Code of Assignment - 2

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EE2025 Independent Project Programming Assignment - 2

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The image(M.S.SubbaLakshmi), in all, contains 400 × 300 = 120000 information bits. We will encode,modulate,transmit,demodulate and decode them using 4-QAM modulation scheme with carrier frequency 2 MHz and symbol duration 1 micro sec, i.e., 2 bits are transmitted per micro second. The receiver will use the optimal demodulator, i.e., the maximum-likelihood detector or the minimum distance detector. We will use and different Channel Encoding Techniques to Encode the bits and Minimum Hamming Distance Decoder to decode it.

The Simulation Results are at the end of pdf/ipynb file.

[1]:
```
# Setting the width of IPython Notebook

from IPython.display import HTML
display(HTML("<style>.container { width:100% !important; }</style>"))
```

```
<IPython.core.display.HTML object>
```

0.1 *Importing Libraries*

[0]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy
from scipy import signal
from sklearn.metrics import mean_squared_error
from numba import jit,cuda
```

0.2 *Functions*

Functions Coded for the given Task

Generates Constellation to encode for 4-QAM

[0]:
```
def Encode_4QAM(Digital_Signal):

    output = np.zeros(Digital_Signal.shape)

    for i in range(Digital_Signal.shape[0]):
        if (Digital_Signal[i] == 0):
            output[i] = 1
        else:
            output[i] = -1

    return output
```

Decodes bits from 4-QAM Constellation

[0]:
```
def Decode_4QAM(Signal):

    output = np.zeros(Signal.shape)

    for i in range(Signal.shape[0]):
        if (Signal[i] == 1):
            output[i] = 0
        else:
            output[i] = 1

    return output.astype(int)
```

Generates a Vector of Analog Signal Transmitted for the Bits Transmitted

[0]:
```
def Analog_Signal_Generator(a,b,i,samples,T,fc,fs,Sampling=False):
    # Generates s(t) for the given input of 2 bits with and without Sampling.
    if Sampling != True:
        # Without Sampling
        t = np.linspace((i-1)*T, i*T, samples,endpoint=False)
        c = np.cos(2*np.pi*fc*t)
        s = np.sin(2*np.pi*fc*t)
        output = a*c + b*s
    else:
        # With Sampling
        t = np.linspace((i-1)*T, i*T, int(T*fs),endpoint=False)
        """
        # to = np.arange((i-1)*T, i*T, 1/fs)
        np.arange has a "Stop Precision Issue so np.linspace is used."
        """
        c = np.cos(2*np.pi*fc*t)
        s = np.sin(2*np.pi*fc*t)
        output = a*c + b*s

    return output
```

Generates White Gaussian Noise

[0]:
```
def WGN(Variance,Nt,samples,T,fs,Sampling=False):
    # Generates White Gaussian Noise with and without Sampling

    if Sampling != True:
        # Without Sampling
        output = np.zeros((Nt,samples))
        mu = 0
        sigma = np.sqrt(Variance)
        for i in range(Nt):
            output[i] = np.random.normal(mu, sigma, samples)
    else:
        # With Sampling
        output = np.zeros((Nt,int(T*fs)))
        mu = 0
        sigma = np.sqrt(Variance)
        for i in range(Nt):
            output[i] = np.random.normal(mu, sigma, int(T*fs))

    return output
```

Generates a Matrix of Analog Signals that need to be Transmitted

[0]:
```
def Analog_Matrix(Digital_Signal,samples,T,fc,fs,Sampling=False):
    # Outputs a matrix of all Transmitted Signals
    s = int(Digital_Signal.shape[0]/2)
    if Sampling != True:
        # Without Sampling
        output = np.zeros((s,samples))
    else:
        # With Sampling
        output = np.zeros((s,int(T*fs)))

    for i in range(s):
        a = Digital_Signal[2*i]
        b = Digital_Signal[2*i + 1]
        output[i] =␣
↪Analog_Signal_Generator(a,b,i+1,samples,T,fc,fs,Sampling=Sampling)

    Nt = s
    return output,Nt
```

To Calculate Energy of each Signal Transmitted

[0]:
```
def Energy_Signal_Matrix(signal_matrix):
    # Total Energy Matrix
    output = np.multiply(signal_matrix,signal_matrix)
```

```
    output = np.mean(output,axis=1)

    return output
```

For Fourier Transform of a Analog Signal Matrix

[0]:
```
def FFT(Signal_Matrix,fs):
    # Gives Fourier Transform of Sampled Analog Noisy Signal Matrix
    FFT_Matrix = np.fft.fft(Signal_Matrix) # /int(Signal_Matrix.shape[-1]/2)
    freq = np.fft.fftfreq(Signal_Matrix.shape[-1])*fs

    return FFT_Matrix,freq
```

For Inverse Fourier Transform of Analog Signal Matrix

[0]:
```
def IFFT(Signal_Matrix):
    # Gives Inverse Fourier Transform of Sampled Analog Noisy Signal Matrix
    IFFT_Matrix = np.fft.ifft(Signal_Matrix).real

    return IFFT_Matrix
```

Low Pass Filter for Matrix of Analog Signals

[0]:
```
def Low_Pass_Filtered_Matrix(Matrix,Cutoff_Freq,fs,T,Nt,Order=8):
    # Applies Low Pass Filter to each Signal in Matrix
    Output = np.zeros((Nt,int(fs*T)))

    for i in range(Matrix.shape[0]):

        if (Cutoff_Freq*2 == fs):
            w = 1 - 1e-9
        else:
            w = Cutoff_Freq*2/fs

        b, a = signal.butter(Order, w)
        x = np.array(list(Matrix[i]))
        output = signal.filtfilt(b,a, x)

        # Decimating or Downsampling Signal
        Output[i] = signal.decimate(output,1)


    return Output
```

Total no.of Waveforms for Transmission

[0]:
```
def Waveforms(M,fs,T,fc):
    # Different Waveforms that are Transmitted by Transmitter
```

```
    Waveforms = np.zeros((M,int(fs*T)))
    a = np.array([0,0,1,1])
    b = np.array([0,1,0,1])
    a_encoded = Encode_4QAM(a)
    b_encoded = Encode_4QAM(b)
    t = np.linspace(0, T, int(fs*T),endpoint=False)
    c = np.cos(2*np.pi*fc*t)
    s = np.sin(2*np.pi*fc*t)
    i = 0

    Directory = {}

    for x,y in zip(a_encoded,b_encoded):
        Waveforms[i] = x*c +y*s
        Directory[i] = np.array([x,y])
        i = i+1

    return Waveforms,Directory
```

Total no.of Distint Possibilites of Encoded bits sent

[0]:
```
def Bit_Transmission_Encoded(Signal,Channel_No):
    BitPoss = {}
    Bits = np.array([])
    Bits_4 = np.array([])
    t = 4

    if Channel_No == 1:
        M = np.matrix('1 1 1 1 0 0 0 0; 1 1 0 0 1 1 0 0; 1 0 1 0 1 0 1 0; 0 1 1␣
↪0 1 0 0 1')
    elif Channel_No == 3:
        M = np.matrix('1 0 0 0 0 1 1 1 1 0 1 0; 0 1 0 0 1 0 1 1 0 1 1 0; 0 0 1␣
↪0 1 1 1 0 1 1 1 1; 0 0 0 1 0 0 0 1 1 1 1 1')

    for i in range(0,Signal.shape[0],t):
        x = np.dot(Signal[i:i+t],M)%2
        x = np.squeeze(np.asarray(x))
        c = x.shape[0]
        BitPoss[np.array_str(x)] = 1

    r = len(list(BitPoss.keys()))

    for i in range(0,Signal.shape[0],t):
        x = np.dot(Signal[i:i+t],M)%2
        x = np.squeeze(np.asarray(x))
        y = np.array_str(x)
```

```
        if y in BitPoss.keys():
            del BitPoss[y]
            Bits = np.concatenate([Bits,x])
            Bits_4 = np.concatenate([Bits_4,Signal[i:i+t]])


    Bits = np.reshape(Bits,(r,c))
    Bits_4 = np.reshape(Bits_4,(r,t))

    return Bits_8,Bits_4
```

[0]:
```
def Bit_Possibilities_4_BitEncoder(Channel_No):

    if Channel_No == 1:
        M = np.matrix('1 1 1 1 0 0 0 0; 1 1 0 0 1 1 0 0; 1 0 1 0 1 0 1 0; 0 1 1␣
↪0 1 0 0 1')
    elif Channel_No == 3:
        M = np.matrix('1 0 0 0 0 1 1 1 1 0 1 0; 0 1 0 0 1 0 1 1 0 1 1 0; 0 0 1␣
↪0 1 1 1 0 1 1 1 1; 0 0 0 1 0 0 0 1 1 1 1 1')

    Bits_4 = np.matrix(np.c_[tuple(i.ravel() for i in np.mgrid[:2,:2,:2,:2])])
    Bits = Bits_4 * M

    return Bits,Bits_4
```

[15]:
```
(np.matrix(np.c_[tuple(i.ravel() for i in np.mgrid[:2,:2,:2,:2])]) * np.
↪matrix('1 1 1 1 0 0 0 0; 1 1 0 0 1 1 0 0; 1 0 1 0 1 0 1 0; 0 1 1 0 1 0 0␣
↪1')).shape
```

[15]: (16, 8)

Decodes the Analog Signal Matrix and returns Bits

[0]:
```
def Decode(Signal_Matrix,Waveforms,Directory,M):
    # Returns Array of Bits decoded at the Reciever
    Index = np.zeros((Signal_Matrix.shape[0],2))
    Error = np.zeros((Signal_Matrix.shape[0],Waveforms.shape[0]))

    for i in range(M):
        Error[:,i] = np.mean(np.multiply((Signal_Matrix -␣
↪Waveforms[i]),(Signal_Matrix - Waveforms[i])),axis=1)

    x = np.argmin(Error,axis=1)

    for i in range(x.shape[0]):
```

```
        Index[i] = np.array(Directory[x[i]])

    Output = Index.flatten()

    return Output.astype(int)
```

Plot BER vs $\frac{E_b}{N_0}$

[0]:
```
def Plot_BERGraph(Eb_per_No_dB,BER,label):
    plt.plot(Eb_per_No_dB,BER)
    plt.ylabel('BER')
    plt.xlabel(r'$\frac{E_{\mathrm{b}}}{N_{0}}$')
    plt.title(r'BER vs $\frac{E_{\mathrm{b}}}{N_{0}}$ Simulation Results for␣
 ↪'+label)
    plt.scatter(Eb_per_No_dB,BER)
    plt.grid()
    plt.show()
```

Plotting Image

[0]:
```
def Plot_Image(Decoded_Array,shape):
    #Plotting the Signal
    Img = Decoded_Array.reshape(shape)
    plt.imshow(Img,'gray')
    plt.show()
    print␣
 ↪("-------------------------------------------------------------------------------------------")
```

Bit Error Rate

[0]:
```
def Bit_Error_Rate(Decoded_Array,Digital_Signal):
    Error_Bits = np.sum(np.abs(Decoded_Array - Digital_Signal))
    print ("No.of Wrong Bits",Error_Bits)
    Fraction_of_Error = Error_Bits /(Decoded_Array.shape[0])
    print ("Fraction of Error",Fraction_of_Error)

    return Fraction_of_Error
```

0.3 *Encode, Transmit, Receieve and Decode*

Function to Encode, Transmit and Decode Signals

0.3.1 *Channel Encoding Techniques*

[0]:
```
def Channel_EncodeTech1(Signal):
    G1 = np.matrix('1 1 1 1 0 0 0 0; 1 1 0 0 1 1 0 0; 1 0 1 0 1 0 1 0; 0 1 1 0␣
↪1 0 0 1')

    S = np.transpose(np.reshape(Signal,(-1,4)))
    X = np.transpose(np.dot(G1.T,S))%2

    output = X.flatten()

    return np.squeeze(np.asarray(output))
```

[0]:
```
def Channel_EncodeTech2(Signal):

    output = np.transpose(np.reshape(np.
↪concatenate([Signal,Signal,Signal]),(3,-1))).flatten()

    return output
```

[0]:
```
def Channel_EncodeTech3(Signal):
    G3 = np.matrix('1 0 0 0 0 1 1 1 1 0 1 0; 0 1 0 0 1 0 1 1 0 1 1 0; 0 0 1 0 1␣
↪1 1 0 1 1 1 1; 0 0 0 1 0 0 0 1 1 1 1 1')

    S = np.transpose(np.reshape(Signal,(-1,4)))
    X = np.transpose(np.dot(G3.T,S))%2

    output = X.flatten()

    return np.squeeze(np.asarray(output))
```

0.3.2 *Modulation Scheme:*

Carrier Frequency = 2 MHz Symbol Duration T = 1 sec.

$s(t) = x_{2i-1}\cos(2\pi f_c t) + x_{2i}\sin(2\pi f_c t)$, for $(i-1)T \leq t < iT$

[0]:
```
def␣
↪Modulation(Digital_Signal,fc,T,M,fs,n,k,Cutoff_Freq,Ratio=0,Variance=0,samples=1000,Varianc
↪

    # Different Waveforms Transmited and Corresponding Directory
    Waveforms_Transmitted,Directory = Waveforms(M,fs,T,fc)

    '''
```

```
    Examples of Non-Sampled Signals ("The Context Non-Sampled implies that they␣
↪are not samples with fs = 50MHz")
    '''

    # Encoding Signal
    Digital_Signal_Encoded = Encode_4QAM(Digital_Signal)
    Analog_Signal_Matrix,Nt =␣
↪Analog_Matrix(Digital_Signal_Encoded,samples,T,fc,fs,Sampling=True)
    print (Analog_Signal_Matrix.shape)

    # Energy for Waveforms_Transmitted
    Average_Energy = T*n # By Integrating Analog Signal
    print ("Average Energy of Transmitted Signal",Average_Energy)
    Energy_per_Bit = Average_Energy/k
    print ("Energy per Bit of Transmitted Signal",Energy_per_Bit)

    if VarianceTruth != True:
        # No Calculations
        No = Energy_per_Bit/pow(10,(Ratio/10))
        R = pow(10,(Ratio/10))
        print ("Eb/No Ratio in dB is", Ratio)
        print ("Eb/No Ratio is", R)

        # Variance of White Gaussian Noise/Channel
        Variance = (No/2)*(2*Cutoff_Freq)
        print ("Variance of WGN",Variance)
    else:
        Variance = Variance
        print ("Variance of WGN",Variance)
        No = Variance/Cutoff_Freq
        R = Energy_per_Bit/No
        print ("Eb/No Ratio is", R)
        Ratio = 10*np.log10(R)
        print ("Eb/No Ratio in dB is", Ratio)

    # Transmitting Signal
    Analog_Sampled_Signal_Matrix = Analog_Signal_Matrix +␣
↪WGN(Variance,Nt,samples,T,fs,Sampling=True)

    # Receving Signal
    Filtered_Signal_Matrix =  Analog_Sampled_Signal_Matrix␣
↪#Low_Pass_Filtered_Matrix(Analog_Sampled_Signal_Matrix,Cutoff_Freq,fs,T,Nt)␣
↪# Commented the Line as the Signal Frequency Components are in Low Pass␣
↪Filter Range

    # Decoding Signal
```

```
    Decoded_Array =␣
↪Decode_4QAM(Decode(Filtered_Signal_Matrix,Waveforms_Transmitted,Directory,M))

    """
    # Probability of Error
    Error_Bits = np.sum(np.abs(Decoded_Array - Digital_Signal))
    print ("No.of Wrong Bits",Error_Bits)
    Fraction_of_Error = Error_Bits /(Decoded_Array.shape[0])
    print ("Fraction of Error is",Fraction_of_Error)
    Q = scipy.stats.norm(0, 1).cdf(-np.sqrt(2*R/1))
    print ("Pe(Proballity of Error) =",Q)
    """

    return Decoded_Array,Ratio
```

0.3.3 *Channel Decoding Techniques*

[0]:
```
def Channel_DecodeTech1(Signal,Ref_8,Ref_4):

    Output = np.array([])
    for i in range(0,Signal.shape[0],8):
        s = Signal[i:i+8]
        m = np.argmin(np.sum(np.add(Ref_8,s)%2,axis=1))
        Output = np.concatenate([Output,np.squeeze(np.asarray(Ref_4[m]))])

    return Output.astype(int)
```

[0]:
```
def Channel_DecodeTech2(Signal):

    output = np.floor(np.sum(np.reshape(Signal,(-1,3)),axis=1)/2)

    return output.astype(int)
```

[0]:
```
def Channel_DecodeTech3(Signal,Ref_12,Ref_4):

    Output = np.array([])
    for i in range(0,Signal.shape[0],12):
        s = Signal[i:i+12]
        m = np.argmin(np.sum(np.add(Ref_12,s)%2,axis=1))
        Output = np.concatenate([Output,np.squeeze(np.asarray(Ref_4[m]))])

    return Output.astype(int)
```

0.3.4 *Binary Image*

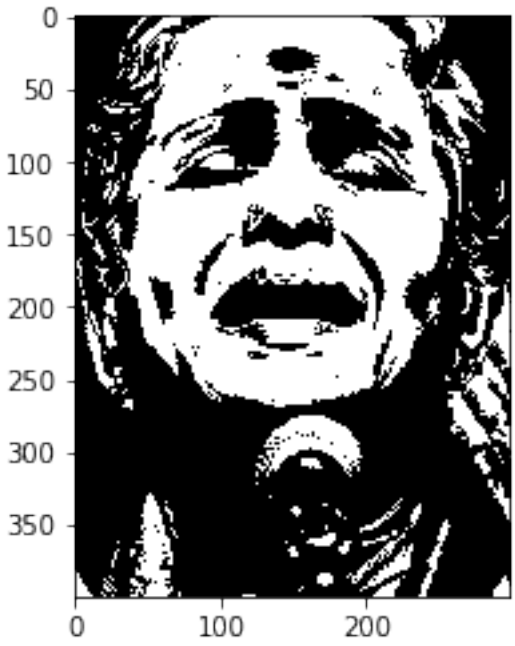
Importing Binary Image file

[27]:
```
MSS = np.load('mss.npy')
shape = MSS.shape
print (shape)
```

```
(400, 300)
```

Displaying Image

[28]:
```
plt.imshow(MSS,'gray')
plt.show()
```



[29]:
```
Digital_Signal = MSS.flatten()
print (Digital_Signal.shape)
```

```
(120000,)
```

0.4 *Results*

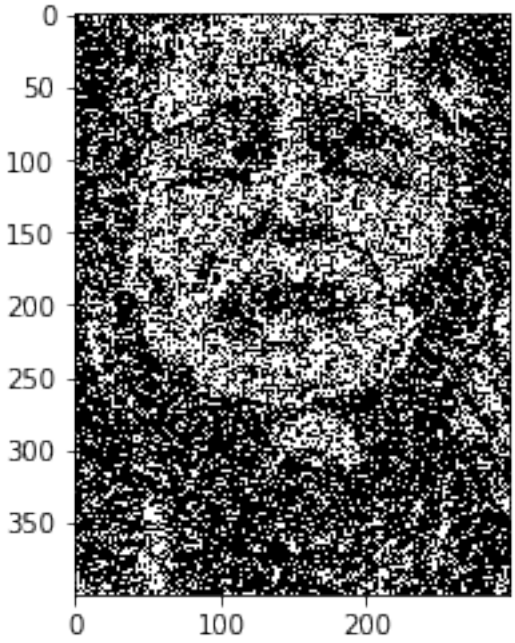
[0]:
```
fc = 2 * 1e6
T = 1e-6
M = 4
fs = 50 * 1e6
```

```
Cutoff_Freq = 25*pow(10,6)
Variance = np.array([20,12,7,5]).astype(int)
Ratio = np.array([-2,0,2,4,6]).astype(int)
```

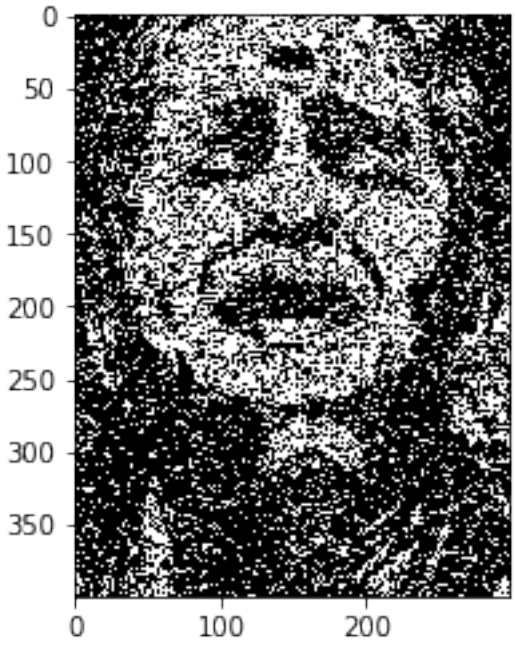
[31]:
```
Variance = np.array([20,12,7,5]).astype(int)
Ratio = np.array([-2,0,2,4,6]).astype(int)

BER = []
Eb_per_No_dB = []

for r in Ratio:
    Encoded_Signal = Channel_EncodeTech1(Digital_Signal)
    Received_Signal,Ratio =␣
 ↪Modulation(Encoded_Signal,fc,T,M,fs,8,4,Cutoff_Freq=Cutoff_Freq,Ratio=r)
    Bits_8,Bits_4 = Bit_Possibilities_4_BitEncoder(1)
    Decoded_Signal = Channel_DecodeTech1(Received_Signal,Bits_8,Bits_4)

    BER.append(Bit_Error_Rate(Decoded_Signal,Digital_Signal))
    Eb_per_No_dB.append(Ratio)

    Plot_Image(Decoded_Signal,shape)

Plot_BERGraph(Eb_per_No_dB,BER,r'Channel Encoding-1 given␣
 ↪$\frac{E_{\mathrm{b}}}{N_{0}}$')


BER = []
Eb_per_No_dB = []

for v in Variance:
    Encoded_Signal = Channel_EncodeTech1(Digital_Signal)
    Received_Signal,Ratio =␣
 ↪Modulation(Encoded_Signal,fc,T,M,fs,8,4,Cutoff_Freq=Cutoff_Freq,Variance=v,VarianceTruth=Tr
    Bits_8,Bits_4 = Bit_Possibilities_4_BitEncoder(1)
    Decoded_Signal = Channel_DecodeTech1(Received_Signal,Bits_8,Bits_4)

    BER.append(Bit_Error_Rate(Decoded_Signal,Digital_Signal))
    Eb_per_No_dB.append(Ratio)

    Plot_Image(Decoded_Signal,shape)

Plot_BERGraph(Eb_per_No_dB,BER,'Channel Encoding-1 given Variance')
```

```
(120000, 50)
Average Energy of Transmitted Signal 8e-06
Energy per Bit of Transmitted Signal 2e-06
Eb/No Ratio in dB is -2
```

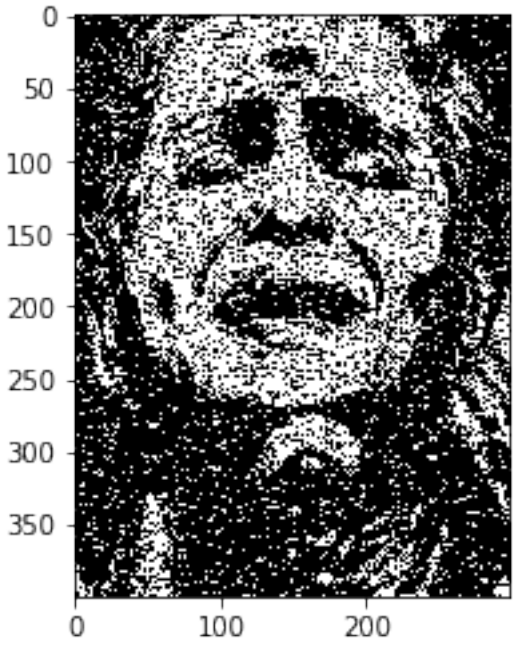
```
Eb/No Ratio is 0.6309573444801932
Variance of WGN 79.24465962305567
No.of Wrong Bits 33879
Fraction of Error 0.282325
```



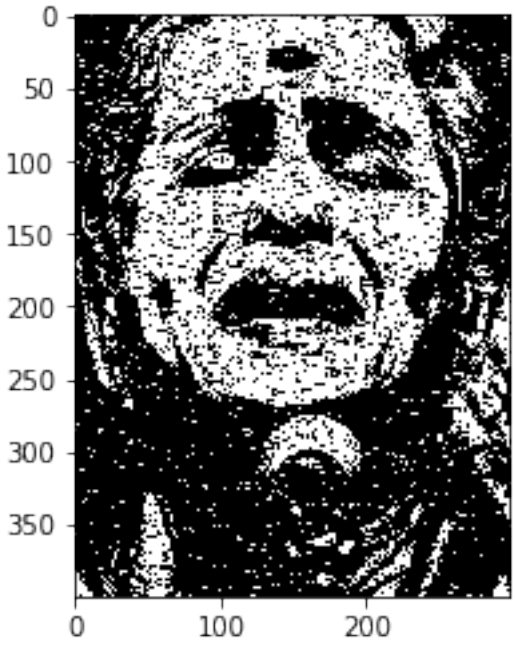

```
--------------------------------------------------------------------------------
-------
(120000, 50)
Average Energy of Transmitted Signal 8e-06
Energy per Bit of Transmitted Signal 2e-06
Eb/No Ratio in dB is 0
Eb/No Ratio is 1.0
Variance of WGN 50.0
No.of Wrong Bits 26983
Fraction of Error 0.22485833333333333
```



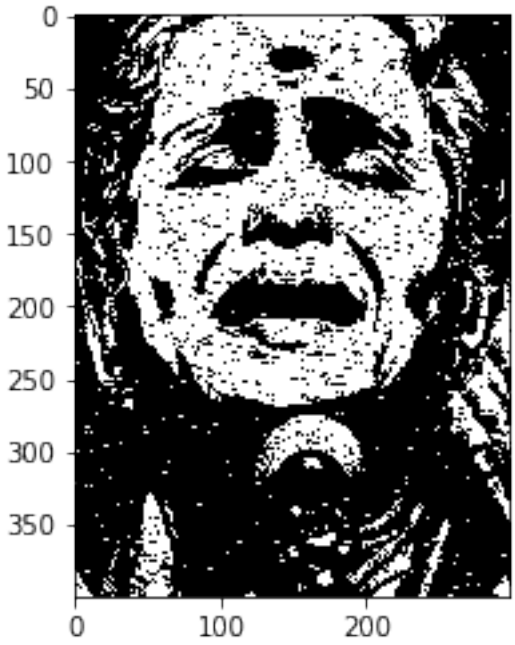

```
--------------------------------------------------------------------------------
-------
(120000, 50)
Average Energy of Transmitted Signal 8e-06
Energy per Bit of Transmitted Signal 2e-06
Eb/No Ratio in dB is 2
Eb/No Ratio is 1.5848931924611136
Variance of WGN 31.547867224009657
No.of Wrong Bits 19174
Fraction of Error 0.15978333333333333
```

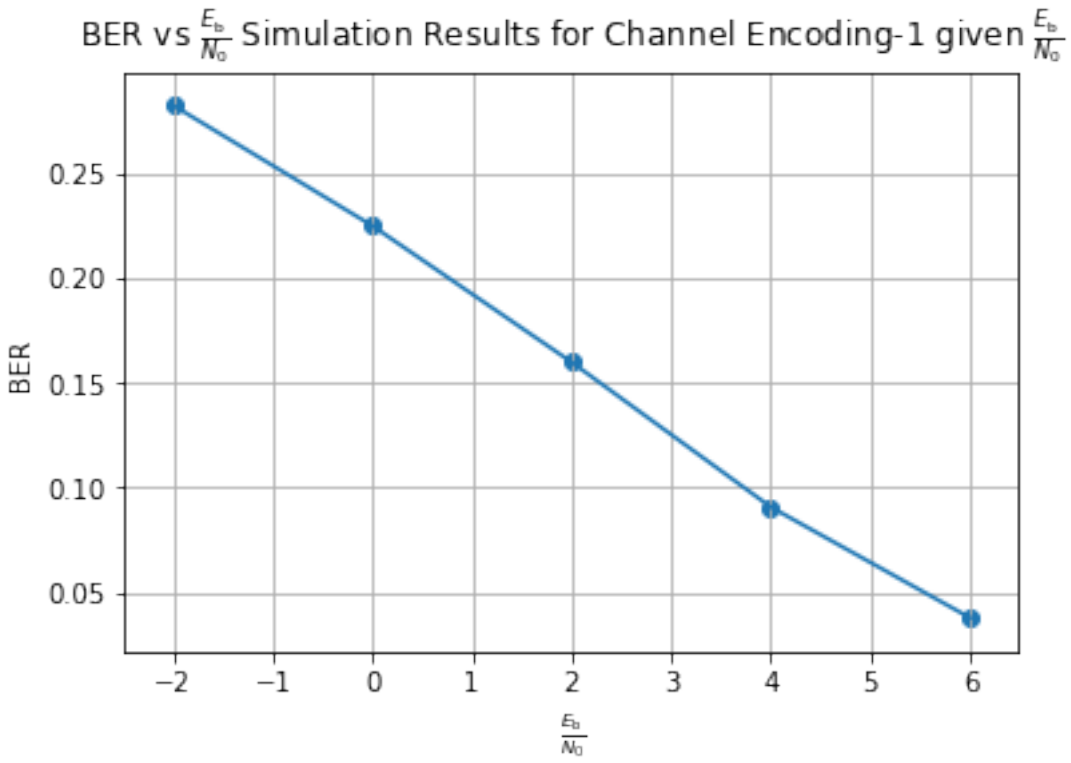



```
--------------------------------------------------------------------------------
-------
(120000, 50)
Average Energy of Transmitted Signal 8e-06
Energy per Bit of Transmitted Signal 2e-06
Eb/No Ratio in dB is 4
Eb/No Ratio is 2.51188643150958
Variance of WGN 19.905358527674863
No.of Wrong Bits 10877
Fraction of Error 0.09064166666666666
```

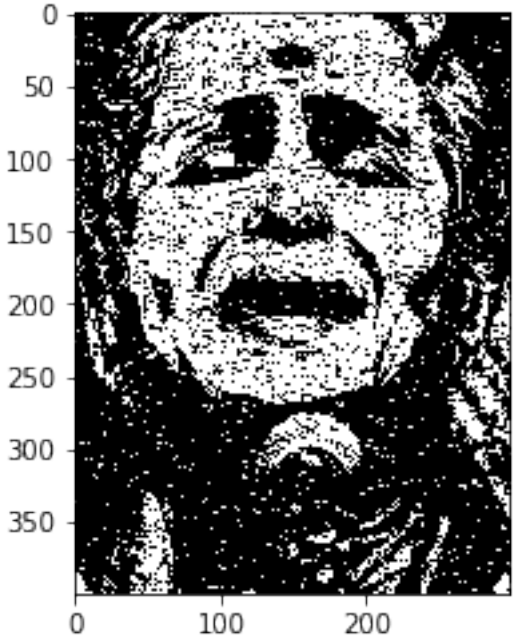



```
--------------------------------------------------------------------------------
-------
(120000, 50)
Average Energy of Transmitted Signal 8e-06
Energy per Bit of Transmitted Signal 2e-06
Eb/No Ratio in dB is 6
Eb/No Ratio is 3.9810717055349722
Variance of WGN 12.559432157547901
No.of Wrong Bits 4536
Fraction of Error 0.0378
```

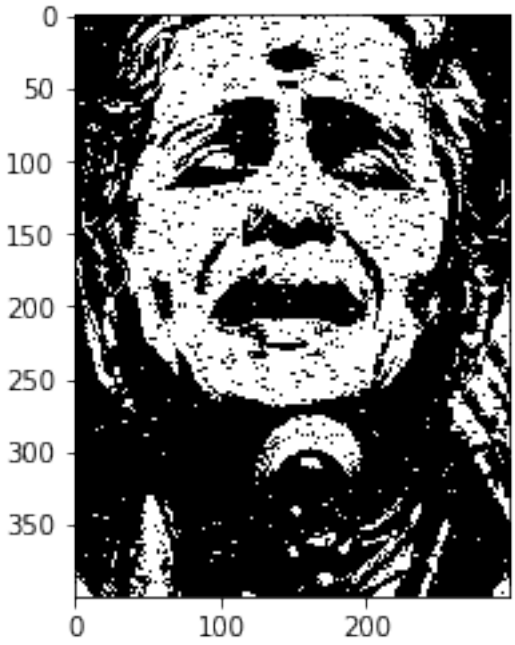

0
50
100
150
200
250
300
350
0
100
200


BER vs $\frac{E_b}{N_0}$ Simulation Results for Channel Encoding-1 given $\frac{E_b}{N_0}$
BER
0.25
0.20
0.15
0.10
0.05
−2
−1
0
1
2
3
4
5
6
$\frac{E_b}{N_0}$

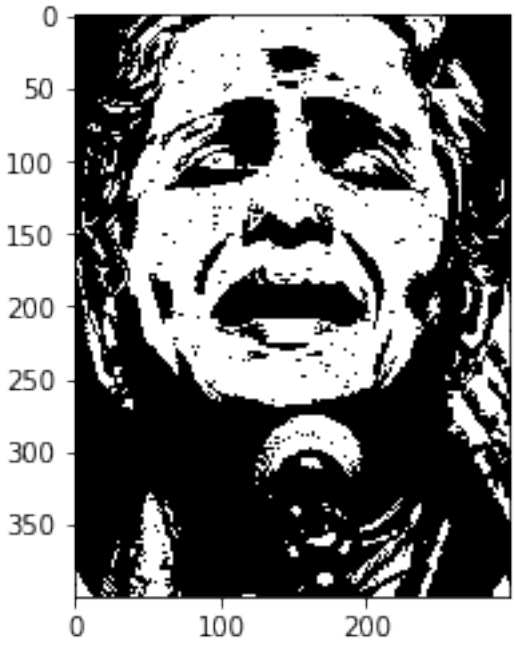
```
(120000, 50)
Average Energy of Transmitted Signal 8e-06
Energy per Bit of Transmitted Signal 2e-06
Variance of WGN 20
Eb/No Ratio is 2.5
Eb/No Ratio in dB is 3.979400086720376
No.of Wrong Bits 10678
Fraction of Error 0.08898333333333333
```



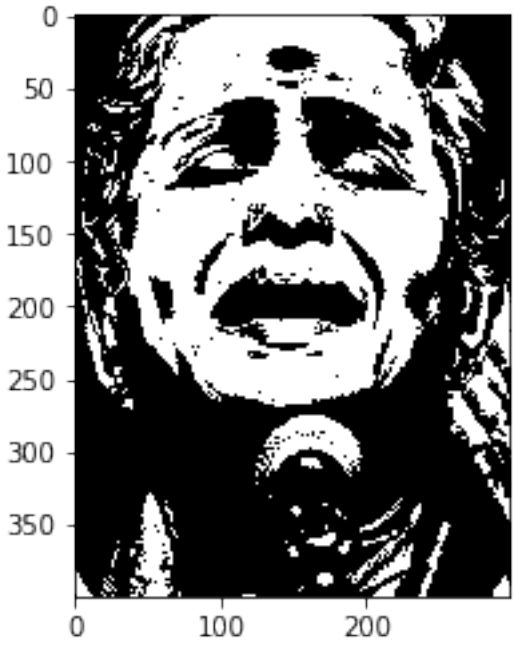

```
--------------------------------------------------------------------------------
-------
(120000, 50)
Average Energy of Transmitted Signal 8e-06
Energy per Bit of Transmitted Signal 2e-06
Variance of WGN 12
Eb/No Ratio is 4.166666666666667
Eb/No Ratio in dB is 6.19788758288394
No.of Wrong Bits 3876
Fraction of Error 0.0323
```

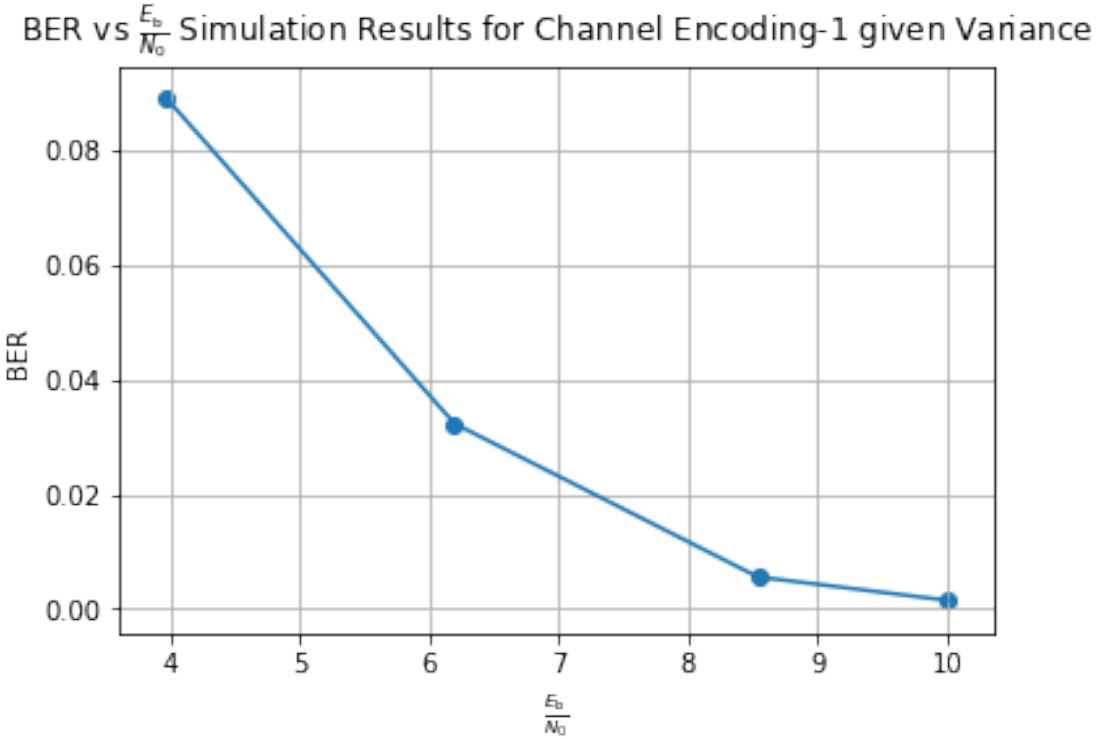



```
--------------------------------------------------------------------------------
-------
(120000, 50)
Average Energy of Transmitted Signal 8e-06
Energy per Bit of Transmitted Signal 2e-06
Variance of WGN 7
Eb/No Ratio is 7.142857142857142
Eb/No Ratio in dB is 8.53871964321762
No.of Wrong Bits 665
Fraction of Error 0.005541666666666667
```



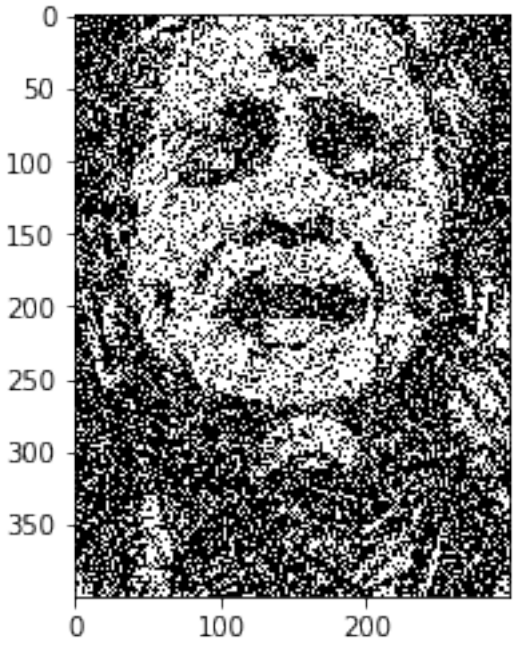

```
--------------------------------------------------------------------------------
-------
(120000, 50)
Average Energy of Transmitted Signal 8e-06
Energy per Bit of Transmitted Signal 2e-06
Variance of WGN 5
Eb/No Ratio is 10.0
Eb/No Ratio in dB is 10.0
No.of Wrong Bits 174
Fraction of Error 0.00145
```


0
50
100
150
200
250
300
350
0
100
200

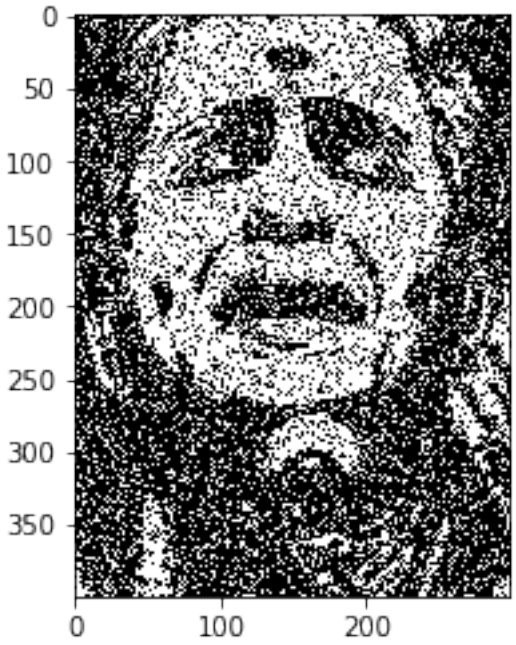

BER vs $\frac{E_b}{N_0}$ Simulation Results for Channel Encoding-1 given Variance
BER
0.08
0.06
0.04
0.02
0.00
4
5
6
7
8
9
10
$\frac{E_b}{N_0}$

[32]:
```
Variance = np.array([20,12,7,5]).astype(int)
Ratio = np.array([-2,0,2,4,6]).astype(int)

BER = []
Eb_per_No_dB = []
for r in Ratio:
    Encoded_Signal = Channel_EncodeTech2(Digital_Signal)
    Received_Signal,Ratio =␣
 ↪Modulation(Encoded_Signal,fc,T,M,fs,3,1,Cutoff_Freq=Cutoff_Freq,Ratio=r)
    Decoded_Signal = Channel_DecodeTech2(Received_Signal)

    BER.append(Bit_Error_Rate(Decoded_Signal,Digital_Signal))
    Eb_per_No_dB.append(Ratio)

    Plot_Image(Decoded_Signal,shape)

Plot_BERGraph(Eb_per_No_dB,BER,r'Channel Encoding-2 given␣
 ↪$\frac{E_{\mathrm{b}}}{N_{0}}$')


BER = []
Eb_per_No_dB = []

for v in Variance:
    Encoded_Signal = Channel_EncodeTech2(Digital_Signal)
    Received_Signal,Ratio =␣
 ↪Modulation(Encoded_Signal,fc,T,M,fs,3,1,Cutoff_Freq=Cutoff_Freq,Variance=v,VarianceTruth=Tr
    Decoded_Signal = Channel_DecodeTech2(Received_Signal)

    BER.append(Bit_Error_Rate(Decoded_Signal,Digital_Signal))
    Eb_per_No_dB.append(Ratio)

    Plot_Image(Decoded_Signal,shape)

Plot_BERGraph(Eb_per_No_dB,BER,'Channel Encoding-2 given Variance')
```

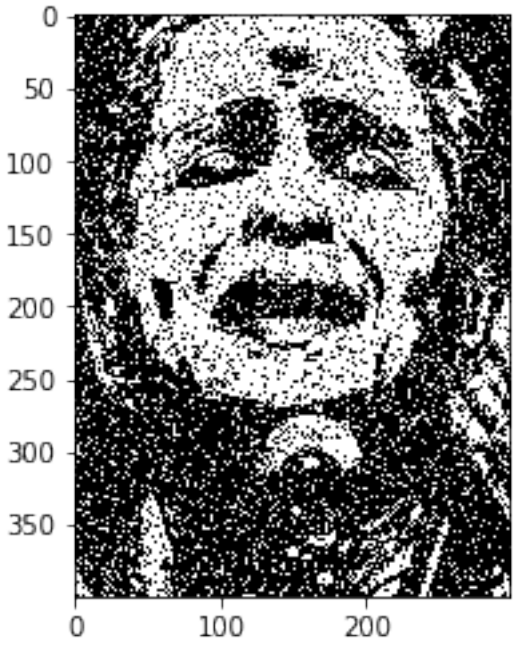
```
(180000, 50)
Average Energy of Transmitted Signal 3e-06
Energy per Bit of Transmitted Signal 3e-06
Eb/No Ratio in dB is -2
Eb/No Ratio is 0.6309573444801932
Variance of WGN 118.8669894345835
No.of Wrong Bits 29281
Fraction of Error 0.24400833333333333
```



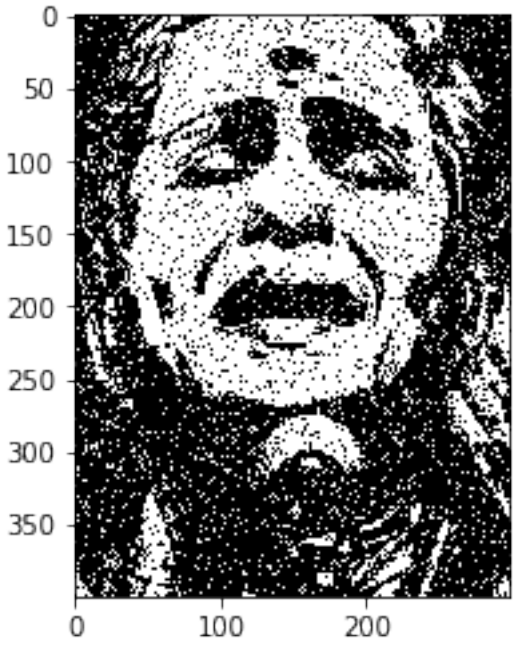

```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 3e-06
Energy per Bit of Transmitted Signal 3e-06
Eb/No Ratio in dB is 0
Eb/No Ratio is 1.0
Variance of WGN 75.0
No.of Wrong Bits 23253
Fraction of Error 0.193775
```



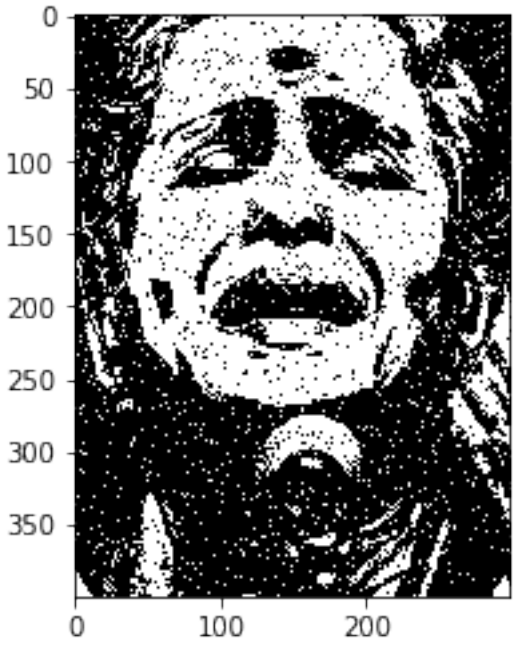

```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 3e-06
Energy per Bit of Transmitted Signal 3e-06
Eb/No Ratio in dB is 2
Eb/No Ratio is 1.5848931924611136
Variance of WGN 47.321800836014496
No.of Wrong Bits 16591
Fraction of Error 0.13825833333333334
```

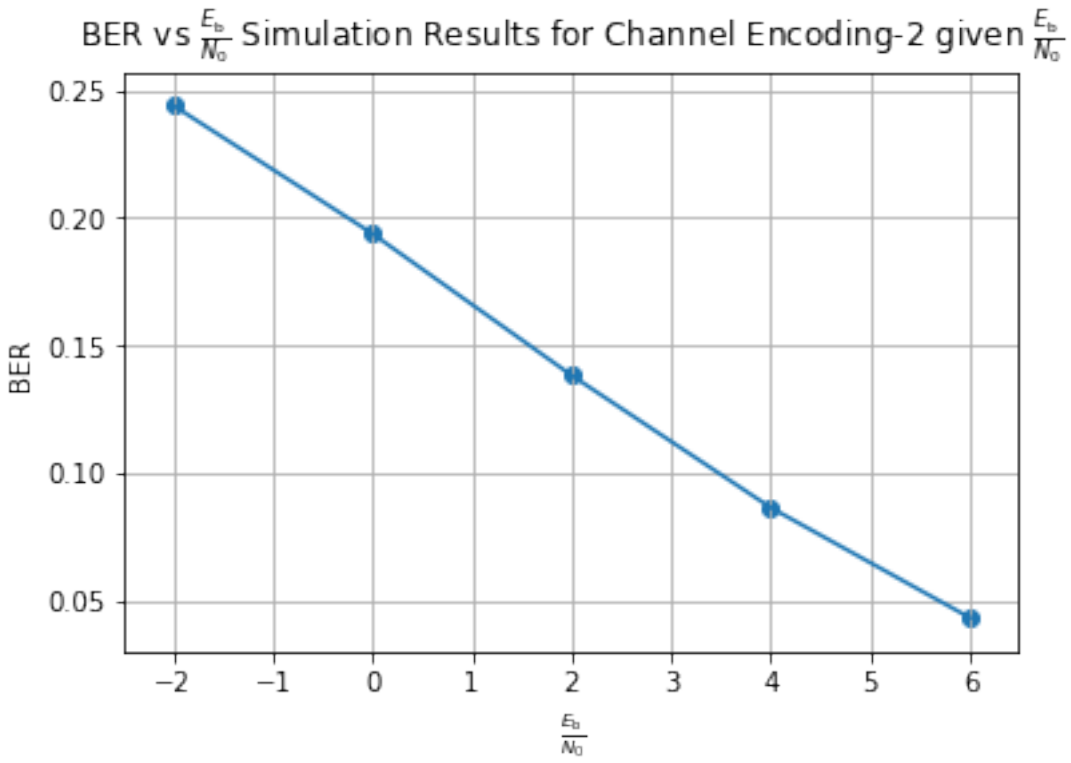



```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 3e-06
Energy per Bit of Transmitted Signal 3e-06
Eb/No Ratio in dB is 4
Eb/No Ratio is 2.51188643150958
Variance of WGN 29.85803779151229
No.of Wrong Bits 10372
Fraction of Error 0.08643333333333333
```

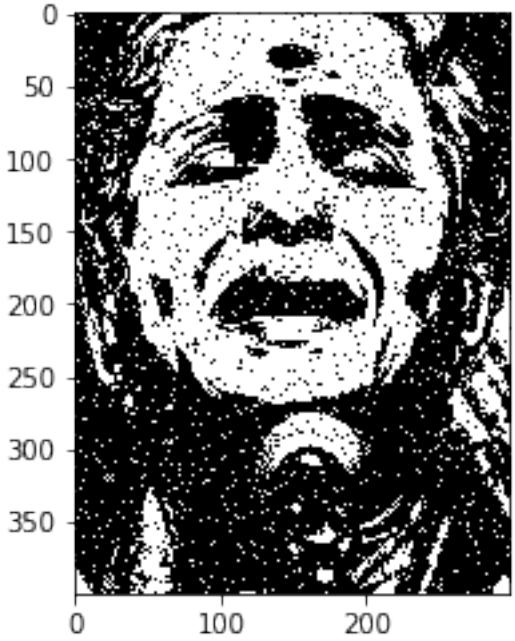



```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 3e-06
Energy per Bit of Transmitted Signal 3e-06
Eb/No Ratio in dB is 6
Eb/No Ratio is 3.9810717055349722
Variance of WGN 18.839148236321854
No.of Wrong Bits 5197
Fraction of Error 0.04330833333333333
```

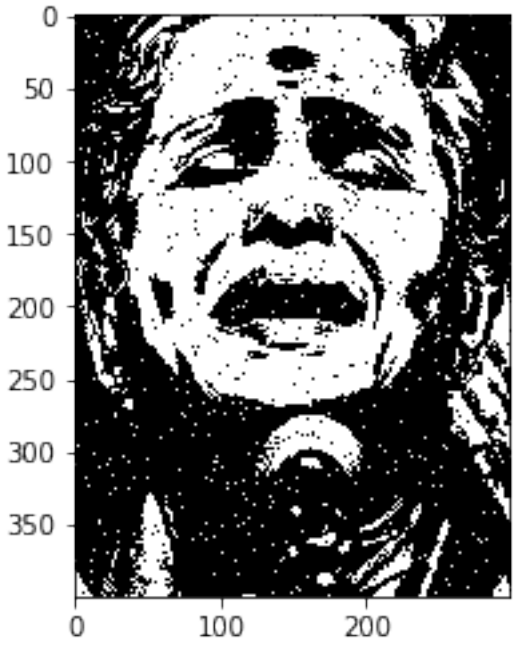

0
50
100
150
200
250
300
350
0
100
200


BER vs $\frac{E_b}{N_0}$ Simulation Results for Channel Encoding-2 given $\frac{E_b}{N_0}$
BER
0.25
0.20
0.15
0.10
0.05
−2
−1
0
1
2
3
4
5
6
$\frac{E_b}{N_0}$

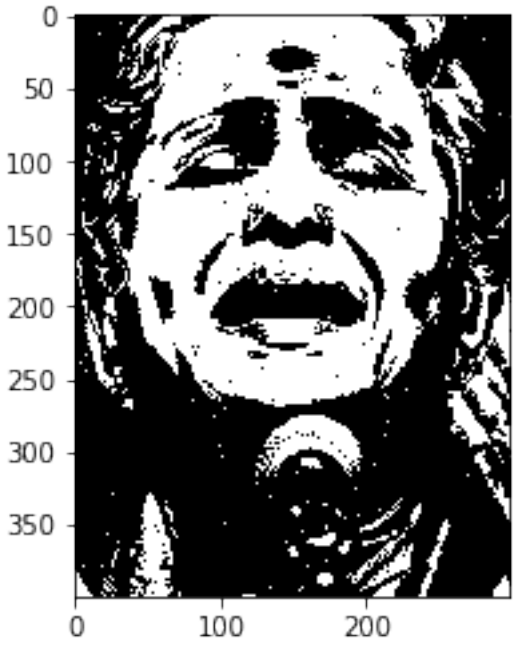
```
(180000, 50)
Average Energy of Transmitted Signal 3e-06
Energy per Bit of Transmitted Signal 3e-06
Variance of WGN 20
Eb/No Ratio is 3.7500000000000004
Eb/No Ratio in dB is 5.740312677277188
No.of Wrong Bits 5773
Fraction of Error 0.048108333333333336
```



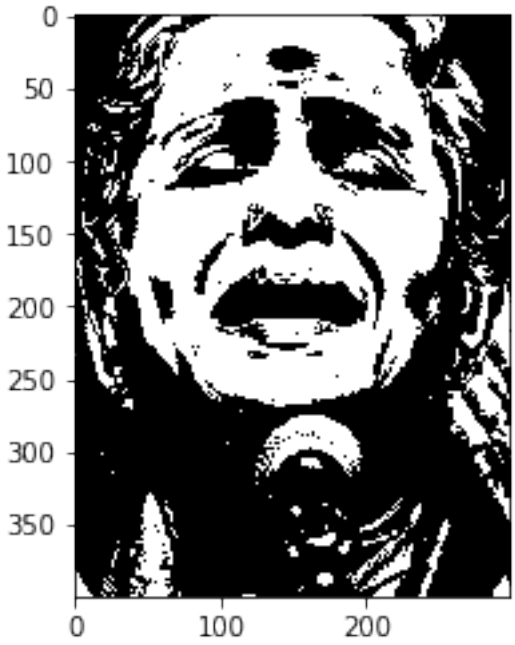

```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 3e-06
Energy per Bit of Transmitted Signal 3e-06
Variance of WGN 12
Eb/No Ratio is 6.250000000000001
Eb/No Ratio in dB is 7.958800173440752
No.of Wrong Bits 1955
Fraction of Error 0.016291666666666666
```

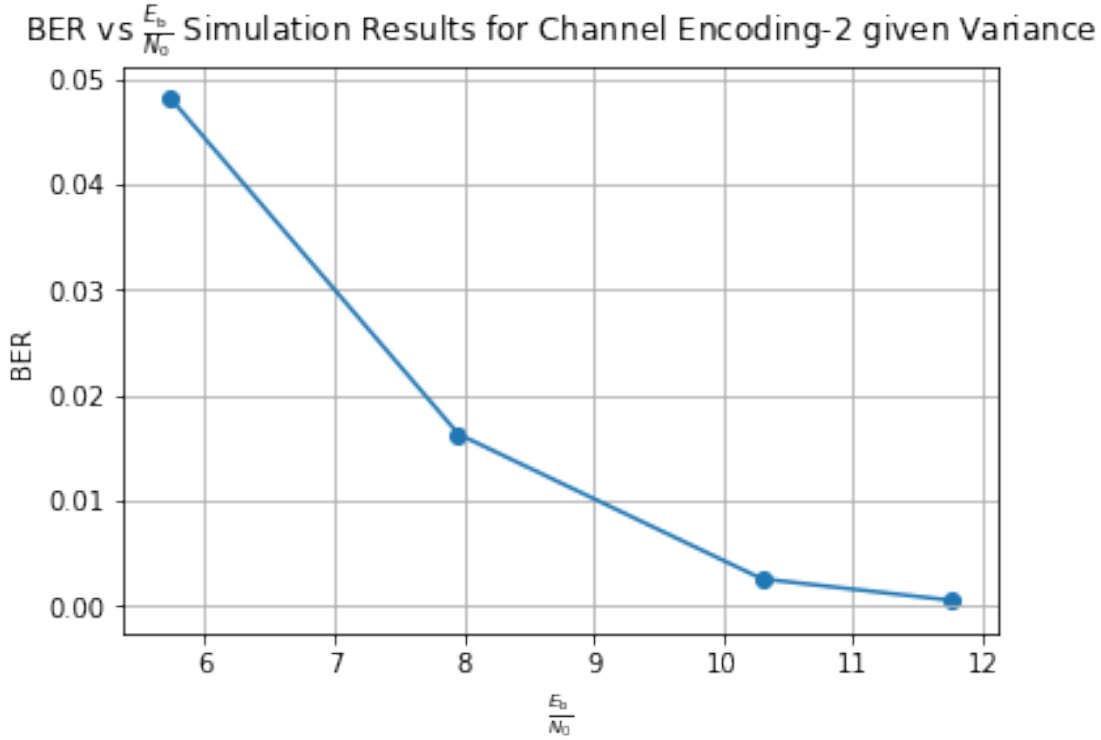



```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 3e-06
Energy per Bit of Transmitted Signal 3e-06
Variance of WGN 7
Eb/No Ratio is 10.714285714285714
Eb/No Ratio in dB is 10.299632233774432
No.of Wrong Bits 308
Fraction of Error 0.0025666666666666667
```



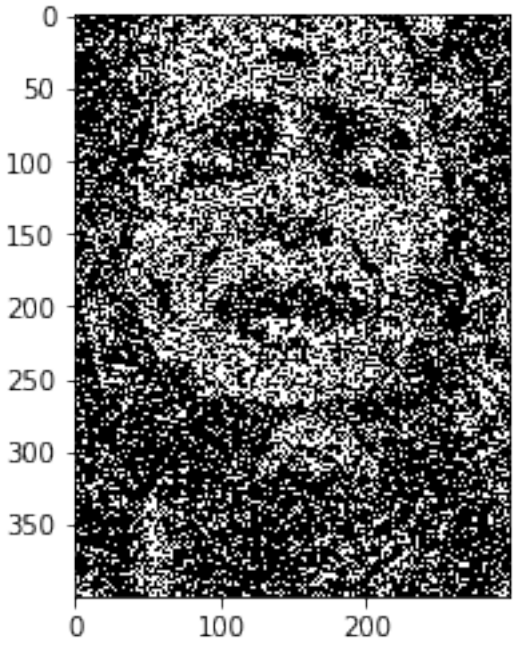

```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 3e-06
Energy per Bit of Transmitted Signal 3e-06
Variance of WGN 5
Eb/No Ratio is 15.000000000000002
Eb/No Ratio in dB is 11.760912590556813
No.of Wrong Bits 63
Fraction of Error 0.000525
```


0
50
100
150
200
250
300
350
0
100
200

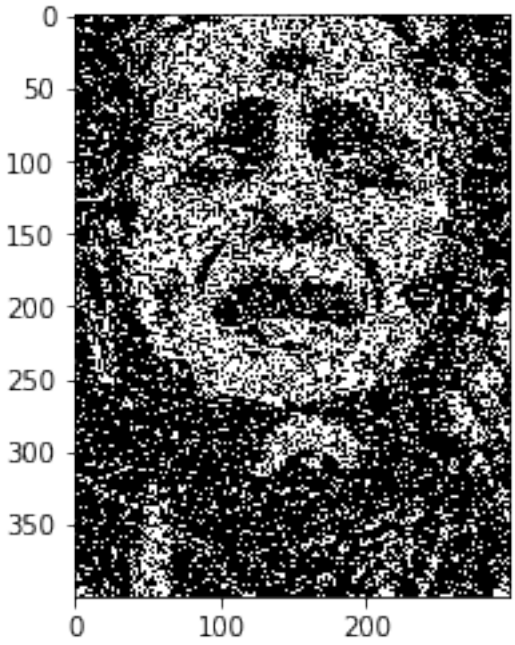

BER vs $\frac{E_b}{N_0}$ Simulation Results for Channel Encoding-2 given Variance
BER
0.05
0.04
0.03
0.02
0.01
0.00
6
7
8
9
10
11
12
$\frac{E_b}{N_0}$

[33]:
```
Variance = np.array([20,12,7,5]).astype(int)
Ratio = np.array([-2,0,2,4,6]).astype(int)

BER = []
Eb_per_No_dB = []
for r in Ratio:
    Encoded_Signal = Channel_EncodeTech3(Digital_Signal)
    Received_Signal,Ratio =␣
 ↪Modulation(Encoded_Signal,fc,T,M,fs,12,4,Cutoff_Freq=Cutoff_Freq,Ratio=r)
    Bits_12,Bits_4 = Bit_Possibilities_4_BitEncoder(3)
    Decoded_Signal = Channel_DecodeTech3(Received_Signal,Bits_12,Bits_4)

    BER.append(Bit_Error_Rate(Decoded_Signal,Digital_Signal))
    Eb_per_No_dB.append(Ratio)

    Plot_Image(Decoded_Signal,shape)

Plot_BERGraph(Eb_per_No_dB,BER,r'Channel Encoding-3 given␣
 ↪$\frac{E_{\mathrm{b}}}{N_{0}}$')


BER = []
Eb_per_No_dB = []

for v in Variance:
    Encoded_Signal = Channel_EncodeTech3(Digital_Signal)
    Received_Signal,Ratio =␣
 ↪Modulation(Encoded_Signal,fc,T,M,fs,12,4,Cutoff_Freq=Cutoff_Freq,Variance=v,VarianceTruth=T
    Bits_12,Bits_4 = Bit_Possibilities_4_BitEncoder(3)
    Decoded_Signal = Channel_DecodeTech3(Received_Signal,Bits_12,Bits_4)

    BER.append(Bit_Error_Rate(Decoded_Signal,Digital_Signal))
    Eb_per_No_dB.append(Ratio)

    Plot_Image(Decoded_Signal,shape)

Plot_BERGraph(Eb_per_No_dB,BER,'Channel Encoding-3 given Variance')
```

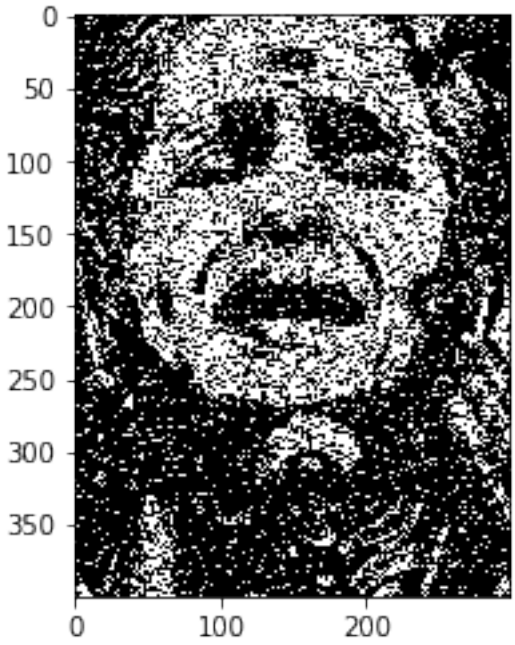
```
(180000, 50)
Average Energy of Transmitted Signal 1.2e-05
Energy per Bit of Transmitted Signal 3e-06
Eb/No Ratio in dB is -2
Eb/No Ratio is 0.6309573444801932
Variance of WGN 118.8669894345835
No.of Wrong Bits 35933
Fraction of Error 0.29944166666666666
```



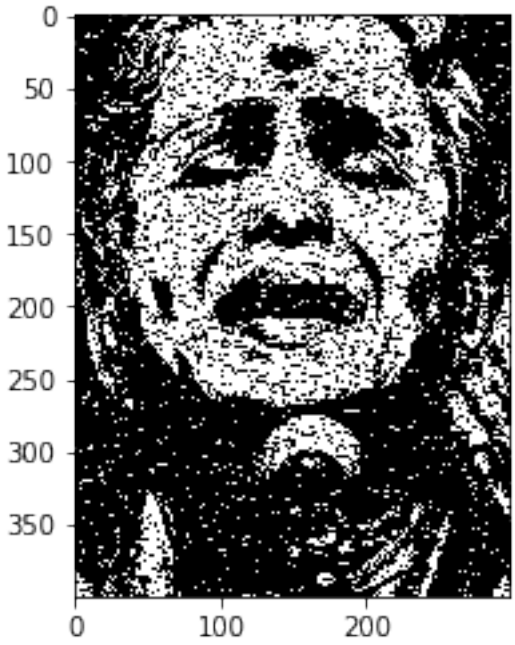

```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 1.2e-05
Energy per Bit of Transmitted Signal 3e-06
Eb/No Ratio in dB is 0
Eb/No Ratio is 1.0
Variance of WGN 75.0
No.of Wrong Bits 28741
Fraction of Error 0.23950833333333332
```



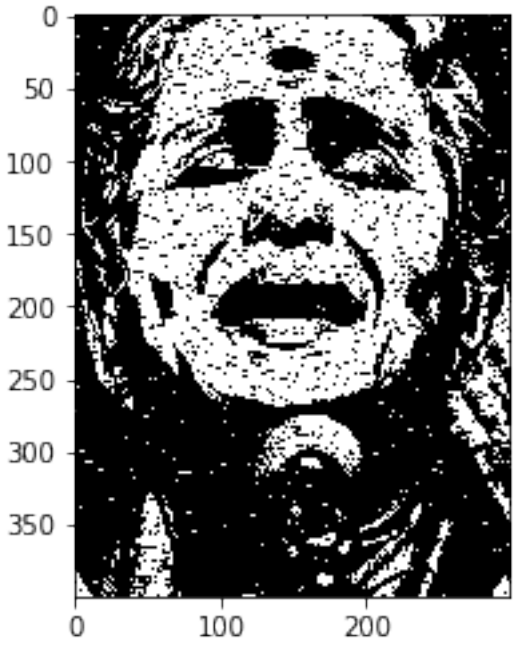

```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 1.2e-05
Energy per Bit of Transmitted Signal 3e-06
Eb/No Ratio in dB is 2
Eb/No Ratio is 1.5848931924611136
Variance of WGN 47.321800836014496
No.of Wrong Bits 21008
Fraction of Error 0.17506666666666668
```

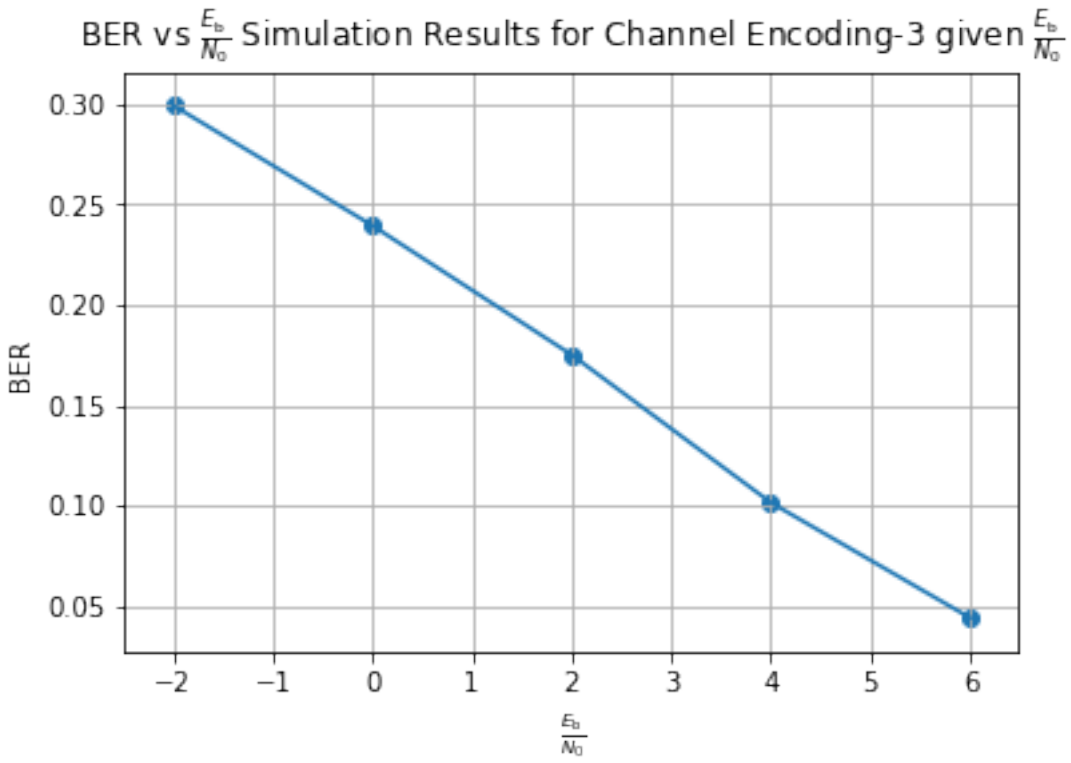



```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 1.2e-05
Energy per Bit of Transmitted Signal 3e-06
Eb/No Ratio in dB is 4
Eb/No Ratio is 2.51188643150958
Variance of WGN 29.85803779151229
No.of Wrong Bits 12165
Fraction of Error 0.101375
```

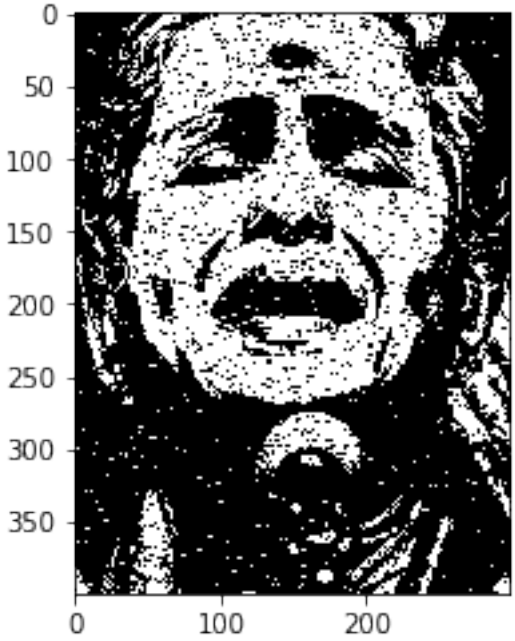



```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 1.2e-05
Energy per Bit of Transmitted Signal 3e-06
Eb/No Ratio in dB is 6
Eb/No Ratio is 3.9810717055349722
Variance of WGN 18.839148236321854
No.of Wrong Bits 5270
Fraction of Error 0.043916666666666666
```

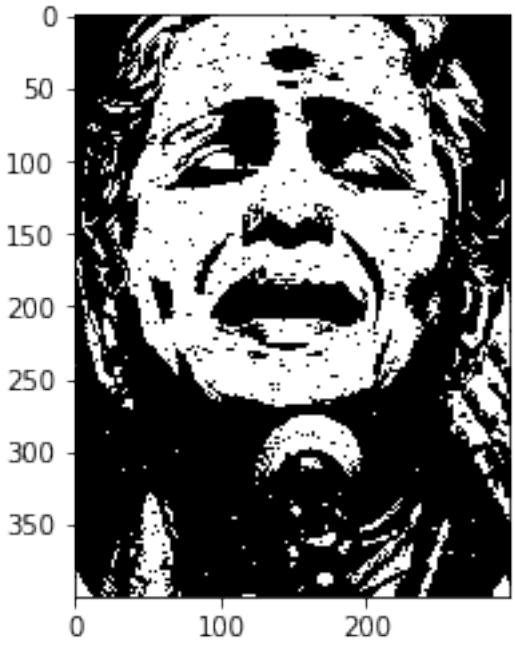

0
50
100
150
200
250
300
350
0
100
200


BER vs $\frac{E_b}{N_0}$ Simulation Results for Channel Encoding-3 given $\frac{E_b}{N_0}$
BER
0.30
0.25
0.20
0.15
0.10
0.05
−2
−1
0
1
2
3
4
5
6
$\frac{E_b}{N_0}$

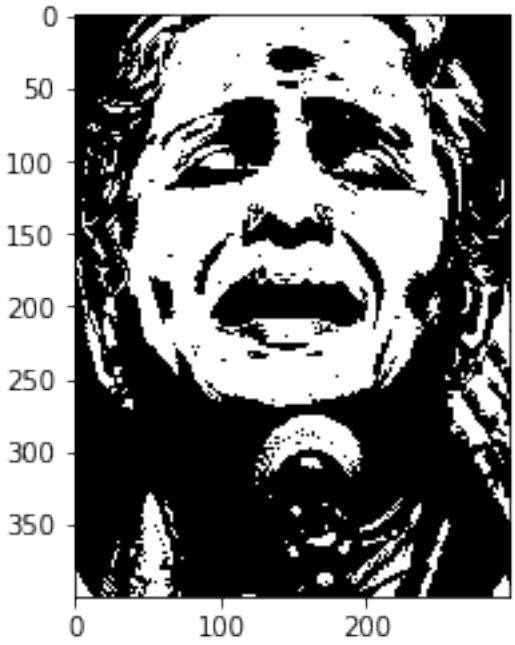
```
(180000, 50)
Average Energy of Transmitted Signal 1.2e-05
Energy per Bit of Transmitted Signal 3e-06
Variance of WGN 20
Eb/No Ratio is 3.7500000000000004
Eb/No Ratio in dB is 5.740312677277188
No.of Wrong Bits 5995
Fraction of Error 0.049958333333333334
```



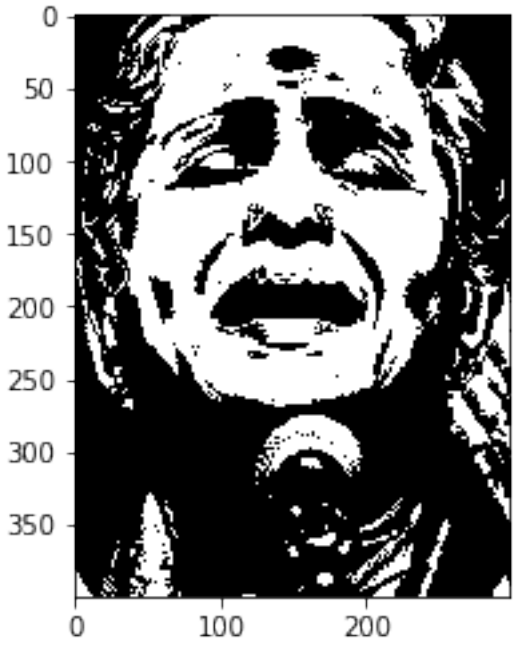

```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 1.2e-05
Energy per Bit of Transmitted Signal 3e-06
Variance of WGN 12
Eb/No Ratio is 6.250000000000001
Eb/No Ratio in dB is 7.958800173440752
No.of Wrong Bits 1347
Fraction of Error 0.011225
```

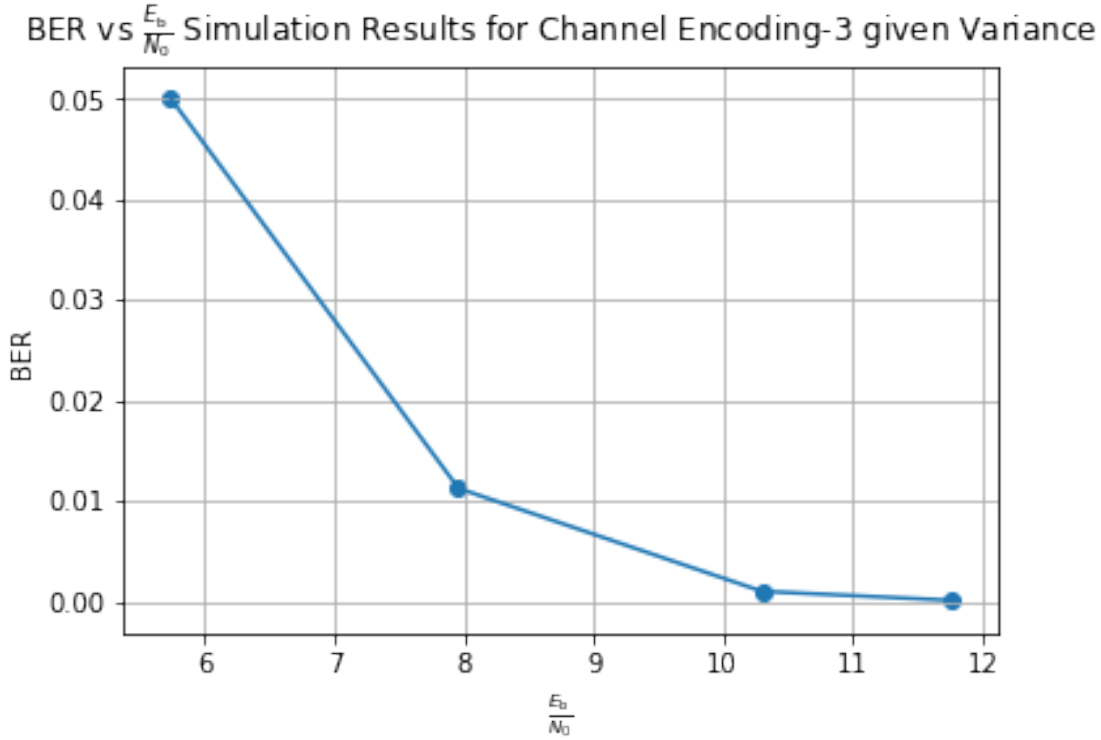



```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 1.2e-05
Energy per Bit of Transmitted Signal 3e-06
Variance of WGN 7
Eb/No Ratio is 10.714285714285714
Eb/No Ratio in dB is 10.299632233774432
No.of Wrong Bits 119
Fraction of Error 0.0009916666666666667
```




```
--------------------------------------------------------------------------------
-------
(180000, 50)
Average Energy of Transmitted Signal 1.2e-05
Energy per Bit of Transmitted Signal 3e-06
Variance of WGN 5
Eb/No Ratio is 15.000000000000002
Eb/No Ratio in dB is 11.760912590556813
No.of Wrong Bits 11
Fraction of Error 9.166666666666667e-05
```


0
50
100
150
200
250
300
350
0
100
200


BER vs $\frac{E_b}{N_0}$ Simulation Results for Channel Encoding-3 given Variance
BER
0.05
0.04
0.03
0.02
0.01
0.00
6
7
8
9
10
11
12
$\frac{E_b}{N_0}$

The Graphs shows us the results of Simulations. X-axis has $\frac{E_b}{N_0}$ in decibal scale and Y-axis as BER(Bit Error Rate). We can observe that as $\frac{E_b}{N_0}$ increases BER decreases.