

Homework 2: 18th Mar 2021

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Instructions: You are encouraged to discuss and collaborate with your classmates. However, you must explicitly mention at the top of your submission who you collaborated with, and all external resources (websites, books) you used, if any. Copying is NOT permitted, and solutions must be written independently and in your own words.

Please scan a copy of your handwritten assignment as pdf with filename `<your ID>_HW<homework no>.pdf`. Example: `EE19BTECH00000_HW1.pdf`.

For programming questions, create separate files. Please use the naming convention `<your ID>_HW<homework no>_problem<problem no>.*`. Example: `EE19BTECH00000_HW1_problem1.c`. You may upload c, cpp, py or m files only. No other format will be allowed.

Finally, upload your submission as separate files on Google classroom. Please do not upload zip files.

Exercise 2.1. Compute the arithmetic code for the two sequences given the shared spreadsheet. The probability mass function is also provided in the spreadsheet.

Exercise 2.2. Compute the LZ78 parsing for the three sequences given in the shared spreadsheet.

Exercise 2.3 (1-bit quantization). In this exercise, you will implement a scalar quantizer that minimizes the mean squared error distortion. For any $x \in \mathbb{R}$, consider a 1-bit/2-level quantizer which does the following:

$$Q(x) = \begin{cases} a & \text{if } x > t \\ b & \text{if } x \leq t \end{cases}$$

where $a > b$ are the two levels, and t is the threshold. We assume that $a > t > b$.

For any real-valued random variable X , the mean squared error of the quantizer for a fixed a, b, t is

$$MSE = \mathbb{E}(X - Q(X))^2$$

Assume that X is uniformly distributed over the interval $[\alpha, \beta]$. Set up the optimization problem and find the a, b, t which minimizes the mean squared error. These would be functions of α, β .

Now use the files mentioned in the shared spreadsheet. Each file contains a set of 100 points uniformly distributed over some $[\alpha, \beta]$. Estimate α, β from the set given to you. Use this, and the optimal 1-bit quantizer that you have designed above. Find $Q(x)$ for all x in the file, and compute the mean squared error $\frac{1}{100} \sum_{x \in \text{file}} (x - Q(x))^2$.