Control Systems

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Contents

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

1.3 Example

2 Gain of Feedback Circuits

2.1 Current Amplifiers

2.1.1. For the feedback current amplifier shown in 2.1.1, Draw the Small-Signal Model

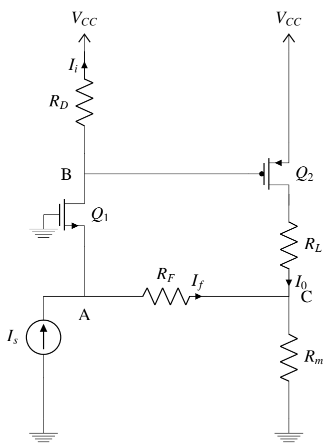

Fig. 2.1.1

Solution: While drawing a Small-Signal Model, we goung all constant voltage sources and open all constant current sources. All Small-Signal paramters are obtained from DC-Analysis of the circuit.

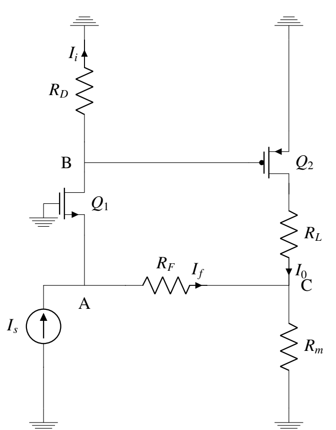

Fig. 2.1.1

2.1.2. Describe the importance of given Amplifier Topology.
Solution: This Feedback Topology is also known as Shunt-Series Feedback because of the parallel (or shunt) resistance at the input, and the series resistance at the output. This topology not only stabilizes the current gain but also results in a lower input resistance, and a higher output resistance, both desirable properties for a current amplifier. The decrease in input resistance results because the feedback current I_f subtracts from the input current I_s , and thus a lower current enters the basic current amplifier. This in turn results in a lower voltage at the amplifier input, that is, across the current source I_s.

2.1.3. Describe how the given circuit is a Negetive Feedback Amplifier.
Solution: For the feedback to be negative, I_f must have the same polarity as I_s. To ascertain that this is the case, we assume an increase in I_s and follow the change around the loop: An increase in I_s causes I_i to increase and the drain voltage of Q_1 will increase. Since this voltage is applied to the gate of the p-channel device Q_2 , its increase will cause I_s , the drain current of Q_2, to decrease. Thus, the voltage across R_M will decrease, which will cause I_f to increase. This is the same polarity assumed for the initial change in I_s, verifying that the feedback is indeed negative.

2.1.4. Find the Expression for the Open-Loop Gain $A = \frac{I_o}{I_i}$, from the Small-Signal Model. For simplicity, neglect the Early effect in Q_1 and Q_2.
Solution: In Small-Signal Model,

$$v_B = I_i R_D \tag{2.1.4.1}$$

$$v_{gs_2} = I_i R_D \tag{2.1.4.2}$$

In Small-Signal Analysis, P-MOSFET is modelled as a current source where current flows from Source to Drain, and the value of current is In Small-Signal Model,

$$I_o = -g_{m_2} v_{gs_2} = -g_{m_2} I_i R_D \tag{2.1.4.3}$$

So, the Open-Circuit Gain is

$$A = \frac{I_o}{I_i} = -g_{m_2} R_D \tag{2.1.4.4}$$

2.1.5. Find the Expression of the Feedback Factor $\beta = \frac{I_f}{I_o}$, from Small-Signal Model. For simplicity, neglect the Early effect in Q_1 and Q_2.
Solution:
To obtain β , we observe that I_o is fed to a current divider formed by R_M and R_F. It is assumed that R_F is a Large Resistance compared to Input resistance of Amplifier and so most of the current flows through it. Hence the voltage at point 'A', $v_A \simeq 0$. So R_F and R_M are parallel

and Voltage Drop across them is same.

$$(I_o + I_f)R_M \simeq -I_f R_o \tag{2.1.5.1}$$

$$\frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \tag{2.1.5.2}$$

So, the Feedback Factor,

$$\beta \equiv \frac{I_f}{I_o} \simeq -\frac{R_M}{R_F + R_M} \tag{2.1.5.3}$$

2.1.6. Find the Expression for the Closed-Loop Gain $A_f = \frac{I_o}{I_s}$. For simplicity, neglect the Early effect in Q_1 and Q_2.

Solution:

From Open-Loop Gain and Feedback Factor,

$$I_s = I_i + I_f \tag{2.1.6.1}$$

$$I_s = \frac{I_o}{A} + \beta I_o \tag{2.1.6.2}$$

$$AI_s = I_o(1 + A\beta) \tag{2.1.6.3}$$

$$\frac{I_o}{I_s} = \frac{A}{1 + A\beta} \tag{2.1.6.4}$$

$$\frac{I_o}{I_s} = -\frac{g_{m_2} R_D}{1 + g_{m2} R_D / \left(1 + \frac{R_F}{R_M}\right)} \tag{2.1.6.5}$$

So, the value of Closed-Loop Gain is

$$A_f = \frac{I_o}{I_s} = -\frac{g_{m2} R_D}{1 + g_{m2} R_D / \left(1 + \frac{R_F}{R_M}\right)} \tag{2.1.6.6}$$