Matrix Project

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Question:

In Geometry

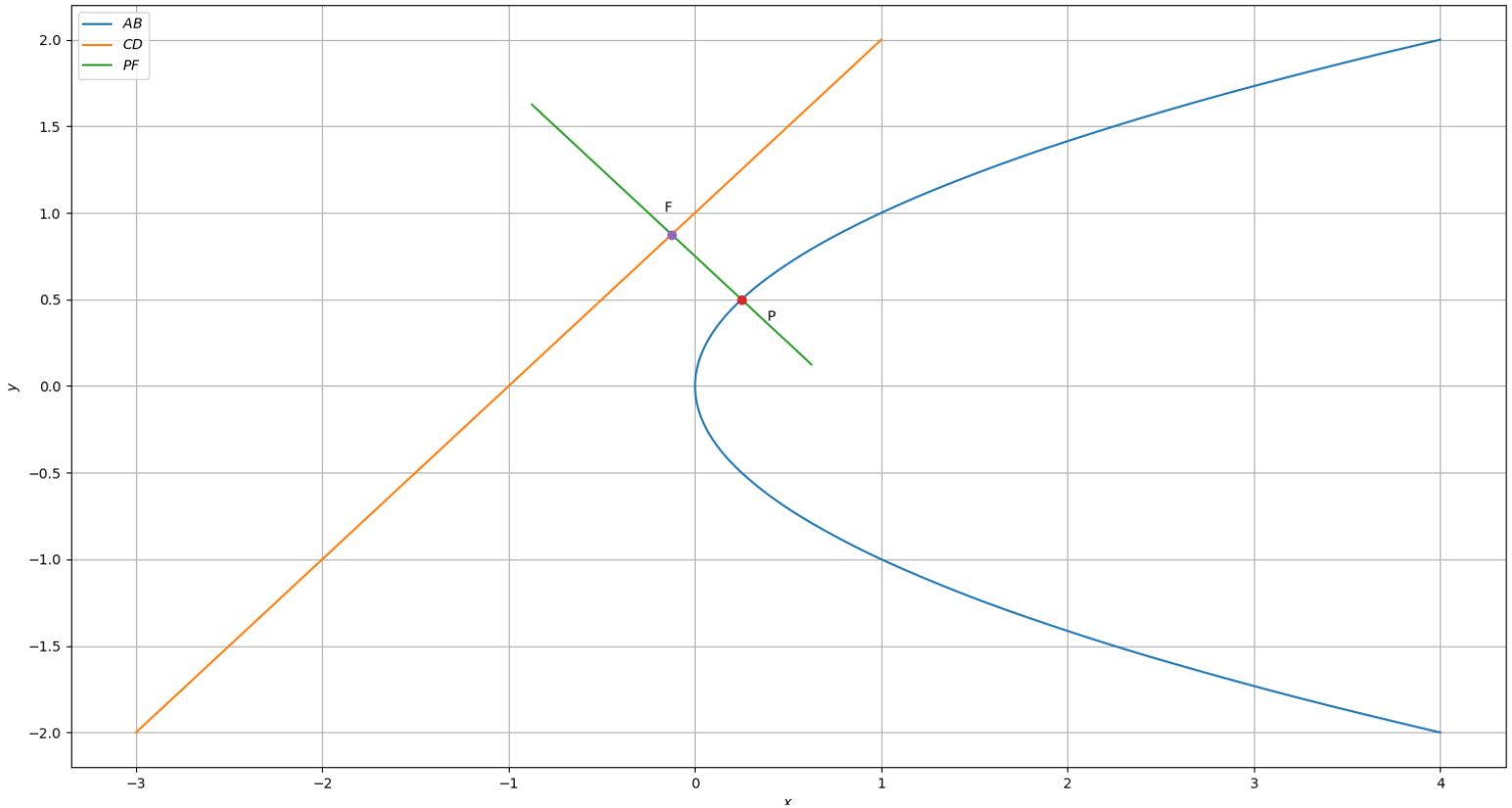
Q.Find the shortest distance between the line $y = x + 1$ and the parabola $y^2 = x$(JEE-MAIN 2009).

In Matrices

Q. Find the shortest distance between the line $\begin{bmatrix} 1 & -1 \end{bmatrix} x + 1 = 0$ and the parabola $x^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} x$ - $\begin{bmatrix} 1 & 0 \end{bmatrix} x = 0$

Ans: $\frac{3\sqrt{2}}{8} = 0.53$

Graphical Representation:



Solution:

Sol) Let 'n' be the direction vector matrix of the given line $x - y + 1 = 0$

$$n = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Let P is a point on the parabola having tangent parallel to given line then the perpendicular drawn from P to the give line has the shortest distance.

$$P = \begin{bmatrix} a \\ b \end{bmatrix}$$

Equation of Parabola

General equation of a conic is $x^T Vx + 2u^T x + F = 0$. Given equation of parabola $x^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} x$ - $\begin{bmatrix} 1 & 0 \end{bmatrix} x = 0$

On comparing the two equations V $= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, u $= \begin{bmatrix} -0.5 & 0 \end{bmatrix}$ and F $= 0$.

Equations of Tangents

- The equation of Tangent at P to the parabola is given as:

$$X^T n + c = 0$$

which can also be written as

$$n^T X + c = 0 \text{ (Since } n^T X = X^T n)$$

General equation of tangent at P to the conic is $(P^T V + u^T) + P^T x + F = 0$ and by substituting the values of P,V,F and u we will get the equation of tangent as

$$\begin{bmatrix} 0.5 & b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \frac{-a}{2} = 0$$

Calculating the Co-ordinates of point P

On comparing the equations of tangents we will get

$\mathrm{n}^T = k(\begin{bmatrix} -0.5 & 0 \end{bmatrix})$.

We know that $\mathrm{n} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ so $(-0.5)k = 1 \Rightarrow k = 2$

$\Rightarrow b = 0.5$

$\Rightarrow a = 0.25$(As P lie on parabola $y^2 = x$)

$\Rightarrow P = \begin{bmatrix} 0.25 \\ 0.5 \end{bmatrix}$

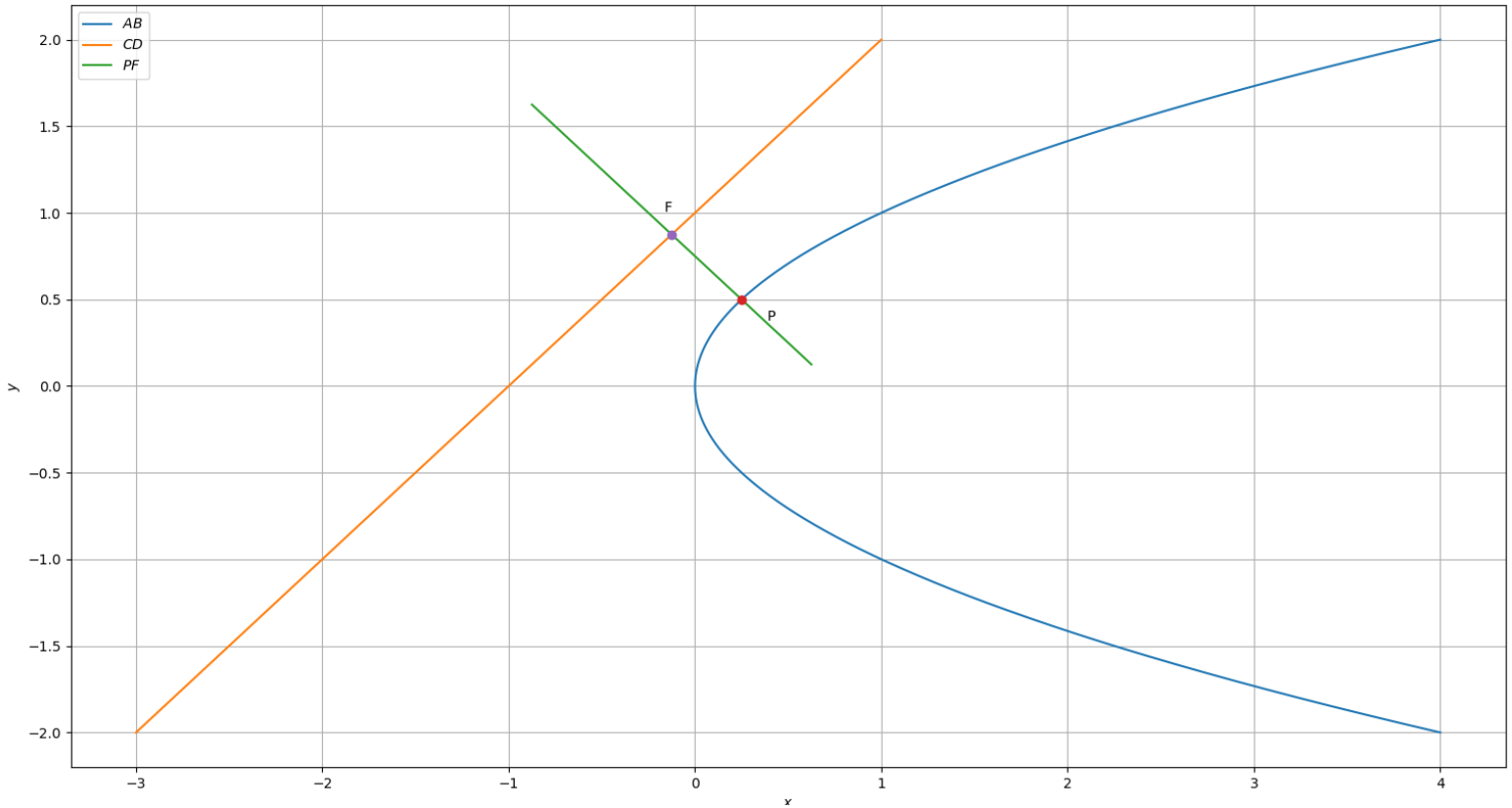
Formula for shortest distance:

Shortest distance(d) from a point to line is the perpendicular distance drawn from the point to the line, which is given by

$$d = \frac{|P^T n+1|}{||n||}; \text{ where } ||n|| = \sqrt{n[0]^2 + n[1]^2}$$

$$d = \frac{3\sqrt{2}}{8}$$

Graphical Representation:



So, the shortest distance between parabola and line $x - y + 1 = 0$ is equal to $\frac{3\sqrt{2}}{8}$ units.