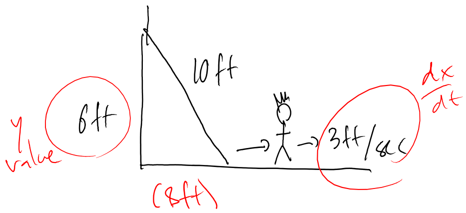
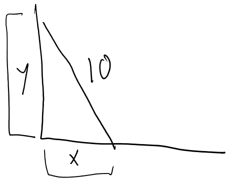
Lecture 8: Related rates, intro to graphing and optimization

Tuesday, September 22, 2015 9:28 AM



how fast is the top of the ladder sliding down the wall?



want an equation that relates

x (whose rate we know about)

& y (whose rate we want)

$$\boxed{x^2+y^2=10^2}$$

want $\frac{dy}{dt}$, so take $\frac{d}{dt}$ both sides

$$\frac{d}{dt}(x^2+y^2)=\frac{d}{dt}(10^2)$$

$$\frac{d}{dt}\left(x(t)^2+y(t)^2\right)$$

implicit differentiation

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

solve for $\frac{dy}{dt}$ $\Rightarrow$ $\frac{dy}{dt}=-\frac{x}{y}\frac{dx}{dt}$

have y value $=6$

$\frac{dx}{dt}=3$

x value $=$?

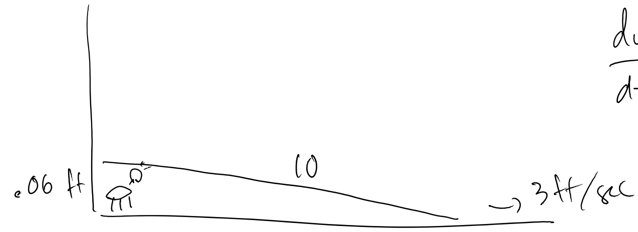
$x^2+y^2=100$ $\quad y=6$

$x^2+36=100$

$x^2=64$

$x=8$ (not negative by picture)

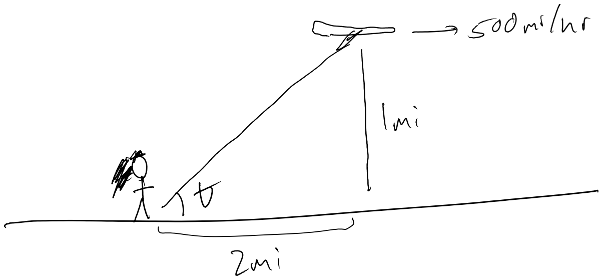
$$\frac{dy}{dt}=-\frac{x}{y}\frac{dx}{dt}=-\frac{8}{6}(3)=-4\text{ ft/sec}$$



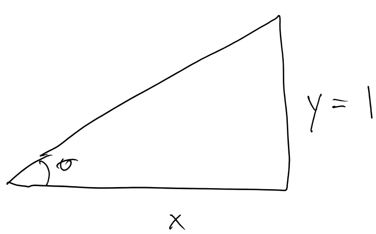
$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} \approx -\frac{10}{.06}(3)$$

$$\approx -\frac{1000}{6} \cdot 3$$

$$= -500 \text{ ft/sec}$$



want $\frac{d\theta}{dt}$



want $\frac{d\theta}{dt}$

know: $\frac{dx}{dt} = 500$

$\frac{dy}{dt} = 0$ ($y = 1$ is constant)

$$\tan\theta = \frac{1}{x}$$

$\frac{d}{dt}$ both sides

$x^{-1} \to -1x^{-2}$

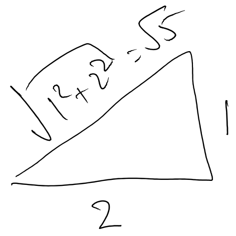
$$\sec^2\theta \boxed{\frac{d\theta}{dt}} = -\frac{1}{x^2}\frac{dx}{dt} \longrightarrow \frac{d\theta}{dt} = \frac{-\frac{1}{x^2}\frac{dx}{dt}}{\sec^2\theta} = -\frac{\cos^2\theta}{x^2}\frac{dx}{dt}$$

$\frac{dx}{dt} = 500$

$x = 2$

$\cos\theta$?

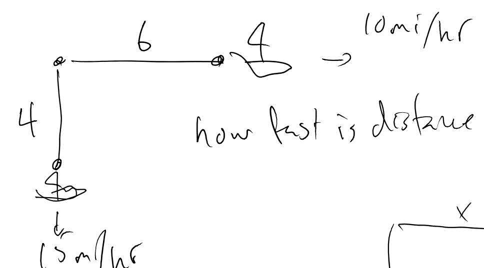
$= \frac{adj}{hyp} = \frac{2}{\sqrt{5}}$ x value



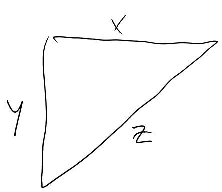
$\cos^2\theta = \frac{4}{5}$

$$= \frac{-\left(\frac{4}{5}\right)}{2}\,\frac{(500)}{2} = -\frac{4}{5\cdot 4}(500)$$

$$= -100 \text{ rad/hr}$$



how fast is distance between boats changing?



$x^2 + y^2 = z^2$

$\downarrow \frac{d}{dt}$

$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(z^2)$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

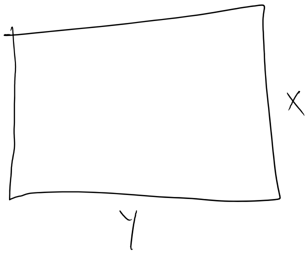
x value = 6

y value = 4

$\frac{dx}{dt} = 10 \quad \frac{dy}{dt} = 15$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z} = \frac{6\cdot 10 + 4\cdot 15}{\sqrt{52}} = \frac{120}{\sqrt{52}}$$

z value: $z^2 = x^2 + y^2 = 36 + 16 = 52 \quad z = \sqrt{52}$



Rectangular enclosure
100 ft of fence
maximize area.

$A = xy$ $\quad$ $100 = P = 2x + 2y$

$2y = 100 - 2x$

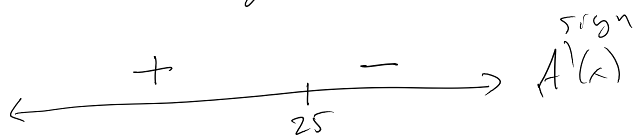
$y = 50 - x$

$A(x) = x(50 - x)$

$= 50x - x^2$

$A'(x) = 50 - 2x$

sign chart



$A'(x) = 0$ when $x = 25$