Independent Project EE 2015

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1 Synthesis of Analog Filters

1a)

Cut-Off Frequency:

It is defined as the frequency at which Output Power is half of Input Power. $V_{Output} = \frac{V_{Input}}{\sqrt{2}}$

The filter provided for us is an Low-Pass filter with cut-off frequency equal to 42.08 KHz. Low-Pass filters High frequencies form input signal. It can be said that higher frequency terms get attenuated.

The filter provided has $\frac{V_{Output}}{V_{Input}} = 0.669$ at low frequency. So, at corresponding ratio for Cut-Off Frequency is 0.473.

The circuit diagram of given filter is

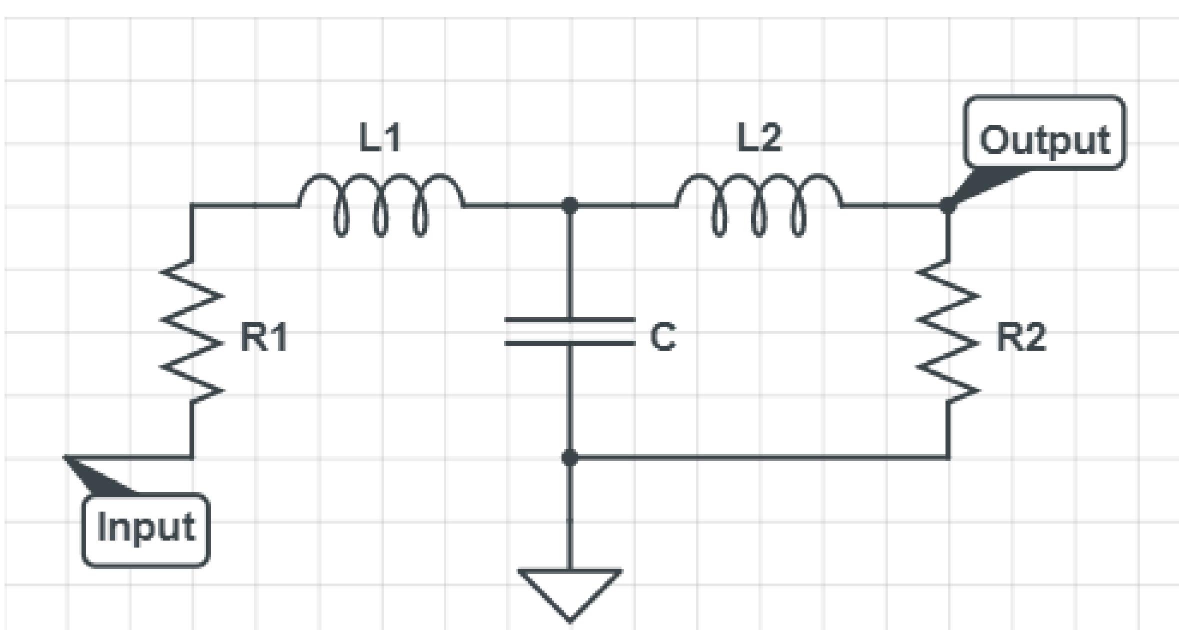

Figure 1: Circuit Diagram of given Analog Design

The Frequency response of the filter is drawn with the ratio "$\frac{V_{Output}}{V_{Input}}$" on y-axis and frequency "f" on x-axis.

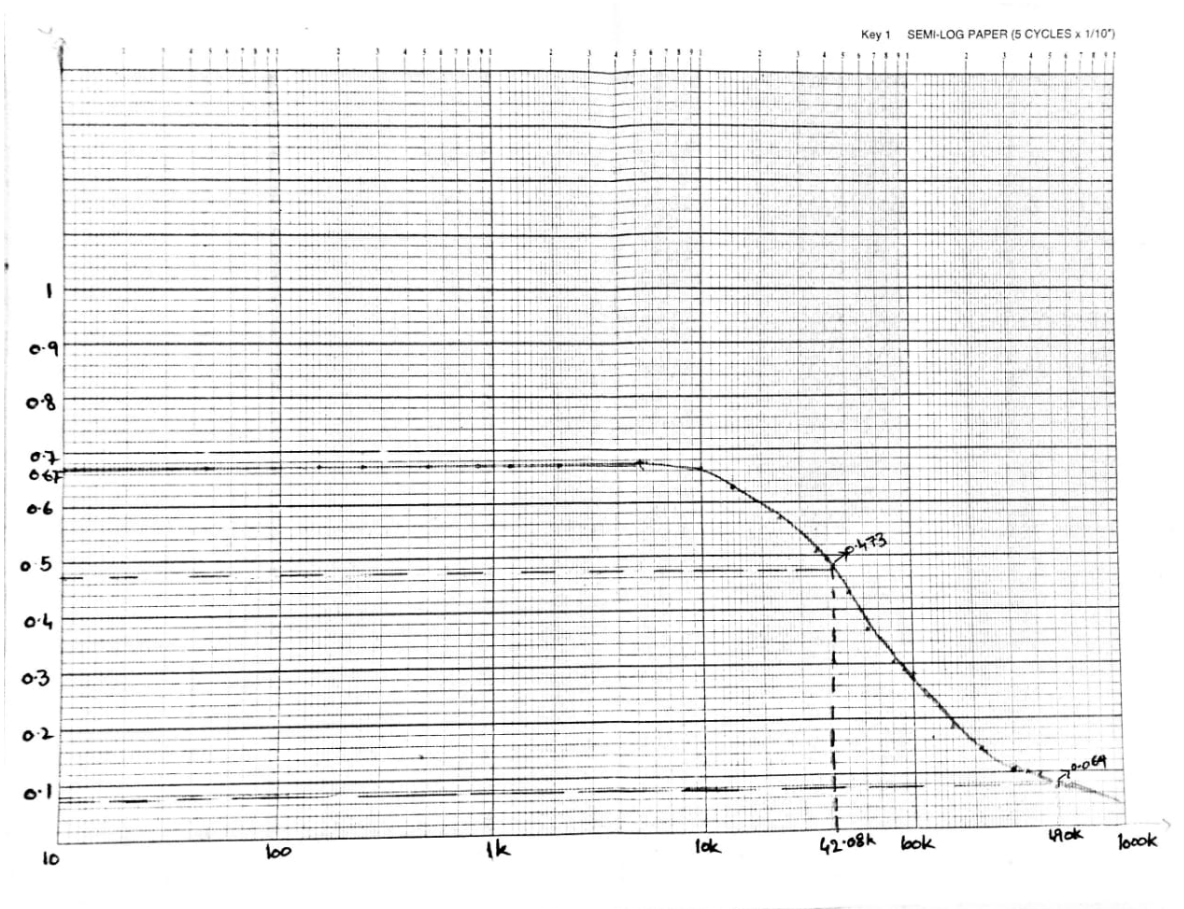

Figure 2: Transfer Function of given Filter

The values of Resistors, Capacitors, Inductors and Frequency are responsible for the obtained Transfer function and its Gain Ratio. The given Low-pass filter is a Second Order Low-Pass Filter. So The nature of the filter might be same but the gains need not be same as First-Order Low-Pass Filter.

1b)

Designing given Transfer Function using Butterworth and Chebyshev type I filters designs:

Fig.2 shows the transfer function We know that Cut-Off Frequency is 42.08KHz. The Gain factor of 0.669 is a scaling factor in-order to scale the voltage. The design remains the same with or without the scaling factor.

So, ignoring the Scaling Factor Butterworth and Chebyshev type I filters are designed. Later to the designs the Scaling Factor is multiplied back(In Frequency domain). So, corresponding change in time domain is adding to resistors it the circuit design.

The solution is as follows.

1b) Given $\delta_p = 3dB$ $\delta_s = 10dB$

Pass - Band frequency $w_p = 42.08\ KHz$

Stop- Band frequency $w_s = 490\ KHz$

For Butterworth Filter,

$$H(jw) = \frac{1}{\sqrt{1+(w/w_c)^{2N}}} \qquad N = \frac{\frac{1}{2}\left(\log\left(\frac{10^{0.1\delta_p}-1}{10^{0.1\delta_s}-1}\right)\right)}{\log\left(\frac{w_p}{w_s}\right)}$$

$\Rightarrow N = 0.45$

$\Rightarrow$ Order of the filter $= 1$ $\Rightarrow \boxed{N=1}$

$$H(jw) = \frac{1}{\sqrt{1+(w/w_c)^2}} \Rightarrow \boxed{w_c = \frac{1}{RC}}$$

We know that $\frac{w_c}{2\pi} = 42.08\ KHz = \frac{w_p}{2\pi}$

$\Rightarrow \boxed{w_c = w_p = 2.64396 \times 10^5\ rad/sec}$

$$\cancel{H(jw) = \frac{1}{\sqrt{1+(w/w_c)^2}}} \qquad \boxed{H(jw) = \frac{1}{\sqrt{1+\left(\frac{w}{w_c}\right)^2}}}$$

For Chebyshev Filter,

At $-3dB$ $|H(jw)| = \frac{1}{\sqrt{2}}$

$-10dB$ $|H(jw)| = 0.1$

$$C = \begin{cases} \cos(N\cos^{-1}(\frac{\omega}{\omega_p})) & |\frac{\omega}{\omega_p}| \leq 1 \\ \cosh^{-1}(N\cosh(\frac{\omega}{\omega_p})) & |\frac{\omega}{\omega_p}| > 1 \end{cases}$$

At $-3dB$, $\frac{1}{\sqrt{1+\epsilon^2}} = \frac{1}{\sqrt{2}}$

$$H(jw) = \frac{1}{\sqrt{1+\epsilon^2 C^2}}$$

$\Rightarrow \boxed{\epsilon = 1}$

At $-10dB$, $\frac{1}{\sqrt{1+C^2}} = \frac{1}{\sqrt{10}}$

$\Rightarrow \boxed{C = 3}$

$$\cosh(N\cosh^{-1}(\frac{\omega_s}{\omega_p})) = 3$$

$$N = \frac{\cosh^{-1}(3)}{\cosh^{-1}\left(\frac{490 \times 10^3}{42.08 \times 10^3}\right)} = 0.56$$

$\Rightarrow \boxed{N = 1}$

$\Rightarrow$ General value of $C = \frac{\omega}{\omega_p}$ $\forall \frac{\omega}{\omega_p}$

$$\Rightarrow H(jw) = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_p}\right)^2}}$$

$\omega_c = \omega_p = 2.64396 \times 10^{15}$ rad/sec

R and C values are adjusted to match the cutoff frequency.

$\omega_p = \omega_c = \frac{1}{RC} = 2.64396 \times 10^{15}$ rad/sec

1c)

Same RC filter is formed by both Cheybshev and Butterworth filter(Scaling Factor is ignored).

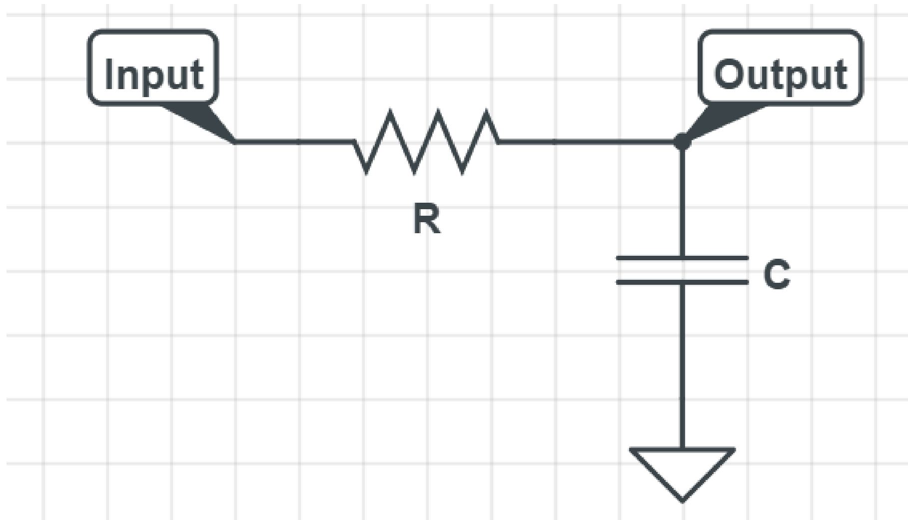

Figure 3: Filter without Scaling Factor

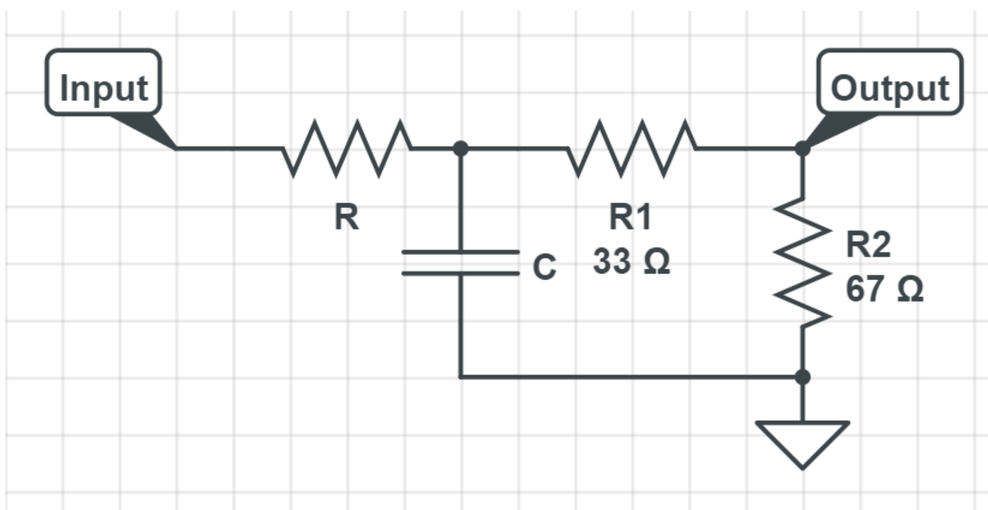

Figure 4: Filter with Scaling Factor

1d)

When a square wave is passed to the given filter the following cases are observed:

1. At Low frequencies/ in Pass-Band Output Square Wave shape is similar to that of Input Square Wave. But amplitude of Output is 0.67 times of that of Input.

2. At Moderate Frequencies amplitude of Output drastically decreases with compared to Input. And also slowly the shape of Output is deviating from Input. Slowing the shape of Output is becoming similar to Triangle wave.

3. At High frequencies the amplitude of Output is negligibly small compared to Input. The shape of Output is almost a Triangle wave.

Reason:

At high frequencies the time period of square wave decreases. So, instead of getting a Steady-State analysis we are getting a Transient-State analysis. So, this is the reason for the output to be similar to that of a Triangle Wave.

1e)

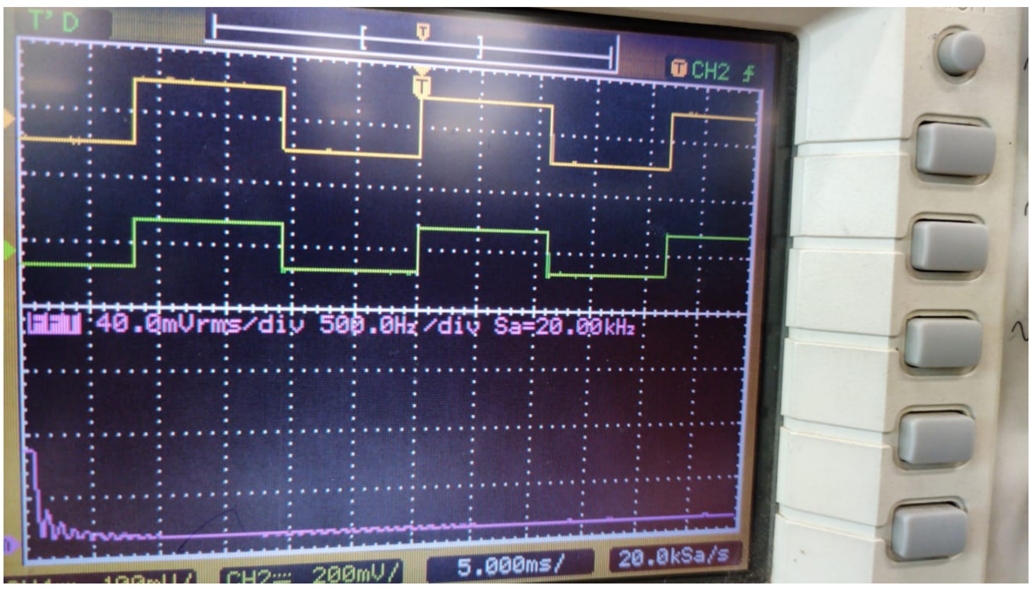

Figure 5: Input FFT at Low Frequency

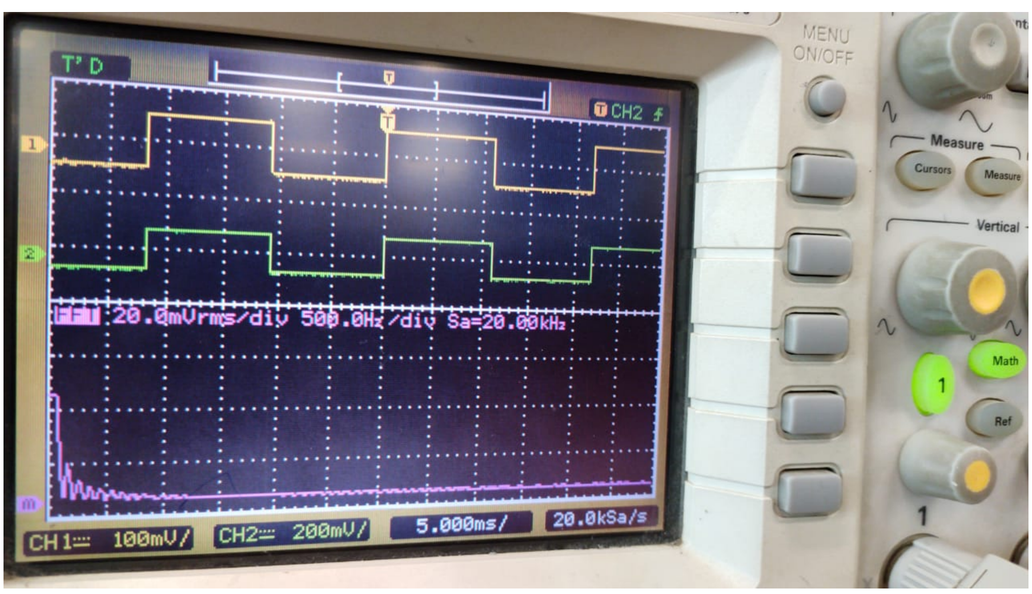

Figure 6: Output FFT at Low Frequency

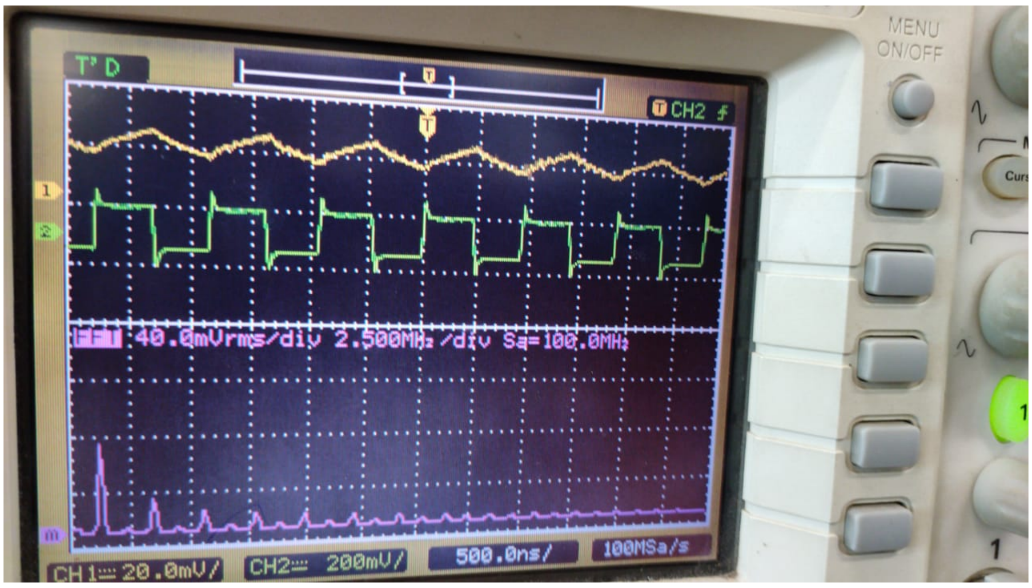

Figure 7: Input FFT at High Frequency

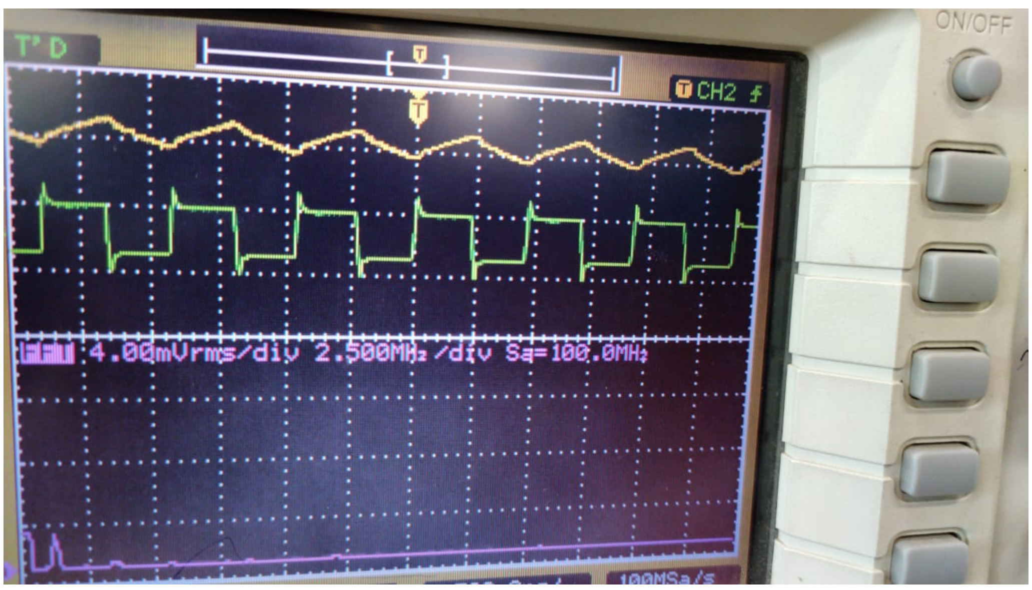

Figure 8: Output FFT at High Frequency

1f)

In order to calculate the phase difference using oscilloscope the Input and Output signal projected on the oscilloscope must be adjusted that they have same horizontal axis. Then depending on the difference in x co-ordinate of both the graphs and scale of oscilloscope phase difference can be calculated.

Phase Difference $\phi = \frac{\Delta t}{T} * 360$