Lecture 14: integrating various things

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = -\int \frac{1}{u} \, du = -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

$$u = \cos x$$
$$du = -\sin x \, dx$$
$$-du = \sin x \, dx$$

$$= \ln|\cos x|^{-1} + C$$

$$= \ln|\sec x| + C$$

$$\int x \sec^2 x \, dx = x \tan x - \int \tan x \, dx$$

$$\int u \, dv = uv - \int v \, du$$

$$u = x \qquad\qquad du = dx$$
$$dv = \sec^2 x \, dx \qquad v = \tan x$$

$$\text{''} \quad x \tan x - \ln|\sec x| + C$$

$$\text{or} \quad x \tan x + \ln|\cos x| + C$$

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int e^{\sqrt{2x+1}} \, dx = \int u e^u \, du = u e^u - \int e^u \, du$$

$$u = \sqrt{2x+1}$$

$$v = u \qquad\qquad dv = du$$
$$dw = e^u \, du \qquad w = e^u$$

$$du = \frac{1}{2}(2x+1)^{-1/2} \cdot 2 = \frac{1}{\sqrt{2x+1}} \, dx = \frac{1}{u} \, dx$$
$$\frac{}{dx} \quad \sqrt{2x+1}$$

$$dx \quad \sqrt{2x+1}$$

directly:

$$u^2 = 2x+1$$

$$2u\,du = 2\,dx$$

$$u\,du = dx$$

$$= ue^u - e^u + C$$

$$= \sqrt{2x+1}\, e^{\sqrt{2x+1}} - e^{\sqrt{2x+1}} + C$$

Trig functions

$\int \sin x\,dx$ ✓ $\qquad$ $\int \cos x\,dx$ ✓ $\qquad$ $\int \tan x\,dx$ ✓

$$\ln|\sec x| + C$$

$$\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx = \ln|\sin x| + C$$

$$u = \sin x$$
$$du = \cos x\,dx$$

$$\int \sec^2 x = \tan x + C \qquad \int \sec x \tan x\,dx = \sec x + C$$

$$\int \sec x\,dx = \int \sec x \,\frac{\sec x + \tan x}{\text{sec} + \tan x}\,dx$$

$$\sec x + \tan x$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\tan x + \sec x}\, dx = \int \frac{1}{u}\, du$$

$$= \ln |u| + C$$

$$u = \tan x + \sec x$$

$$du = (\sec^2 x + \sec x \tan x)\, dx$$

$$= \ln |\sec x + \tan x| + C$$

$$\int \csc x\, dx = \int \csc x\, \frac{\csc x + \cot x}{\csc x + \cot x}\, dx$$

$$\frac{d}{dx} \csc x = -\csc x \cot x \qquad \frac{d}{dx} \cot x = -\csc^2 x$$

$$= \int \frac{\csc^2 x + \csc x \cot x}{\cot x + \csc x}\, dx = -\int \frac{1}{u}\, du$$
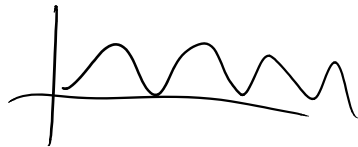
$$u = \csc x + \cot x$$

$$du = -(\text{top})\, dx$$

$$= -\ln |\csc x + \cot x| + C$$

$$\int \csc^2 x\, dx = -\cot x + C$$

$$\int \sin^2 x \, dx =$$

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

look at videos on Integrating trig fcns