

$$\text{vec}(x \star k) = K_{\text{conv}} \cdot \text{vec}(x)$$

x_1	x_2	x_3
x_4	x_5	x_6
x_7	x_8	x_9

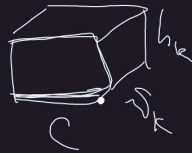
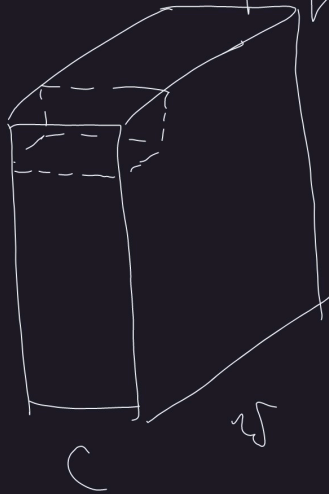
\star

k_1	k_2
k_3	k_4

$=$

y_1	y_2
y_3	y_4

$$y_{ij} = \sum_{p,q=0}^{h_k, w_k} k_{pq} x_{i+p, j+q}$$



$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}$$

$$= \begin{pmatrix} k_1 & k_2 & 0 & k_3 & k_4 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & 0 & k_3 & k_4 & 0 & 0 \\ 0 & 0 & 0 & k_1 & k_2 & 0 & k_3 & k_4 \\ 0 & 0 & 0 & 0 & k_1 & k_2 & 0 & k_3 & k_4 \end{pmatrix}$$

K_{conv}

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_g \end{pmatrix}$$

$$y_{ij} = \sum_{p,q} k_{pq} x_{i+p, j+q}$$

$$\begin{bmatrix} 0 & 0 & \frac{\partial L}{\partial y_0} & \frac{\partial L}{\partial y_1} & \frac{\partial L}{\partial y_2} & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} k_2 & k_1 & k_0 \end{bmatrix}$$

$$\nabla_k L = \underbrace{X} + \underbrace{\nabla_y L}$$

$$\nabla_x L = \nabla_y L + \text{resaped}(k)$$

$$dL = \sum_{i,j} \frac{\partial L}{\partial y_{ij}} dy_{ij} = \sum_{i,j} \frac{\partial L}{\partial y_{ij}} \sum_{p,q} (dk_{pq} x_{i+p, j+q} +$$

$$+ k_{pq} dx_{i+p, j+q}) = \sum_{p,q} dk_{pq} \left(\sum_{i,j} \frac{\partial L}{\partial y_{ij}} x_{i+p, j+q} \right) = \frac{\partial L}{\partial k_{pq}} +$$

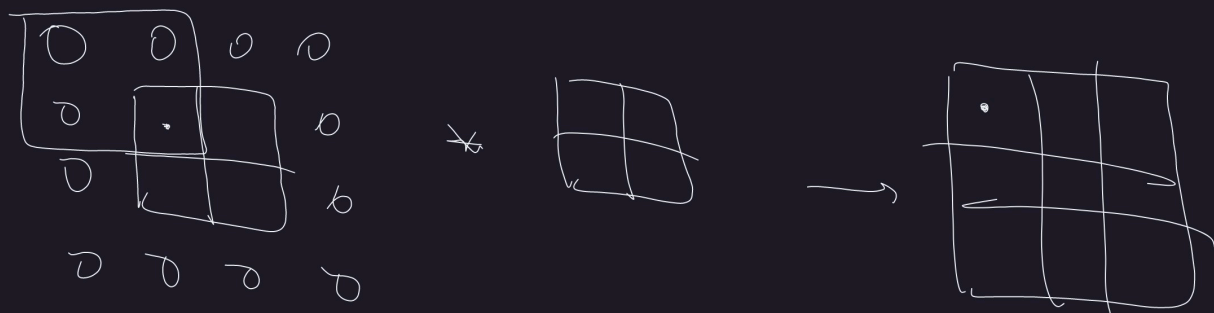
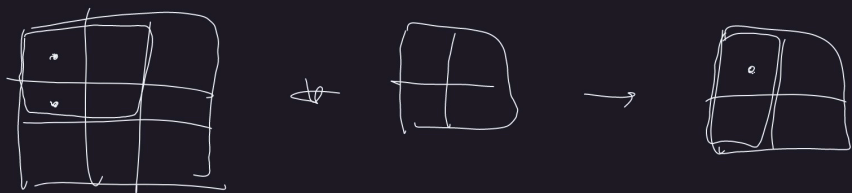
$$+ \left\{ \begin{array}{l} u = i+p \\ \delta = j+q \end{array} \right\} \sum_{u,\delta} dx_{u,\delta} \left(\sum_{p,q} k_{pq} \frac{\partial L}{\partial y_{u-p, \delta-q}} \right) = \frac{\partial L}{\partial x_{u,\delta}}$$

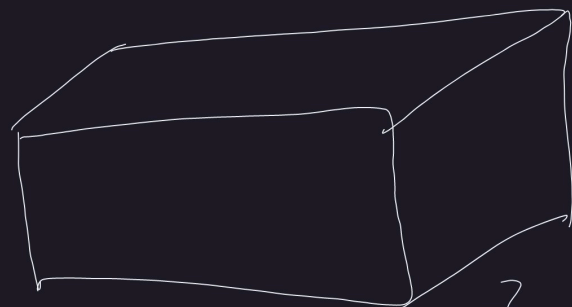
$$\frac{\partial L}{\partial x_0} = k_0 \frac{\partial L}{\partial y_0} + k_1 \frac{\partial L}{\partial y_{-1}} + k_2 \frac{\partial L}{\partial y_{-2}} + \dots$$

$$\frac{\partial L}{\partial x_1} = k_0 \frac{\partial L}{\partial y_1} + k_1 \frac{\partial L}{\partial y_0} + k_2 \frac{\partial L}{\partial y_{-1}} + \dots$$

$$\frac{\partial L}{\partial x_2} = k_0 \frac{\partial L}{\partial y_2} + k_1 \frac{\partial L}{\partial y_1} + k_2 \frac{\partial L}{\partial y_0} + \dots$$

	Linear	Conv 2d
Forward	$z = Wx + b$	$z = x * k + b$
Backward	$\nabla_x L = W^T \nabla_z L$ $\nabla_w L = \nabla_z L \cdot x^T$ $\nabla_b L = \nabla_z L$	$\nabla_k L = x \otimes \nabla_z L^{\text{IR}}$ $\nabla_x L = \nabla_z L \otimes \text{resized}(k)$ $\nabla_b L = \text{sum}(\nabla_z L)$
input size	$(n_{\text{batch}}, n_{\text{input}})$	$(n_{\text{batch}}, n_{\text{in-channels}}, w_{\text{input}}, h_{\text{input}})$
output size	$(n_{\text{batch}}, n_{\text{output}})$	$(n_{\text{batch}}, n_{\text{out-channels}}, w_{\text{output}}, h_{\text{output}})$
# params	$n_{\text{inputs}} \cdot n_{\text{outputs}} + n_{\text{outputs}}$	$(n_{\text{in-ch}} \cdot h_k \cdot w_k + 1) \cdot n_{\text{out-channels}}$
comp. complexity	$\mathcal{O}(n_{\text{batch}} \cdot n_{\text{inputs}} \cdot n_{\text{outputs}})$	$\mathcal{O}(n_{\text{in-ch}} \cdot h_k \cdot w_k \cdot n_{\text{batch}} \cdot n_{\text{out-ch}} \cdot w_{\text{output}} \cdot h_{\text{output}})$





7

AvPool



512