# Control Systems

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Contents


**1 Signal Flow Graph** 1
- 1.1 Mason's Gain Formula . . . 1
- 1.2 Matrix Formula . . . . . 1
- 1.3 Example . . . . . 1

**2 Gain of Feedback Circuits** 1
- 2.1 Current Amplifiers . . . . . 1

**3 Bode Plot** 4
- 3.1 Introduction . . . . . 4
- 3.2 Example . . . . . 4
- 3.3 Phase . . . . . 4
- 3.4 Example . . . . . 4

**4 Second order System** 4
- 4.1 Damping . . . . . 4
- 4.2 Peak Overshoot . . . . . 4
- 4.3 Settling Time . . . . . 4
- 4.4 Example . . . . . 4

**5 Routh Hurwitz Criterion** 4
- 5.1 Routh Array . . . . . 4
- 5.2 Marginal Stability . . . . . 4
- 5.3 Stability . . . . . 4
- 5.4 Example . . . . . 4

**6 State-Space Model** 4
- 6.1 Controllability and Observability . . . . . 4
- 6.2 Second Order System . . . . 4
- 6.3 Example . . . . . 4

**7 Nyquist Plot** 4
- 7.1 Introduction . . . . . 4
- 7.2 Example . . . . . 4

**8 Compensators** 4
- 8.1 Phase Lead . . . . . 4
- 8.2 Lead Circuit . . . . . 4
- 8.3 Lag Lead . . . . . 4
- 8.4 Example . . . . . 4

**9 Gain Margin** 4
- 9.1 Introduction . . . . . 4
- 9.2 Example . . . . . 4

**10 Phase Margin** 4
- 10.1 Intoduction . . . . . 4
- 10.2 Example . . . . . 4

**11 Oscillator** 4
- 11.1 Introduction . . . . . 4
- 11.2 Example . . . . . 4

**12 Root Locus** 4
- 12.1 Introduction . . . . . 4
- 12.2 Example . . . . . 4

**13 Polar Plot** 4
- 13.1 Introduction . . . . . 4

**14 PID Controller** 4
- 14.1 Introduction . . . . . 4


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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

## 1 Signal Flow Graph

### *1.1 Mason's Gain Formula*

### *1.2 Matrix Formula*

### *1.3 Example*

## 2 Gain of Feedback Circuits

### *2.1 Current Amplifiers*

2.1.1. For the feedback current amplifier shown in 2.1.1, Draw the Small-Signal Model

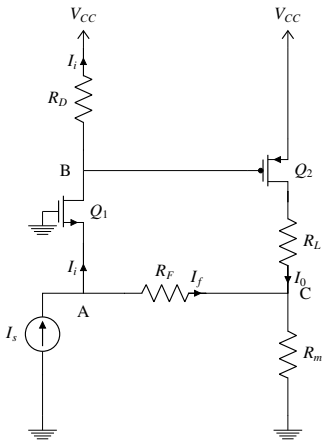


Fig. 2.1.1

**Solution:** While drawing a Small-Signal Model, we ground all constant voltage sources and open all constant current sources. All Small-Signal paramters are obtained from DC-Analysis of the circuit.

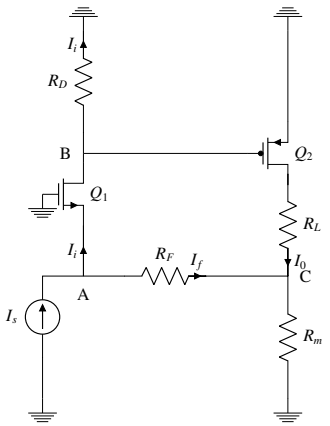


Fig. 2.1.1

2.1.2. Describe the importance of given Amplifier Topology.
**Solution:** This Feedback Topology is also known as Shunt-Series Feedback because of the parallel (or shunt) resistance at the input, and the series resistance at the output. This topology not only stabilizes the current gain but also results in a lower input resistance, and a higher output resistance, both desirable properties for a current amplifier. The decrease in input resistance results because the feedback current $I_f$ subtracts from the input current $I_s$ , and thus a lower current enters the basic current amplifier. This in turn results in a lower voltage at the amplifier input, that is, across the current source $I_s$.

2.1.3. Describe how the given circuit is a Negetive Feedback Amplifier.
**Solution:** For the feedback to be negative, $I_f$ must have the same polarity as $I_s$. To ascertain that this is the case, we assume an increase in $I_s$ and follow the change around the loop: An increase in $I_s$ causes $I_i$ to increase and the drain voltage of $Q_1$ will increase. Since this voltage is applied to the gate of the p-channel device $Q_2$ , its increase will cause $I_o$ , the drain current of $Q_2$, to decrease. Thus, the voltage across $R_M$ will decrease, which will cause $I_f$ to increase. This is the same polarity assumed for the initial change in $I_s$, verifying that the feedback is indeed negative.

2.1.4. Find the Expression for the Open-Loop Gain $G = \frac{I_o}{I_i}$, from the Small-Signal Model. For simplicity, neglect the Early effect in $Q_1$ and $Q_2$.
**Solution:** In Small-Signal Model,

$$v_B = I_i R_D \tag{2.1.4.1}$$

$$v_{gs_2} = v_B = I_i R_D \tag{2.1.4.2}$$

In Small-Signal Analysis, P-MOSFET is modelled as a current source where current flows from Source to Drain. So, the value of current flowing from Source to Drain in P-MOSFET is,

$$I_o = -g_{m_2} v_{gs_2} = -g_{m_2} I_i R_D \tag{2.1.4.3}$$

So, the Open-Circuit Gain is

$$G = \frac{I_o}{I_i} = -g_{m_2} R_D \tag{2.1.4.4}$$

2.1.5. Find the Expression of the Feedback Factor $H = \frac{I_f}{I_o}$, from Small-Signal Model. For simplicity, neglect the Early effect in $Q_1$ and $Q_2$.
**Solution:**
$I_o$ is fed to a current divider formed by $R_M$ and $R_F$. $R_F$ is a Large Resistance compared to Input resistance of Amplifier and so most of the current flows through it leaving a small current as input to Amplifier. Hence the voltage at point 'A' is very small and is considered, $v_A \simeq 0$. So $R_F$ and $R_M$ are parallel and Voltage

Drop across them is same.

$$(I_o + I_f)R_M \simeq -I_f R_o \tag{2.1.5.1}$$

$$\frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \tag{2.1.5.2}$$

So, the Feedback Factor,

$$H \equiv \frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \tag{2.1.5.3}$$

2.1.6. Find the Expression for the Closed-Loop Gain $T = \frac{I_o}{I_s}$. For simplicity, neglect the Early effect in $Q_1$ and $Q_2$.

**Solution:**

From Open-Loop Gain and Feedback Factor,

$$I_s = I_i + I_f \tag{2.1.6.1}$$

$$I_s = \frac{I_o}{G} + HI_o \tag{2.1.6.2}$$

$$GI_s = I_o(1 + GH) \tag{2.1.6.3}$$

$$\frac{I_o}{I_s} = \frac{G}{1 + GH} \tag{2.1.6.4}$$

$$\frac{I_o}{I_s} = -\frac{g_{m_2} R_D}{1 + g_{m2} R_D / \left(1 + \frac{R_F}{R_M}\right)} \tag{2.1.6.5}$$

So, the value of Closed-Loop Gain is

$$T = \frac{I_o}{I_s} = -\frac{g_{m2} R_D}{1 + g_{m2} R_D / \left(1 + \frac{R_F}{R_M}\right)} \tag{2.1.6.6}$$

3 Bode Plot
*3.1 Introduction*
*3.2 Example*
*3.3 Phase*
*3.4 Example*
4 Second order System
*4.1 Damping*
*4.2 Peak Overshoot*
*4.3 Settling Time*
*4.4 Example*
5 Routh Hurwitz Criterion
*5.1 Routh Array*
*5.2 Marginal Stability*
*5.3 Stability*
*5.4 Example*
6 State-Space Model
*6.1 Controllability and Observability*
*6.2 Second Order System*
*6.3 Example*
7 Nyquist Plot
*7.1 Introduction*
*7.2 Example*
8 Compensators
*8.1 Phase Lead*
*8.2 Lead Circuit*
*8.3 Lag Lead*
*8.4 Example*
9 Gain Margin
*9.1 Introduction*
*9.2 Example*
10 Phase Margin
*10.1 Intoduction*
*10.2 Example*
11 Oscillator
*11.1 Introduction*
*11.2 Example*
12 Root Locus
*12.1 Introduction*
*12.2 Example*
13 Polar Plot
*13.1 Introduction*
14 PID Controller
*14.1 Introduction*