Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

1 Signal Flow Graph

2 Gain of Feedback Circuits

2.1 Current Amplifiers

2.1.1. For the feedback current amplifier shown in 2.1.1, Draw the Small-Signal Model.eglect the Early effect in Q_1 and Q_2.

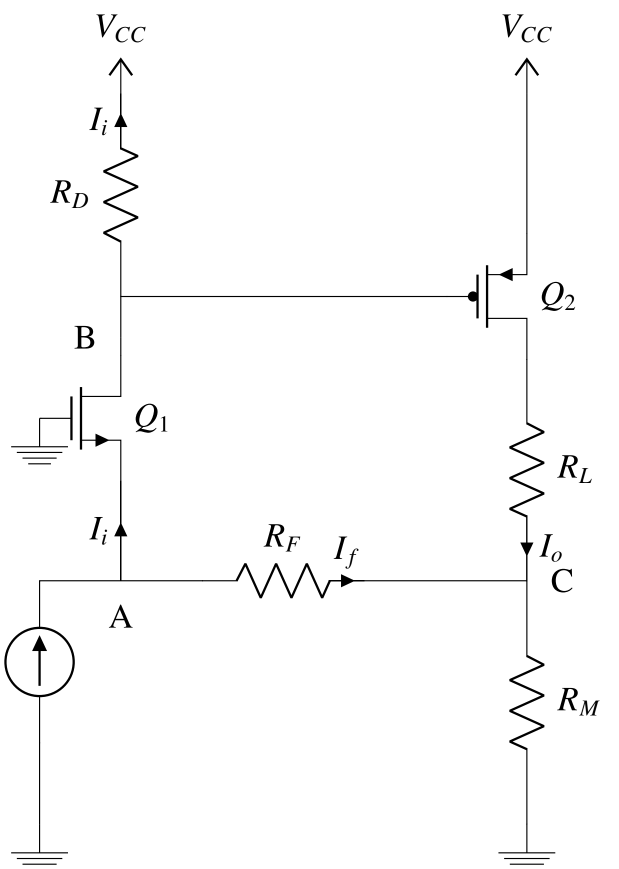

Fig. 2.1.1

Solution: While drawing a Small-Signal Model, we ground all constant voltage sources and open all constant current sources. All Small-Signal paramters are obtained from DC-Analysis of the circuit. Neglecting Early effect, in Small-Signal Analysis a N-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{gs}$ flowing from Drain to

Source. Whereas a P-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{sg}$ flowing from Source to Drain.

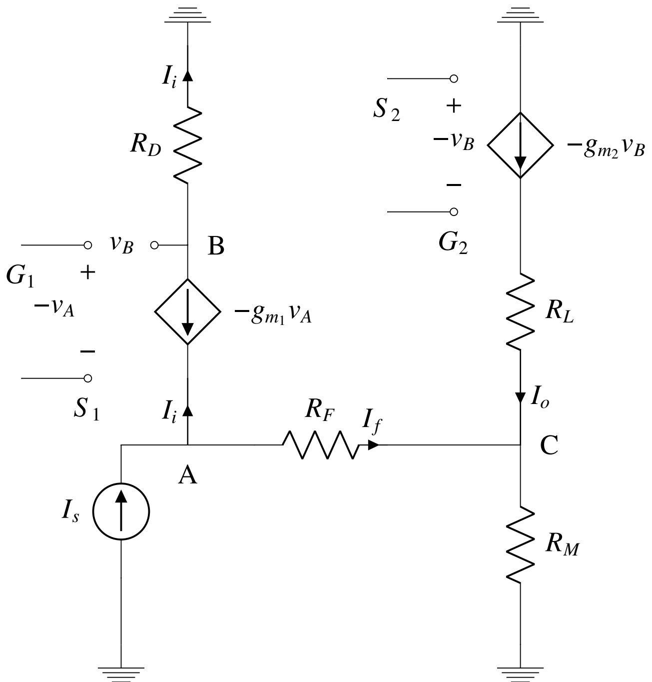

Fig. 2.1.1: Small Signal Model

2.1.2. Describe how the given circuit is a Negetive Feedback Current Amplifier.
Solution: For the feedback to be negative, I_f must have the same polarity as I_s. To ascertain that this is the case, we assume an increase in I_s and follow the change around the loop: An increase in I_s causes I_i to increase and the drain voltage of Q_1 will increase. Since this voltage is applied to the gate of the p-channel device Q_2 , its increase will cause I_o , the drain current of Q_2, to decrease. Thus, the voltage across R_M will decrease, which will cause I_f to increase. This is the same polarity assumed for the initial change in I_s, verifying that the feedback is indeed negative.

2.1.3. Write all KVL and KCL Equations
Solution:

$$I_i = g_{m_1} v_A \tag{2.1.3.1}$$
$$v_B = I_i R_D \tag{2.1.3.2}$$
$$I_o = -g_{m_2} v_B \tag{2.1.3.3}$$
$$v_A = I_F R_F - (I_F + I_o) R_M \tag{2.1.3.4}$$

2.1.4. Find the Expression for the Open-Loop Gain $G = \frac{I_o}{I_i}$, from the Small-Signal Model. For simplicity, neglect the Early effect in Q_1 and Q_2.
Solution: In Small-Signal Model,

$$v_B = I_i R_D \tag{2.1.4.1}$$
$$v_{gs_2} = v_B = I_i R_D \tag{2.1.4.2}$$

In Small-Signal Analysis, P-MOSFET is modelled as a current source where current flows from Source to Drain. So, the value of current flowing from Source to Drain in P-MOSFET is,

$$I_o = -g_{m_2} v_{gs_2} = -g_{m_2} I_i R_D \tag{2.1.4.3}$$

So, the Open-Circuit Gain is

$$G = \frac{I_o}{I_i} = -g_{m_2} R_D \tag{2.1.4.4}$$

2.1.5. Find the Expression of the Feedback Factor $H = \frac{I_f}{I_o}$, from Small-Signal Model. For simplicity, neglect the Early effect in Q_1 and Q_2.
Solution:
I_o is fed to a current divider formed by R_M and R_F. Assuming system has Good Gain, Most part of I_s flows as I_F leaving behind small I_i. As I_i is small, the voltage at point 'A' is very small and is considered, $v_A \simeq 0$. So R_F and R_M are parallel and Voltage Drop across them is same.
From (2.1.3.4),

$$(I_o + I_f) R_M \simeq -I_f R_F \tag{2.1.5.1}$$
$$\frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \tag{2.1.5.2}$$

So, the Feedback Factor,

$$H \equiv \frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \tag{2.1.5.3}$$

2.1.6. Find the Expression for the Closed-Loop Gain $T = \frac{I_o}{I_s}$. For simplicity, neglect the Early effect in Q_1 and Q_2.
Solution:

From Open-Loop Gain and Feedback Factor,

$$I_s = I_i + I_f \tag{2.1.6.1}$$

$$I_s = \frac{I_o}{G} + HI_o \tag{2.1.6.2}$$

$$GI_s = I_o(1 + GH) \tag{2.1.6.3}$$

$$\frac{I_o}{I_s} = \frac{G}{1 + GH} \tag{2.1.6.4}$$

$$\frac{I_o}{I_s} = -\frac{g_{m_2} R_D}{1 + g_{m_2} R_D / \left(1 + \frac{R_F}{R_M}\right)} \tag{2.1.6.5}$$

So the Block Diagram of Feedback Current Amplifier is

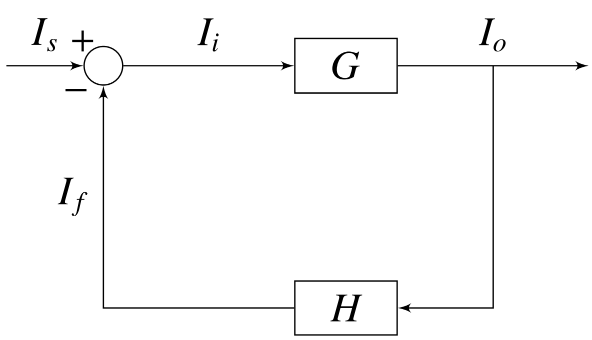

Fig. 2.1.6

where $G = -g_{m_2} R_D$ and $H = -\frac{R_M}{R_F + R_M}$

So, the value of Closed-Loop Gain is

$$T = \frac{I_o}{I_s} = -\frac{g_{m_2} R_D}{1 + g_{m_2} R_D / \left(1 + \frac{R_F}{R_M}\right)} \tag{2.1.6.6}$$

2.1.7. Draw the Circuit Diagram of Feedback Network.
Solution: The Circuit Diagram of Feedback Network is 2.1.7

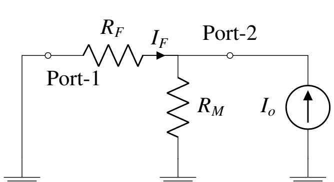

Fig. 2.1.7: Feedback Network

By KVL and KCL,

$$(I_o + I_f) R_M = -I_f R_F \tag{2.1.7.1}$$

$$\frac{I_f}{I_o} = -\frac{R_M}{R_F + R_M} \tag{2.1.7.2}$$

So, Gain of Feedback Network is

$$H = -\frac{R_M}{R_F + R_M} \tag{2.1.7.3}$$

The Block Diagram of Feedback Network is

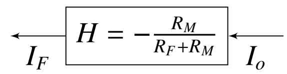

Fig. 2.1.7: Feedback Block Diagram

2.1.8. Find R_{11} and R_{22} of Feedback Network where R_{11} is input resistance through Port-1 and R_{22} is Input Resistance through Port-2.
Solution:
While calculating R_{11}, Port-2 should be Opened. So, Net-Resistance seen through Port-1 is

$$R_{11} = R_F + R_M \tag{2.1.8.1}$$

While calculating R_{22}, Port-1 should be Shorted. So, Net-Resistance seen through Port-2 is

$$R_{22} = R_F \| R_M \tag{2.1.8.2}$$

$$R_{22} = \frac{R_F R_M}{R_F + R_M} \tag{2.1.8.3}$$

So,

$$R_{11} = R_F + R_M \tag{2.1.8.4}$$

$$R_{22} = \frac{R_F R_M}{R_F + R_M} \tag{2.1.8.5}$$

2.1.9. Draw the Circuit Diagram of Open-Loop Network.
Solution:
As the ciruit is Shunt-Series Topology, R_{11} is shunted across the Input Terminal and R_{22} is added in series to Output Terminal.
Current Flowing through R_D is approximately same as I_i, as R_F is a Large Resistance.
The Circuit Diagram of Open-Loop Network is 2.1.9
By KVL and KCL,

$$v_B = I_i R_D \tag{2.1.9.1}$$

$$v_{gs_2} = v_B = I_i R_D \tag{2.1.9.2}$$

$$I_o = -g_{m_2} v_{gs_2} = -g_{m_2} I_i R_D \tag{2.1.9.3}$$

$$\frac{I_o}{I_i} = -g_{m_2} R_D \tag{2.1.9.4}$$

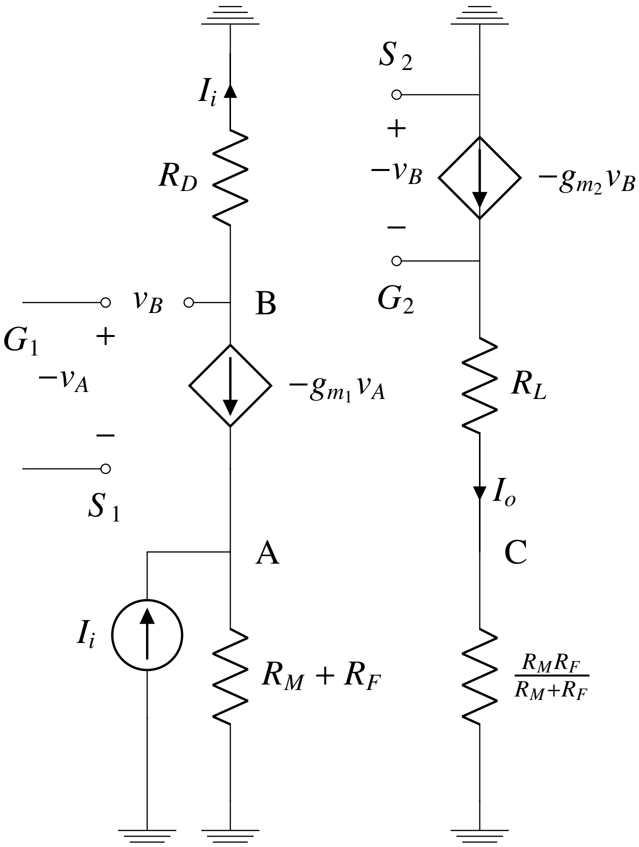

Fig. 2.1.9: Open-Loop Network

So, Open-Loop Gain is

$$G = \frac{I_o}{I_i} = -g_{m_2}R_D \tag{2.1.9.5}$$

The Block Diagram of Open-Loop Network is

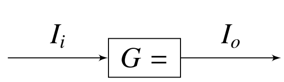

Fig. 2.1.9: Open-Loop Block Diagram