Previously defined étale morphism
(flat, unramified, locally finite presentation)

étale covering: $\{U_i \to U\}$ étale, universal epimorphism.
$\iff$ surjects set theoretically on scheme theoretic pts
$\iff \coprod U_i \to U$ faithfully flat

X scheme

$\{U_i \xrightarrow{\text{étale}} U\}$
étale / étale
X

$X_{ét}$
cat is étale morphisms to X
$U \to X$ étale.

X_{Et} same def of covers, different cats
cat is all X-schemes
$U \to X$ arb.

Flat topologies:

fppf = fidelement plat of presentation finie
"
fffp = faithfully flat and finitely presented

covers consist of $\{U_i \to U\}$ each morphism is flat & finitely presented

$\iff$ univ. epi as above.

$\coprod U_i \to U$
surjects on schemepts
$\Leftarrow$
faithfully flat.

X_{Ffpf}
X_{fppf}

fpqc = Fidelement plat et quasi-compact

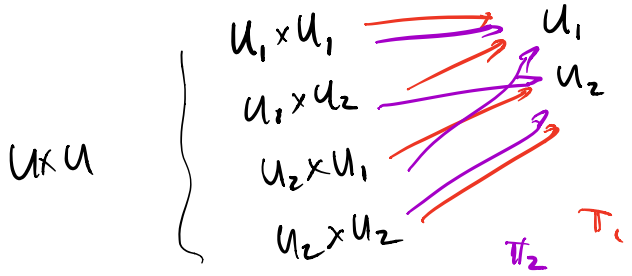
ffqc = faithfully flat & quasi-compact

cons $\{U_i \to U\}$ (more to this?)

flat, q-compact cov if surj on schem pts $\Leftarrow$ univ. epi ...

$U = \coprod U_i \longrightarrow X$

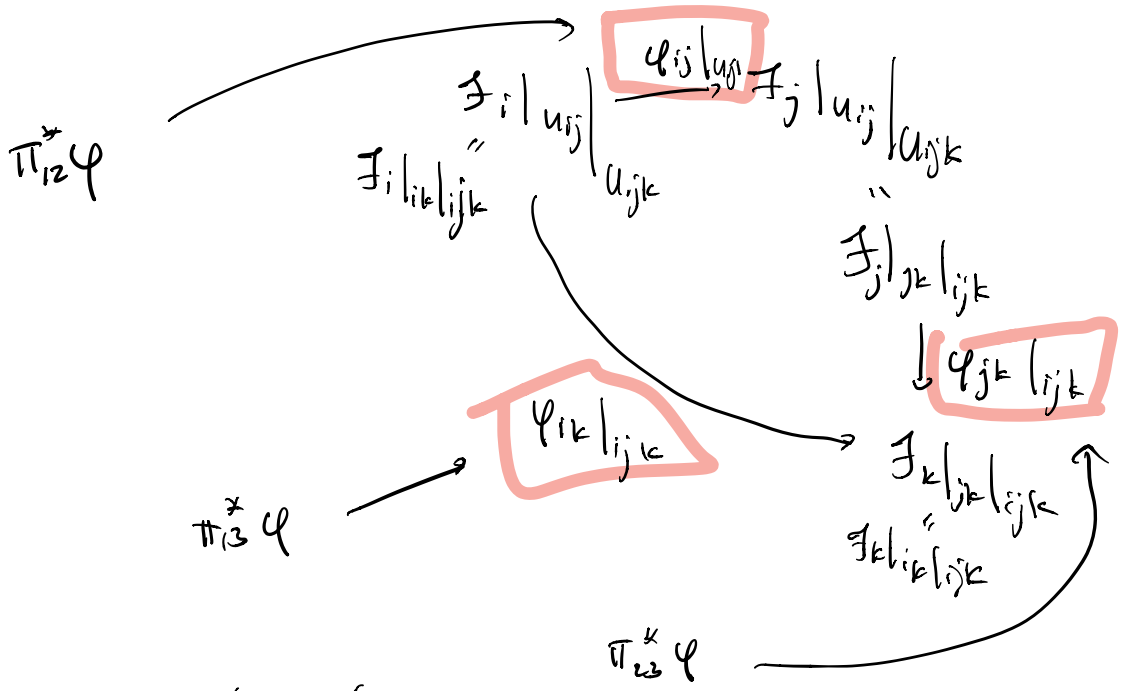
$U \times_X U = \coprod U_i \times_X U_j \;\underset{\pi_2}{\overset{\pi_1}{\rightrightarrows}}\; \coprod U_i$



$\varphi_{ij} : \mathcal{F}_i|_{U_{ij}} \xrightarrow{\sim} \mathcal{F}_j|_{U_{ij}}$ $\qquad \mathcal{F}/U$

$\mathcal{F}_i|_{U_{ij}} \xrightarrow{\pi_1^*} U_{ij} \xleftarrow{\pi_2^*} \mathcal{F}_j|_{U_{ij}}$

$\varphi_{ij} \longleftrightarrow \varphi : \pi_1^*\mathcal{F} \to \pi_2^*\mathcal{F}$



$$\pi_{13}^* \varphi = \pi_{23}^* \varphi \; \pi_{12}^* \varphi$$