Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

1.3 Example

2 Gain of Feedback Circuits

2.1 Current Amplifiers

2.1.1. For the feedback current amplifier shown in 2.1.1, Draw the Small-Signal Model

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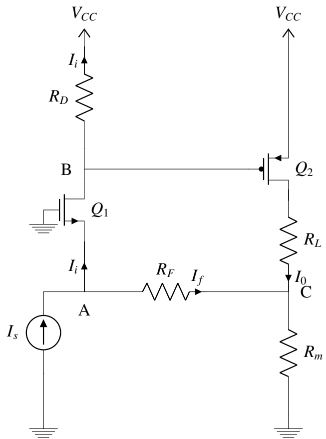

Fig. 2.1.1

Solution: While drawing a Small-Signal Model, we ground all constant voltage sources and open all constant current sources. All Small-Signal paramters are obtained from DC-Analysis of the circuit.

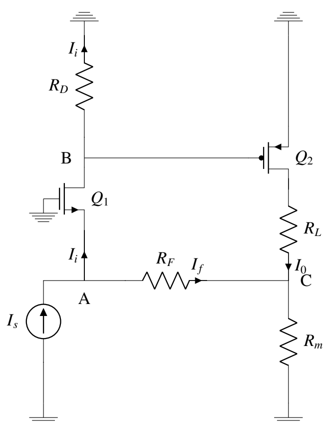

Fig. 2.1.1

2.1.2. Describe how the given circuit is a Negetive Feedback Amplifier.
Solution: For the feedback to be negative, I_f must have the same polarity as I_s. To ascertain that this is the case, we assume an increase in I_s and follow the change around the loop: An increase in I_s causes I_i to increase and the drain voltage of Q_1 will increase. Since this voltage is applied to the gate of the p-channel device Q_2 , its increase will cause I_o , the drain current of Q_2, to decrease. Thus, the voltage across R_M will decrease, which will cause I_f to increase. This is the same polarity assumed for the initial change in I_s, verifying that the feedback is indeed negative.

2.1.3. Find the Expression for the Open-Loop Gain $G = \frac{I_o}{I_i}$, from the Small-Signal Model. For simplicity, neglect the Early effect in Q_1 and Q_2.
Solution: In Small-Signal Model,

$$v_B = I_i R_D \tag{2.1.3.1}$$

$$v_{gs_2} = v_B = I_i R_D \tag{2.1.3.2}$$

In Small-Signal Analysis, P-MOSFET is modelled as a current source where current flows from Source to Drain. So, the value of current flowing from Source to Drain in P-MOSFET is,

$$I_o = -g_{m_2} v_{gs_2} = -g_{m_2} I_i R_D \tag{2.1.3.3}$$

So, the Open-Circuit Gain is

$$G = \frac{I_o}{I_i} = -g_{m_2} R_D \tag{2.1.3.4}$$

2.1.4. Find the Expression of the Feedback Factor $H = \frac{I_f}{I_o}$, from Small-Signal Model. For simplicity, neglect the Early effect in Q_1 and Q_2.
Solution:
I_o is fed to a current divider formed by R_M and R_F. R_F is a Large Resistance compared to Input resistance of Amplifier and so most of the current flows through it leaving a small current as input to Amplifier. Hence the voltage at point 'A' is very small and is considered, $v_A \simeq 0$. So R_F and R_M are parallel and Voltage Drop across them is same.

$$(I_o + I_f) R_M \simeq -I_f R_F \tag{2.1.4.1}$$

$$\frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \tag{2.1.4.2}$$

So, the Feedback Factor,

$$H \equiv \frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \tag{2.1.4.3}$$

2.1.5. Find the Expression for the Closed-Loop Gain $T = \frac{I_o}{I_s}$. For simplicity, neglect the Early effect in Q_1 and Q_2.
Solution:

From Open-Loop Gain and Feedback Factor,

$$I_s = I_i + I_f \qquad (2.1.5.1)$$

$$I_s = \frac{I_o}{G} + HI_o \qquad (2.1.5.2)$$

$$GI_s = I_o(1 + GH) \qquad (2.1.5.3)$$

$$\frac{I_o}{I_s} = \frac{G}{1 + GH} \qquad (2.1.5.4)$$

$$\frac{I_o}{I_s} = -\frac{g_{m_2} R_D}{1 + g_{m_2} R_D / \left(1 + \frac{R_F}{R_M}\right)} \qquad (2.1.5.5)$$

So the Block Diagram of Feedback Current Amplifier is

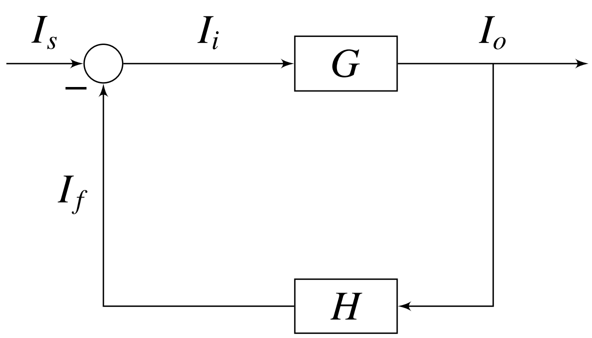

Fig. 2.1.5

where $G = -g_{m_2} R_D$ and $H = -\frac{R_M}{R_F + R_M}$

So, the value of Closed-Loop Gain is

$$T = \frac{I_o}{I_s} = -\frac{g_{m_2} R_D}{1 + g_{m_2} R_D / \left(1 + \frac{R_F}{R_M}\right)} \qquad (2.1.5.6)$$

3 Bode Plot

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3.2 Example

3.3 Phase

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