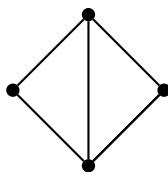


## Graph Theory, Spring 2016, Homework 5

1. Suppose  $G$  is a graph with no cycles of odd length and no loops. Show that every circuit in  $G$  must have even length.
2. Suppose  $G$  is a graph with no cycles of odd length and no loops. Let  $v, w \in V_G$ . Show that either every  $(v, w)$ -walk has odd length or every  $(v, w)$ -walk has even length.
3. Calculate the chromatic polynomial  $\chi_G(x)$  of the graph shown below:



4. Let  $G$  be a 5-regular graph, and suppose that  $G$  has two Hamiltonian cycles  $C_1, C_2$  which are edge disjoint. Show that  $G$  is 5-colorable.