Lecture 20: more parts

Tuesday, November 3, 2015 9:59 AM

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C$$

$u = x \qquad du = dx$

$dv = \cos x\,dx \qquad v = \sin x$

$$\int x\cos x^2\,dx = \frac{1}{2}\int \cos u\,du = \frac{1}{2}\sin u + C = \frac{1}{2}\sin x^2 + C$$

$u = x^2$

$du = 2x\,dx \longrightarrow \frac{1}{2}du = x\,dx$

$$\int x^2\sin x\,dx = -x^2\cos x + 2\underbrace{\int x\cos x\,dx} = -x^2\cos x + 2\left[x\sin x - \int \sin x\,dx\right]$$

$u = x^2 \qquad du = 2x\,dx \qquad u = x \qquad du = dx$

$dv = \sin x\,dx \qquad v = -\cos x \qquad dv = \cos x\,dx \qquad v = \sin x$

$$\int u\,dv = uv - \int v\,du$$

$$= -x^2\cos x + 2x\sin x - 2\int \sin x\,dx$$

$$= -x^2\cos x + 2x\sin x + 2\cos x + C$$

$$\int x^n\sin x\,dx = -x^n\cos x + n\int x^{n-1}\cos x\,dx$$

$u = x^n \qquad du = nx^{n-1}\,dx$

$dv = \sin x\,dx \qquad v = -\cos x$

$$\int x^n \cos x\,dx = x^n \sin x - n\int x^{n-1}\sin x\,dx$$

$u = x^n \qquad du = nx^{n-1}$

$dv = \cos x\,dx \qquad v = \sin x$

More substitution

originally:

$u(x) = u$

$u'(x)\,dx = du$

$u = \ln x$

$du = \frac{1}{x}dx$

"implicit"

$e^u = x$

$e^u\,du = dx$

$$\int \frac{1}{1+x^2}dx = \int \frac{1}{1+\tan^2 u}\sec^2 u\,du = \int \frac{\sec^2 u}{\sec^2 u}du = \int du$$

$1+\tan^2 x = \sec^2 x$

$x = \tan u \longleftrightarrow u = \arctan x$

$dx = \sec^2 u\,du$

$= u + C$

$= \arctan x + C$

"Trig substitution"

$$\int \frac{1}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-\sin^2 u}}\cos u\,du = \int \frac{\cos u}{\cos u}du$$

$= \int du = u + C$

$x = \sin u \longleftrightarrow u = \arcsin x$

$x = \sin u \longleftrightarrow u = \arcsin x$

$= \int u \cdots$

$dx = \cos u\, du$

$= \arcsin x + C$

$$\int \frac{1}{(\sqrt{x^2-1})^3} dx = \int \frac{1}{(\sqrt{\sec^2 y - 1})^3} \sec y \tan y \, dy$$

$\sec^2 y - 1 = \tan^2 y$

$$= \int \frac{1}{\tan^3 y} \sec y \tan y \, dy$$

$x = \sec y$

$dx = \sec y \tan y \, dy$

$$= \int \frac{\sec y}{\tan^2 y} dy$$

$$= \int \frac{1}{\cos y} \frac{\cos^2 y}{\sin^2 y} dy$$

$$= \int \frac{\cos y}{\sin^2 y} dy$$

$u = \sin y$

$du = \cos y \, dy$

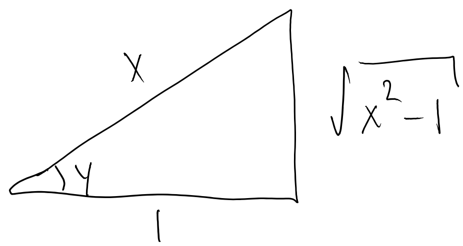
$$= \int \frac{1}{u^2} du = \int u^{-2} du = \frac{1}{-2+1} u^{-2+1} + C$$

$$= -\frac{1}{u} + C$$

know:

$\sec y = x$

$$= -\frac{1}{\sin y} + C$$



$$\sin y = \frac{\sqrt{x^2-1}}{x}$$

$$= -\frac{1}{\left(\frac{\sqrt{x^2-1}}{x}\right)} + C$$

$$= \boxed{-\frac{x}{\sqrt{x^2-1}} + C}$$