Math 477, Lecture 4 class work

Name: _____

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since we'll be talking about some of these next time as well, I'm only putting in the solution to the first problem!

1. Suppose have the dice in a box only can roll 6's and the others are normal. If one is chosen at random and we first roll a 6, what is the probability that the next roll is a 6?

   Let F be the event that the first roll is a 6 and N the event that the next roll is a 6. Let L be the event that the die can only roll 6's.

   We know that $P(L) = 1/2 = P(L^c)$. We also know that $P(F|L) = 1 = P(N|L) = P(FN|L)$. That is to say, if we know that the die can only roll 6's, then we know with certainty that the first roll must be a 6 and the second roll must be a 6. We also know that $P(F|L^c) = 1/6$ and $P(FN|L^c) = 1/36$. That is, if we have a normal die, then there is a 1/6 chance of the first roll being a 6, and a 1/36 chance that both rolls are 6's.

   Now, let's answer the question. We are trying to find $P(N|F)$. By definition, this is $P(FN)/P(F)$. We have

   $$P(F) = P(FL) + P(FL^c) = P(F|L)P(L) + P(F|L^c)P(L^c) = (1)(1/2) + (1/6)(1/2) = 7/12$$

   we also have

   $$P(FN) = P(FNL) + P(FNL^c) = P(FN|L)P(L) + P(FN|L^c)P(L^c) = (1)(1/2) + (1/36)(1/2) = 37/72$$

   Therefore, we get:

   $$P(N|F) = P(FN)/P(F) = (37/72)/(7/12) = \frac{37}{(7)(6)} = 37/42,$$

   so there is a pretty good chance that the second roll will be a 6.

2. Suppose that 30% of the population is accident prone and will has a 40% chance of having an accident in any given year, while 70% of the population has only a 5% chance of having an accident in a given year. If a certain person has an accident in one year, what's the probability that they are accident prone? What's the probability that they'll have an accident the following year as well?

3. Suppose we have a blood test for a given disease. If you are sick, there is a 95% chance that the blood test will detect it. If you are well, there is a 1% chance that the blood test will falsely show that you are sick. Suppose only 0.1% of the population has the disease. If a random person is given the test, and it results in showing that they are sick, what is the probability that they are actually sick?