Lecture 16: definite integrals

Monday, October 26, 2015 10:22 AM

Practice / Review of u-substitutions

1. $\int \sin(2x+1)\,dx$

2. $\int e^x \cos e^x\,dx$

3. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

4. $\int \frac{x}{\sqrt{x-1}}\,dx$

$u = e^x$
$du = e^x\,dx$

$u = 2x+1$
$du = 2\,dx$
$\frac{1}{2}du = dx$

$$\int \sin(2x+1)\,dx = \frac{1}{2}\int \sin\underbrace{(2x+1)}_{u}\,\underbrace{2\,dx}_{du}$$

$$= \frac{1}{2}\int \sin u\,du = \frac{-\cos(2x+1)}{2} + C$$

$$\int \sin(2x+1)\,dx = \int \sin(u)\,\tfrac{1}{2}\,du$$

$u = \sqrt{x}$
$du = \frac{1}{2\sqrt{x}}\,dx$

$u = x-1$
$du = dx$
$u+1 = x$

$$\int \frac{u+1}{\sqrt{u}}\,du$$

$$= \int \left(\frac{u}{\sqrt{u}} + \frac{1}{\sqrt{u}}\right)du = \int \left(u^{1/2} + u^{-1/2}\right)du$$

$u \cdot u^{-1/2}$ $\quad u^{-1/2}$

$$\frac{2}{3}(x-1)^{3/2} + 2(x-1)^{1/2} + C$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

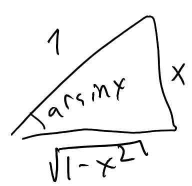
$(\sqrt{x})^2 = x$

$\sin(\arcsin x) = x$

$\ln e^x = x$

$e^{\ln x} = x$

$$\cos(\arcsin x)\cdot \frac{d}{dx}\arcsin x = 1$$



$$\frac{d}{dx}\arcsin x = \frac{1}{\cos(\arcsin x)}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}$$

$$\frac{d}{dx}(\ln x)^{\sec x}$$

$$(\ln x)^{\sec x} = f(x)$$

$$\ln\left((\ln x)^{\sec x}\right) = \ln f(x)$$

$\ln(A^B) = B\ln A \rightarrow$

$$\sec x \ln(\ln x) = \ln f(x)$$

$\downarrow$ d/dx both sides

$$\left[\sec x \tan x \ln(\ln x) + \sec x \cdot \frac{1}{\ln x}\frac{1}{x}\right]$$

$$= \frac{1}{f(x)} \cdot f'(x)$$

$$\overset{=}{\swarrow}$$

$$e^{\sec x \ln(\ln x)} = f(x)$$

$\downarrow$ d/dx

$$e^{\sec x \ln(\ln(x))} \cdot \underbrace{\frac{d}{dx}(\sec x \ln(\ln x))}_{\sec x \tan x \ln(\ln x) + \sec x \underbrace{\frac{d}{dx}(\ln(\ln x))}_{\frac{1}{\ln x} \cdot \underbrace{\frac{d}{dx}(\ln x)}_{\frac{1}{x}}}}$$

$$e^{\sec x \ln(\ln x)}\left[\sec x \tan x \ln(\ln x) + \sec x \cdot \frac{1}{\ln x} \cdot \frac{1}{x}\right]$$

$$A^B = e^{?} \qquad A^B = e^{B\ln A}$$

$$\ln(A^B) = ?$$

$$B\ln A = ?$$

$$\frac{d}{dx}x^x = \frac{d}{dx}e^{x\ln x} = e^{x\ln x} \cdot (x\ln x)'$$

$$= e^{x\ln x}\left(\ln x + x\frac{1}{x}\right)$$

$$\int \tan^2 x \sec^2 x \, dx = \int u^2 \, du = \frac{1}{3}u^3 + C = \frac{1}{3}\tan^3 x + C$$

$u = \tan x$

$du = \sec^2 x \, dx$

$$\int \tan^2 x \sec^2 x \, dx = \int \underbrace{\sec x}_{u} \tan x \underbrace{\sec x \tan x \, dx}_{du}$$

$u = \sec x$

$du = \sec x \tan x \, dx$

$$= \int u \sqrt{u^2 - 1} \, du$$

$v = u^2 - 1 \ldots$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

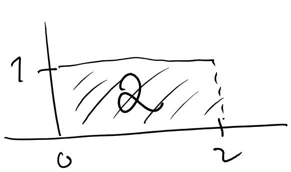
$\tan x = \sqrt{\sec^2 x - 1}$

$= \sqrt{u^2 - 1}$

~~$\frac{du}{\sec x \, dx} = \tan x$~~ evil. DON'T

Definite Integrals

definite integral = (signed) area under a curve (graph)



$f(x) = 1$

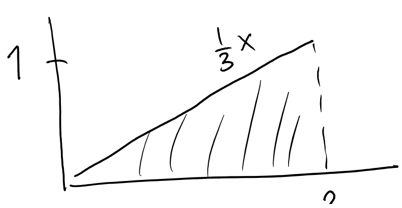
The definite integral of $f(x) = 1$ from $x = 0$ to $x = 2$ is 2.

Notation: $$\int_{x=0}^{x=2} 1 \, dx = 2$$

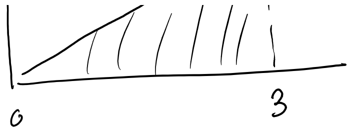
''

$$\int_0^2 1 \, dx$$

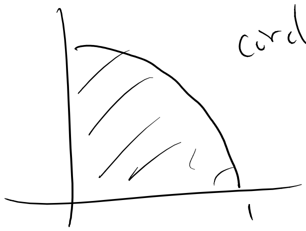
ex



$$\int^3 \frac{1}{3}x \, dx = \frac{3}{2} \quad \left(\frac{1}{2}bh\right)$$

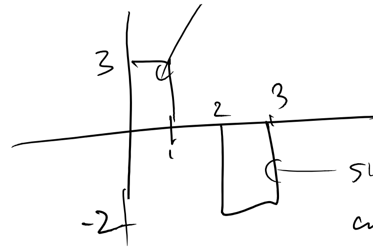


$$\int_0^3 \frac{1}{3}x\,dx = \frac{3}{2} \quad \left(\tfrac{1}{2}bh\right)$$



circle

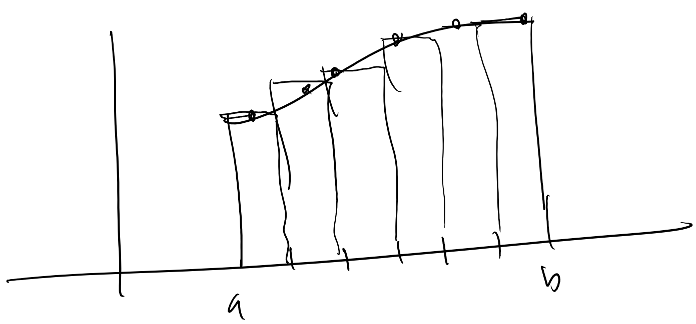
$$\int_0^1 f(x)\,dx = \frac{\pi}{4}$$



signed area 3

signed area: -2

Riemann Integral 1850's.



"Riemann Sum = area in all rectangles"