

Graph Theory, Spring 2016, Homework 8

1. Let D be a digraph and $x, y \in V_D$. We say that $S \subset A_D$ is an (x, y) -arrow cut if $D - S$ contains no directed (x, y) -paths. We say that is minimal if there is no (x, y) -arrow cut S' with S' a proper subset of S .

Suppose that in the digraph D , for every vertex $v \in V_D$, there exists both a directed (x, v) -path, and a directed (v, y) -path. Show that if K is a minimal (x, y) -arrow cut, if and only if we can partition the vertices of V_D as $V_D = X \cup Y$ with $x \in X, y \in Y$ and $X \cap Y = \emptyset$, and with $K = [X, Y]$.

2. Suppose that D is as in the previous problem, and that K is a minimal (x, y) arrow cut. Give an algorithm for producing the sets X and Y as above.