ADSP Assignment-5 Question-4

EE18BTECH11014

1 Z-Transform

Z-transform converts a discrete-time signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation. It can be considered as a discrete-time equivalent of the Laplace Transform.

$$X(z) = \frac{N(z)}{D(z)} = \frac{\sum_{k=0}^{M} b_k z^k}{\sum_{k=0}^{N} a_k z^k} = \frac{b_M}{a_N} \frac{\prod_{k=1}^{M}(z - z_{z_k})}{\prod_{k=1}^{N}(z - z_{p_k})}$$

z_{z_k} : Zeros

z_{p_k} : Poles.

2 Moving Average Filter

The Moving Average Filter operates by averaging a number of points from the input signal to produce each point in the output signal.

Equation of Moving Average Filter is

$$y[n] = \frac{1}{M} \sum_{i=0}^{M-1} x[n+i]$$

x[n] : Input

y[n] : Output.

Zeros of a Moving Average Filter lie on a unit circle of radius equal to 1. Pole of a Moving Average Filter lie at origin.

3 Notch Filter

A Notch Filter is a very narrow bandwidth band-stop filter. It has a gain which drops and then rises very steeply with increasing frequency and so the magnitude response has the shape of a notch.

Poles and Zeros of a Notch Filter will be very close to each other. This is because for any point farther to Zeros and Poles distance to Poles and Zeros is almost same. And for any point nearer to zero the amplitude drops drastically resulting in a Band-Stop Filter.

4 Effects of applying Notch Filter and Moving Average Filters to Noisy Signals

When a Moving Average Filter is applied to Noisy Signals, high frequency components gets attenuated and low frequency components pass through the filter by a fractional change in Amplitude. The high frequency components in a noisy signal are mainly due to noise which gets attenuated when passed through filter leaving behind the original signal with a slightly lower amplitude. This is because when Sampling Frequency is fixed, at high frequencies the no.of samples per cycle decreases as frequency increases leading to less contribution from High Frequency Components. This is similar to that of Nyquist Sampling Theorem which states that Sampling Frequency should be atleast twice of maximum frequency of Input. It can be understood that at high frequencies Nyquist Sampling Theorem is violated.

When a Notch Filter is applied to Noisy Signal, the filter stops all frequencies in its bandwidth resulting in loss of small part of Input signal along with Noise.

Amplitude doesn't get affected when Noisy Signal is passed through Notch Filter. As distance from Pole and Zero almost remains same for any point farther from Poles and Zeros. So their ratio becomes equal to one.

Amplitude gets affected in case of Moving Average System as distance of any point from Zeros and Poles changes for each and every point. As Amplitude depends on Distance from Poles and Zeros, it also depends on M-Point Moving Average Filter used.

5 Results

5.1 Notch Filter

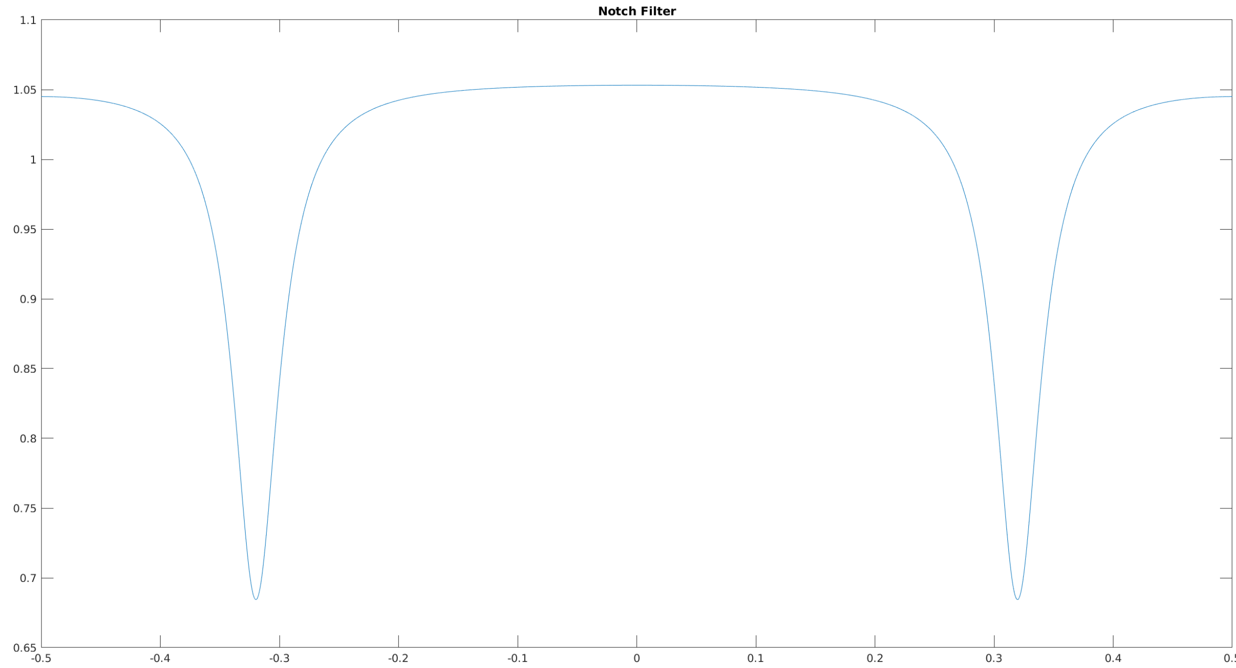
The following is a Magnitude of Frequency Response of Notch Filter with

Zeros $= 0.9e^{j\omega}, 0.9e^{-j\omega}$

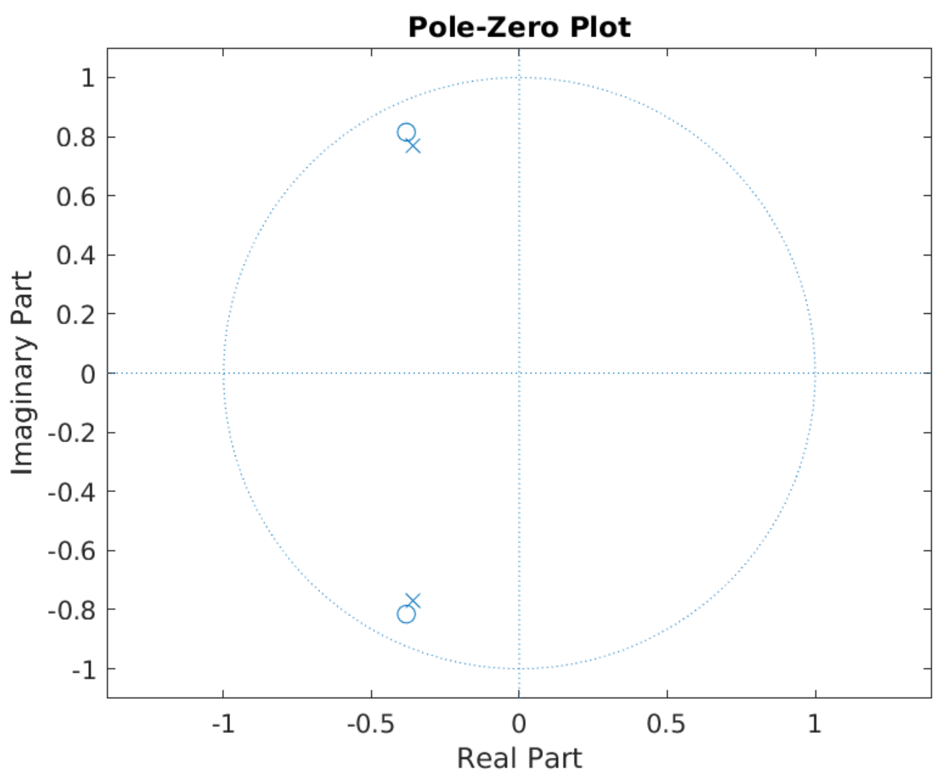
Poles $= 0.8e^{j\omega}, 0.8e^{-j\omega}$

where $\omega = 4.274$

Transfer Function $= \frac{(z-0.9e^{j\omega})(z-0.9e^{-j\omega})}{(z-0.8e^{j\omega})(z-0.8e^{-j\omega})}$



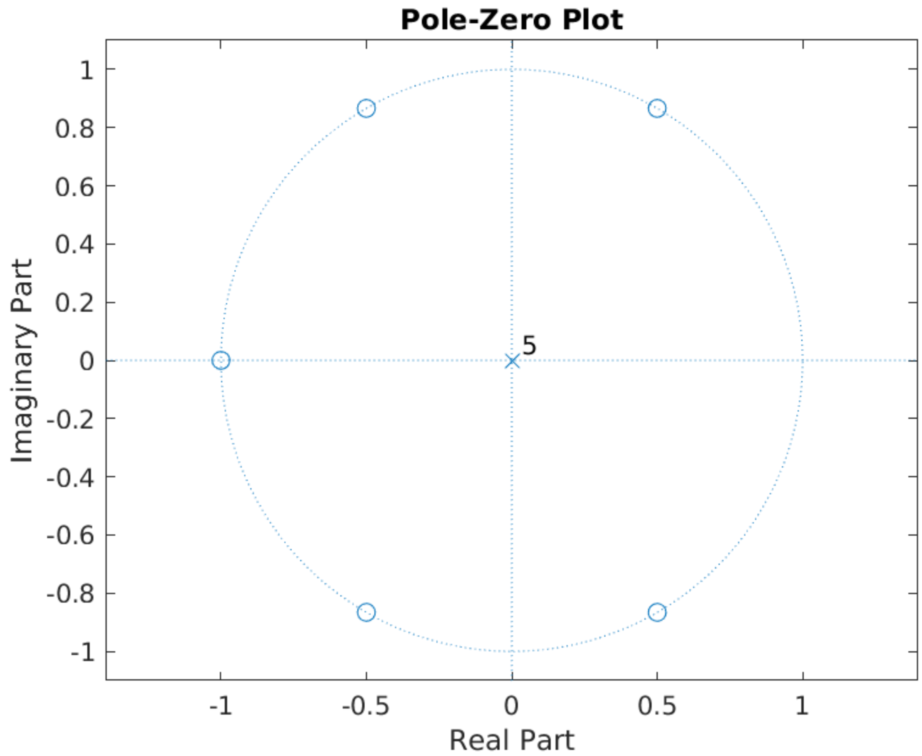
The Notch Filter has the Pole-Zero Plot as follows



It can be observed that when z is close to a Zero, magnitude drops and when it is far away from Zeros and Poles Magnitude almost remains as 1. This happens due to closeness of Zeros and Poles.

5.2 Moving Average Filter

The Moving Average Filter has the Pole-Zero Plot as follows



Here 6-Point Moving Average Filter is considered. So it has 5-Zeros and 1-Pole.

Transfer Function $= \frac{z^5+z^4+z^3+z^2+z^1+1}{z^6}$

5.3 Code

Matlab Code is written to demonstrate the results. It takes a Audio Sample as Input, adds Sinusoidal High Frequency Noise and passes the resultant signal through both Notch Filter and Moving Average Filter respectively.

6 Acknowledgements

https://www.youtube.com/watch?v=hewTwm5P0Gg