Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

1 Signal Flow Graph

2 Gain of Feedback Circuits

2.1 Estimation of Voltage Gain

2.1.1. Consider the Magnitude Bode Plot and Phase Bode Plot 2.1.1 of Open-Loop Transfer Function of an Amplifier. Estimate the Open-Loop Transfer Function. (Assume $'A'$ as $'G'$ and $'\beta'$ as $'H'$)

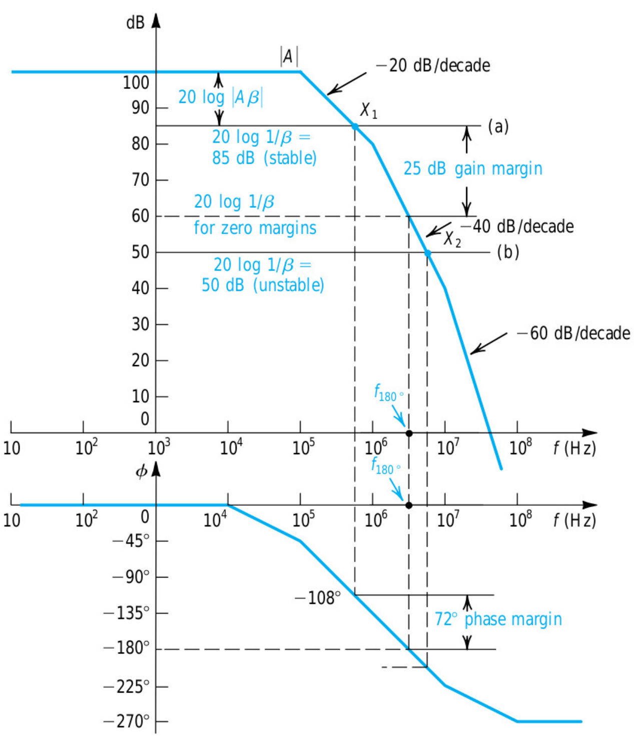

Fig. 2.1.1: Magnitude and Phase Bode Plot

Solution: Let $G(f)$ be the Open-Loop Transfer Function,

$$G(f) = \begin{cases} 100 & 0 < f < 10^5 \\ 200 - 20log(f) & 10^5 < f < 10^6 \\ 320 - 40log(f) & 10^6 < f < 10^7 \\ 460 - 60log(f) & 10^7 < f \end{cases} \tag{2.1.1.1}$$

$$\nabla G(f) = \frac{d(G(f))}{d(log(f))} = \begin{cases} 0 & 0 < f < 10^5 \\ -20 & 10^5 < f < 10^6 \\ -40 & 10^6 < f < 10^7 \\ -60 & 10^7 < f \end{cases} \tag{2.1.1.2}$$

As we know that, **When a pole is encountered the slope always decreases by 20 dB/decade** and **When a zero is encountered the slope always increases by 20 dB/decade**. So, by observing 2.1.1 it can be concluded that we are having Poles at $f = 10^5 Hz, 10^6 Hz, 10^7 Hz$ and No Zeros.

So, the Open-Loop Transfer Function $G(f)$ is

$$G(f) = \frac{10^5}{\left(1 + j\frac{f}{10^5}\right)\left(1 + j\frac{f}{10^6}\right)\left(1 + j\frac{f}{10^7}\right)} \tag{2.1.1.3}$$

2.1.2. Calculate the Phase of Open-Loop Transfer Function.
Solution:
Phase of Open-Loop Transfer Function = ϕ

$$\phi = -\left[\tan^{-1}\left(f/10^5\right) + \tan^{-1}\left(f/10^6\right) + \tan^{-1}\left(f/10^7\right)\right] \tag{2.1.2.1}$$

2.1.3. Determine the Closed-Loop Voltage Gain of the System assuming $|GH| \gg 1$ and also assuming the block diagram of Control System is 2.1.3

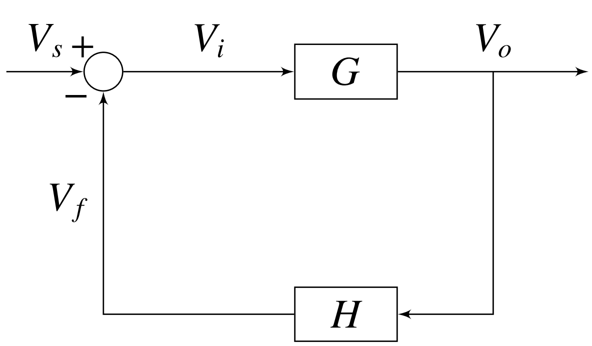

Fig. 2.1.3

Solution:
The Closed-Loop Voltage Gain of the Control System is

$$T = \frac{V_o}{V_s} = \frac{G}{1 + GH} \tag{2.1.3.1}$$

$$20\log(T) = 20\log(G) - 20\log(1 + GH) \tag{2.1.3.2}$$

Considering the assumption $|GH| \gg 1$, It can be written as

$$20\log(1 + GH) = 20\log(GH) \tag{2.1.3.3}$$

So,

$$20\log(T) = 20\log(G) - 20\log(GH) \tag{2.1.3.4}$$
$$20\log(T) = -20\log(H) \tag{2.1.3.5}$$
$$20\log(T) = 20\log(\frac{1}{H}) \tag{2.1.3.6}$$
$$T = \frac{1}{H} \tag{2.1.3.7}$$

So, The value of Closed-Loop Voltage Gain of the Control System under the assumption, $|GH| \gg 1$ is $T = \frac{1}{H}$

2.1.4. What is the value of Loop-Gain?
Solution:
The value of Loop-Gain can be calculated by the difference of 2-curves $20\log|A|$ and $20log(\frac{1}{H})$. The difference between the two curves will be

$$20\log|G| - 20\log\frac{1}{H} = 20\log|GH| \tag{2.1.4.1}$$

2.1.5. Define Phase-Margin
Solution:
Phase-Margin: The phase margin is defined as the angle in degrees by which the phase angle is smaller than $-180°$ at the gain crossover, the gain crossover being the frequency at which the open-loop gain first reaches 1.

2.1.6. Find the frequencies for which phase margins are 90° and 45° respectively?
Solution:
Let Phase Margin be $\alpha = 90°$. Then,

$$\alpha = \phi - (-180°) \tag{2.1.6.1}$$
$$\phi = -180° + \alpha \tag{2.1.6.2}$$
$$\phi = -90° \tag{2.1.6.3}$$

So, by the definition of Phase-Margin, at $\phi = -90^\circ$, $|GH| = 1$. The value of $\phi = -90^\circ$ between poles $f = 10^5 Hz, 10^6 Hz$. Assuming the Poles are farther apart,

$$\tan^{-1}(\frac{f}{10^7}) \approx 0 \quad (2.1.6.4)$$

where $10^5 < f < 10^6$
So,

$$-\tan^{-1}\left(f/10^5\right) - \tan^{-1}\left(f/10^6\right) = -90 \quad (2.1.6.5)$$

$$\tan^{-1}\left(f/10^5\right) + \tan^{-1}\left(f/10^6\right) = 90 \quad (2.1.6.6)$$

$$\tan^{-1}\left(f/10^5\right) = 90 - \tan^{-1}\left(f/10^6\right) \quad (2.1.6.7)$$

$$\tan^{-1}\left(f/10^5\right) = \cot^{-1}\left(f/10^6\right) \quad (2.1.6.8)$$

$$\tan^{-1}\left(f/10^5\right) = \tan^{-1}\left(10^6/f\right) \quad (2.1.6.9)$$

$$f^2 = 10^{11} \quad (2.1.6.10)$$

$$f = 3.162 \times 10^5 \quad (2.1.6.11)$$

So, the approximate value of f at which Phase Margin is 90° is $f = 3.162 \times 10^5 Hz$.

Similarly let Phase Margin be $\alpha = 45^\circ$. Then,

$$\alpha = \phi - (-180^\circ) \quad (2.1.6.12)$$

$$\phi = -180^\circ + \alpha \quad (2.1.6.13)$$

$$\phi = -135^\circ \quad (2.1.6.14)$$

So, by the definition of Phase-Margin, at $\phi = -135^\circ$, $|GH| = 1$. The value of $\phi = -135^\circ$ aproximately at poles $f = 10^6 Hz$.
So, the approximate value of f at which Phase Margin is 45° is $f = 10^6$.

2.1.7. Find the minimum values of Closed-Loop Voltage Gain for which phase margins are 90° and 45° respectively
Solution:
For $\alpha = 90^\circ$,

$$f = 3.162 \times 10^5 \quad (2.1.7.1)$$

By substituting f in Open-Loop Gain $G(f)$ (assuming poles are far part),

$$G(f) = 200 - 20log(3.162 \times 10^5) \quad (2.1.7.2)$$

$$G(f) = 90dB \quad (2.1.7.3)$$

$$G = 3.1625 \times 10^4 \quad (2.1.7.4)$$

At that $f = 3.162 \times 10^5$,

$$H = \frac{1}{G} \quad (2.1.7.5)$$

$$H = 3.162 \times 10^{-5} \quad (2.1.7.6)$$

The minimum value of Closed-Loop Gain occurs at $|GH| \gg 1$ and the value of Closed-Loop Gain is $T = \frac{1}{H}$

$$T = \frac{1}{H} = 3.1625 \times 10^4 \quad (2.1.7.7)$$

So, The minimum value of Closed-Loop Gain with Phase Margin equal to $\alpha = 90^\circ$ is $T_{min} = 3.1625 \times 10^4$.

For $\alpha = 45^\circ$,

$$f = 10^6 \quad (2.1.7.8)$$

By substituting f in Open-Loop Gain $G(f)$ (assuming poles are far part),

$$G(f) = 200 - 20log(10^6) \quad (2.1.7.9)$$

$$G(f) = 80dB \quad (2.1.7.10)$$

$$G = 10^4 \quad (2.1.7.11)$$

At that $f = 10^6$,

$$H = \frac{1}{G} \quad (2.1.7.12)$$

$$H = 10^{-4} \quad (2.1.7.13)$$

The minimum value of Closed-Loop Gain occurs at $|GH| \gg 1$ and the value of Closed-Loop Gain is $T = \frac{1}{H}$

$$T = \frac{1}{H} = 10^4 \quad (2.1.7.14)$$

So, The minimum value of Closed-Loop Gain with Phase Margin equal to $\alpha = 45^\circ$ is $T_{min} = 10^4$.

3 Bode Plot

4 Second order System

5 Routh Hurwitz Criterion

6 State-Space Model

7 Nyquist Plot

8 Compensators

9 Gain Margin

10 Phase Margin

11 Oscillator

12 Root Locus

13 Polar Plot

14 PID Controller