# Control Systems

Durbha Krishna Srikar*

## Contents



## 1 Circuit Design from Bode Plot

1.1. Consider the Magnitude Bode Plot and Phase Bode Plot 1.1 of Open-Loop Transfer Function of an Amplifier. Estimate the Open-Loop Transfer Function. (Assume $'A'$ as $'G'$ and $'\beta'$ as $'H'$)

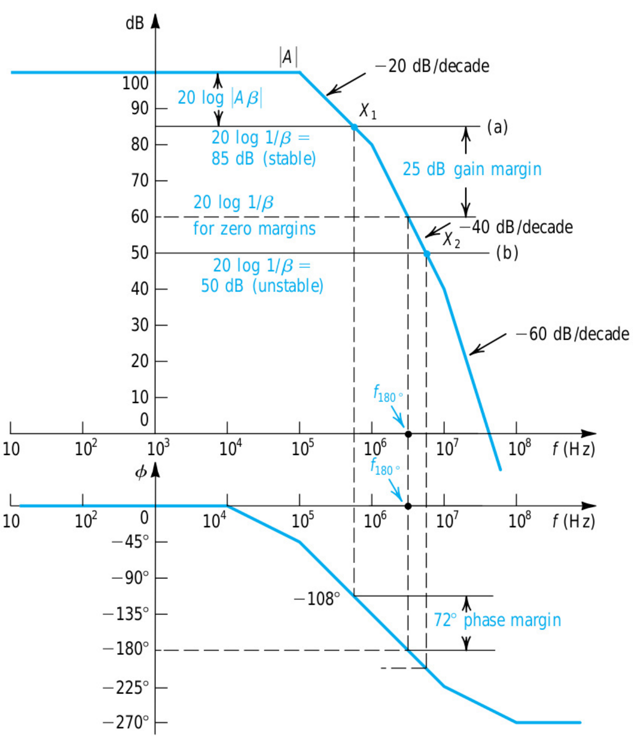


Fig. 1.1: Magnitude and Phase Bode Plot

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India. 

**Solution:** Let $G(f)$ be the Open-Loop Transfer Function,

$$G(f) = \begin{cases} 100 & 0 < f < 10^5 \\ 200 - 20\log(f) & 10^5 < f < 10^6 \\ 320 - 40\log(f) & 10^6 < f < 10^7 \\ 460 - 60\log(f) & 10^7 < f \end{cases} \tag{1.1.1}$$

$$\nabla G(f) = \frac{d(G(f))}{d(\log(f))} = \begin{cases} 0 & 0 < f < 10^5 \\ -20 & 10^5 < f < 10^6 \\ -40 & 10^6 < f < 10^7 \\ -60 & 10^7 < f \end{cases} \tag{1.1.2}$$

As we know that, **When a pole is encountered the slope always decreases by 20 dB/decade** and **When a zero is encountered the slope always increases by 20 dB/decade**. So, by observing Fig. 1.1 it can be concluded that we are having Poles at $f = 10^5 Hz, 10^6 Hz, 10^7 Hz$ and No Zeros.

So, the Open-Loop Transfer Function $G(f)$ is

$$G(f) = \frac{10^5}{\left(1 + j\frac{f}{10^5}\right)\left(1 + j\frac{f}{10^6}\right)\left(1 + j\frac{f}{10^7}\right)} \tag{1.1.3}$$

1.2. Calculate the Phase of Open-Loop Transfer Function.
**Solution:**

$$\phi(f) = -\left[\tan^{-1}\left(\frac{f}{10^5}\right) + \tan^{-1}\left(\frac{f}{10^6}\right) + \tan^{-1}\left(\frac{f}{10^7}\right)\right] \tag{1.2.1}$$

1.3. Verify (1.1.3) by plotting the Magnitude and Phase Bode Plots of $G(f)$ and comparing with (1.1.1)
**Solution:** See Fig. 1.3

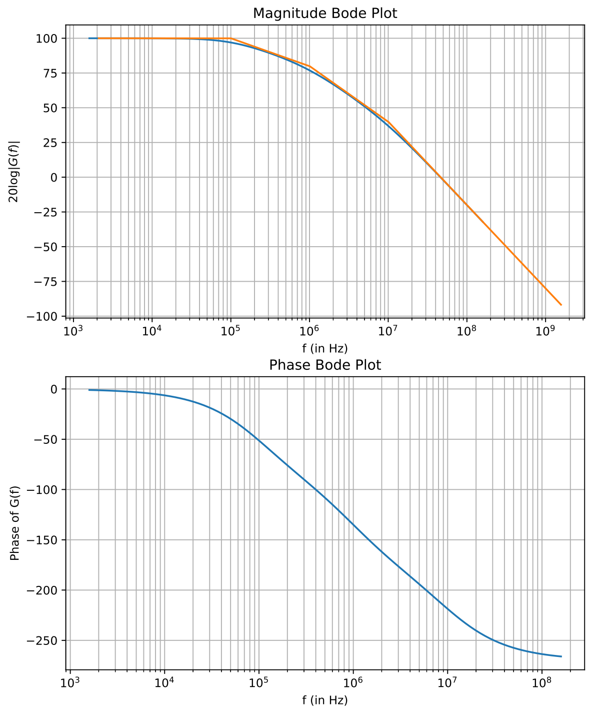


Fig. 1.3: Magnitude Bode Plot

Python Code for Bode Plot is at

```
codes/ee18btech11014/Bode_Plot.py
```

1.4. Find the PM from Fig. 1.1, given that he feedback gain $H(f)$ is constant and given by

$$20\log\left(\frac{1}{H(f)}\right) = 85dB \tag{1.4.1}$$

$$\text{or, } H(f) = 5.623 \times 10^{-5}. \tag{1.4.2}$$

**Solution:** From the figure,

$$20\log|G(f_1)| = 85dB \tag{1.4.3}$$

$$\implies 20\log|G(f_1)| = 20\log\left(\frac{1}{H(f_1)}\right) \tag{1.4.4}$$

$$\text{or, } |G(f_1)H(f_1)| = 1 \tag{1.4.5}$$

and

$$f_1 = 0.493MHz, \tag{1.4.6}$$

from (1.4.3) and (1.1.3). Also,

$$\because \angle H(f) = 0, \forall f \tag{1.4.7}$$

$$\angle G(f_1)H(f_1) = \angle G(f_1) = -108^\circ \tag{1.4.8}$$

$$\implies PM = 180^\circ - 108^\circ = 72^\circ \tag{1.4.9}$$

using (1.4.6) in (1.2.1).

1.5. Find the GM.

**Solution:** The crossover frequency $f_\pi$ is defined as

$$\angle G(f_\pi)H(f_\pi) = 180^\circ \tag{1.5.1}$$

$$\implies \angle G(f_\pi) = 180^\circ \tag{1.5.2}$$

$$\implies f_\pi = 3.34MHz \tag{1.5.3}$$

by solving (1.2.1). From Fig. 1.1,

$$20\log|G(f_\pi)| = 60dB \tag{1.5.4}$$

$$\implies 20\log|G(f_\pi)| - 20\log\left(\frac{1}{H(f_\pi)}\right) = (60-85)\,dB \tag{1.5.5}$$

$$\implies GM = \left|20\log|G(f_\pi)H(f_\pi)|\right| = 25dB \tag{1.5.6}$$

1.6. Break the Transfer Function $G(s)$ into Simple Blocks and create a block diagram.

**Solution:** From (1.1.3)

$$G(s) = \frac{10^5}{\left(1+\frac{s}{2\pi 10^5}\right)\left(1+\frac{s}{2\pi 10^6}\right)\left(1+\frac{s}{2\pi 10^7}\right)} \tag{1.6.1}$$

The block diagram is available in Fig. 1.6

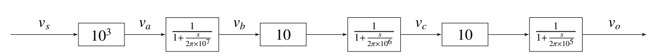


Fig. 1.6

1.7. Find the Gain of RC-Circuit in Fig. 1.7 and identify the pole location.

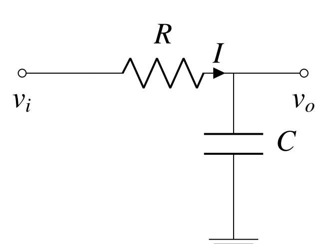


Fig. 1.7

**Solution:**

$$v_o = v_i \frac{\frac{1}{sc}}{R + \frac{1}{sC}} \tag{1.7.1}$$

$$\implies \frac{v_o}{v_i} = \frac{1}{1 + sCR} \tag{1.7.2}$$

Thus, there is a pole at

$$s = -\frac{1}{RC} \tag{1.7.3}$$

1.8. Design a circuit for $G(s)$.
**Solution:** (1.6.1) can be expressed as

$$\because G(s) = \frac{G_1 G_2 G_3}{(1 + sC_1R_1)(1 + sC_2R_2)(1 + sC_3R_3)} \tag{1.8.1}$$

where the parameters are available in Table 1.8. Choosing an OPAMP of gain $G_1$,$G_2$ and $G_3$ and noting from (1.7.2) that each of the blocks in Fig. 1.6 can be realised through the RC circuit in Fig. 1.7 with parameters in Table 1.8, the circuit design is available in Fig. 1.8.

| Circuit Element | Value |
|---|---|
| $G_1$ | $60dB$ |
| $G_2$ | $20dB$ |
| $G_3$ | $20dB$ |
| $R_1$ | $100\Omega$ |
| $R_2$ | $1k\Omega$ |
| $R_3$ | $10k\Omega$ |
| $C_1$ | $\frac{1}{2\pi}nF$ |
| $C_2$ | $\frac{1}{2\pi}nF$ |
| $C_3$ | $\frac{1}{2\pi}nF$ |

TABLE 1.8

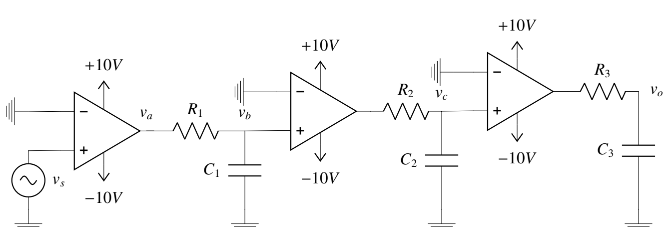


Fig. 1.8

1.9. Design a circuit for $H(s)$.
**Solution:** From (1.4.2), $H$ is constant and should not involve any Reactive Elements. The simplest way to realise $H$ is through a voltage divider as shown in Fig. 1.9. Thus,

$$H = \frac{R_F}{R_F + R_M} \tag{1.9.1}$$

with resistance values available in Table 1.9.

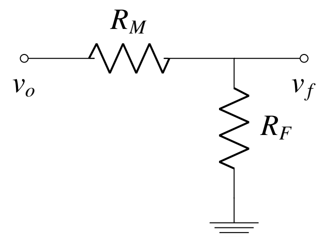


Fig. 1.9

| Circuit Element | Value |
|---|---|
| $R_M$ | $1.778 \times 10^5\Omega$ |
| $R_F$ | $10\Omega$ |

TABLE 1.9

1.10. Find the closed loop transfer function $T(s)$ and draw the equivalent circuit.
**Solution:** The closed loop circuit is easily obtained from Figs. 1.8 and 1.9 as shown in Fig. 1.10

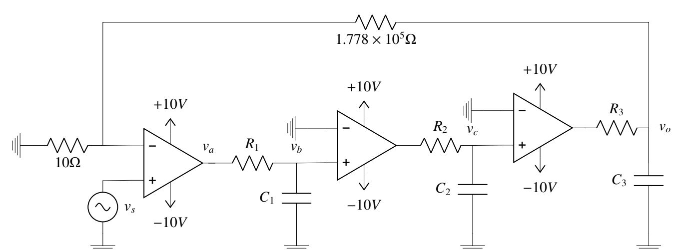


Fig. 1.10

The closed loop gain,

$$T(s) = \frac{G(s)}{1 + G(s)H} \tag{1.10.1}$$

$$= \frac{10^5}{\left(1 + \frac{s}{2\pi 10^5}\right)\left(1 + \frac{s}{2\pi 10^6}\right)\left(1 + \frac{s}{2\pi 10^7}\right) + 5.623} \tag{1.10.2}$$

1.11. Using ngspice, find the output of Fig. 1.10 for a DC input and verify that $T(s)$ in (1.10.2) is stable.
**Solution:**

Frequency of a DC Input is 0. So, the Gain of the system for DC input should be

$$f = 0 \tag{1.11.1}$$
$$s = 0 \tag{1.11.2}$$
$$T_0 = T(0) = 15098.8 \tag{1.11.3}$$

Check the following spice file for circuit.

spice/ee18btech11014/ee18btech11014_a.net

Following are the instructions to run the spice file.

spice/ee18btech11014/README.md

Observe the results by running the Python Code

spice/ee18btech11014/
EE18BTECH11014_Simulation−a.py

The Response of Closed-Loop System for DC Signal as Input is Fig.1.11

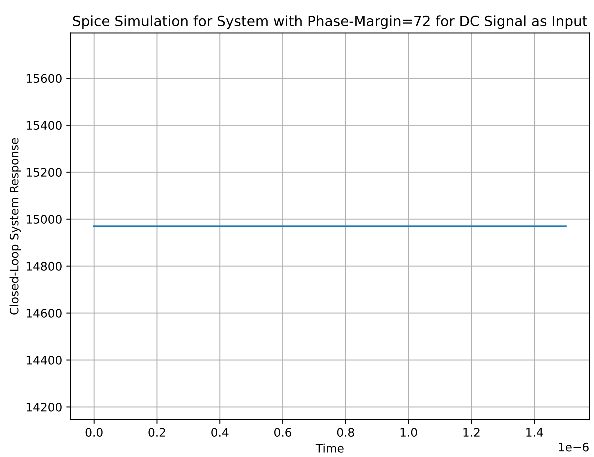


Fig. 1.11

The value of Gain($T_0$) from simulation is 14969.4763. The Error with theoritically calculated value is 0.8645%

## 2 Stabilty

2.1. Discuss the relation between Stability and Phase Margin (PM).
**Solution:** Let the loop gain

$$L(s) = G(s)H(s) \tag{2.1.1}$$

Then the closed loop gain

$$T(\jmath\omega) = \frac{G(\jmath\omega)}{1 + L(\jmath\omega)} \tag{2.1.2}$$

and If

$$\left|L(\jmath\omega_0)\right| = 1, \tag{2.1.3}$$
$$T(\jmath\omega_0) = \frac{G(\jmath\omega_0)}{1 + L(\jmath\omega_0)} \tag{2.1.4}$$
$$\tag{2.1.5}$$
$$= \frac{G(\jmath\omega_0)}{1 + \exp\{\angle L(\jmath\omega_0)\}}, \tag{2.1.6}$$

and

$$PM = 180^{\circ} - \angle L(\jmath\omega_0) \tag{2.1.7}$$

and for

$$PM \begin{cases} > 0 & T(s) \text{ stable} \\ = 0 & T(s) \text{ marginally stable} \\ < 0 & T(s) \text{ unstable} \end{cases} \tag{2.1.8}$$

2.2. For constant $H$, find the frequency at which $\angle L(\jmath\omega) = -180^{\circ}$ and determine the region for Stability. **Solution:**

$$\angle G(f)H(f) = \angle G(f) \tag{2.2.1}$$

$$\implies \angle G(f) = -180^{\circ}$$
$$= -\left[\tan^{-1}\left(\frac{f}{10^5}\right) + \tan^{-1}\left(\frac{f}{10^6}\right) + \tan^{-1}\left(\frac{f}{10^7}\right)\right] \tag{2.2.2}$$

or,

$$f = f_{\pi} = 3.34MHz. \tag{2.2.3}$$

So, for

- $f > 3.34MHz$, System is Unstable
- $f = 3.34MHz$, System is Marginally Stable
- $f < 3.34MHz$, System is Stable

2.3. Determine the range of $H$ for Stability.
**Solution:**

$$|G(f_{\pi})| = 320 - 40\log(f_{\pi}) \tag{2.3.1}$$
$$= 59dB = 896 \tag{2.3.2}$$
$$H = 1.11 \times 10^{-3} \ (\because |G(f_{\pi})H| = 1) \tag{2.3.3}$$

Thus,

- $H > 1.11 \times 10^{-3}$, System is Unstable

- $H = 1.11\times10^{-3}$, System is Marginally Stable
- $H < 1.11\times10^{-3}$, System is Stable

2.4. Verify the stablity from the value of $H = 9.9\times10^{-3}$
**Solution:**
**System is Unstable** as $H > 1.11\times10^{-3}$.
Run the following code to verify the stablity of the system

```
codes/ee18btech11014/Stability.py
```

2.5. Using ngspice, find the Unit-Step Response for System for $H = 9.9\times10^{-3}$
**Solution:**
At $H = 9.9\times10^{-3}$, the Phase-Margin($\alpha$) of the system is

$$\alpha \approx -38.5^\circ \tag{2.5.1}$$

So the system is **Unstable**.
Check the following spice file for circuit.

```
spice/ee18btech11014/ee18btech11014_3.net
```

Following are the instructions to run the spice file.

```
spice/ee18btech11014/README.md
```

Run the following code to see the Unit Step Response of Closed-Loop System for $H = 9.9\times10^{-3}$.

```
spice/ee18btech11014/
    EE18BTECH11014_Simulation-3.py
```

The Unit-Step Response is Fig.2.5

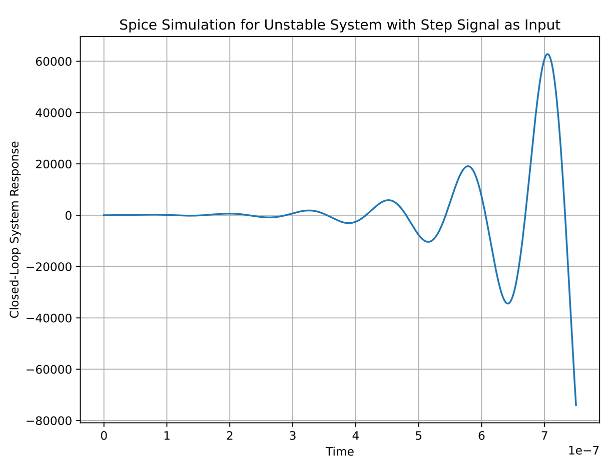


Fig. 2.5

## 3 Phase Margin

3.1. Find the frequency for which $PM = 90^\circ$. Assume $H$ to be constant.
**Solution:** $\because \angle H(f) = 1$,

$$\angle G(f_{90})H(f_{90}) = \angle G(f_{90}) = 90^\circ - 180^\circ \tag{3.1.1}$$
$$= -90^\circ \tag{3.1.2}$$

The Bode plot in Fig. 1.1 shows that

$$|G(f)| < 1, \quad f > 10^8 \tag{3.1.3}$$

Also,

$$\tan^{-1}\left(\frac{f}{10^7}\right) \approx 0, \quad f < 10^8 \tag{3.1.4}$$

Thus, from (1.2.1) and (3.1.2),

$$\phi(f) \approx -\left[\tan^{-1}\left(\frac{f}{10^5}\right) + \tan^{-1}\left(\frac{f}{10^6}\right)\right] \tag{3.1.5}$$
$$= -90^\circ \tag{3.1.6}$$
$$\implies f_{90} = 3.162\times10^5 \tag{3.1.7}$$

after simplification.

3.2. Find $H$ when the $PM = 90^\circ$.
**Solution:** By definition of the PM,

$$|G(f_{90})H(f_{90})| = 1 \tag{3.2.1}$$
$$\implies |H(f_{90})| = \frac{1}{|G(f_{90})|} \tag{3.2.2}$$

From (1.1.1),

$$20\log|G(f)| = 200 - 20\log(3.162\times10^5) \tag{3.2.3}$$
$$= 90dB \tag{3.2.4}$$
$$\implies |G(f)| = 3.1625\times10^4 \tag{3.2.5}$$
$$\implies H = 3.162\times10^{-5} \tag{3.2.6}$$

using (3.2.2).

3.3. Design the closed loop circuit for $PM = 90^\circ$
**Solution:** See Fig. 3.3, where Fig. 1.9 is used for the feedback $H$ with $R_M = 0.3162M\Omega$ and $R_F = 10\Omega$.

3.4. Using ngspice, find the output of the 3.3 for Unit-Step Signal and Sinusoidal Signal as Input.
**Solution:**

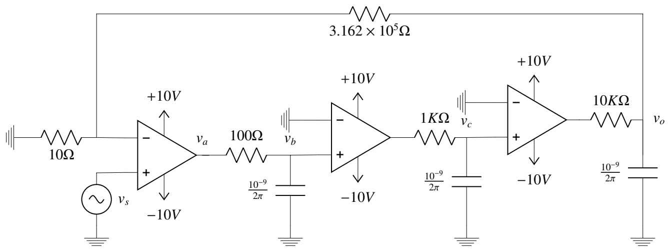


Fig. 3.3

For Unit-Step Signal as Input,

$$T = 24026.91 \tag{3.4.1}$$

For Sinusoidal Signal as Input,

$$T = 354.5655 \tag{3.4.2}$$

Check the following spice file for circuits for the inputs Unit-Step and Sinusoidal Signals respectively.

```
spice/ee18btech11014/ee18btech11014_1.net
spice/ee18btech11014/ee18btech11014_2.net
```

Following are the instructions to run the spice file.

```
spice/ee18btech11014/README.md
```

Run the following Python Code for Visualising the Responses of the System for both the Inputs.

```
spice/ee18btech11014/
    EE18BTECH11014_Simulation−1,2.py
```

The Responses are shown in Fig.3.4
From Simulation, for Unit-Step Signal as Input,

$$T = 23842.97 \tag{3.4.3}$$

From Simulation, for Sinusoidal Signal as Input,

$$T = 349.01 \tag{3.4.4}$$

Percentage of Error from Theoritical and Simulation results when Unit-Step Signal is given as Input is 0.77%.
Percentage of Error from Theoritical and Simulation results when Sinusoidal Signal is given as Input is 1.59%.

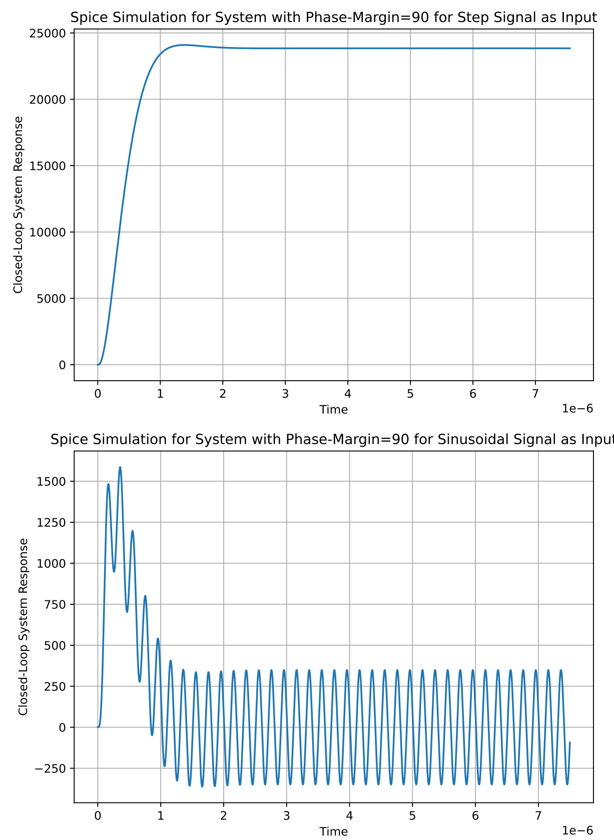


Fig. 3.4

3.5. Repeat all the above for $PM = 45°$.