TEACHING STATEMENT

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In the most broad sense, my teaching philosophy could be labeled as "guided self-discovery". More specifically, I believe that the best learning outcomes typically arise when a student is able to grapple with most of the material themselves, with guidance and encouragement for when they face difficulties. My belief in this approach stems from a large volume of personal experience in both being a student and also as a teacher. My first experience with this style of teaching was at my undergraduate institution. I attended a large state university, and due to its size, experiences such as reading courses or even relationships with professors beyond the formal student-teacher relationship were uncommon. Eventually, I became friends with a graduate student who took me on as an informal mentee. By letting me explore a subject as far as I could go, and then giving me problems to think about or suggestions to overcome a sticking point, they helped me explore my various interests in mathematics, allowing me to gain exposure and mathematical maturity far beyond what I would have obtained through normal coursework. To this day, I credit much of my success and passion for the subject to their willingness to engage with me, as well as their approach to being a mentor.

Keeping in mind my experiences in undergraduate, the first opportunity I had to implement such a discovery-based approach was when I was the instructor for the course "Math of Decision and Beauty" (M106). This course, intended for general education students, tackles a number of topics which are not seen in typical math curricula, such as game theory, voting theory, and even elementary projective geometry. During the entirety of the course, I was especially careful to try to structure a more individually motivated approach to the material. For example, during the game theory portion, I gave the students monopoly money, and had them play some of the various games that we were modeling in lecture. To give them some incentive to win, the amount of monopoly money they had at the end translated into some amount of extra credit points. As another example, during the voting theory portion I allowed the students to vote for a class president using a voting system of their choice. To make things more interesting, they were also encouraged to try to rig this election using a handful of strategic voting tactics that I taught them. I believe that this approach had a very positive impact, both on their relation to the material, as they were one of my highest performing classes I've taught, but also in their experiences in the classroom, as I believe that it helped create a positive and encouraging environment for them to learn. As an example, one student wrote the following in their review after the semester

> "... In class he allowed us to ask questions and let us feel comfortable to be involved. I think teachers that help you add reason/logic when you're doing math are great because I learn best by not memorizing but being able to logically understand why the answer is what it is, which makes math fun instead of scary. I've always feared getting answers wrong but he reminded us that practicing is the only way to get better."

Perhaps one of my most valuable teaching experiences in graduate school was actually an assisting/grading assignment. I had been assigned to grade for a Euclidean geometry course intended for secondary education majors with Dr. Chris Judge as the instructor. Dr. Judge was running a class loosely based on Moore's method, where the students were split into small groups, had assigned reading, and from that reading they were expected to pose and answer their own questions. They started from only primitive notions, so the sophisticated tools they had learned in other classes were (at least temporarily) off the table. My job was to guide them in the right direction by asking probing questions, giving suggestions, and sometimes just by being encouraging. At first the students were reluctant, but after a few weeks they began to ask more and more questions and express a genuine desire to have them answered. This experience has taught me how to properly guide students in a more exploratory-format beyond just group work detailed in the previous paragraph. I learned the hard way on several occasions that this can be challenging. You do not want to give them the answer and prevent them from finding their own explanation, but also one cannot let them drift too far down paths that lead to dead-ends.

One particular problem which I think demonstrated their creativity and was one of my successes as a guide was the class' eventual justification for the geometric construction of the centroid of a triangle. One takes a line from a vertex of the triangle to the midpoint of the opposite edge. They were asked to prove that the resulting three lines always intersect in a point, called the centroid. Their solution was related to Archimedes' method of exhaustion: one recursively constructs smaller triangles by joining the midpoints of the edges of the previous triangle, and the original lines join the vertices to the midpoints on the smaller triangles. They argued that since the triangles eventually came to a point, the lines would have to intersect in a point. This arose as Dr. Judge and I had given them an example on how to estimate the area of a circle by approximating it with regular polygons. However the students recognized on their own that such a technique could be applied to the centroid problem, and it only took a few nudges from myself before they had the full solution on hand. I was particulary proud of them for coming up with such a creative solution.

The students, who we had asked to describe their experiences beforehand, had come in with a relatively bleak view of mathematics, seeing it as formulaic and static. By the end of the course however, they realized that mathematics is dynamic and very much alive, rooted in human curiosity rather then sterile logic more suited for machines. The experience of working with students who were, in some sense, discovering the geometry of the ancient Greeks for the first time further cemented my belief that mathematics is best learned by inventing it yourself (perhaps with some generous help by the instructor).

At Indiana University Bloomington, I have been the instructor of record for a course on eight different occasions, which has afforded me a generous amount of teaching experience at a wide variety of mathematical levels, from basic algebra to calculus. For many of them, I was responsible for assigning homework, creating quizzes and exams, and structuring classroom time. My previous experiences have demonstrated to me that completely self-discovery based course can have practical drawbacks. It is very time intensive on the students, usually not enough material can be covered in the time alloted, and without enough support the students can even become overwhelmed and discouraged. This is why with many of the courses that I teach, I have adopted a middle-of-the-road approach, where I address the salient features, give context and motivation, and several examples before allowing students to explore the intricacies of the topic themselves. For example, in the "Math of Decision and Beauty", I like to justify why the complete bipartite graph $K_{3,3}$

is non-planar in a sequence of steps. We begin by talking about bipartite graphs, and in particular, which cycle graphs are bipartite. The answer is simple, but allowing them to discover it gives confidence and willingness to continue. Then we relate this to subgraphs of planar bipartite graphs. After some probing questions, they begin to realize that every face must be surrounded by a cycle of even length. It then takes a bit of encouragement, but most classes manage to deduce the needed inequality between the number of edges and faces, and then it is short work to apply Euler's identity to conclude that $K_{3,3}$ cannot be planar. All of this work is mostly done by the students by breaking it down into small manageable chunks. Once they understand all of the moving parts, they often find it very easy to see the larger picture.

Another place where my teaching philosophy is important is in my undergraduate mentoring. Among what I consider to be my most important contributions to the mathematics department at Indiana University is my involvement to the Directed Reading Program (DRP). The DRP pairs an undergraduate with a graduate student for an independent study lasting the duration of the semester. This provides the undergraduate valuable experience learning in a less-structured environment, and also provides the graduate student valuable experience being a mentor. In light of my undergraduate experiences detailed above, this program means a great deal to me. Upon entering graduate school at IU, I immediately recognized the enormous boost this afforded me, and so I wanted to ensure that I was able to provide a similar positive experience for other undergraduates. The directed reading program at that point was newly established at IU, and I was extremely grateful to be an active part of its formation.

An additional impact that this mentoring program has is that of diversity in mathematics. For any individual, a positive mentor-mentee relationship provides the mentee with an enormous amount of mathematical and personal confidence in their abilities. In particular, such a strong relationship is known to be one of the most important factors in the retention of minority and underrepresented groups in mathematics, and so I expect that a proactive approach to mentoring such individuals will help create a positive and enriching atmosphere for them to thrive.

During my involvement, I have mentored two projects, both with the same student. The first was an introduction to algebraic geometry using a problem-based approach, and the second was an advanced number theory project on elliptic curves. During these projects I proactively tried to develop a positive relationship with my mentee, as I was well aware of how important and impactful such a relationship could be. I can say proudly that I believe I succeeded, as he went from an undergraduate who had little idea of what he was interested in to a graduate student pursuing a thesis in algebraic geometry. We are good enough friends that I have since attended his wedding, and I am very proud of what he has accomplished so far. After mentoring these two very successful projects and at the request of the previous coordinator, I took over as the coordinator of the DRP at IU, and have ran the program every semester since.

In the future, wherever I end up, I plan to bring some version of this program with me. At many liberal arts institutions however, the exact same format will not work for obvious reasons, but there are several alternative formats that I think will work just as effectively. For one, having a "learning seminar" with supervision from myself is an excellent way to introduce several undergraduates at once to new material. Coupled with the tried and true method of individualized reading courses and undergraduate research, such a program could have a very positive impact for many.