Graph Theory, Spring 2016, Practice Sheet for Exam 1

1. Show that every tree has at least 2 leaves.

2. Show that a simple connected graph G contains a cycle if and only if it contains more than one spanning tree.

3. Suppose that G is a forest. Show that $v(G) = e(G) + c(G)$.

4. Suppose that G is a simple connected graph with $e(G) = v(G)$. Show that G must contain a unique cycle.

5. Suppose that G is a simple graph with $e(G) = v(G)$. Must G be connected? If so, prove it. If not, give a counter-example.

6. Give an example of a graph G with $\kappa(G) = 1$ and $\kappa'(G) = 2$.

7. Give an example of a graph G with $\kappa'(G) = 3$ and $\delta(G) = 4$.

8. Suppose that G and G' are connected simple graphs with the same number of vertices, and such that every vertex has degree 2. Show that $G \cong G'$.

9. Give examples of graphs G and G' are simple with 6 vertices, where each vertex has degree 2, but $G \not\cong G'$.

10. Give an example of a graph with a bridge, but with no cut vertex

11. Show that if G is a simple 3-regular graph with a bridge, then G has a cut vertex.