Ideas in Mathematics, Fall 2023, Weekly worksheet 4

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1. Prove that any positive integer greater than or equal to 9 can be written as a sum of the form $a + b$ where a is a multiple of 5 and b is even.

2. Prove that there is no rational number $q = a/b$ with $q^2 = 7$.

3. Suppose a has a remainder of 8 when divided by 12 and b has a remainder of 7 when divided by 12. What will be the remainder of ab when it is divided by 12? Explain why this is true

   (you don't need to give a formal proof, but give an explanation based on the ideas from class).

4. Suppose a has a remainder of 1 when divided by 73. What will be the remainder of a^{27} when divided by 73?

5. Find the last digit of the following numbers (in their standard base 10 decimal expansions). Give some explanation for your answers.

   (a) 231^{297}

   (b) 99^{100}

   (c) 77^{99}

6. The factorial of a number n, written as $n!$ denotes the product of all integers from 1 to n. For example, $5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$.

   Show that if n and m are integers such that $2 \leq m \leq n$, then $n! + m$ is never a prime number (that is, it is a composite number).