Algebraic Geometry 2½ supplimentary worksheet 4

Proj, and properties of schemes

Critical Hartshorne problems in Chapter II

- (short term) 2.19, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6
- (for later in semester) 3.7, 3.8, 3.10, 3.12

1. Let A be a commutative ring and consider the polynomial rings $A[x], A[y]$, and the ring $B = A[x,y]/(xy-1)$. We may identify B with the localizations $A[x]_x$ and $A[y]_y$ via the natural inclusions. Let X be the scheme obtained by gluing $\operatorname{Spec} A[x]$ and $\operatorname{Spec} A[y]$ along $\operatorname{Spec} B$, thought of as the open subschemes D_x and D_y in $\operatorname{Spec} A[x]$ and $\operatorname{Spec} B[x]$ respectively. Show that $X \cong \operatorname{Proj} A[s,t]$.