EE6317: Channel coding **2021**

Homework 1: April 2021

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Instructions: *You are encouraged to discuss and collaborate with your classmates. However, you* must explicitly *mention at the top of your submission who you collaborated with. Copying is NOT permitted, and solutions must be written independently and in your own words.*

Homeworks must be submitted on Google classroom. Please scan a copy of your handwritten assignment and upload as pdf with filename <your ID>_HW<homework no>.pdf. Example: EEB19BTECH00000_HW1.pdf.

For programming questions, submit as separate files. Please use the naming convention <your ID>_HW<homework no>_problem<problem no>.. Example: EEB19BTECH00000_HW1_problem1.c*

Exercise 1.1 (25 points)**.** For the channel given to you in the spreadsheet, calculate the capacity. Also derive the maximum likelihood decoder.

Exercise 1.2 (25 points)**.** You will now simulate the performance of a given code over the channel specified in the previous problem by transmitting a black-and-white image.

The image file is given to you in numpy and Matlab formats. This is a 400×300 array, where each element is a 0 or a 1. Here, a 0 represents a black pixel and 1 is a white pixel. You can load and render the image in python and Matlab as follows. Essentially, this is a sequence of 120000 (message) bits.

Python:

```
import numpy as np
from matplotlib import pyplot as plt
mss = np.load(mss.npy')
plt.imshow(mss,'gray'), plt.show()
```

Matlab:

```
load mss.mat
imshow(mss, [0 1])
```

Simulate the transmission of this message in the uncoded form by corrupting each bit independently according to the distribution specified in the previous problem. Let $\underline{x} = (x_1, x_2, \ldots, x_k)$ denote the transmitted sequence (i.e., the actual image), and $\underline{y} = (y_1, \ldots, y_k)$ be the recieved sequence.

The decoder estimates $\hat{\underline{x}}$ using the maximum likelihood decoder, i.e., for a specific received sequence $\underline{y}$,

$$\hat{x}_i = \arg\max_{x\in\{0,1\}} p_{Y|X}(y_i|x)$$

for $i = 1, 2, \ldots, k$. Compute the bit error rate, i.e., the fraction of bits that are erroneously decoded.

Exercise 1.3 (50 points)**.** Repeat what you did in problem 2, but using coded transmission with the following codes:

1. **Rate 1/3 repetition code:** Each message bit m_i is transmitted thrice. If $m_1, \ldots, m_k$ are the message bits, and $x_1, x_2, \ldots, x_n$ is the transmitted codeword, then $n = 3k$, and $x_{3i-2} = x_{3i-1} = x_{3i} = m_i$ for $i = 1, 2, \ldots, k$. The receiver decodes each bit using the maximum likelihood decoder.

2. **Rate** $2/5$ **block code**: the message is partitioned into blocks of 2 bits each, i.e., $(m_1, m_2), (m_3, m_4), \ldots$ and each block is separately encoded as follows

$$\begin{aligned} ENC(00) &= 00000 \\ ENC(01) &= 00111 \\ ENC(10) &= 11100 \\ ENC(11) &= 11011 \end{aligned}$$

   Therefore the k-bit message is converted to a $(5/2)k$-bit codeword and transmitted across the channel. The receiver uses a maximum likelihood decoder for each block.

For each of the above, simulate and find the bit error rate.