Control Systems

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Contents

Abstract—**This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

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1 Signal Flow Graph

2 Gain of Feedback Circuits

2.1 Current Amplifiers

2.1.1. For the feedback current amplifier shown in 2.1.1, Draw the Small-Signal Model.eglect the Early effect in Q_1 and Q_2.
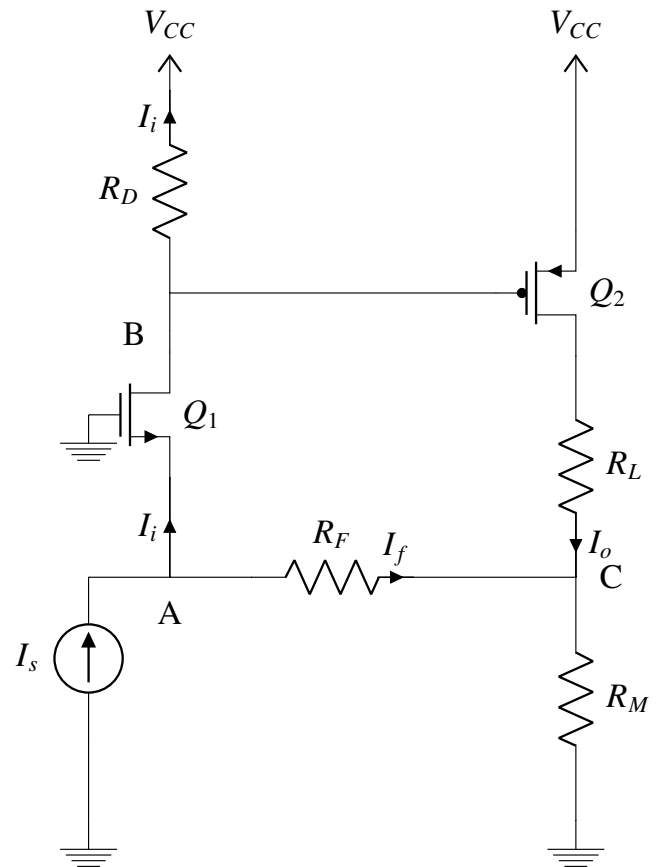

Fig. 2.1.1

Solution: While drawing a Small-Signal Model, we ground all constant voltage sources and open all constant current sources. All Small-Signal paramters are obtained from DC-Analysis of the circuit. Neglecting Early effect, in Small-Signal Analysis a N-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{gs}$ flowing from Drain to

Source. Whereas a P-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{sg}$ flowing from Source to Drain.
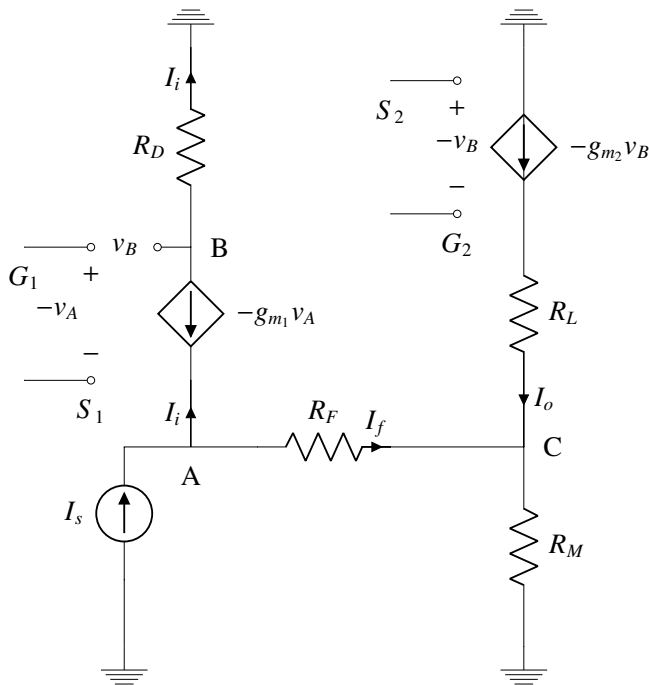

Fig. 2.1.1: Small Signal Model

2.1.2. Describe how the given circuit is a Negetive Feedback Current Amplifier.
Solution: For the feedback to be negative, I_f must have the same polarity as I_s. To ascertain that this is the case, we assume an increase in I_s and follow the change around the loop: An increase in I_s causes I_i to increase and the drain voltage of Q_1 will increase. Since this voltage is applied to the gate of the p-channel device Q_2, its increase will cause I_o, the drain current of Q_2, to decrease. Thus, the voltage across R_M will decrease, which will cause I_f to increase. This is the same polarity assumed for the initial change in I_s, verifying that the feedback is indeed negative.

2.1.3. Write all KVL and KCL Equations
Solution:

$$I_i = g_{m_1} v_A \qquad (2.1.3.1)$$
$$v_B = I_i R_D \qquad (2.1.3.2)$$
$$I_o = -g_{m_2} v_B \qquad (2.1.3.3)$$
$$v_A = I_F R_F - (I_F + I_o) R_M \qquad (2.1.3.4)$$

2.1.4. Find the Expression for the Open-Loop Gain

$G = \frac{I_o}{I_i}$, from the Small-Signal Model. For simplicity, neglect the Early effect in Q_1 and Q_2.
Solution: In Small-Signal Model,

$$v_B = I_i R_D \qquad (2.1.4.1)$$
$$v_{gs_2} = v_B = I_i R_D \qquad (2.1.4.2)$$

In Small-Signal Analysis, P-MOSFET is modelled as a current source where current flows from Source to Drain. So, the value of current flowing from Source to Drain in P-MOSFET is,

$$I_o = -g_{m_2} v_{gs_2} = -g_{m_2} I_i R_D \qquad (2.1.4.3)$$

So, the Open-Circuit Gain is

$$G = \frac{I_o}{I_i} = -g_{m_2} R_D \qquad (2.1.4.4)$$

2.1.5. Find the Expression of the Feedback Factor $H = \frac{I_f}{I_o}$, from Small-Signal Model. For simplicity, neglect the Early effect in Q_1 and Q_2.
Solution:
I_o is fed to a current divider formed by R_M and R_F. Assuming system has Good Gain, Most part of I_s flows as I_F leaving behind small I_i. As I_i is small, the voltage at point 'A' is very small and is considered, $v_A \simeq 0$. So R_F and R_M are parallel and Voltage Drop across them is same.
From (2.1.3.4),

$$(I_o + I_f) R_M \simeq -I_f R_F \qquad (2.1.5.1)$$
$$\frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \qquad (2.1.5.2)$$

So, the Feedback Factor,

$$H \equiv \frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \qquad (2.1.5.3)$$

2.1.6. Find the Expression for the Closed-Loop Gain $T = \frac{I_o}{I_s}$. For simplicity, neglect the Early effect in Q_1 and Q_2.
Solution:

From Open-Loop Gain and Feedback Factor,

$$I_s = I_i + I_f \quad\quad (2.1.6.1)$$

$$I_s = \frac{I_o}{G} + HI_o \quad\quad (2.1.6.2)$$

$$GI_s = I_o(1 + GH) \quad\quad (2.1.6.3)$$

$$\frac{I_o}{I_s} = \frac{G}{1 + GH} \qu\quad (2.1.6.4)$$

$$\frac{I_o}{I_s} = -\frac{g_{m_2}R_D}{1 + g_{m_2}R_D/\left(1 + \frac{R_F}{R_M}\right)} \quad\quad (2.1.6.5)$$

So the Block Diagram of Feedback Current Amplifier is
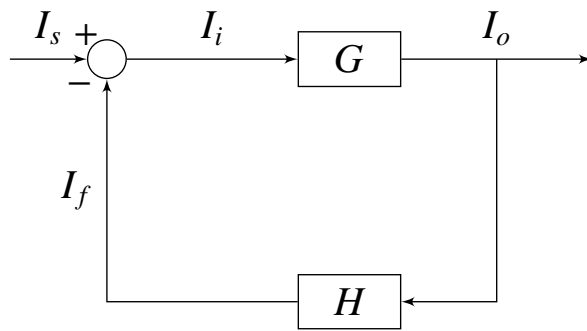

Fig. 2.1.6

where $G = -g_{m_2}R_D$ and $H = -\frac{R_M}{R_F+R_M}$

So, the value of Closed-Loop Gain is

$$T = \frac{I_o}{I_s} = -\frac{g_{m_2}R_D}{1 + g_{m_2}R_D/\left(1 + \frac{R_F}{R_M}\right)} \quad\quad (2.1.6.6)$$

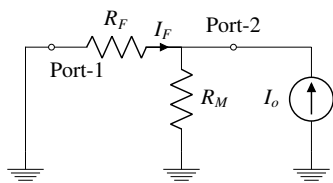2.1.7. Draw the Circuit Diagram of Feedback Network.
Solution:

Fig. 2.1.7

So, Gain of Feedback Network is

$$(I_o + I_f)R_M = -I_f R_F \quad\quad (2.1.7.1)$$

$$\frac{I_f}{I_o} = -\frac{R_M}{R_F + R_M} \quad\quad (2.1.7.2)$$

$$H = -\frac{R_M}{R_F + R_M} \quad\quad (2.1.7.3)$$

2.1.8. Find R_{11} and R_{22} of Feedback Network where R_{11} is input resistance through Port-1 and R_{22} is Input Resistance through Port-2.
Solution:
While calculating R_{11}, Port-2 should be Opened. So, Net-Resistance seen through Port-1 is

$$R_{11} = R_F + R_M \quad\quad (2.1.8.1)$$

While calculating R_{22}, Port-1 should be Shorted. So, Net-Resistance seen through Port-2 is

$$R_{22} = R_F \| R_M \quad\quad (2.1.8.2)$$

$$R_{22} = \frac{R_F R_M}{R_F + R_M} \quad\quad (2.1.8.3)$$

2.1.9. Draw the Circuit Diagram of Open-Loop Network.
Solution:
As the ciruit is Shunt-Series Topology, R_{11} is shunted across the Input Terminal and R_{22} is added in series to Output Terminal.
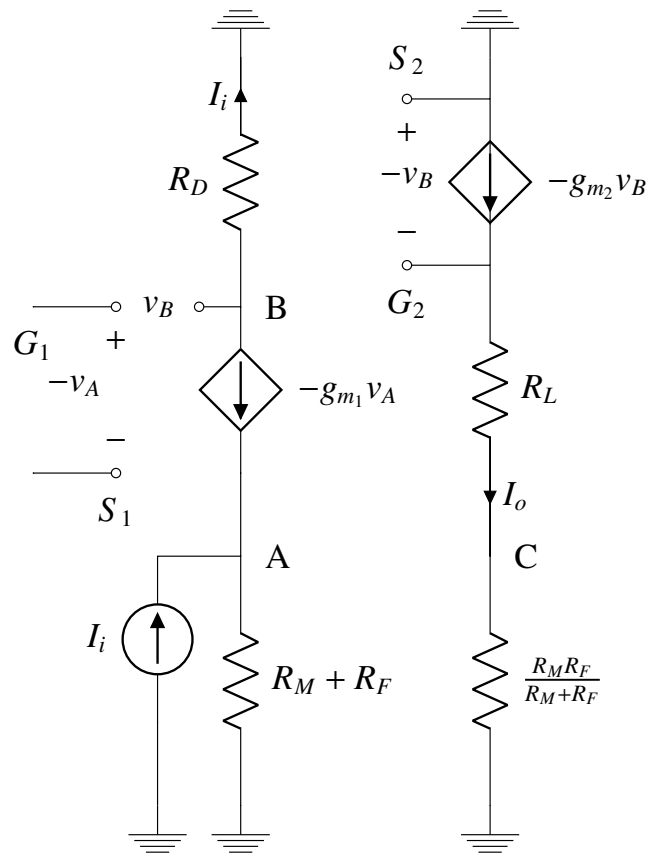

Fig. 2.1.9

Current Flowing through R_D is approximately

same as I_i, as R_F is a Large Resistance. So,

$$v_B = I_i R_D \qquad (2.1.9.1)$$

$$v_{gs_2} = v_B = I_i R_D \qquad (2.1.9.2)$$

$$I_o = -g_{m_2} v_{gs_2} = -g_{m_2} I_i R_D \qquad (2.1.9.3)$$

$$G = \frac{I_o}{I_i} = -g_{m_2} R_D \qquad (2.1.9.4)$$