

Ex. round 6.

Task 1. Fundamental matrix and essential matrix

a) Fundamental matrix is  $3 \times 3$  matrix rank 2. It is used for defining points in space from a different view. If  $x$  = first view  $x'$  = second view, image points give relation  $x'^T F x = 0$ .

Essential matrix is a specialized case of fundamental matrix, using normalized image coordinates. Compared to the fundamental matrix, the essential matrix has less degrees of freedom and some additional properties such as the normalized coordinates.

b)  $E = [t]_x R = R [R^T t]_x$  (Fundamental matrix corresponding to the pair of normalized cameras)  
 Essential Matrix  $\hat{x}'^T E \hat{x} = 0$  (substitution of  $\hat{x}$  and  $\hat{x}' \Rightarrow x'^T K'^{-T} E K^{-1} x = 0$ )  
 $E = K'^T F K$  (needed normalized camera matrix)  
 $K^{-1} p = [R | t]$

c) A  $3 \times 3$  homogenous matrix has nine elements.

$\rightarrow$  It has 8 independent ratios.

As the fundamental matrix is according to the constraint  $F=0$ , one degree is removed

$\rightarrow$  7 degrees of freedom.

d) The essential matrix has five degrees of freedom.

- Rotation matrix: 3

- Translation: 3

As the essential matrix is homogenous and there is scale ambiguity  $\rightarrow$  5 degrees of freedom.