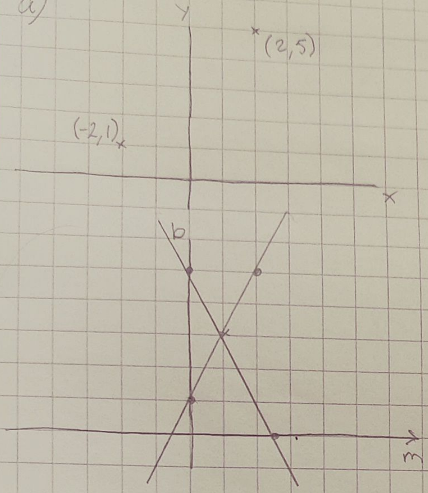
Ex round 4 Daniel Kusnetsoff

Task 1.

a)



$(x_0, y_0) = (-2, 1)$

$(x_1, y_1) = (2, 5)$

$b = -x_0 m + y_0 = -(-2) \cdot m + 1$

$b = -x_1 m + y_1 = -(2) \cdot m + 5$

$(m', b') = (1, 3)$

b)

$$y = \frac{-\cos(\theta)}{\sin(\theta)} x + \frac{\rho}{\sin(\theta)}$$

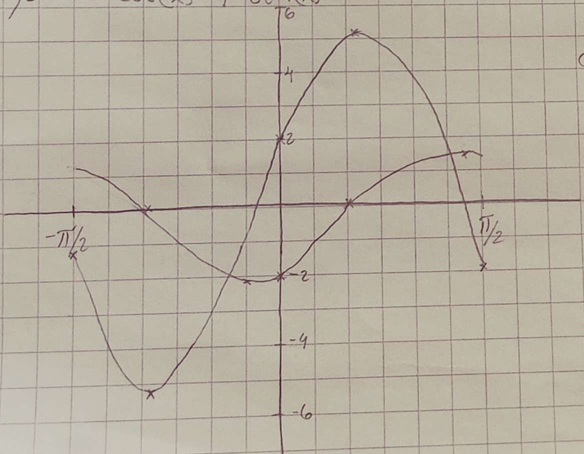
$$\rho = x \cdot \cos(\theta) + y \cdot \sin(\theta)$$

ρ = norm distance from line of origin

θ = angle between the norm and x-axis

$\rho = 2 \cdot \cos(x) + 5 \cdot (\sin(x))$

$\rho = -2 \cdot \cos(x) + 1 \cdot \sin(x)$



crosses at (-0,79; -2,12)

(2,35; 2,1)

task 1.

c)

In cartesian cordinate system the value of the slope is undefined. This leads to the issue, that the vertical lines require infinite values of m. Polar coordinate form does not have the issue

Another issue that the cartesian coordinates have is that it has unbounded parameter domains. The polar form is also therefore a good alternative to cartesian form.

EX round 4

February 6, 2022

[ ]:

```
Daniel Kusnetsoff
#Task 2
```

[ ]:

```
from linefitlsq import linefitlsq
import numpy as np
import matplotlib.pyplot as plt

# Load and plot points
data = np.load('points.npy')
x, y = data[0, :], data[1, :]
plt.figure(1, (10, 10))
plt.plot(x, y, 'kx')
plt.axis('scaled')

# RANSAC parameters
# m is the number of data points
m = np.size(x) * 1.0
# s is the size of the random sample
s = 2
# t is the inlier distance threshold
t = np.sqrt(3.84) * 2
# e is the expected outlier ratio
e = 0.8
# at least one random sample should be free
# from outliers with probability p
p = 0.999
# required number of samples
N_estimated = np.log(1 - p) / np.log(1 - (1 - e) ** s)

############### RANSAC loop ######################

# First initialize some variables
N = np.inf
sample_count = 0
max_inliers = 0
best_line = np.zeros((3, 1))
```

```
# Data points in homogeneous coordinates
points_h = np.vstack((x, y, np.ones((int(m)))))

while N > sample_count:
    # Pick two random samples
    samples = np.random.choice(np.arange(len(x)), 2, replace=False)
    id1 = samples[0]  # sample id 1
    id2 = samples[1]  # sample id 2

    # Determine the line crossing the points with the cross product of the␣
↪points (in homogeneous coordinates).
    # Also normalize the line by dividing each element by sqrt(a^2+b^2), where␣
↪a and b are the line coefficients

    ##-your-code-starts-here-##
    l = np.cross(points_h[:,id1],points_h[:,id2])

    ##-your-code-ends-here-##

    # Determine inliers by finding the indices for the line and data point dot
    # products (absolute value) that are less than inlier distance threshold.

    ##-your-code-starts-here-##
    inliers =[]

    for i in range(int(m)):
        distance = np.abs(np.dot(l, points_h[:,i]))
        if (distance <= t):
            inliers.append(i)


    ##-your-code-ends-here-##

    # Store the line in best_line and update max_inliers if the number of
    # inliers is the best so far
    inlier_count = np.size(inliers)
    if inlier_count > max_inliers:
        best_line = l
        max_inliers = inlier_count

    # Update the estimate of the outlier ratio
    e = 1 - inlier_count / m
    # Update also the estimate for the required number of samples
    N = np.log(1 - p) / np.log(1 - (1 - e) ** s)

    sample_count += 1
```

```python
# Least squares fitting to the inliers of the best hypothesis, i.e
# find the inliers similarly as above but this time for the best line.

##-your-code-starts-here-##
for i in range(int(m)):
    distance = np.abs(np.dot(best_line, points_h[:,i]))
    if (distance <= t):
        inliers.append(i)

x_inliers = x[inliers]
y_inliers = y[inliers]


##-your-code-ends-here-##

# Fit a line to the given points (non-homogeneous)
l = linefitlsq(x_inliers, y_inliers)

# Plot the resulting line and the inliers
k = -l[0] / l[1]
b = -l[2] / l[1]
plt.plot(np.arange(1, 101), k * np.arange(1, 101) + b, 'm-')
plt.plot(x[inliers], y[inliers], 'ro', markersize=7)
plt.show()
```

[ ]:
```
Task 3
```

[ ]:
```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.ndimage import maximum_filter
from scipy.ndimage.interpolation import map_coordinates
from scipy.ndimage.filters import convolve as conv2
from skimage.io import imread
from utils import gaussian2, maxinterp


# Familiarize yourself with the harris function
def harris(im, sigma=1.0, rel_thresh=0.0001, k=0.04):
    im = im.astype(np.float)  # Make sure im is float

    # Get smoothing and derivative filters
    g, _, _, _, _, _, = gaussian2(sigma)
    _, gx, gy, _, _, _, = gaussian2(np.sqrt(0.5))

    # Partial derivatives
    Ix = conv2(im, -gx, mode='constant')
    Iy = conv2(im, -gy, mode='constant')
```

```
    # Components of the second moment matrix
    Ix2Sm = conv2(Ix**2, g, mode='constant')
    Iy2Sm = conv2(Iy**2, g, mode='constant')
    IxIySm = conv2(Ix*Iy, g, mode='constant')

    # Determinant and trace for calculating the corner response
    detC = (Ix2Sm*IxIySm)-(Iy2Sm**2)
    traceC = Ix2Sm+IxIySm

    # Corner response function R
    # "Corner": R > 0
    # "Edge": R < 0
    # "Flat": |R| = small
    R = detC-k*traceC**2
    maxCornerValue = np.amax(R)

    # Take only the local maxima of the corner response function
    fp = np.ones((3,3))
    fp[1,1] = 0
    maxImg = maximum_filter(R, footprint=fp, mode='constant')

    # Test if cornerness is larger than neighborhood
    cornerImg = R>maxImg

    # Threshold for low value maxima
    y, x = np.nonzero((R > rel_thresh * maxCornerValue) * cornerImg)

    # Convert to float
    x = x.astype(np.float)
    y = y.astype(np.float)

    # Remove responses from image borders to reduce false corner detections
    r, c = R.shape
    idx = np.nonzero((x<2)+(x>c-3)+(y<2)+(y>r-3))[0]
    x = np.delete(x,idx)
    y = np.delete(y,idx)

    # Parabolic interpolation
    for i in range(len(x)):
        _,dx=maxinterp((R[int(y[i]), int(x[i])-1], R[int(y[i]), int(x[i])], R[int(y[i]), int(x[i])+1]))
        _,dy=maxinterp((R[int(y[i])-1, int(x[i])], R[int(y[i]), int(x[i])], R[int(y[i])+1, int(x[i])]))
        x[i]=x[i]+dx
        y[i]=y[i]+dy
```

```python
    return x, y, cornerImg


# Let's try to do Harris corner extraction and matching using our own
# implementation in a less black-box manner.

# Load images
I1 = imread('Boston1.png')/255.
I2 = imread('Boston2m.png')/255.

# Harris corner extraction, take a look at the source code above
x1, y1, cimg1 = harris(I1)
x2, y2, cimg2 = harris(I2)

# Pre-allocate the memory for the 15*15 image patches extracted
# around each corner point from both images
patch_size = 15
npts1 = x1.shape[0]
npts2 = x2.shape[0]
patches1 = np.zeros((patch_size, patch_size, npts1))
patches2 = np.zeros((patch_size, patch_size, npts2))

# The following part extracts the patches using bilinear interpolation
k = (patch_size-1)/2.
xv, yv = np.meshgrid(np.arange(-k, k+1), np.arange(-k, k+1))
for i in range(npts1):
    patch = map_coordinates(I1, (yv + y1[i], xv + x1[i]))
    patches1[:, :, i] = patch
for i in range(npts2):
    patch = map_coordinates(I2, (yv + y2[i], xv + x2[i]))
    patches2[:, :, i] = patch


############################ SSD MEASURE ######################################
# Compute the sum of squared differences (SSD) of pixels' intensities
# for all pairs of patches extracted from the two images
distmat = np.zeros((npts1, npts2))
for i1 in range(npts1):
    for i2 in range(npts2):
        distmat[i1, i2] = np.sum((patches1[:,:,i1]-patches2[:,:,i2])**2)

# Next, compute pairs of patches that are mutually nearest neighbors
# according to the SSD measure
ss1 = np.amin(distmat, axis=1)
ids1 = np.argmin(distmat, axis=1)
ss2 = np.amin(distmat, axis=0)
ids2 = np.argmin(distmat, axis=0)
```

```python
pairs = []
for k in range(npts1):
    if k == ids2[ids1[k]]:
        pairs.append(np.array([k, ids1[k], ss1[k]]))
pairs = np.array(pairs)

# We sort the mutually nearest neighbors based on the SSD
sorted_ssd = np.sort(pairs[:,2], axis=0)
id_ssd = np.argsort(pairs[:,2], axis=0)

# Visualize the 40 best matches which are mutual nearest neighbors
# and have the smallest SSD values
Nvis = 40
montage = np.concatenate((I1, I2), axis=1)

plt.figure(figsize=(16, 8))
plt.suptitle("The best 40 matches according to SSD measure", fontsize=20)
plt.imshow(montage, cmap='gray')
plt.title('The best 40 matches')
for k in range(np.minimum(len(id_ssd), Nvis)):
    l = id_ssd[k]
    plt.plot(x1[int(pairs[l, 0])], y1[int(pairs[l, 0])], 'rx')
    plt.plot(x2[int(pairs[l, 1])] + I1.shape[1], y2[int(pairs[l, 1])], 'rx')
    plt.plot([x1[int(pairs[l, 0])], x2[int(pairs[l, 1])]+I1.shape[1]],
         [y1[int(pairs[l, 0])], y2[int(pairs[l, 1])]])


############################ NCC MEASURE ######################################
# Now, your task is to do matching in similar manner but using normalised
# cross-correlation (NCC) instead of SSD. You should also report the
# number of correct correspondences for NCC as shown above for SSD.
#
# HINT: Compared to the previous SDD-based implementation, all you need
# to do is to modify the lines performing the 'distmat' calculation
# from SSD to NCC.
# Thereafter, you can proceed as above but notice the following details:
# You need to determine the mutually nearest neighbors by
# finding pairs for which NCC is maximized (i.e. not minimized like SSD).
# Also, you need to sort the matches in descending order in terms of NCC
# in order to find the best matches (i.e. not ascending order as with SSD).

##-your-code-starts-here-##

distmat = np.zeros((npts1, npts2))

g_a = np.mean(patches1)
```

```
f_a = np.mean(patches2)
for i1 in range(npts1):
    for i2 in range(npts2):
        number = np.sum((patches1[:,:,i1] - g_a) * (patches2[:,:,i2] - f_a))
        den1 = np.sqrt(np.sum((patches1[:,:,i1] - g_a)**2) * np.sum((patches2[:
 ↪,:,i2] - f_a)**2))
        distmat[i1, i2] = number / den1

ss1 = np.amax(distmat, axis=1)
ids1 = np.argmax(distmat, axis=1)
ss2 = np.amax(distmat, axis=0)
ids2 = np.argmax(distmat, axis=0)

pairs = []
for k in range(npts1):
    if k == ids2[ids1[k]]:
        pairs.append(np.array([k, ids1[k], ss1[k]]))
pairs = np.array(pairs)

sorted_ncc = np.sort(pairs[:,2], axis=0)
sorted_ncc = np.flip(sorted_ncc) # Flip to desc. order
id_ncc = np.argsort(pairs[:,2], axis=0)
id_ncc = np.flip(id_ncc) # Flip to desc. order

##-your-code-ends-here-##


# Next we visualize the 40 best matches which are mutual nearest neighbors
# and have the smallest SSD values
Nvis = 40
montage = np.concatenate((I1, I2), axis=1)

plt.figure(figsize=(16, 8))
plt.suptitle("The best 40 matches according to NCC measure", fontsize=20)
plt.imshow(montage, cmap='gray')
plt.title('The best 40 matches')
for k in range(np.minimum(len(id_ncc), Nvis)):
    l = id_ncc[k]
    plt.plot(x1[int(pairs[l, 0])], y1[int(pairs[l, 0])], 'rx')
    plt.plot(x2[int(pairs[l, 1])] + I1.shape[1], y2[int(pairs[l, 1])], 'rx')

    plt.plot([x1[int(pairs[l, 0])], x2[int(pairs[l, 1])]+I1.shape[1]],
          [y1[int(pairs[l, 0])], y2[int(pairs[l, 1])]])
plt.show()


# b.
```

```
# The NCC measure outperforms the ssd as it takes the local average intensity␣
↪into account.
```

[ ]:
```
task 4
```

[ ]:
```
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    # Determinant and trace for calculating the corner response
    detC = (Ix2Sm*IxIySm)-(Iy2Sm**2)
    traceC = Ix2Sm+IxIySm

    # Corner response function R
    # "Corner": R > 0
    # "Edge": R < 0
    # "Flat": |R| = small
    R = detC-k*traceC**2
    maxCornerValue = np.amax(R)

    # Take only the local maxima of the corner response function
    fp = np.ones((3,3))
    fp[1,1] = 0
    maxImg = maximum_filter(R, footprint=fp, mode='constant')
```

```
    # Test if cornerness is larger than neighborhood
    cornerImg = R>maxImg

    # Threshold for low value maxima
    y, x = np.nonzero((R > rel_thresh * maxCornerValue) * cornerImg)

    # Convert to float
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    # Remove responses from image borders to reduce false corner detections
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    for i in range(len(x)):
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        _,dy=maxinterp((R[int(y[i])-1, int(x[i])], R[int(y[i]), int(x[i])],␣
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        x[i]=x[i]+dx
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```

```python
# The following part extracts the patches using bilinear interpolation
k = (patch_size-1)/2.
xv, yv = np.meshgrid(np.arange(-k, k+1), np.arange(-k, k+1))
for i in range(npts1):
    patch = map_coordinates(I1, (yv + y1[i], xv + x1[i]))
    patches1[:, :, i] = patch
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ids1 = np.argmin(distmat, axis=1)
ss2 = np.amin(distmat, axis=0)
ids2 = np.argmin(distmat, axis=0)

pairs = []
for k in range(npts1):
    if k == ids2[ids1[k]]:
        pairs.append(np.array([k, ids1[k], ss1[k]]))
pairs = np.array(pairs)

# We sort the mutually nearest neighbors based on the SSD
sorted_ssd = np.sort(pairs[:,2], axis=0)
id_ssd = np.argsort(pairs[:,2], axis=0)

# Visualize the 40 best matches which are mutual nearest neighbors
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plt.figure(figsize=(16, 8))
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plt.imshow(montage, cmap='gray')
plt.title('The best 40 matches')
for k in range(np.minimum(len(id_ssd), Nvis)):
    l = id_ssd[k]
```

```
    plt.plot(x1[int(pairs[l, 0])], y1[int(pairs[l, 0])], 'rx')
    plt.plot(x2[int(pairs[l, 1])] + I1.shape[1], y2[int(pairs[l, 1])], 'rx')
    plt.plot([x1[int(pairs[l, 0])], x2[int(pairs[l, 1])]+I1.shape[1]],
         [y1[int(pairs[l, 0])], y2[int(pairs[l, 1])]])


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# Now, your task is to do matching in similar manner but using normalised
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# You need to determine the mutually nearest neighbors by
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# Also, you need to sort the matches in descending order in terms of NCC
# in order to find the best matches (i.e. not ascending order as with SSD).

##-your-code-starts-here-##

distmat = np.zeros((npts1, npts2))

g_a = np.mean(patches1)
f_a = np.mean(patches2)
for i1 in range(npts1):
    for i2 in range(npts2):
        number = np.sum((patches1[:,:,i1] - g_a) * (patches2[:,:,i2] - f_a))
        den1 = np.sqrt(np.sum((patches1[:,:,i1] - g_a)**2) * np.sum((patches2[:
 ↪,:,i2] - f_a)**2))
        distmat[i1, i2] = number / den1

ss1 = np.amax(distmat, axis=1)
ids1 = np.argmax(distmat, axis=1)
ss2 = np.amax(distmat, axis=0)
ids2 = np.argmax(distmat, axis=0)

pairs = []
for k in range(npts1):
    if k == ids2[ids1[k]]:
        pairs.append(np.array([k, ids1[k], ss1[k]]))
pairs = np.array(pairs)

sorted_ncc = np.sort(pairs[:,2], axis=0)
sorted_ncc = np.flip(sorted_ncc) # Flip to desc. order
id_ncc = np.argsort(pairs[:,2], axis=0)
```
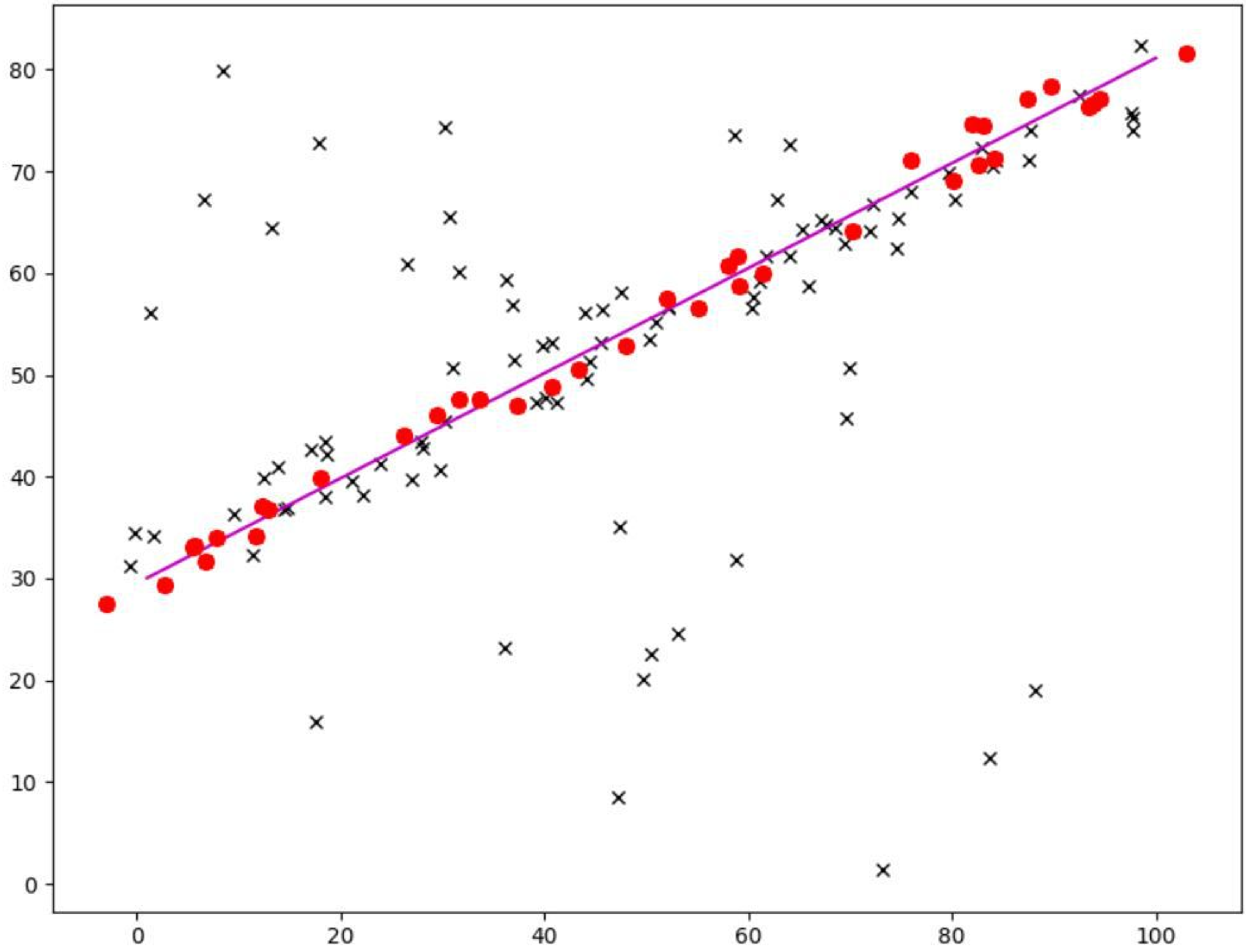
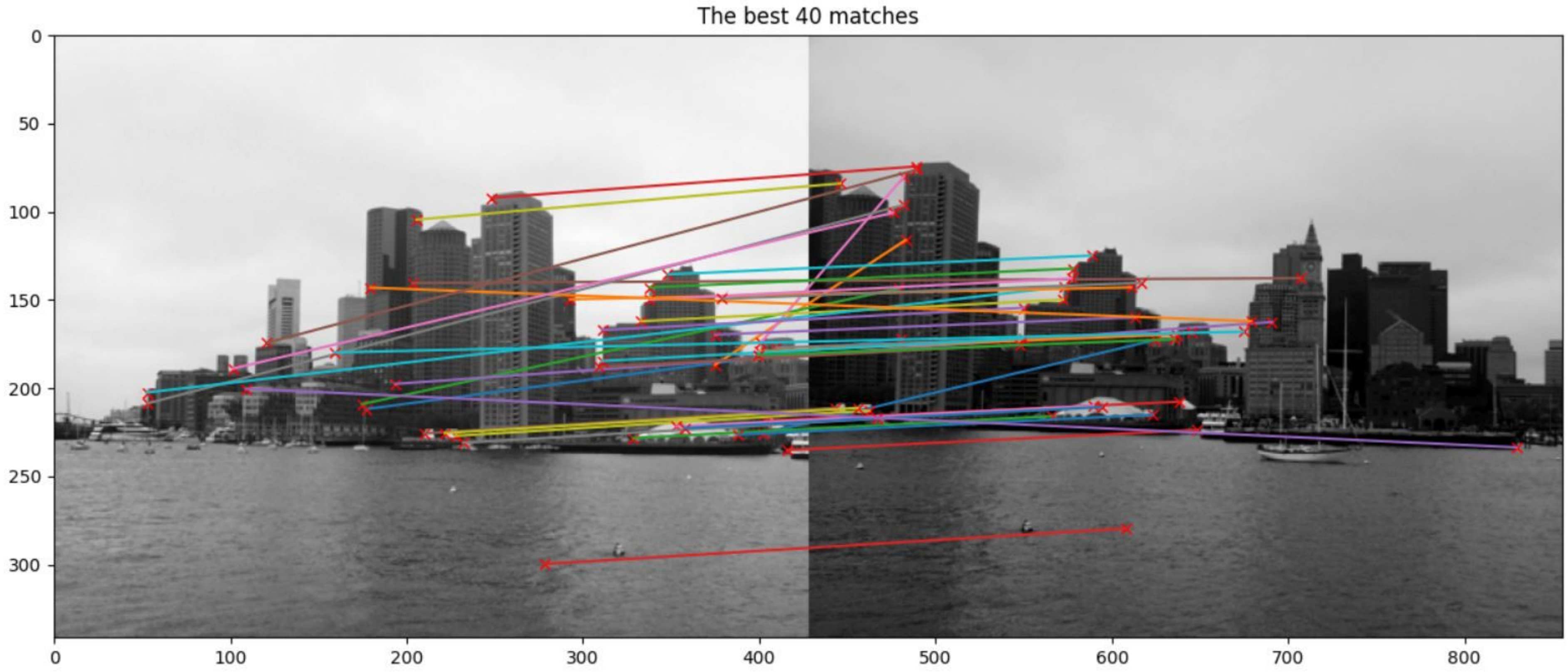

```python
id_ncc = np.flip(id_ncc) # Flip to desc. order

##-your-code-ends-here-##


# Next we visualize the 40 best matches which are mutual nearest neighbors
# and have the smallest SSD values
Nvis = 40
montage = np.concatenate((I1, I2), axis=1)

plt.figure(figsize=(16, 8))
plt.suptitle("The best 40 matches according to NCC measure", fontsize=20)
plt.imshow(montage, cmap='gray')
plt.title('The best 40 matches')
for k in range(np.minimum(len(id_ncc), Nvis)):
    l = id_ncc[k]
    plt.plot(x1[int(pairs[l, 0])], y1[int(pairs[l, 0])], 'rx')
    plt.plot(x2[int(pairs[l, 1])] + I1.shape[1], y2[int(pairs[l, 1])], 'rx')

    plt.plot([x1[int(pairs[l, 0])], x2[int(pairs[l, 1])]+I1.shape[1]],
         [y1[int(pairs[l, 0])], y2[int(pairs[l, 1])]])
plt.show()


# b.
# The NCC measure outperforms the ssd as it takes the local average intensity␣
 ↪into account.
```



80
70
60
50
40
30
20
10
0
0
20
40
60
80
100

The best 40 matches according to SSD measure



The best 40 matches according to NCC measure

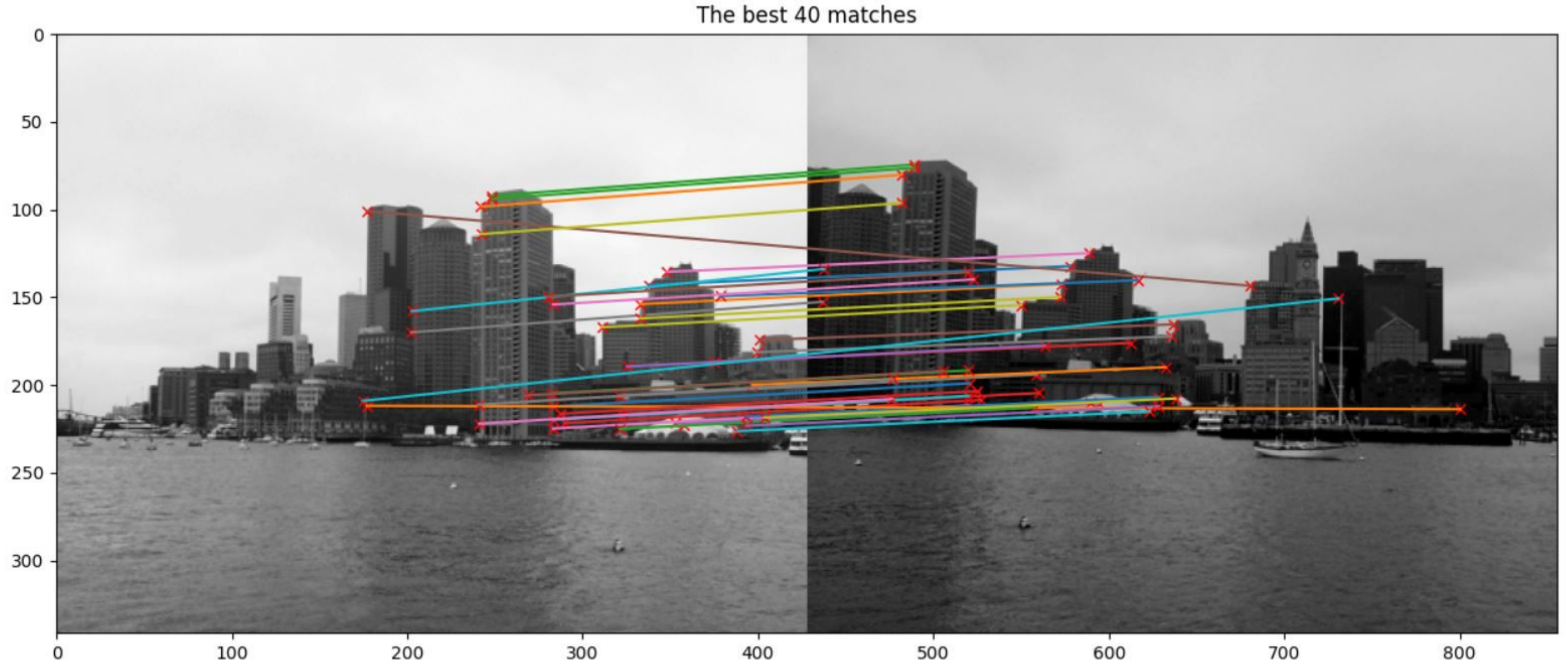


Figure 2

SURF matching with NNDR

Top 5 mutual nearest neighbors of SURF features

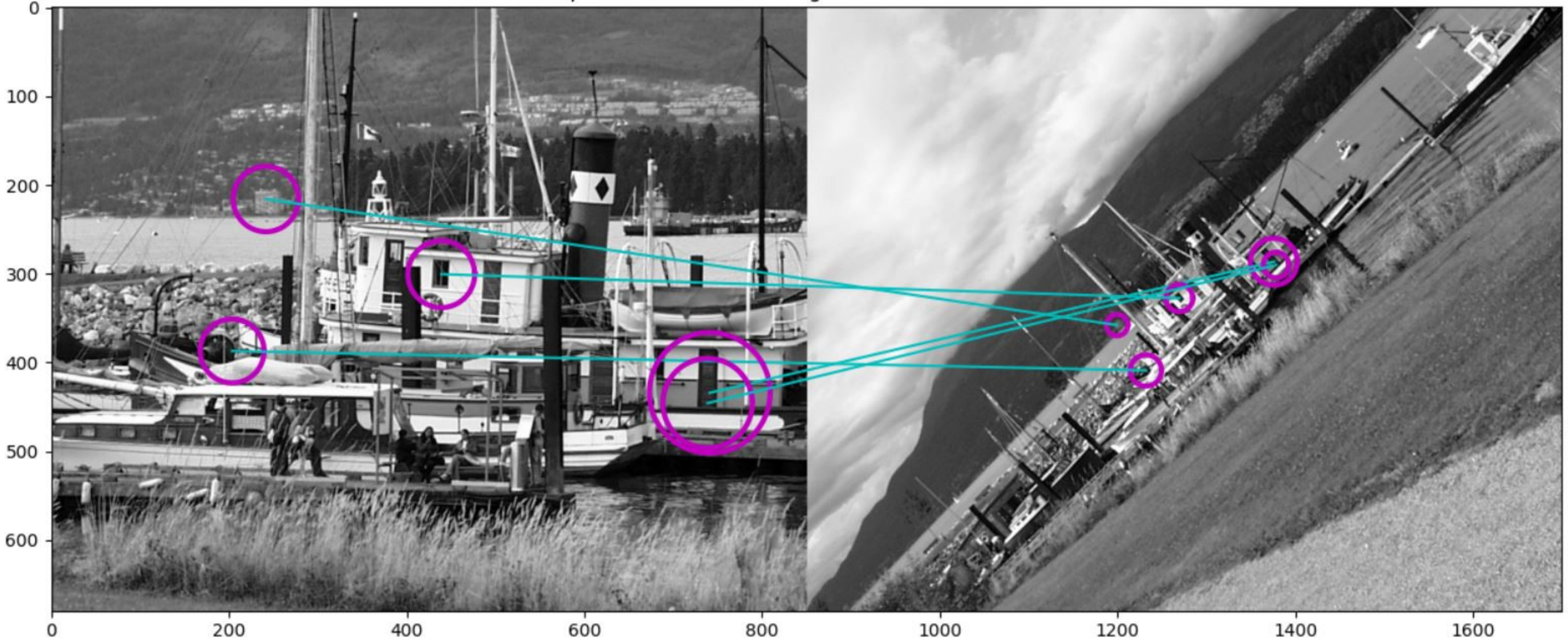


Figure 1

Top 5 mutual nearest neigbors of SURF features