

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/codes>

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1 SIGNAL FLOW GRAPH

2 GAIN OF FEEDBACK CIRCUITS

2.1 Estimation of Voltage Gain

2.1.1. Consider the Magnitude Bode Plot and Phase Bode Plot 2.1.1 of Open-Loop Transfer Function of an Amplifier. Estimate the Open-Loop Transfer Function. (Assume 'A' as 'G' and 'β' as 'H')

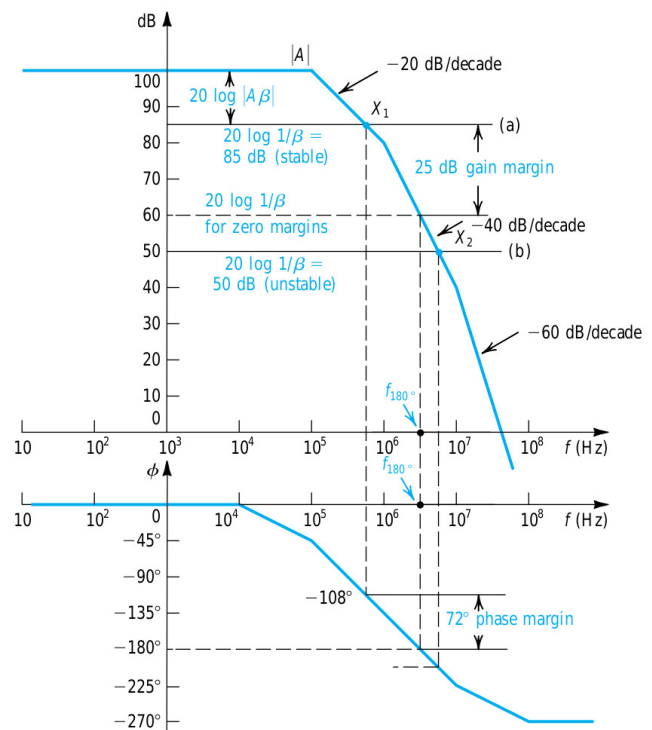


Fig. 2.1.1: Magnitude and Phase Bode Plot

Solution: Let $G(f)$ be the Open-Loop Transfer Function,

$$G(f) = \begin{cases} 100 & 0 < f < 10^5 \\ 200 - 20\log(f) & 10^5 < f < 10^6 \\ 320 - 40\log(f) & 10^6 < f < 10^7 \\ 460 - 60\log(f) & 10^7 < f \end{cases} \quad (2.1.1.1)$$

$$\nabla G(f) = \frac{d(G(f))}{d(\log(f))} = \begin{cases} 0 & 0 < f < 10^5 \\ -20 & 10^5 < f < 10^6 \\ -40 & 10^6 < f < 10^7 \\ -60 & 10^7 < f \end{cases} \quad (2.1.1.2)$$

As we know that, **When a pole is encountered the slope always decreases by 20 dB/decade** and **When a zero is encountered the slope always increases by 20 dB/decade**. So, by observing 2.1.1 it can be concluded that we are having Poles at $f = 10^5 \text{Hz}, 10^6 \text{Hz}, 10^7 \text{Hz}$ and No Zeros.

So, the Open-Loop Transfer Function $G(f)$ is

$$G(f) = \frac{10^5}{(1 + j\frac{f}{10^5})(1 + j\frac{f}{10^6})(1 + j\frac{f}{10^7})} \quad (2.1.1.3)$$

2.1.2. Calculate the Phase of Open-Loop Transfer Function.

Solution:

Phase of Open-Loop Transfer Function = ϕ

$$\phi = -\left[\tan^{-1}\left(f/10^5\right) + \tan^{-1}\left(f/10^6\right) + \tan^{-1}\left(f/10^7\right)\right] \quad (2.1.2.1)$$

2.1.3. Determine the Closed-Loop Voltage Gain of the System assuming $|GH| \gg 1$ and also assuming the block diagram of Control System is 2.1.3

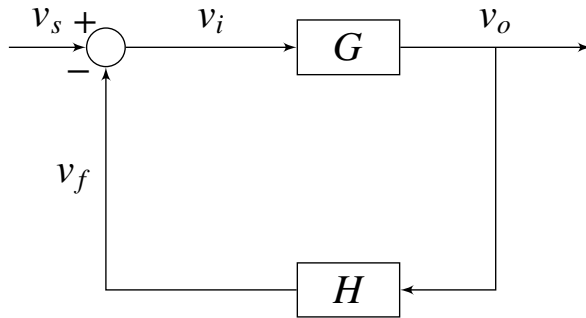


Fig. 2.1.3

Solution:

The Closed-Loop Voltage Gain of the Control System is

$$T = \frac{V_o}{V_s} = \frac{G}{1 + GH} \quad (2.1.3.1)$$

$$20 \log(T) = 20 \log(G) - 20 \log(1 + GH) \quad (2.1.3.2)$$

Considering the assumption $|GH| \gg 1$, It can be written as

$$20 \log(1 + GH) = 20 \log(GH) \quad (2.1.3.3)$$

So,

$$20 \log(T) = 20 \log(G) - 20 \log(GH) \quad (2.1.3.4)$$

$$20 \log(T) = -20 \log(H) \quad (2.1.3.5)$$

$$20 \log(T) = 20 \log\left(\frac{1}{H}\right) \quad (2.1.3.6)$$

$$T = \frac{1}{H} \quad (2.1.3.7)$$

So, The value of Closed-Loop Voltage Gain of the Control System under the assumption, $|GH| \gg 1$ is $T = \frac{1}{H}$

2.1.4. What is the value of Loop-Gain?

Solution:

The value of Loop-Gain can be calculated by the difference of 2-curves $20 \log |A|$ and $20 \log(\frac{1}{H})$. The difference between the two curves will be

$$20 \log |G| - 20 \log \frac{1}{H} = 20 \log |GH| \quad (2.1.4.1)$$

2.1.5. Define Phase-Margin

Solution:

Phase-Margin: The phase margin is defined as the angle in degrees by which the phase angle is smaller than -180° at the gain crossover, the gain crossover being the frequency at which the open-loop gain first reaches 1.

2.1.6. Find the frequencies for which phase margins are 90° and 45° respectively?

Solution:

Let Phase Margin be $\alpha = 90^\circ$. Then,

$$\alpha = \phi - (-180^\circ) \quad (2.1.6.1)$$

$$\phi = -180^\circ + \alpha \quad (2.1.6.2)$$

$$\phi = -90^\circ \quad (2.1.6.3)$$

So, by the definition of Phase-Margin, at $\phi = -90^\circ$, $|GH| = 1$. The value of $\phi = -90^\circ$ between poles $f = 10^5 \text{ Hz}$, 10^6 Hz . Assuming the Poles are farther apart,

$$\tan^{-1}\left(\frac{f}{10^7}\right) \approx 0 \quad (2.1.6.4)$$

where $10^5 < f < 10^6$

So,

$$-\tan^{-1}\left(f/10^5\right) - \tan^{-1}\left(f/10^6\right) = -90 \quad (2.1.6.5)$$

$$\tan^{-1}\left(f/10^5\right) + \tan^{-1}\left(f/10^6\right) = 90 \quad (2.1.6.6)$$

$$\tan^{-1}\left(f/10^5\right) = 90 - \tan^{-1}\left(f/10^6\right) \quad (2.1.6.7)$$

$$\tan^{-1}\left(f/10^5\right) = \cot^{-1}\left(f/10^6\right) \quad (2.1.6.8)$$

$$\tan^{-1}\left(f/10^5\right) = \tan^{-1}\left(10^6/f\right) \quad (2.1.6.9)$$

$$f^2 = 10^{11} \quad (2.1.6.10)$$

$$f = 3.162 \times 10^5 \quad (2.1.6.11)$$

So, the approximate value of f at which Phase Margin is 90° is $f = 3.162 \times 10^5 \text{ Hz}$.

Similarly let Phase Margin be $\alpha = 45^\circ$. Then,

$$\alpha = \phi - (-180^\circ) \quad (2.1.6.12)$$

$$\phi = -180^\circ + \alpha \quad (2.1.6.13)$$

$$\phi = -135^\circ \quad (2.1.6.14)$$

So, by the definition of Phase-Margin, at $\phi = -135^\circ$, $|GH| = 1$. The value of $\phi = -135^\circ$ approximately at poles $f = 10^6 \text{ Hz}$.

So, the approximate value of f at which Phase Margin is 45° is $f = 10^6$.

2.1.7. Find the minimum values of Closed-Loop Voltage Gain for which phase margins are 90° and 45° respectively

Solution:

For $\alpha = 90^\circ$,

$$f = 3.162 \times 10^5 \quad (2.1.7.1)$$

By substituting f in Open-Loop Gain $G(f)$

(assuming poles are far part),

$$G(f) = 200 - 20\log(3.162 \times 10^5) \quad (2.1.7.2)$$

$$G(f) = 90 \text{ dB} \quad (2.1.7.3)$$

$$G = 3.1625 \times 10^4 \quad (2.1.7.4)$$

At that $f = 3.162 \times 10^5$,

$$H = \frac{1}{G} \quad (2.1.7.5)$$

$$H = 3.162 \times 10^{-5} \quad (2.1.7.6)$$

The minimum value of Closed-Loop Gain occurs at $|GH| \gg 1$ and the value of Closed-Loop Gain is $T = \frac{1}{H}$

$$T = \frac{1}{H} = 3.1625 \times 10^4 \quad (2.1.7.7)$$

So, The minimum value of Closed-Loop Gain with Phase Margin equal to $\alpha = 90^\circ$ is $T_{min} = 3.1625 \times 10^4$.

For $\alpha = 45^\circ$,

$$f = 10^6 \quad (2.1.7.8)$$

By substituting f in Open-Loop Gain $G(f)$ (assuming poles are far part),

$$G(f) = 200 - 20\log(10^6) \quad (2.1.7.9)$$

$$G(f) = 80 \text{ dB} \quad (2.1.7.10)$$

$$G = 10^4 \quad (2.1.7.11)$$

At that $f = 10^6$,

$$H = \frac{1}{G} \quad (2.1.7.12)$$

$$H = 10^{-4} \quad (2.1.7.13)$$

The minimum value of Closed-Loop Gain occurs at $|GH| \gg 1$ and the value of Closed-Loop Gain is $T = \frac{1}{H}$

$$T = \frac{1}{H} = 10^4 \quad (2.1.7.14)$$

So, The minimum value of Closed-Loop Gain with Phase Margin equal to $\alpha = 45^\circ$ is $T_{min} = 10^4$.

2.1.8. Break the Transfer Function $G(f)$ into Simple Blocks and Create a Block Diagram for $G(f)$.

Solution:

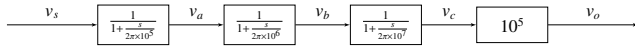


Fig. 2.1.8

2.1.9. Find the Gain of RC-Circuit shown below 2.1.9 and also identify the location of Poles.

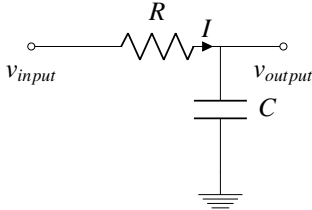


Fig. 2.1.9

Solution:

$$I = \frac{v_{input}}{R + \frac{1}{Cs}} \quad (2.1.9.1)$$

$$v_{output} = I \times \frac{1}{Cs} \quad (2.1.9.2)$$

$$v_{output} = \frac{v_{input} \times \frac{1}{Cs}}{R + \frac{1}{Cs}} \quad (2.1.9.3)$$

$$\frac{v_{output}}{v_{input}} = \frac{1}{RCs + 1} \quad (2.1.9.4)$$

$$s = j2\pi f \quad (2.1.9.5)$$

$$Gain = \frac{v_{output}}{v_{input}} = \frac{1}{j2\pi RCf + 1} \quad (2.1.9.6)$$

So, there is a Pole at frequency $f = \frac{1}{2\pi RC}$ for the Transfer Function of Gain.

2.1.10. Find the Gain of Small-Signal of N-MOSFET. The circuit diagram of Small-Signal Model is 2.1.10.

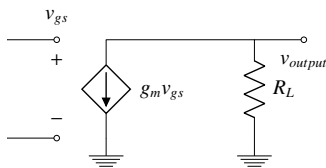


Fig. 2.1.10

Solution:

$$v_{output} = -g_m v_{gs} R_L \quad (2.1.10.1)$$

So, Gain of the above circuit is $-g_m v_{gs} R_L$.

2.1.11. Design a Circuit Model that follows the Transfer Function $G(f)$

Solution:

Our Design for Modelling the Transfer Function is based on Poles of RC-Circuit and Gain of N-MOSFET. The cascade of RC Circuits are used to introduce poles in the circuit and N-MOSFETs are used to achieve the Gain required.

So, the Circuit Diagram is,

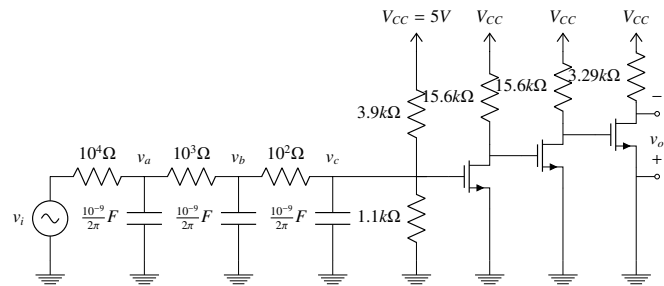


Fig. 2.1.11

Assuming, N-MOSFETs are identical and properties of N-MOSFET are,

$$\mu_n C_{ox} = 100 \mu A/V^2 \quad (2.1.11.1)$$

$$\frac{W}{L} = 500 \quad (2.1.11.2)$$

$$V_T = 1V \quad (2.1.11.3)$$

V_{CC} is divided across $4.9 \times 10^3 \Omega$ and $1.1 \times 10^3 \Omega$

$$V_{GS} = 1.1V \quad (2.1.11.4)$$

Small Signal Parameter g_m ,

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \quad (2.1.11.5)$$

$$g_m = 5 \times 10^{-3} \Omega^{-1} \quad (2.1.11.6)$$

$$I_D = \frac{g_m}{2} (V_{GS} - V_{TH}) \quad (2.1.11.7)$$

$$I_D = 2.5 \times 10^{-4} A \quad (2.1.11.8)$$

Small Signal Model of the circuit is

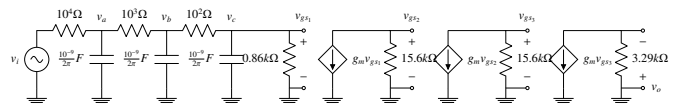


Fig. 2.1.11

At the first RC-Circuit,

$$2\pi RC = 10^{-5} \quad (2.1.11.9)$$

$$v_a = \frac{v_i}{1 + j\frac{f}{10^5}} \quad (2.1.11.10)$$

At the second RC-Circuit,

$$2\pi RC = 10^{-6} \quad (2.1.11.11)$$

$$v_b = \frac{v_a}{1 + j\frac{f}{10^6}} \quad (2.1.11.12)$$

$$v_b = \frac{v_i}{(1 + j\frac{f}{10^5})(1 + j\frac{f}{10^6})} \quad (2.1.11.13)$$

At the third RC-Circuit,

$$2\pi RC = 10^{-7} \quad (2.1.11.14)$$

$$v_c = \frac{v_b}{1 + j\frac{f}{10^7}} \quad (2.1.11.15)$$

$$v_{gs1} = v_c \quad (2.1.11.16)$$

$$v_{gs1} = \frac{v_i}{(1 + j\frac{f}{10^5})(1 + j\frac{f}{10^6})(1 + j\frac{f}{10^7})} \quad (2.1.11.17)$$

The RC Circuits introduces poles at $f = 10^5 \text{ Hz}$, 10^6 Hz , 10^7 Hz respectively from left to right. So, the value of v_{gs1} is

$$v_{gs1} = \frac{v_i}{(1 + j\frac{f}{10^5})(1 + j\frac{f}{10^6})(1 + j\frac{f}{10^7})} \quad (2.1.11.18)$$

The MOSFETs used in the circuit are Self-Biased with $V_{GS} = 1.1 \text{ V}$ and $I_D = 2.5 \times 10^{-4} \text{ A}$ for all 3 N-MOSFETs. 3 N-MOSFETs are used to achieve a large gain of 10^5 . At 3rd N-MOSFET, Terminals of output are swaped, inorder to compensate for the negative sign of gain.

At the first N-MOSFET,

$$v_{gs2} = -g_m v_{gs1} \times 15.6 \times 10^3 \quad (2.1.11.19)$$

$$v_{gs2} = -78 \times v_{gs1} \quad (2.1.11.20)$$

$$v_{gs2} = \frac{-78 \times v_i}{(1 + j\frac{f}{10^5})(1 + j\frac{f}{10^6})(1 + j\frac{f}{10^7})} \quad (2.1.11.21)$$

Similarly, at the second N-MOSFET,

$$v_{gs3} = -g_m v_{gs2} \times 15.6 \times 10^3 \quad (2.1.11.22)$$

$$v_{gs3} = -78 \times v_{gs2} \quad (2.1.11.23)$$

$$v_{gs3} = \frac{6084 \times v_i}{(1 + j\frac{f}{10^5})(1 + j\frac{f}{10^6})(1 + j\frac{f}{10^7})} \quad (2.1.11.24)$$

Similarly, at the third N-MOSFET,

$$-v_o = -g_m v_{gs3} \times 3.29 \times 10^3 \quad (2.1.11.25)$$

$$v_o = 16.45 \times v_{gs3} \quad (2.1.11.26)$$

$$v_o = \frac{10^5 \times v_i}{(1 + j\frac{f}{10^5})(1 + j\frac{f}{10^6})(1 + j\frac{f}{10^7})} \quad (2.1.11.27)$$

$$\frac{v_o}{v_i} = \frac{10^5}{(1 + j\frac{f}{10^5})(1 + j\frac{f}{10^6})(1 + j\frac{f}{10^7})} \quad (2.1.11.28)$$

$$G(f) = \frac{10^5}{(1 + j\frac{f}{10^5})(1 + j\frac{f}{10^6})(1 + j\frac{f}{10^7})} \quad (2.1.11.29)$$

2.1.12. Design a Circuit Model that follows the Transfer Function $H(f)$

Solution:

On Bode Plot is H is independent of frequency. So, H should not involve any Reactive Elements. So, H is a combination of Resistors or a Voltage Divider.

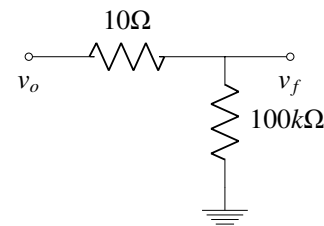


Fig. 2.1.12

$$v_f = \frac{10}{10 + 10^5} \times v_o \quad (2.1.12.1)$$

$$v_f \approx 10^{-4} v_o \quad (2.1.12.2)$$

$$\frac{v_f}{v_o} \approx 10^{-4} \quad (2.1.12.3)$$

$$H(f) = 10^{-4} \quad (2.1.12.4)$$

- 3 BODE PLOT
- 4 SECOND ORDER SYSTEM
- 5 ROUTH HURWITZ CRITERION
- 6 STATE-SPACE MODEL
- 7 NYQUIST PLOT
- 8 COMPENSATORS
- 9 GAIN MARGIN
- 10 PHASE MARGIN
- 11 OSCILLATOR
- 12 ROOT LOCUS
- 13 POLAR PLOT
- 14 PID CONTROLLER