Graph Theory, Homework 1

Due Thursday, February 6

Recall that the binomial coefficient $\binom{n}{m}$ for $m < n$ nonnegative integers, represents the number of distinct ways to choose m elements from a set with n elements.

By "graph" below, we mean "simple graph" unless specified otherwise!

1. Explain why the following statement is true: In any group of two or more people, there are always at least two people within the group who have the same number of friends within the group.
2. Explain why for any graph G with n vertices, the number of edges is no more than $\binom{n}{2}$.
3. Explain why, if a graph has n vertices at more than $\binom{n-1}{2}$ vertices, it must be connected.
4. Is it possible to have a graph with 6 vertices whose degrees are $6, 3, 3, 2, 2, 1$? Why or why not? What about a graph with 4 vertices whose degrees are $3, 3, 1, 1$?