Lecture 10: sketching and optimization

Monday, September 28, 2015 10:17 AM

Basic Mechanical skill:

Sign charts

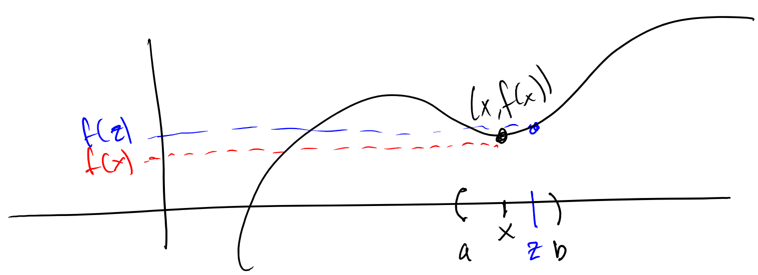
$f(x)$: positive or neg (above or below x-axis) (least important)

$f'(x)$: when is $f(x)$ increasing or decreasing (max & mins, very important)

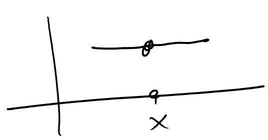
$f''(x)$: concavity / curvature (occasionally useful)

Extrema (minima & maxima)

Definition a point on a graph $(x, f(x))$ is a local minimum if for some interval $a < x < b$, we have $f(x) \le f(z)$ for every $a < z < b$.



if $f(x)$ is constant



is also a local min

Def $(x, f(x))$ a local maximum if for some $a < x < b$, we have, for every $a < z < b$, $f(z) \le f(x)$

Main point:

if $(x, f(x))$ is a local min or max, then " $f(x)$ is not increasing or decreasing at x."

in particular, $f'(x)$ not positive or negative.

For this to happen, either

$f'(x) = 0$ or $f'(x)$ D.N.E.

<u>Definition</u> If a is within (not a boundary pt) the domain of $f(x)$, and either

- $f'(a) = 0$ or
- $f'(a)$ not defined then we say

a is a critical point for $f(x)$.

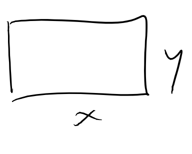
<u>Theorem (Fermat)</u> If $f(x)$ has a local extremum (min or max) at $x=a$, then a must be a critical point.

<u>Farmer & fence:</u>

100 ft of fence, no side can be smaller than 1 ft.

we want to minimize the area!

$A = xy$



$P = 100 = 2x + 2y$

$50 = x + y$

$y = 50 - x$

$A(x) = x(50-x) = 50x - x^2$

<u>critical points:</u> $A'(x) = 50 - 2x$

$A'(x) = 0$ or not defined

$\underbrace{A'(x) = 0 \text{ or not defined}}_{\text{(always defined)}}$

$0 = A'(x) = 50 - 2x$

$\boxed{x = 25} \rightarrow y = 50 - 25 = 25$

in domain.

actually not answer – turns out this is a max, not a min.

sign chart for $f'(x)$

Next time: Sketch a bunch of graphs