$

$[i/n, i+1/n] \leftarrow (i+1/n)^2$

$[n-1/n, n/n] \leftarrow (n/n)^2$

$n/n = 1$

area: $\frac{1}{n}(1/n)^2 + \frac{1}{n}(2/n)^2 + \cdots + \frac{1}{n}(n/n)^2$

$$\sum_{i=1}^{n} \frac{1}{n}(i/n)^2 = \sum_{i=1}^{n} \frac{1}{n} \frac{i^2}{n^2} = \frac{1}{n^3} \sum_{i=1}^{n} i^2$$

$$= \frac{1}{n^3} \frac{n(n+1)(2n+1)}{6}$$

$$\frac{n(n+1)(2n+1)}{} = \cdots = 1$$

$$\lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6n^3} = \cdots = \frac{1}{3}$$