Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

1 Signal Flow Graph

2 Gain of Feedback Circuits

2.1 Estimation of Voltage Gain

2.1.1. Consider the Magnitude Bode Plot and Phase Bode Plot 2.1.1 of Open-Loop Transfer Function of an Amplifier. Estimate the Open-Loop Transfer Function. (Assume $'A'$ as $'G'$ and $'\beta'$ as $'H'$)

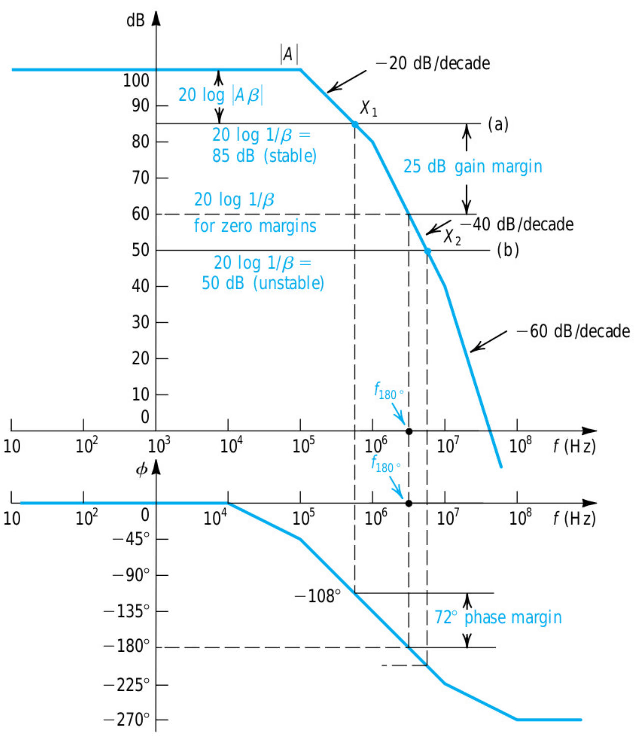

Fig. 2.1.1: Magnitude and Phase Bode Plot

Solution: Let $G(f)$ be the Open-Loop Transfer Function,

$$G(f) = \begin{cases} 100 & 0 < f < 10^5 \\ 200 - 20log(f) & 10^5 < f < 10^6 \\ 320 - 40log(f) & 10^6 < f < 10^7 \\ 460 - 60log(f) & 10^7 < f \end{cases} \tag{2.1.1.1}$$

$$\nabla G(f) = \frac{d(G(f))}{d(log(f))} = \begin{cases} 0 & 0 < f < 10^5 \\ -20 & 10^5 < f < 10^6 \\ -40 & 10^6 < f < 10^7 \\ -60 & 10^7 < f \end{cases} \tag{2.1.1.2}$$

As we know that, **When a pole is encountered the slope always decreases by 20 dB/decade** and **When a zero is encountered the slope always increases by 20 dB/decade**. So, by observing 2.1.1 it can be concluded that we are having Poles at $f = 10^5 Hz, 10^6 Hz, 10^7 Hz$ and No Zeros.

So, the Open-Loop Transfer Function $G(f)$ is

$$G(f) = \frac{10^5}{\left(1 + j\frac{f}{10^5}\right)\left(1 + j\frac{f}{10^6}\right)\left(1 + j\frac{f}{10^7}\right)} \tag{2.1.1.3}$$

2.1.2. Calculate the Phase of Open-Loop Transfer Function.
Solution:
Phase of Open-Loop Transfer Function = ϕ

$$\phi = -\left[\tan^{-1}\left(f/10^5\right) + \tan^{-1}\left(f/10^6\right) + \tan^{-1}\left(f/10^7\right)\right] \tag{2.1.2.1}$$

2.1.3. Determine the Closed-Loop Voltage Gain of the System assuming $|GH| \gg 1$ and also assuming the block diagram of Control System is 2.1.3

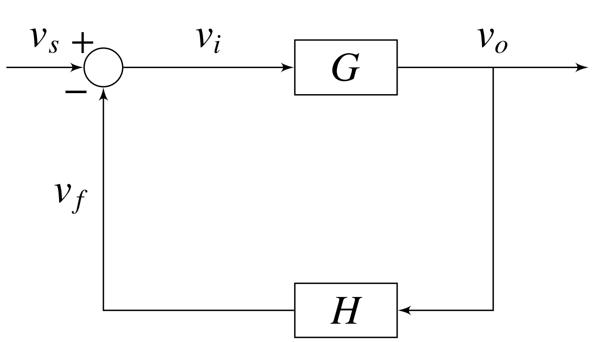

Fig. 2.1.3

Solution:
The Closed-Loop Voltage Gain of the Control System is

$$T = \frac{V_o}{V_s} = \frac{G}{1 + GH} \tag{2.1.3.1}$$

$$20\log(T) = 20\log(G) - 20\log(1 + GH) \tag{2.1.3.2}$$

Considering the assumption $|GH| \gg 1$, It can be written as

$$20\log(1 + GH) = 20\log(GH) \tag{2.1.3.3}$$

So,

$$20\log(T) = 20\log(G) - 20\log(GH) \tag{2.1.3.4}$$

$$20\log(T) = -20\log(H) \tag{2.1.3.5}$$

$$20\log(T) = 20\log(\frac{1}{H}) \tag{2.1.3.6}$$

$$T = \frac{1}{H} \tag{2.1.3.7}$$

So, The value of Closed-Loop Voltage Gain of the Control System under the assumption, $|GH| \gg 1$ is $T = \frac{1}{H}$

2.1.4. What is the value of Loop-Gain?
Solution:
The value of Loop-Gain can be calculated by the difference of 2-curves $20\log|A|$ and $20log(\frac{1}{H})$. The difference between the two curves will be

$$20\log|G| - 20\log\frac{1}{H} = 20\log|GH| \tag{2.1.4.1}$$

2.1.5. Define Phase-Margin
Solution:
Phase-Margin: The phase margin is defined as the angle in degrees by which the phase angle is smaller than −180° at the gain crossover, the gain crossover being the frequency at which the open-loop gain first reaches 1.

2.1.6. Find the frequencies for which phase margins are 90° and 45° respectively?
Solution:
Let Phase Margin be $\alpha = 90°$. Then,

$$\alpha = \phi - (-180°) \tag{2.1.6.1}$$

$$\phi = -180° + \alpha \tag{2.1.6.2}$$

$$\phi = -90° \tag{2.1.6.3}$$

So, by the definition of Phase-Margin, at $\phi = -90^\circ$, $|GH| = 1$. The value of $\phi = -90^\circ$ between poles $f = 10^5 Hz, 10^6 Hz$. Assuming the Poles are farther apart,

$$\tan^{-1}(\frac{f}{10^7}) \approx 0 \tag{2.1.6.4}$$

where $10^5 < f < 10^6$
So,

$$-\tan^{-1}\left(f/10^5\right) - \tan^{-1}\left(f/10^6\right) = -90 \tag{2.1.6.5}$$

$$\tan^{-1}\left(f/10^5\right) + \tan^{-1}\left(f/10^6\right) = 90 \tag{2.1.6.6}$$

$$\tan^{-1}\left(f/10^5\right) = 90 - \tan^{-1}\left(f/10^6\right) \tag{2.1.6.7}$$

$$\tan^{-1}\left(f/10^5\right) = \cot^{-1}\left(f/10^6\right) \tag{2.1.6.8}$$

$$\tan^{-1}\left(f/10^5\right) = \tan^{-1}\left(10^6/f\right) \tag{2.1.6.9}$$

$$f^2 = 10^{11} \tag{2.1.6.10}$$

$$f = 3.162 \times 10^5 \tag{2.1.6.11}$$

So, the approximate value of f at which Phase Margin is 90° is $f = 3.162 \times 10^5 Hz$.

Similarly let Phase Margin be $\alpha = 45^\circ$. Then,

$$\alpha = \phi - (-180^\circ) \tag{2.1.6.12}$$

$$\phi = -180^\circ + \alpha \tag{2.1.6.13}$$

$$\phi = -135^\circ \tag{2.1.6.14}$$

So, by the definition of Phase-Margin, at $\phi = -135^\circ$, $|GH| = 1$. The value of $\phi = -135^\circ$ aproximately at poles $f = 10^6 Hz$.
So, the approximate value of f at which Phase Margin is 45° is $f = 10^6$.

2.1.7. Find the minimum values of Closed-Loop Voltage Gain for which phase margins are 90° and 45° respectively
Solution:
For $\alpha = 90^\circ$,

$$f = 3.162 \times 10^5 \tag{2.1.7.1}$$

By substituting f in Open-Loop Gain $G(f)$ (assuming poles are far part),

$$G(f) = 200 - 20log(3.162 \times 10^5) \tag{2.1.7.2}$$

$$G(f) = 90dB \tag{2.1.7.3}$$

$$G = 3.1625 \times 10^4 \tag{2.1.7.4}$$

At that $f = 3.162 \times 10^5$,

$$H = \frac{1}{G} \tag{2.1.7.5}$$

$$H = 3.162 \times 10^{-5} \tag{2.1.7.6}$$

The minimum value of Closed-Loop Gain occurs at $|GH| \gg 1$ and the value of Closed-Loop Gain is $T = \frac{1}{H}$

$$T = \frac{1}{H} = 3.1625 \times 10^4 \tag{2.1.7.7}$$

So, The minimum value of Closed-Loop Gain with Phase Margin equal to $\alpha = 90^\circ$ is $T_{min} = 3.1625 \times 10^4$.

For $\alpha = 45^\circ$,

$$f = 10^6 \tag{2.1.7.8}$$

By substituting f in Open-Loop Gain $G(f)$ (assuming poles are far part),

$$G(f) = 200 - 20log(10^6) \tag{2.1.7.9}$$

$$G(f) = 80dB \tag{2.1.7.10}$$

$$G = 10^4 \tag{2.1.7.11}$$

At that $f = 10^6$,

$$H = \frac{1}{G} \tag{2.1.7.12}$$

$$H = 10^{-4} \tag{2.1.7.13}$$

The minimum value of Closed-Loop Gain occurs at $|GH| \gg 1$ and the value of Closed-Loop Gain is $T = \frac{1}{H}$

$$T = \frac{1}{H} = 10^4 \tag{2.1.7.14}$$

So, The minimum value of Closed-Loop Gain with Phase Margin equal to $\alpha = 45^\circ$ is $T_{min} = 10^4$.

2.1.8. Design a Circuit Model that follows the Transfer Function $G(f)$
Solution:

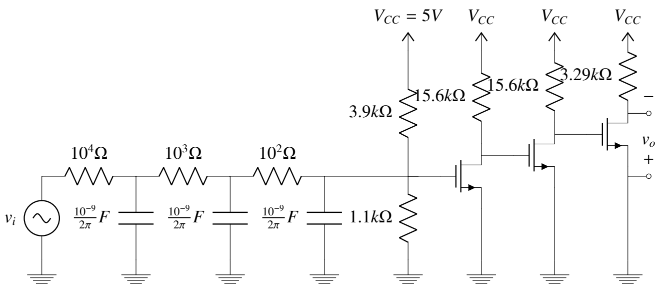

Fig. 2.1.8

Assuming, N-MOSFETs are identical and properties of N-MOSFET are,

$$\mu_n C_{ox} = 100\mu A/V^2 \tag{2.1.8.1}$$

$$\frac{W}{L} = 500 \tag{2.1.8.2}$$

$$V_T = 1V \tag{2.1.8.3}$$

V_{CC} is divided across $4.9\times10^3\Omega$ and $1.1\times10^3\Omega$

$$V_{GS} = 1.1V \tag{2.1.8.4}$$

Small Signal Parameter g_m,

$$g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH}) \tag{2.1.8.5}$$

$$g_m = 5 \times 10^{-3}\Omega^{-1} \tag{2.1.8.6}$$

$$I_D = \frac{g_m}{2}(V_{GS} - V_{TH}) \tag{2.1.8.7}$$

$$I_D = 2.5 \times 10^{-4}A \tag{2.1.8.8}$$

Small Signal Model of the circuit is

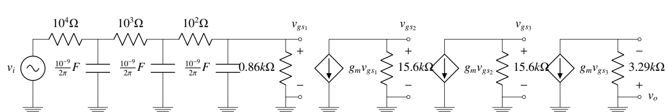

Fig. 2.1.8

The cascade of RC Circuits introduces poles in the circuit. The RC Circuits introduces poles at $f = 10^5Hz, 10^6Hz, 10^7Hz$ respectively from left to right. So, the value of v_{gs_1} is

$$v_{gs_1} = \frac{v_s}{\left(1 + j\frac{f}{10^5}\right)\left(1 + j\frac{f}{10^6}\right)\left(1 + j\frac{f}{10^7}\right)} \tag{2.1.8.9}$$

The MOSFETs used in the circuit are Self-Biased with $V_{GS} = 1.1V$ and $I_D = 2.5 \times 10^{-4}A$ for all 3 N-MOSFETs. 3 N-MOSFETs are used to achieve a large gain of 10^5. At 3rd N-MOSFET, Terminals of output are swaped, inorder to compensate for the negetive sign of gain.

At the first N-MOSFET,

$$v_{gs_2} = -g_m v_{gs_1} \times 15.6 \times 10^3 \tag{2.1.8.10}$$

$$v_{gs_2} = -78 \times v_{gs_1} \tag{2.1.8.11}$$

$$v_{gs_2} = \frac{-78 \times v_s}{\left(1 + j\frac{f}{10^5}\right)\left(1 + j\frac{f}{10^6}\right)\left(1 + j\frac{f}{10^7}\right)} \tag{2.1.8.12}$$

Similarly, at the second N-MOSFET,

$$v_{gs_3} = -g_m v_{gs_2} \times 15.6 \times 10^3 \tag{2.1.8.13}$$

$$v_{gs_3} = -78 \times v_{gs_2} \tag{2.1.8.14}$$

$$v_{gs_3} = \frac{6084 \times v_s}{\left(1 + j\frac{f}{10^5}\right)\left(1 + j\frac{f}{10^6}\right)\left(1 + j\frac{f}{10^7}\right)} \tag{2.1.8.15}$$

$$\tag{2.1.8.16}$$

Similarly, at the third N-MOSFET,

$$-v_o = -g_m v_{gs_3} \times 3.29 \times 10^3 \tag{2.1.8.17}$$

$$v_o = 16.45 \times v_{gs_3} \tag{2.1.8.18}$$

$$v_o = \frac{10^5 \times v_s}{\left(1 + j\frac{f}{10^5}\right)\left(1 + j\frac{f}{10^6}\right)\left(1 + j\frac{f}{10^7}\right)} \tag{2.1.8.19}$$

$$\frac{v_o}{v_s} = \frac{10^5}{\left(1 + j\frac{f}{10^5}\right)\left(1 + j\frac{f}{10^6}\right)\left(1 + j\frac{f}{10^7}\right)} \tag{2.1.8.20}$$

$$G(f) = \frac{10^5}{\left(1 + j\frac{f}{10^5}\right)\left(1 + j\frac{f}{10^6}\right)\left(1 + j\frac{f}{10^7}\right)} \tag{2.1.8.21}$$

2.1.9. Design a Circuit Model that follows the Transfer Function $H(f)$

Solution:

On Bode Plot is H is independent of frequency. So, H should not involve any Reactive Elements. So, H is a combination of Resistors or a Voltage Divider.

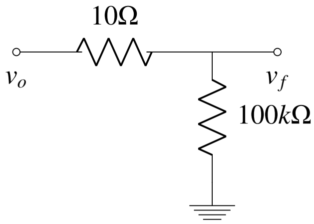

Fig. 2.1.9

$$v_f = \frac{10}{10 + 10^5} \times v_o \tag{2.1.9.1}$$

$$v_f \approx 10^{-4} v_o \tag{2.1.9.2}$$

$$\frac{v_f}{v_o} \approx 10^{-4} \tag{2.1.9.3}$$

$$H(f) = 10^{-4} \tag{2.1.9.4}$$

3 Bode Plot

4 Second order System

5 Routh Hurwitz Criterion

6 State-Space Model

7 Nyquist Plot

8 Compensators

9 Gain Margin

10 Phase Margin

11 Oscillator

12 Root Locus

13 Polar Plot

14 PID Controller