

MATH 3120 — Homework 2

1. Linear transformations and matrices (11 points).

Consider the following linear transformations from \mathbb{R}^2 to \mathbb{R}^2 :

- T takes a vector and rotates it 90° counterclockwise.
- S takes a vector (a, b) and swaps the entries, giving (b, a) .
- U takes a vector (a, b) and subtracts the second entry from the first, giving $(a - b, b)$.

1. (2 points) What is the matrix representing S ?

$$\begin{pmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{pmatrix}$$

2. (2 points) What is the matrix representing T ?

$$\begin{pmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{pmatrix}$$

3. (2 points) What is the matrix representing U ?

$$\begin{pmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{pmatrix}$$

4. (2 points) What is the matrix representing T^{-1} ?

$$\begin{pmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{pmatrix}$$

5. (3 points) Suppose A is the matrix given by the product UTS . Find the matrix A^{-1} .

$$\begin{pmatrix} \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} & \underline{\hspace{1cm}} \end{pmatrix}$$

2. Interpreting a matrix column (3 points).

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear transformation with matrix A .

What does the first column of A represent? Circle one.

- (a) the image of $(0, 0, 1)$ under T
- (b) the image of $(1, 0, 0)$ under T
- (c) the x -coordinate of the output of T
- (d) the determinant of T

3. Matrix-action multiple choice (3 points).

Suppose

$$A = \begin{pmatrix} 2 & 5 & -1 \\ 3 & 4 & 7 \end{pmatrix}$$

is the matrix of a linear transformation

$$T : \mathbb{R}^3 \rightarrow \mathbb{R}^2.$$

What is $T(0, 1, 0)$? Circle one.

- (a) $(2, 3)$
- (b) $(5, 4)$
- (c) $(-1, 7)$
- (d) $(3, 4)$

4. Squaring a transformation (3 points).

Let $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation that swaps the entries of a vector:

$$S(a, b) = (b, a).$$

What transformation does S^2 represent? Circle one.

- (a) reflection across the x -axis
- (b) rotation by 90°
- (c) the identity transformation
- (d) projection onto the line $y = x$