

# Control Systems

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**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/control/codes
```

## 1 SIGNAL FLOW GRAPH

### 1.1 Mason's Gain Formula

### 1.2 Matrix Formula

### 1.3 Example

## 2 GAIN OF FEEDBACK CIRCUITS

### 2.1 Current Amplifiers

- 2.1.1. For the feedback current amplifier shown in 2.1.1, Draw the Small-Signal Model

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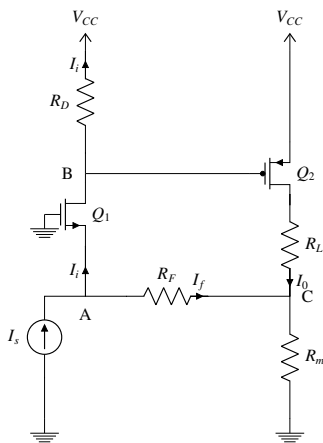


Fig. 2.1.1

**Solution:** While drawing a Small-Signal Model, we ground all constant voltage sources and open all constant current sources. All Small-Signal parameters are obtained from DC-Analysis of the circuit.

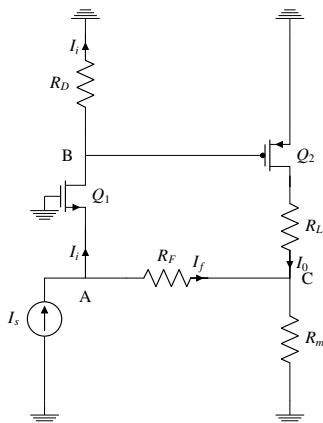


Fig. 2.1.1

2.1.2. Describe how the given circuit is a Negative Feedback Current Amplifier.

**Solution:** For the feedback to be negative,  $I_f$  must have the same polarity as  $I_s$ . To ascertain that this is the case, we assume an increase in  $I_s$  and follow the change around the loop: An increase in  $I_s$  causes  $I_i$  to increase and the drain voltage of  $Q_1$  will increase. Since this voltage is applied to the gate of the p-channel device  $Q_2$ , its increase will cause  $I_o$ , the drain current of  $Q_2$ , to decrease. Thus, the voltage across  $R_M$  will decrease, which will cause  $I_f$  to increase. This is the same polarity assumed for the initial change in  $I_s$ , verifying that the

feedback is indeed negative.

2.1.3. Find the Expression for the Open-Loop Gain  $G = \frac{I_o}{I_i}$ , from the Small-Signal Model. For simplicity, neglect the Early effect in  $Q_1$  and  $Q_2$ .

**Solution:** In Small-Signal Model,

$$v_B = I_i R_D \quad (2.1.3.1)$$

$$v_{gs2} = v_B = I_i R_D \quad (2.1.3.2)$$

In Small-Signal Analysis, P-MOSFET is modelled as a current source where current flows from Source to Drain. So, the value of current flowing from Source to Drain in P-MOSFET is,

$$I_o = -g_{m2} v_{gs2} = -g_{m2} I_i R_D \quad (2.1.3.3)$$

So, the Open-Circuit Gain is

$$G = \frac{I_o}{I_i} = -g_{m2} R_D \quad (2.1.3.4)$$

2.1.4. Find the Expression of the Feedback Factor  $H = \frac{I_f}{I_o}$ , from Small-Signal Model. For simplicity, neglect the Early effect in  $Q_1$  and  $Q_2$ .

**Solution:**

$I_o$  is fed to a current divider formed by  $R_M$  and  $R_F$ .  $R_F$  is a Large Resistance compared to Input resistance of Amplifier and so most of the current flows through it leaving a small current as input to Amplifier. Hence the voltage at point 'A' is very small and is considered,  $v_A \approx 0$ . So  $R_F$  and  $R_M$  are parallel and Voltage Drop across them is same.

$$(I_o + I_f) R_M \approx -I_f R_F \quad (2.1.4.1)$$

$$\frac{I_f}{I_o} \approx -\frac{R_M}{R_F + R_M} \quad (2.1.4.2)$$

So, the Feedback Factor,

$$H \equiv \frac{I_f}{I_o} \approx -\frac{R_M}{R_F + R_M} \quad (2.1.4.3)$$

2.1.5. Find the Expression for the Closed-Loop Gain  $T = \frac{I_o}{I_s}$ . For simplicity, neglect the Early effect in  $Q_1$  and  $Q_2$ .

**Solution:**

From Open-Loop Gain and Feedback Factor,

$$I_s = I_i + I_f \quad (2.1.5.1)$$

$$I_s = \frac{I_o}{G} + HI_o \quad (2.1.5.2)$$

$$GI_s = I_o(1 + GH) \quad (2.1.5.3)$$

$$\frac{I_o}{I_s} = \frac{G}{1 + GH} \quad (2.1.5.4)$$

$$\frac{I_o}{I_s} = -\frac{g_{m2}R_D}{1 + g_{m2}R_D / \left(1 + \frac{R_F}{R_M}\right)} \quad (2.1.5.5)$$

So the Block Diagram of Feedback Current Amplifier is

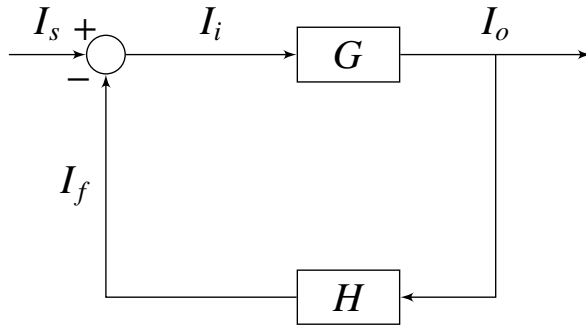


Fig. 2.1.5

where  $G = -g_{m2}R_D$  and  $H = -\frac{R_M}{R_F + R_M}$

So, the value of Closed-Loop Gain is

$$T = \frac{I_o}{I_s} = -\frac{g_{m2}R_D}{1 + g_{m2}R_D / \left(1 + \frac{R_F}{R_M}\right)} \quad (2.1.5.6)$$

### 3 BODE PLOT

#### 3.1 *Introduction*

#### 3.2 *Example*

#### 3.3 *Phase*

#### 3.4 *Example*

### 4 SECOND ORDER SYSTEM

#### 4.1 *Damping*

#### 4.2 *Peak Overshoot*

#### 4.3 *Settling Time*

#### 4.4 *Example*

### 5 ROUTH HURWITZ CRITERION

#### 5.1 *Routh Array*

#### 5.2 *Marginal Stability*

#### 5.3 *Stability*

#### 5.4 *Example*

### 6 STATE-SPACE MODEL

#### 6.1 *Controllability and Observability*

#### 6.2 *Second Order System*

#### 6.3 *Example*

### 7 NYQUIST PLOT

#### 7.1 *Introduction*

#### 7.2 *Example*

### 8 COMPENSATORS

#### 8.1 *Phase Lead*

#### 8.2 *Lead Circuit*

#### 8.3 *Lag Lead*

#### 8.4 *Example*

### 9 GAIN MARGIN

#### 9.1 *Introduction*

#### 9.2 *Example*

### 10 PHASE MARGIN

#### 10.1 *Intoduction*

#### 10.2 *Example*

### 11 OSCILLATOR

#### 11.1 *Introduction*

#### 11.2 *Example*

### 12 ROOT LOCUS

#### 12.1 *Introduction*

#### 12.2 *Example*

### 13 POLAR PLOT

#### 13.1 *Introduction*

### 14 PID CONTROLLER

#### 14.1 *Introduction*