Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

1 Mason's Gain Formula

2 Bode Plot

2.1 Introduction

2.1. For an LTI system, the Bode plot for its gain defined as

$$G(s) = 20\log|H(s)| \tag{2.1.1}$$

is as illustrated in the Fig. 2.1. Express $G(f)$ in terms of f.

Solution: Let us consider a generalized transfer gain

$$H(s) = k\frac{(s-z_1)(s-z_2)...(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)....(s-p_{n-1})(s-p_n)} \tag{2.1.2}$$

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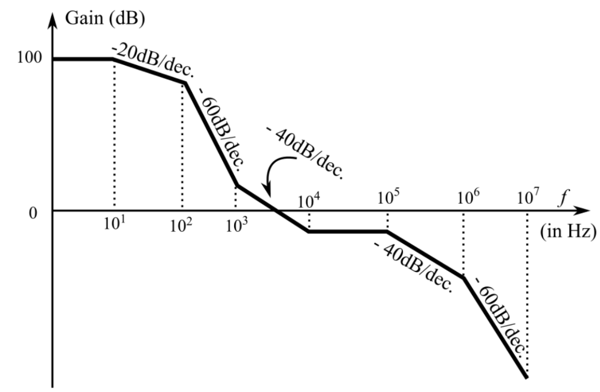

Fig. 2.1

$$Gain = 20\log|H(s)| = 20\log|k|+20\log|s-z_1| \\ +20\log|s-z_2|+\cdots+20\log|s-z_m|-20\log|s-p_1| \\ -20\log|s-p_2|-\cdots-20\log|s-z_n| \tag{2.1.3}$$

Let us consider the term $20\log|s-z_1|$ and let $s = j\omega$

$$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| \tag{2.1.4}$$

Based on log scale plot approximations,to the left of z_1 $\omega << z_1$ and towards right $\omega >> z_1$
For $\omega < z_1$

$$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| = 20\log|z_1| \tag{2.1.5}$$

$$= constant \tag{2.1.6}$$

i.e. Slope = 0

For $\omega > z_1$

$$20\log|s - z_1| = 20\log\left|\sqrt{\omega^2 + z_1^2}\right| = 20\log|\omega| \tag{2.1.7}$$

i.e Slope = 20

When a zero is encountered the slope always increases by 20 dB/decade
By performing similar analysis for $-20\log|s - p_1|$, we conclude that
When a pole is encountered the slope always decreases by 20 dB/decade
So, Poles are encountered at $f = 10, 10^2, 10^5, 10^6$
Similarly Zeros are encountered at $f = 0, 10^3, 10^4$
Final Transfer function is

$$H(f) = \frac{K(f + 10^3)(f + 10^4)^2}{(f + 10^1)(f + 10^2)^2(f + 10^5)^2(f + 10^6)} \tag{2.1.8}$$

$$G(f) = 20\log\frac{K(f + 10^3)(f + 10^4)^2}{(f + 10^1)(f + 10^2)^2(f + 10^5)^2(f + 10^6)} \tag{2.1.9}$$

2.2. Express the slope of $G(f)$ in terms of f.
Solution:

$$Slope = \nabla G(f) = \frac{d(G(f))}{df} \tag{2.2.1}$$

$$\nabla G(f) = \begin{cases} 0 & 0 < f < 10^1 \\ -20 & 10 < f < 10^2 \\ -60 & 10^2 < f < 10^3 \\ -40 & 10^3 < f < 10^4 \\ 0 & 10^4 < f < 10^5 \\ -40 & 10^5 < f < 10^6 \\ -60 & 10^6 < f < 10^7 \end{cases} \tag{2.2.2}$$

2.3. Express the change of slope of $G(f)$ in terms of f.
Solution:
$\Delta(\nabla G(f))$ = Change of slope G(f) at f

$$\Delta(\nabla G(f)) = \begin{cases} -20 & f = 10^1 \\ -40 & f = 10^2 \\ +20 & f = 10^3 \\ +40 & f = 10^4 \\ -40 & f = 10^5 \\ -20 & f = 10^6 \end{cases} \tag{2.3.1}$$

2.4. Find the number of poles and zeros of $H(s)$.
Solution:
When a zero is encountered the slope always increases by 20 dB/decade
When a pole is encountered the slope always decreases by 20 dB/decade

$$N_p = 6 \tag{2.4.1}$$

$$N_z = 3 \tag{2.4.2}$$

2.5. Find the location of the poles and zeros of $H(s)$.The number of system poles N_p and number of system zeros N_z in the frequency range 1 Hz ≤ f ≤ 10^7 Hz is.
Solution:
When a zero is encountered the slope always increases by 20 dB/decade
When a pole is encountered the slope always decreases by 20 dB/decade
So, Poles are encountered at
$f = 10, 10^2, 10^5, 10^6$
Similarly Zeros are encountered at
$f = 0, 10^3, 10^4$

2.6. Obtain the transfer function of $H(s)$.
Solution: $s = j\omega = 2\pi f$

$$H(s) = \frac{K(f + 2\pi 10^3)(s + 2\pi 10^4)^2}{(s + 2\pi 10^1)(s + 2\pi 10^2)^2(s + 2\pi 10^5)^2(s + 2\pi 10^6)} \tag{2.6.1}$$

2.7. Obtain the Bode plot and the slope plot for $H(s)$ and verify with Fig. 2.1
Solution: Bode Plot of obtained Transfer Function is

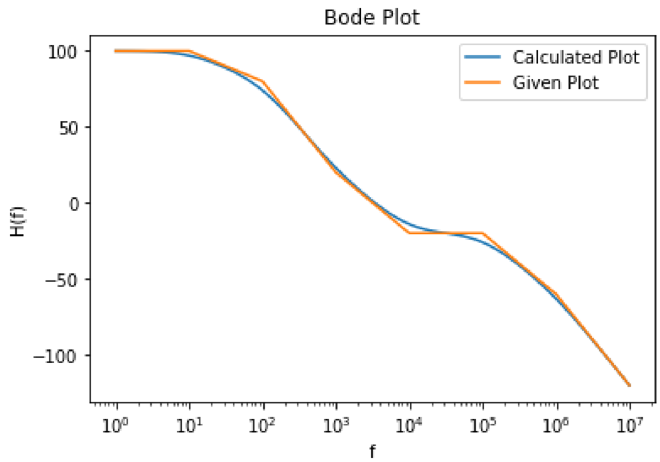

Fig. 2.7

3 Second order System

4 Routh Hurwitz Criterion

5 State-Space Model

6 Nyquist Plot

7 Compensators