Graph Theory, Spring 2016, Homework 3

1. Show that if T is a tree and T has a vertex of degree d, then T has at least d leaves.

2. Show that if a vertex v has degree 1, it cannot be a cut vertex.

3. Suppose that G is a graph and H is a connected spanning subgraph. Show that if $v \in V_G = V_H$ is a cut vertex for G then it is also a cut vertex for H.

4. Show that every nontrivial graph (i.e. with more than one vertex) has at least 2 vertices which are not cut vertices.

5. Let G be a connected graph with a cut edge e. Show that either G has less than 3 vertices or G has at least one cut vertex incident to e.

6. A simple connected graph G has 10 vertices and contains exactly 2 cycles. How many edges does it have? Rigorously justify your answer.

7. Show that a simple connected graph contains a cycle if and only if it has more than one spanning subtree.

8. (6000 level) Suppose that G is a bridgeless graph with at least 3 vertices, and with the same number of vertices as edges. Show that G is a cycle – that is, G is isomorphic to C_n for some n.

9. (6000 level) Let G be a graph, and suppose that e, f, g are edges such that there exists a cycle C containing both the edges e and f and a cycle C' containing both the edges f and g. Show that there exists a cycle containing both e and g.