Graph Theory, Spring 2016, Homework 4

1. Let G be a graph and $e \in E_G$. If e is incident to vertices u, v (not necessarily distinct), let G/e denote the graph $G/\{u, v\}$.

   Show that if G is a graph is k-connected, then G/e is $(k-1)$-connected.

2. Show that if a simple connected graph contains a cycle, then it must contain at least 3 distinct spanning subtrees.

3. Suppose that a simple graph G contains 10 vertices and 11 edges. Show it is possible for number of cycles in G to be 2 or 3 but that no other value is possible.

4. A graph G is called bipartite if we may partition the set of vertices into two nonempty disjoint parts $V(G) = V_1 \cup V_2$, such that whenever two vertices v, w are in the same part V_i, we have that v and w are nonadjacent. In other words, the only edges in the graph are those which connect vertices in different parts. Show that if G is a tree with at least 2 vertices then G is bipartite.