# 1 PS 5.17

## 1.1

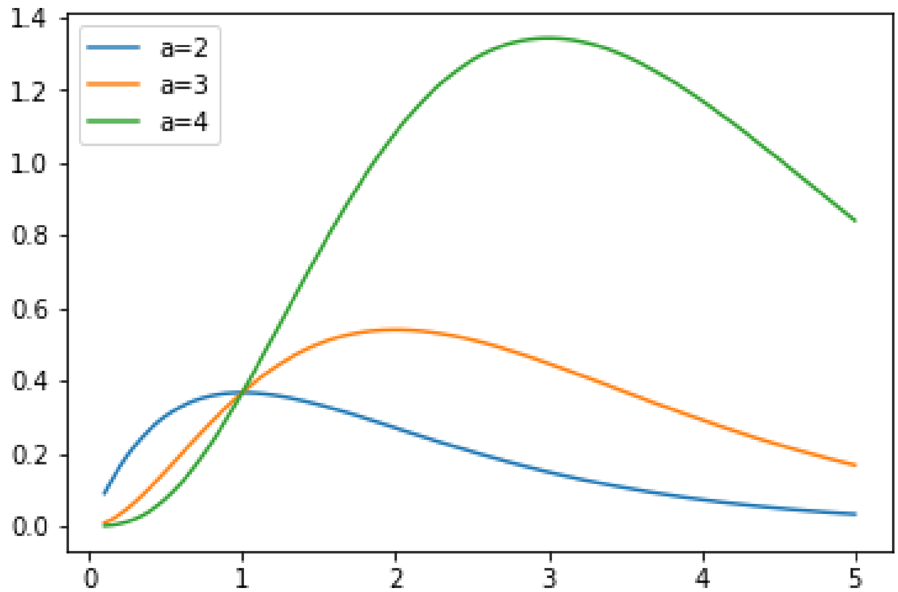


Figure 1: Integrand at a=2,3,4

## 1.2

$$
\begin{aligned}
&\frac{d}{dx}x^{a-1}e^{-x} \\
=&(a-1)x^{a-2}e^{x} - x^{a-1}e^{-x} \\
=&x^{a-2}e^{-x}\left(a-1-x\right)
\end{aligned}
$$

which has a root at $x = a - 1$ and at $x = 0$. However, at all higher derivatives, there will be some factor of $x^{-n}$, so the root a $x = 0$ can correspond to a maxima. However if $a - 1$ is negative, there is no peak in the defined range. An approximation would be the first moment i.e.

$$\int_0^{\inf} xe^{-x}x^{a-1} = \int_0^{\inf} e^{-x}x^{a} = \Gamma(a+1)$$

## 1.3

To peak $x$ around $x = \frac{1}{2}$, $\frac{1}{2} = \frac{a\pm1}{c+a\pm1} \Rightarrow c = a \pm 1$

### 1.4

$\ln(x) - x$ is roughly linear so there difference is roughly linear. Therefor, we wouldn't expect machine precision errors except for very large numbers. Very small numbers may have some precision error as $\ln(x) \to -\infty$ but exponentiation regulates errors to order of unity instead.

## 2

Time is scaled to a dimensionless parameter given by

$$\frac{t_i - \bar{t}}{\sum t_i^2}$$

where $\bar{t}$ is its mean.

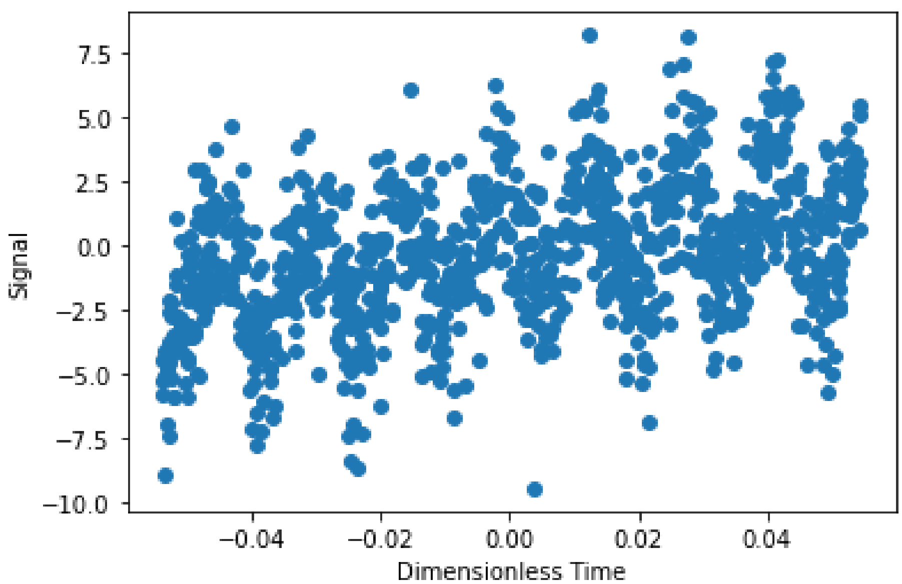


Figure 2: Signal

### 2.1

Assuming a gaussian like distribution, the residuals have a standard deviation of $\sim 3$ which means $\sim \%45$ of the date is outside the error bounds of the model which is unacceptable.

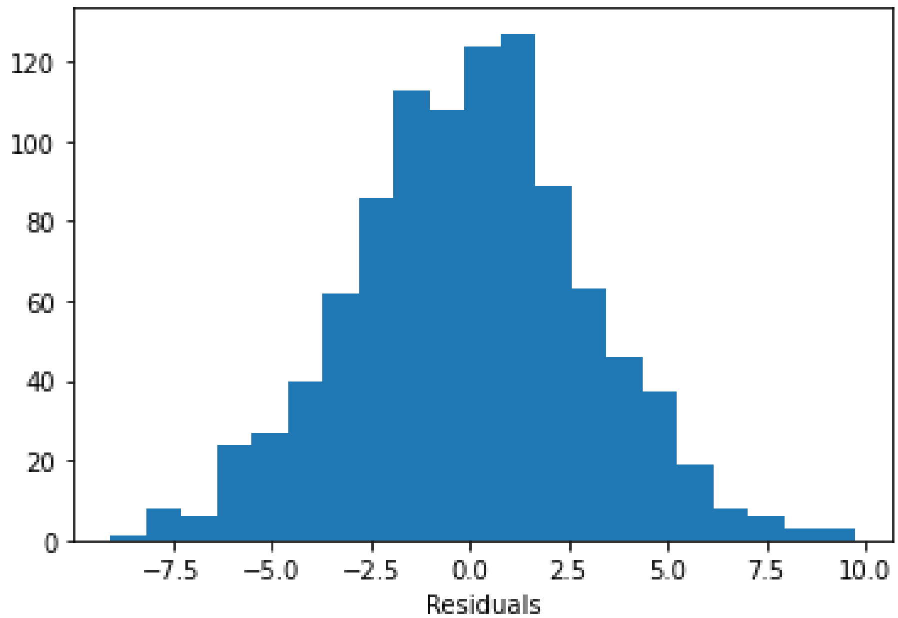


Figure 3: Residuals of 3rd order polynomial

### 2.2

The highest order polynomial within machine precision is at order 6 with condition number $7.4 \times 10^9$ which means calculations are done within 6 orders of magnitude of machine precision while getting the least possible residuals.

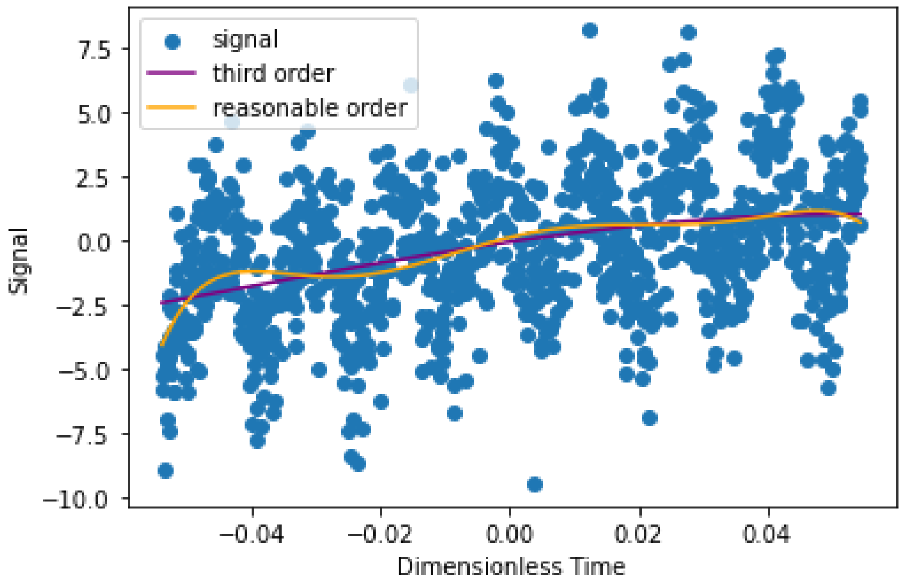


Figure 4: 3rd and 6th order plots

### 2.3

Both cos and sin fits seem to fare poorly with residuals $\sim 3000$. Both however clearly had a natural frequency of 4 times the base frequency.

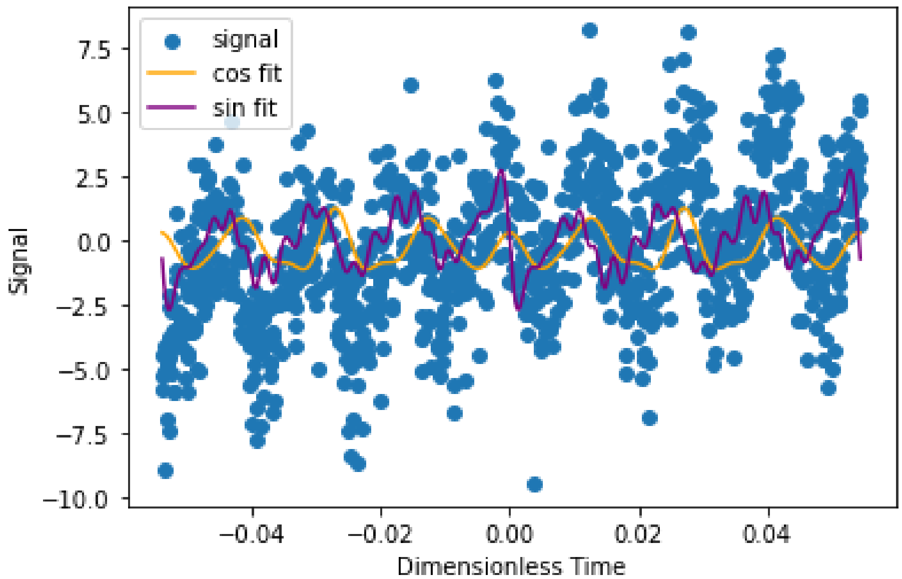


Figure 5: Trig Plots