Lecture 24: optimization

Thursday, November 19, 2015 9:35 AM

F = foxes
R = rabbits
} functions of time.

$$\frac{dR}{dt} = bR - eFR$$

e = eating coefficient

$$\frac{dF}{dt} = kFR - dF$$

<u>Problem</u>: maximum value of R happens when?

$$R' = bR - eFR = 0$$

$$bR - eFR = 0$$

$$R(b - eF) = 0$$

$$R = 0 \text{ or } b = eF \rightsquigarrow F = \frac{b}{e}$$

max or min? second derivative!

$$R' = bR - eFR$$

$$R'' = bR' - eF'R - eFR'$$ want this when $R' = 0$

$$R'' = -eF'R$$ if $F' > 0$ (foxes inc.) Rabbits at max

$$\alpha A + \beta B \underset{k_-}{\overset{k_+}{\rightleftharpoons}} \gamma C + \delta D$$

reacting, remove C at some const rate, ask: max concentration of B?

k_- ... of B?

rate $\rightarrow$ $k_+ A^\alpha B^\beta = \frac{\text{Reacts}}{\text{time}}$ each reaction $\rightarrow$ gives γ C's & δ D's

rate $\leftarrow$ $k_- C^\gamma D^\delta = \frac{\text{react}}{\text{time}}$ $\rightarrow$ $\frac{dC}{dt} = k_+ A^\alpha B^\beta \gamma$

$\leftarrow \frac{dC}{dt} = -k_- C^\gamma D^\delta \gamma$

in total : $\rightleftarrows$ $\frac{dC}{dt} = k_+ A^\alpha B^\beta \gamma - k_- C^\gamma D^\delta \gamma$

$\frac{dC}{dt} = 0 \implies k_+ A^\alpha B^\beta \gamma = k_- C^\gamma D^\delta \gamma$

$$1 = \frac{k_+}{k_-} \frac{A^\alpha B^\beta}{C^\gamma D^\delta}$$

$$\frac{k_-}{k_+} = \frac{A^\alpha B^\beta}{C^\gamma D^\delta} \qquad \boxed{\frac{C^\gamma D^\delta}{A^\alpha B^\beta} = \frac{k_+}{k_-}}$$

instead:

$$\frac{\partial C}{\partial t} = k_+ A^\alpha B^\beta \gamma - k_- C^\gamma D^\delta \gamma - rC$$

rate of removal

maximize C!

set = 0 $\quad k_+ A^\alpha B^\beta \gamma - k_- C^\gamma D^\delta \gamma - rC = 0$ $\quad$ what is max C?

$$k_+ A^\alpha B^\beta \gamma = k_- C^\gamma D^\delta \gamma - rC$$

solve for C?

$$X = YC^{\gamma} - ZC$$

$\gamma = 1$ factor & solve

$\gamma = 2$ quad formula.

$\gamma =$ cubic formula.

$\gamma = 4$ quartic

$\gamma = 5$ out of luck.