Graph Theory, Spring 2016, Homework 1

1. Given a graph $G = (V, E, \psi)$, we may define a relation $\sim$ on the edges by defining $e \sim f$ if there is some vertex v incident to both edges. Give an example of a graph where this is an equivalence relation. Is this always an equivalence relation? Why or why not?

2. Show that two simple graphs G, H are isomorphic if and only if we may find a bijection $f : V_G \to V_H$ such that vertices $v, w \in V_G$ are adjacent in G if and only if $f(v), f(w) \in V_H$ are adjacent in H.

3. Prove that any simple graph must have at least two vertices with the same degree.

4. Prove that if G is a graph, it must have an even number of vertices whose degree is odd.

5. Is it possible to have a simple graph G with 6 vertices whose degrees are $6, 3, 3, 2, 2, 1$? Why or why not? What about a graph with 4 vertices whose degrees are $3, 3, 1, 1$?

6. (6000 level) Suppose that G is a simple graph with vertices $v_0, v_1, \ldots, v_n$, of degrees $d_0 \geq d_1 \geq d_2 \geq \cdots \geq d_n$. Show that we may find a new graph G' with vertices $v_0', v_1', \ldots, v_n'$ with $deg_G v_i = deg_{G'} v_i'$, and with v_0' adjacent to the vertices $v_1', \ldots, v_{d_0}'$.