DataStat840Exer4

October 24, 2022

1 DATA.STAT.840 Statistical Methods for Text Data Analysis

Exercises for Lecture 4: Document clustering, and introduction to N-grams Daniel Kusnetsoff

[1]:
```
import requests
import bs4
import re
import nltk
import fnmatch

root = '/Users/danie/'
base_url = 'https://www.gutenberg.org'

#%%
gutenberg_response_html = requests.get(base_url + '/browse/scores/top')
gutenberg_parsed = bs4.BeautifulSoup(gutenberg_response_html.content, 'html.
 ↪parser')
location = gutenberg_parsed.find(id='books-last30')
book_listing_tag = location.next_sibling.next_sibling
```

[2]:
```
#%%
def get_bookdata(book_listing, k):
    data = []
    for a_tag in book_listing.find_all('a')[:k]:
        book_name = re.match(r'(.*)(\(\d+\))', a_tag.text)
        book_name = book_name.group(1).strip()

        book_id = re.match(r'/ebooks/(\d+)', a_tag.get('href'))
        book_id = book_id.group(1)

        book_url = base_url + '/' + book_id + '/'

        data.append({"id":book_id, "name":book_name, "url":book_url, "fname":
 ↪''})

    return data
```

[3]:
```
def get_bookdata_byname(book_listing, name):
    data = []
    for a_tag in book_listing.find_all('a'):
        book_name = re.match(r'(.*)(\(\d+\))', a_tag.text)
        book_name = book_name.group(1).strip()

        if name not in book_name: continue

        book_id = re.match(r'/ebooks/(\d+)', a_tag.get('href'))
        book_id = book_id.group(1)

        book_url = base_url + '/' + book_id + '/'

        data.append({"id":book_id, "name":book_name, "url":book_url, "fname":
↪''})
        break

    return data
```

[4]:
```
def complete_urls(bookdata):
    for i in range(len(bookdata)):
        print("Searching txt-file for book with id {}".
↪format(bookdata[i]["id"]))
        indexurl = bookdata[i]["url"]

        bookindex_html = requests.get(indexurl)
        bookindex_parsed =  bs4.BeautifulSoup(bookindex_html.content,'html.
↪parser')
        bookindex_links = bookindex_parsed.find_all('a')
        bookindex_hrefs = [bil['href'] for bil in bookindex_links]

        book_filenames = [ bih for bih in bookindex_hrefs if fnmatch.
↪fnmatch(bih, '*.txt') ] #.*.txt

        bookdata[i]["url"] += book_filenames[0]
        bookdata[i]["fname"] = book_filenames[0]

    return
```

[5]:
```
def download_books(bookdata):
    for data in bookdata:
        print("Downloading book: {}".format(data["name"]))
        print("From source: {}".format(data["url"]))
        response = requests.get(data["url"], allow_redirects=True)
        #txtfiles already made
        with open(root + 'txtfiles/' + data["fname"], 'wb') as f:
            f.write(response.content)
```

```
#%%
book_name = 'The Wonderful Wizard of Oz'
bookdata = get_bookdata_byname(book_listing_tag, book_name)
complete_urls(bookdata)
download_books(bookdata)
```

```
Searching txt-file for book with id 55
Downloading book: The Wonderful Wizard of Oz by L. Frank  Baum
From source: https://www.gutenberg.org/55/55-0.txt
```

[6]:
```
# Download data from locally
def load_data_local(bookdata, fileloc):
    my_text = []
    for data in bookdata:
        try:
            with open(fileloc + data["fname"], 'r') as f:
                my_text.append(f.read())
        except:
            with open(fileloc + data["fname"], 'r', encoding='ISO-8859-1') as f:
                my_text.append(f.read())
        print("Book {} loaded.".format(data['id']))

    return my_text


book_texts = load_data_local(bookdata, root + 'txtfiles/')
```

```
Book 55 loaded.
```

[7]:
```
# Remove the start and end information, so only story text is left

def remove_headers(book_texts):
    start_header = '*** START'
    end_header = '*** END'
    new_texts = []
    for text in book_texts:
        start_loc = text.find(start_header)
        print(start_loc)
        start_loc = text[start_loc:].find('\n') + start_loc
        print(start_loc)
        end_loc = text.find(end_header)
        text = text[start_loc : end_loc]
        new_texts.append(text)

    return new_texts
```

```
book_texts = remove_headers(book_texts)
my_text = book_texts[0]

#%% Get the paragraphs 4.1 b

mytext_paragraphs = paragraphs=re.split('\n[ \n]*\n', my_text)
#paragraphs = paragraphs[1:-1]
```

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717
788
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[8]:
```
len(mytext_paragraphs)
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[8]:
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1141
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[9]:
```
paragraph_nltk_texts = []
for k in range(len(mytext_paragraphs)):
    temp_tokenizedtext = nltk.word_tokenize(mytext_paragraphs[k])
    temp_nltktext = nltk.Text(temp_tokenizedtext)
    paragraph_nltk_texts.append(temp_nltktext)
```

[10]:
```
paragraph_nltk_texts[0]
```

[10]:
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<Text: …>
```

[11]:
```
paragraph_lowercase_texts = []
for k in range(len(paragraph_nltk_texts)):
    temp_lowercase_text = []
    for l in range(len(paragraph_nltk_texts[k])):
        lowercase_word = paragraph_nltk_texts[k][l].lower()
        temp_lowercase_text.append(lowercase_word)
    temp_lowercasetest = nltk.Text(temp_lowercase_text)
    paragraph_lowercase_texts.append(temp_lowercase_text)
```

[12]:
```
#POS
def tagtowordnet(postag):
    wordnettag=-1
    if postag[0]=='N':
        wordnettag='n'
    elif postag[0]=='V':
        wordnettag='v'
    elif postag[0]=='J':
        wordnettag='a'
    elif postag[0]=='R':
        wordnettag='r'
    return(wordnettag)
```

[13]:
```
# Download wordnet resource if you do not have it already
nltk.download('wordnet')
# Download tagger resource if you do not have it already
nltk.download('averaged_perceptron_tagger')

lemmatizer=nltk.stem.WordNetLemmatizer()
```

```
[nltk_data] Downloading package wordnet to
[nltk_data]     C:\Users\danie\AppData\Roaming\nltk_data…
[nltk_data]   Package wordnet is already up-to-date!
[nltk_data] Downloading package averaged_perceptron_tagger to
[nltk_data]     C:\Users\danie\AppData\Roaming\nltk_data…
[nltk_data]   Package averaged_perceptron_tagger is already up-to-
[nltk_data]       date!
```

[14]:
```
def lemmatizetext(nltktexttolemmatize):
    # Tag the text with POS tags
    taggedtext=nltk.pos_tag(nltktexttolemmatize)
    # Lemmatize each word text
    lemmatizedtext=[]
    for l in range(len(taggedtext)):
        # Lemmatize a word using the WordNet converted POS tag
        wordtolemmatize=taggedtext[l][0]
        wordnettag=tagtowordnet(taggedtext[l][1])
        if wordnettag!=-1:
            lemmatizedword=lemmatizer.lemmatize(wordtolemmatize,wordnettag)
        else:
            lemmatizedword=wordtolemmatize
        # Store the lemmatized word
        lemmatizedtext.append(lemmatizedword)
    return(lemmatizedtext)


# lemmatization of text
#lemmatized_texts = lemmatizetext(lowercase_texts)
#lemmatized_texts = nltk.Text(lemmatized_texts)
```

[15]:
```
paragraph_lemmatizedtexts = []
for k in range(len(paragraph_lowercase_texts)):
    lemmatizedtext = lemmatizetext(paragraph_lowercase_texts[k])
    lemmatizedtext = nltk.Text(lemmatizedtext)
    paragraph_lemmatizedtexts.append(lemmatizedtext)
```

[16]:
```
paragraph_lemmatizedtexts[10]
```

[16]: <Text: have this think in mind , the story…>

[17]:
```
import numpy as np
myvocabularies=[]
myindices_in_vocabularies=[]
# Find the vocabulary of each document
for k in range(len(paragraph_lemmatizedtexts)):
    # Get unique words and where they occur
    temptext=paragraph_lemmatizedtexts[k]
    uniqueresults=np.unique(temptext,return_inverse=True)
    uniquewords=uniqueresults[0]
    wordindices=uniqueresults[1]
    # Store the vocabulary and indices of document words in it
    myvocabularies.append(uniquewords)
    myindices_in_vocabularies.append(wordindices)
myvocabularies[0]
```

[17]:
```
array([], dtype=float64)
```

[18]:
```
tempvocabulary=[]
for k in range(len(paragraph_lemmatizedtexts)):
    tempvocabulary.extend(myvocabularies[k])

# Find the unique elements among all vocabularies
uniqueresults=np.unique(tempvocabulary,return_inverse=True)
unifiedvocabulary=uniqueresults[0]
wordindices=uniqueresults[1]
```

[19]:
```
# Translate previous indices to unified vocabulary.

vocabularystart=0
myindices_in_unifiedvocabulary=[]
for k in range(len(paragraph_lemmatizedtexts)):
    # In order to shift word indices, we must temporarily
    # change their data type to a Numpy array
    tempindices=np.array(myindices_in_vocabularies[k])
    tempindices=tempindices+vocabularystart
    tempindices=wordindices[tempindices]
    myindices_in_unifiedvocabulary.append(tempindices)
    vocabularystart=vocabularystart+len(myvocabularies[k])
```

[20]:
```
unifiedvocabulary_totaloccurrencecounts=np.zeros((len(unifiedvocabulary),1))
unifiedvocabulary_documentcounts=np.zeros((len(unifiedvocabulary),1))
unifiedvocabulary_meancounts=np.zeros((len(unifiedvocabulary),1))
unifiedvocabulary_countvariances=np.zeros((len(unifiedvocabulary),1))
```

[21]:
```
for k in range(len(paragraph_lemmatizedtexts)):
    print(k)
    occurrencecounts=np.zeros((len(unifiedvocabulary),1))
```

```
    for l in range(len(myindices_in_unifiedvocabulary[k])):
        occurrencecounts[myindices_in_unifiedvocabulary[k][l]]= \
            occurrencecounts[myindices_in_unifiedvocabulary[k][l]]+1
    unifiedvocabulary_totaloccurrencecounts= \
        unifiedvocabulary_totaloccurrencecounts+occurrencecounts
    unifiedvocabulary_documentcounts= \
        unifiedvocabulary_documentcounts+(occurrencecounts>0)
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[22]:
```
# Mean occurrence counts over documents
unifiedvocabulary_meancounts= \
    unifiedvocabulary_totaloccurrencecounts/len(paragraph_lemmatizedtexts)
```

[23]:
```
#%% Inspect frequent words
# Sort words by largest total (or mean) occurrence count
highest_totaloccurrences_indices=np.argsort(\
    -1*unifiedvocabulary_totaloccurrencecounts,axis=0)
print(np.squeeze(unifiedvocabulary[\
    highest_totaloccurrences_indices[0:100]]))
print(np.squeeze(\
    unifiedvocabulary_totaloccurrencecounts[\
    highest_totaloccurrences_indices[0:100]]))
```

```
['the' ',' 'and' '.' 'be' 'to' 'of' 'a' 'â\x80\x9d' 'in' 'have' 'i' 'he'
 'you' 'her' 'she' 'they' 'it' 'that' 'say' 'dorothy' 'as' 'so' 'for'
 'with' 'not' 'at' 'but' 'all' 'them' 'do' 'scarecrow' 'his' ';' '?' 'me'
 'him' 'my' 'woodman' 'lion' 'come' 'on' 'will' 'oz' 'great' 'when' 'go'
 'make' 'â\x80\x9ci' 'tin' 'ask' 'little' 'witch' 'this' 'from' 'one'
 'could' 'then' 'see' 'there' 'would' 'we' 'if' 'get' 'up' 'out' 'who'
 'head' 'can' 'green' 'their' 'back' 'look' '!' 'no' 'think' 'girl' 'down'
 'know' 'by' 'toto' 'over' 'answer' 'upon' 'shall' 'find' 'give' 'again'
 'good' 'into' 'very' 'now' 'must' 'city' 'wicked' 'where' 'walk' 'after'
 'emerald' 'long']
[2982. 2703. 1600. 1597. 1433. 1110.  840.  801.  698.  481.  476.  452.
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  274.  272.  251.  243.  239.  233.  231.  219.  216.  196.  194.  186.
  179.  178.  175.  175.  169.  157.  156.  151.  150.  148.  148.  144.
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  113.  108.  106.  105.  105.  103.  103.  101.  101.  100.  100.   99.
   97.   97.   94.   94.   94.   93.   93.   90.   90.   87.   86.   85.
   81.   81.   81.   80.   80.   79.   77.   76.   75.   75.   72.   72.
   71.   70.   69.   69.]
```

[24]:
```
nltk.download('stopwords')

#%% Vocabulary pruning
nltkstopwords=nltk.corpus.stopwords.words('english')
pruningdecisions=np.zeros((len(unifiedvocabulary),1))
for k in range(len(unifiedvocabulary)):
    # Rule 1: check the nltk stop word list
    if (unifiedvocabulary[k] in nltkstopwords):
        pruningdecisions[k]=1
    # Rule 2: if the word is in the top 1% of frequent words
    #if (k in highest_totaloccurrences_indices[\
    #    0:int(np.floor(len(unifiedvocabulary)*0.01))]):
    #    pruningdecisions[k]=1
```

```
   # Rule 3: if the word occurs less than 4 times
    if(unifiedvocabulary_totaloccurrencecounts[k] < 4):
        pruningdecisions[k] = 1
    # Rule 4: if the word is too short
    if len(unifiedvocabulary[k])<2:
        pruningdecisions[k]=1
    # Rule 5: if the word is too long
    if len(unifiedvocabulary[k])>20:
        pruningdecisions[k]=1
    # Rule 6: if the word has unwanted characters
    # (here for simplicity only a-z allowed)
    if unifiedvocabulary[k].isalpha()==False:
        pruningdecisions[k]=1
```

```
[nltk_data] Downloading package stopwords to
[nltk_data]     C:\Users\danie\AppData\Roaming\nltk_data…
[nltk_data]   Package stopwords is already up-to-date!
```

[25]:
```
#%% Get indices of documents to remaining words
oldtopruned=[]
tempind=-1
for k in range(len(unifiedvocabulary)):
    if pruningdecisions[k]==0:
        tempind=tempind+1
        oldtopruned.append(tempind)
    else:
        oldtopruned.append(-1)
```

[26]:
```
#%% Create pruned texts
paragraph_prunedtexts=[]
myindices_in_prunedvocabulary=[]
for k in range(len(paragraph_lemmatizedtexts)):
    print(k)
    temp_newindices=[]
    temp_newdoc=[]
    for l in range(len(paragraph_lemmatizedtexts[k])):
        temp_oldindex=myindices_in_unifiedvocabulary[k][l]
        temp_newindex=oldtopruned[temp_oldindex]
        if temp_newindex!=-1:
            temp_newindices.append(temp_newindex)
            temp_newdoc.append(unifiedvocabulary[temp_oldindex])
    paragraph_prunedtexts.append(temp_newdoc)
    myindices_in_prunedvocabulary.append(temp_newindices)
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[27]:
```
print('Top 100 word-list after pruning unified vocabulary:\n')
remaining_indices = np.squeeze(np.where(pruningdecisions==0)[0])
remaining_vocabulary = unifiedvocabulary[remaining_indices]
remainingvocabulary_totaloccurrencecounts =␣
 ↪unifiedvocabulary_totaloccurrencecounts[remaining_indices]
remaining_highest_totaloccurrences_indices = np.
 ↪argsort(-1*remainingvocabulary_totaloccurrencecounts, axis=0)
print(np.
 ↪squeeze(remaining_vocabulary[remaining_highest_totaloccurrences_indices[0:
 ↪100]]))
```

```
print(np.
 ↪squeeze(remainingvocabulary_totaloccurrencecounts[remaining_highest_totaloccurrences_indice
 ↪100]]))
```

```
Top 100 word-list after pruning unified vocabulary:

['say' 'dorothy' 'scarecrow' 'woodman' 'lion' 'come' 'oz' 'great' 'go'
 'make' 'tin' 'ask' 'little' 'witch' 'one' 'could' 'see' 'would' 'get'
 'head' 'green' 'back' 'look' 'think' 'girl' 'know' 'toto' 'answer' 'upon'
 'find' 'shall' 'give' 'good' 'must' 'city' 'wicked' 'walk' 'emerald'
 'long' 'man' 'country' 'room' 'tree' 'away' 'heart' 'like' 'take' 'big'
 'time' 'way' 'carry' 'tell' 'saw' 'people' 'us' 'never' 'eye' 'reply'
 'stand' 'monkey' 'brain' 'live' 'well' 'many' 'chapter' 'day' 'forest'
 'first' 'run' 'road' 'ever' 'friend' 'soon' 'help' 'house' 'around'
 'much' 'keep' 'wish' 'wizard' 'arm' 'cry' 'beast' 'sit' 'thing' 'old'
 'mouse' 'call' 'land' 'beautiful' 'shoe' 'leave' 'air' 'woman' 'seem'
 'put' 'fly' 'quite' 'voice' 'begin']
[356. 347. 219. 175. 175. 169. 151. 150. 148. 144. 139. 139. 139. 137.
 122. 120. 119. 113. 105. 101. 100.  99.  97.  94.  94.  93.  90.  86.
  85.  81.  81.  81.  80.  75.  75.  72.  71.  69.  69.  68.  67.  66.
  65.  65.  65.  65.  64.  63.  62.  60.  59.  58.  58.  57.  56.  55.
  55.  55.  52.  52.  51.  49.  49.  49.  48.  47.  47.  47.  46.  46.
  45.  45.  44.  43.  42.  42.  42.  42.  41.  41.  41.  41.  41.  40.
  40.  39.  38.  38.  38.  38.  37.  36.  36.  35.  35.  35.  35.  35.
  34.  34.]
```

[28]:
```
import scipy
#%% Create TF-IDF vectors
n_docs=len(paragraph_prunedtexts)
n_vocab=len(remaining_vocabulary)
# Matrix of term frequencies
tfmatrix=scipy.sparse.lil_matrix((n_docs,n_vocab))
# Row vector of document frequencies
dfvector=scipy.sparse.lil_matrix((1,n_vocab))
# Loop over documents
for k in range(n_docs):
    # Row vector of which words occurred in this document
    temp_dfvector=scipy.sparse.lil_matrix((1,n_vocab))
    # Loop over words
    for l in range(len(paragraph_prunedtexts[k])):
        # Add current word to term-frequency count and document-count
        currentword=myindices_in_prunedvocabulary[k][l]
        tfmatrix[k,currentword]=tfmatrix[k,currentword]+1
        temp_dfvector[0,currentword]=1
    # Add which words occurred in this document to overall document counts
    dfvector=dfvector+temp_dfvector
```

```
# TF:length-normalized frequency
for i in range(n_docs):
    for j in range(len(tfmatrix.data[i])):
        tfmatrix.data[i][j] = tfmatrix.data[i][j]/len(tfmatrix.data[i])


# smoothed logarithmic idf
idfvector = np.squeeze(np.array(dfvector.todense()))
idfvector = np.log(1 + ((idfvector+1)**-1)*n_docs)

tfidfmatrix = scipy.sparse.lil_matrix((n_docs, n_vocab))
for k in range(n_docs):
    # tf and idf terms
    tfidfmatrix[k,:]=tfmatrix[k,:]*idfvector

# tf-idf matrix
#tfidfmatrix = scipy.sparse.lil_matrix((n_docs, n_vocab))
for k in range(n_docs):
    # find nonzero term frequencies
    tempindices = np.nonzero(tfmatrix[k, :])[1]
    tfterm = np.squeeze(np.array(tfmatrix[k, tempindices].todense()))
    tfidfmatrix[k, tempindices] = tfterm * idfvector[tempindices]
```

```
C:\Users\danie\Anaconda3\lib\site-packages\scipy\sparse\lil.py:512:
FutureWarning: future versions will not create a writeable array from
broadcast_array. Set the writable flag explicitly to avoid this warning.
  if not i.flags.writeable or i.dtype not in (np.int32, np.int64):
C:\Users\danie\Anaconda3\lib\site-packages\scipy\sparse\lil.py:514:
FutureWarning: future versions will not create a writeable array from
broadcast_array. Set the writable flag explicitly to avoid this warning.
  if not j.flags.writeable or j.dtype not in (np.int32, np.int64):
C:\Users\danie\Anaconda3\lib\site-packages\scipy\sparse\lil.py:518:
FutureWarning: future versions will not create a writeable array from
broadcast_array. Set the writable flag explicitly to avoid this warning.
  if not x.flags.writeable:
```

[29]:
```
#%% Use the TF-IDF matrix as data to be clustered
X=tfidfmatrix
# Normalize the documents to unit vector norm
tempnorms=np.squeeze(np.array(np.sum(X.multiply(X),axis=1)))
# If any documents have zero norm, avoid dividing them by zero
tempnorms[tempnorms==0]=1
X=scipy.sparse.diags(tempnorms**-0.5).dot(X)
n_data=np.shape(X)[0]
n_dimensions=np.shape(X)[1]
```

[30]:
```
# Function to initialize the Gaussian mixture model, create component parameters
def initialize_mixturemodel(X,n_components):
    # Create lists of sparse matrices to hold the parameters
    n_dimensions=np.shape(X)[1]
    n_data = np.shape(X)[0]
    mixturemodel_means=scipy.sparse.lil_matrix((n_components,n_dimensions))
    mixturemodel_weights=np.zeros((n_components))
    mixturemodel_covariances=[]
    mixturemodel_inversecovariances=[]
    for k in range(n_components):
        tempcovariance=scipy.sparse.lil_matrix((n_dimensions,n_dimensions))
        mixturemodel_covariances.append(tempcovariance)
        tempinvcovariance=scipy.sparse.lil_matrix((n_dimensions,n_dimensions))
        mixturemodel_inversecovariances.append(tempinvcovariance)
    # Initialize the parameters
    for k in range(n_components):
        mixturemodel_weights[k]=1/n_components
        # Pick a random data point as the initial mean
        tempindex=scipy.stats.randint.rvs(low=0,high=n_data)
        mixturemodel_means[k]=X[tempindex,:].toarray()
        # Initialize the covariance matrix to be spherical
        for l in range(n_dimensions):
            mixturemodel_covariances[k][l,l]=1
            mixturemodel_inversecovariances[k][l,l]=1

    ␣
↪return(mixturemodel_weights,mixturemodel_means,mixturemodel_covariances,mixturemodel_invers
```

[31]:
```
def␣
↪run_estep(X,mixturemodel_means,mixturemodel_covariances,mixturemodel_inversecovariances,mix
↪
    # For each component, compute terms that do not involve data
    meanterms=np.zeros((n_components))
    logdeterminants=np.zeros((n_components))
    logconstantterms=np.zeros((n_components))

    for k in range(n_components):
        # Compute mu_k*inv(Sigma_k)*mu_k
        meanterms[k]=(mixturemodel_means[k,:
↪]*mixturemodel_inversecovariances[k]*mixturemodel_means[k,:].T)[0,0]
        # Compute determinant of Sigma_k. For a diagonal matrix
        # this is just the product of the main diagonal
        logdeterminants[k]=np.sum(np.log(mixturemodel_covariances[k].
↪diagonal(0)))
        # Compute constant term beta_k * 1/(|Sigma_k|^1/2)
        # Omit the (2pi)^d/2 as it cancels out
        logconstantterms[k]=np.log(mixturemodel_weights[k]) - 0.
↪5*logdeterminants[k]
```

```
    print('E-step part2 ')
    # Compute terms that involve distances of data from components
    xnorms=np.zeros((n_data,n_components))
    xtimesmu=np.zeros((n_data,n_components))

    for k in range(n_components):
        #print(k)
        xnorms[:,k]=(X*mixturemodel_inversecovariances[k]*X.T).diagonal(0)
        xtimesmu[:,k]=np.squeeze((X*mixturemodel_inversecovariances[k]*␣
↪mixturemodel_means[k,:].T).toarray())

    xdists=xnorms+np.matlib.repmat(meanterms,n_data,1)-2*xtimesmu
    # Substract maximal term before exponent (cancels out) to maintain␣
↪computational precision
    numeratorterms=logconstantterms-xdists/2
    numeratorterms-=np.matlib.repmat(np.
↪max(numeratorterms,axis=1),n_components,1).T
    numeratorterms=np.exp(numeratorterms)
    mixturemodel_componentmemberships=numeratorterms/np.matlib.repmat(np.
↪sum(numeratorterms,axis=1),n_components,1).T
    return(mixturemodel_componentmemberships)
```

[32]:
```
def run_mstep_sumweights(mixturemodel_componentmemberships):
    # Compute total weight per component
    mixturemodel_weights=np.sum(mixturemodel_componentmemberships,axis=0)
    return(mixturemodel_weights)
```

[33]:
```
def run_mstep_means(X,mixturemodel_componentmemberships,mixturemodel_weights):
    # Update component means
    mixturemodel_means=scipy.sparse.lil_matrix((n_components,n_dimensions))
    for k in range(n_components):
        mixturemodel_means[k,:]=\
            np.sum(scipy.sparse.diags(mixturemodel_componentmemberships[:,k]).
↪dot(X),axis=0)
        mixturemodel_means[k,:]/=mixturemodel_weights[k]
    return(mixturemodel_means)
```

[34]:
```
def␣
↪run_mstep_covariances(X,mixturemodel_componentmemberships,mixturemodel_weights,mixturemodel
↪
    # Update diagonal component covariance matrices
    n_dimensions=np.shape(X)[1]
    n_components=np.shape(mixturemodel_componentmemberships)[1]
    tempcovariances=np.zeros((n_components,n_dimensions))
    mixturemodel_covariances=[]
```

```
    mixturemodel_inversecovariances=[]

    for k in range(n_components):
        tempcovariances[k,:]= np.sum(scipy.sparse.diags(
            mixturemodel_componentmemberships[:,k]).dot(
            X.multiply(X)),axis=0)-mixturemodel_means[k,:].
↪multiply(mixturemodel_means[k,:])*mixturemodel_weights[k]
        tempcovariances[k,:]/=mixturemodel_weights[k]
        # Convert to sparse matrices
        tempepsilon=1e-10
        # Add a small regularization term
        temp_covariance=scipy.sparse.diags(tempcovariances[k,:]+tempepsilon)
        temp_inversecovariance=scipy.sparse.diags((tempcovariances[k,:
↪]+tempepsilon)**-1)
        mixturemodel_covariances.append(temp_covariance)
        mixturemodel_inversecovariances.append(temp_inversecovariance)
    return(mixturemodel_covariances,mixturemodel_inversecovariances)
```

[35]:
```
def run_mstep_normalizeweights(mixturemodel_weights):
    # Update mixture-component prior probabilities
    mixturemodel_weights/=sum(mixturemodel_weights)
    return(mixturemodel_weights)
```

[36]:
```
#%% Perform the EM algorithm iterations
def perform_emalgorithm(X,n_components,n_emiterations):
    mixturemodel_weights,mixturemodel_means,mixturemodel_covariances,\
    mixturemodel_inversecovariances=initialize_mixturemodel(X,n_components)
    for t in range(n_emiterations):
        # ====== E-step: Compute the component membership
        # probabilities of each data point ======
        print('E-step ' + str(t))
       ␣
↪mixturemodel_componentmemberships=run_estep(X,mixturemodel_means,mixturemodel_covariances,\
        mixturemodel_inversecovariances,mixturemodel_weights)
        # ====== M-step: update component parameters======
        print('M-step ' + str(t))
        print('M-step part1 ' + str(t))
       ␣
↪mixturemodel_weights=run_mstep_sumweights(mixturemodel_componentmemberships)
        print('M-step part2 ' + str(t))
       ␣
↪mixturemodel_means=run_mstep_means(X,mixturemodel_componentmemberships,mixturemodel_weights
        print('M-step part3 ' + str(t))
       ␣
↪mixturemodel_covariances,mixturemodel_inversecovariances=run_mstep_covariances(X,\
```

```
      ␣
 ↪mixturemodel_componentmemberships,mixturemodel_weights,mixturemodel_means)
       print('M-step part4 ' + str(t))
       mixturemodel_weights=run_mstep_normalizeweights(mixturemodel_weights)
    return(mixturemodel_weights,mixturemodel_means,mixturemodel_covariances,\
mixturemodel_inversecovariances)
# Try out the functions we just defined on the data
n_components=10
n_emiterations=100
mixturemodel_weights,mixturemodel_means,mixturemodel_covariances,\
mixturemodel_inversecovariances =␣
 ↪perform_emalgorithm(X,n_components,n_emiterations)
```

```
C:\Users\danie\Anaconda3\lib\site-packages\scipy\sparse\lil.py:512:
FutureWarning: future versions will not create a writeable array from
broadcast_array. Set the writable flag explicitly to avoid this warning.
  if not i.flags.writeable or i.dtype not in (np.int32, np.int64):
C:\Users\danie\Anaconda3\lib\site-packages\scipy\sparse\lil.py:514:
FutureWarning: future versions will not create a writeable array from
broadcast_array. Set the writable flag explicitly to avoid this warning.
  if not j.flags.writeable or j.dtype not in (np.int32, np.int64):
```

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[37]:
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for k in range(n_components):
    print(k)
    highest_dimensionweight_indices=np.argsort(-np.
 ↪squeeze(mixturemodel_means[k,:].toarray()),axis=0)

    print(' '.join(remaining_vocabulary[highest_dimensionweight_indices[1:10]]))
```

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0
stranger gravely grateful gown street stretch gold strike glass
1
party pass pat pave perch person pick part piece
2
nearly nearer near mouth monster milkmaid mile midst middle
3
place play pleasant please pleased plenty point pole poor
4
often oil open order others ought palace offer part
5
ought paint party pat patch pave perch pick piece
6
mistake tire tip monster morning mouth move tiny munchkin
7
rock gently gaze gayelette gather roof funny rope fruit
8
inquire ruler remark dear spider figure five scare inside
9
inquire indeed immediately imagine spider hung humbug spite spot
```

[38]:
```
# Version 2 - Get documents closest to component mean, i.e. highest p(d|k).
# ---The computation of distances here is the same as done in the E-step of␣
 ↪EM---
# For each component, compute terms that do not involve data
meanterms=np.zeros((n_components))
logdeterminants=np.zeros((n_components))
logconstantterms=np.zeros((n_components))
for k in range(n_components):
    # Compute mu_k*inv(Sigma_k)*mu_k
    meanterms[k]=(mixturemodel_means[k,:
 ↪]*mixturemodel_inversecovariances[k]*mixturemodel_means[k,:].T)[0,0]

# Compute terms that involve distances of data from components
xnorms=np.zeros((n_data,n_components))
xtimesmu=np.zeros((n_data,n_components))
```

```
for k in range(n_components):
    xnorms[:,k]=(X*mixturemodel_inversecovariances[k]*X.T).diagonal(0)
    xtimesmu[:,k]=np.
 ↪squeeze((X*mixturemodel_inversecovariances[k]*mixturemodel_means[k,:].T).
 ↪toarray())

xdists=xnorms+np.matlib.repmat(meanterms,n_data,1)-2*xtimesmu

for k in range(n_components):
    tempdists=np.array(np.squeeze(xdists[:,k]))
    highest_componentprob_indices=np.argsort(-1*tempdists,axis=0)
    print(k)
    print(highest_componentprob_indices[0:10])
    print(' '.join(paragraph_nltk_texts[highest_componentprob_indices[0]]))
```

```
0
[   0  876  635  633  631  420   11 1140    1    7]

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[   0  876  635  633  631  420   11 1140    7    1]

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[   0  876  635  633  631  420   11 1140    7    3]

3
[   0  876  635  633  631  420   11 1140    7    3]

4
[   0  876  635  633  631  420   11 1140    3    7]

5
[   0  876  635  633  631  420   11 1140    7    3]

6
[   0  876  635  633  631  420   11 1140    7    1]

7
[   0  876  635  633  631  420   11 1140    1    7]

8
[   0  876  635  633  631  420   11 1140    3    7]

9
[   0  876  635  633  631  420   11 1140    1    7]
```

g) In every mixture component, the highest membership probability does not match with the current top-10 words in the same mixture component.

The result might be due to that the paragraph in question might be very near the cluster center. This might happen as it is built out of rather generic words that might be common in every cluster.

A few of the paragraphs with the highest membership probability gave the same result. Is this correct.

For some reason, the words dissappeared in the last part of the task, even though they were prior visible.

2 EX 4.2

[39]:
```
#Find out the amout of words in paragraphs
word_count = np.zeros((len(paragraph_lemmatizedtexts), 1))
for k in range(len(paragraph_lemmatizedtexts)):
    counting = len(paragraph_lemmatizedtexts[k])
    word_count[k] = counting
```

[40]:
```
max(word_count)
```

[40]:
```
array([234.])
```

[41]:
```
no_longest_paragraph = np.argsort(-1*word_count, axis=0)[0]
```

[42]:
```
no_longest_paragraph
```

[42]:
```
array([505], dtype=int64)
```

[43]:
```
#The longest paragraph
print(' '.join(paragraph_lemmatizedtexts[505]))
```

```
she leave dorothy alone and go back to the others . these she also lead to room
, and each one of them find himself lodge in a very pleasant part of the palace
. of course this politeness be waste on the scarecrow ; for when he find himself
alone in his room he stand stupidly in one spot , just within the doorway , to
wait till morning . it would not rest him to lie down , and he could not close
his eye ; so he remain all night star at a little spider which be weave its web
in a corner of the room , just as if it be not one of the most wonderful room in
the world . the tin woodman lay down on his bed from force of habit , for he
remember when he be make of flesh ; but not be able to sleep , he pass the night
move his joint up and down to make sure they keep in good work order . the lion
would have prefer a bed of dried leaf in the forest , and do not like be shut up
in a room ; but he have too much sense to let this worry him , so he spring upon
the bed and roll himself up like a cat and purr himself asleep in a minute .
```

[44]:
```
longestpara=(' '.join(paragraph_lemmatizedtexts[505]))
longestpara
```

[44]:
```
'she leave dorothy alone and go back to the others . these she also lead to room
, and each one of them find himself lodge in a very pleasant part of the palace
. of course this politeness be waste on the scarecrow ; for when he find himself
alone in his room he stand stupidly in one spot , just within the doorway , to
wait till morning . it would not rest him to lie down , and he could not close
his eye ; so he remain all night star at a little spider which be weave its web
in a corner of the room , just as if it be not one of the most wonderful room in
the world . the tin woodman lay down on his bed from force of habit , for he
remember when he be make of flesh ; but not be able to sleep , he pass the night
move his joint up and down to make sure they keep in good work order . the lion
would have prefer a bed of dried leaf in the forest , and do not like be shut up
in a room ; but he have too much sense to let this worry him , so he spring upon
the bed and roll himself up like a cat and purr himself asleep in a minute .'
```

[45]:
```
longestmyvocabularies=[]
longestmyindices_in_vocabularies=[]
# Find the vocabulary of each document
for k in range(len(longestpara)):
    # Get unique words and where they occur
    temptext=longestpara[k]
    uniqueresults=np.unique(temptext,return_inverse=True)
    uniquewords=uniqueresults[0]
    wordindices=uniqueresults[1]
    # Store the vocabulary and indices of document words in it
    longestmyvocabularies.append(uniquewords)
    longestmyindices_in_vocabularies.append(wordindices)
longestmyvocabularies[0]
```

[45]:
```
array(['s'], dtype='<U1')
```

[46]:
```
tempvocabulary=[]
for k in range(len(paragraph_lemmatizedtexts)):
    tempvocabulary.extend(myvocabularies[k])

# Find the unique elements among all vocabularies
uniqueresults=np.unique(tempvocabulary,return_inverse=True)
unifiedvocabulary=uniqueresults[0]
wordindices=uniqueresults[1]
```

[47]:
```
# Translate previous indices to unified vocabulary.

vocabularystart=0
myindices_in_unifiedvocabulary=[]
for k in range(len(paragraph_lemmatizedtexts)):
    # In order to shift word indices, we must temporarily
    # change their data type to a Numpy array
    tempindices=np.array(myindices_in_vocabularies[k])
```

```
    tempindices=tempindices+vocabularystart
    tempindices=wordindices[tempindices]
    myindices_in_unifiedvocabulary.append(tempindices)
    vocabularystart=vocabularystart+len(myvocabularies[k])
```

[48]:
```
unifiedvocabulary_totaloccurrencecounts=np.zeros((len(unifiedvocabulary),1))
unifiedvocabulary_documentcounts=np.zeros((len(unifiedvocabulary),1))
unifiedvocabulary_meancounts=np.zeros((len(unifiedvocabulary),1))
unifiedvocabulary_countvariances=np.zeros((len(unifiedvocabulary),1))
```

[49]:
```
for k in range(len(longestpara)):
    print(k)
    occurrencecounts=np.zeros((len(unifiedvocabulary),1))
    for l in range(len(myindices_in_unifiedvocabulary[k])):
        occurrencecounts[myindices_in_unifiedvocabulary[k][l]]= \
            occurrencecounts[myindices_in_unifiedvocabulary[k][l]]+1
    unifiedvocabulary_totaloccurrencecounts= \
        unifiedvocabulary_totaloccurrencecounts+occurrencecounts
    unifiedvocabulary_documentcounts= \
        unifiedvocabulary_documentcounts+(occurrencecounts>0)
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[50]:
```
# Mean occurrence counts over documents
unifiedvocabulary_meancounts= \
    unifiedvocabulary_totaloccurrencecounts/len(longestpara)
```

[51]:
```
#%% Inspect frequent words
# Sort words by largest total (or mean) occurrence count
highest_totaloccurrences_indices=np.argsort(\
    -1*unifiedvocabulary_totaloccurrencecounts,axis=0)
print(np.squeeze(unifiedvocabulary[\
    highest_totaloccurrences_indices[0:100]]))
print(np.squeeze(\
    unifiedvocabulary_totaloccurrencecounts[\
    highest_totaloccurrences_indices[0:100]]))
```

```
['the' ',' '.' 'and' 'be' 'to' 'of' 'a' 'â\x80\x9d' 'in' 'have' 'i' 'he'
 'you' 'her' 'she' 'they' 'it' 'that' 'say' 'dorothy' 'as' 'so' 'for'
 'not' 'with' 'at' 'but' 'do' 'all' 'them' 'scarecrow' 'his' ';' '?' 'him'
 'my' 'me' 'woodman' 'come' 'on' 'lion' 'oz' 'great' 'make' 'will' 'go'
 'â\x80\x9ci' 'when' 'little' 'tin' 'witch' 'ask' 'this' 'one' 'could'
 'from' 'see' 'would' 'then' 'there' 'we' 'if' 'who' 'get' 'green' 'out'
 'up' 'can' 'their' 'head' 'look' 'know' 'no' 'think' 'girl' 'back' 'toto'
 '!' 'down' 'by' 'upon' 'answer' 'find' 'shall' 'again' 'city' 'give'
 'into' 'over' 'very' 'must' 'wicked' 'now' 'emerald' 'man' 'good' 'where'
 'after' 'walk']
[2709. 2494. 1485. 1458. 1324. 1009.  760.  737.  633.  441.  424.  422.
  410.  401.  369.  361.  357.  356.  342.  323.  321.  290.  282.  278.
  256.  252.  237.  228.  217.  217.  212.  203.  198.  185.  178.  172.
  172.  170.  168.  153.  151.  150.  148.  139.  139.  138.  137.  136.
  136.  133.  133.  128.  126.  118.  114.  113.  111.  110.  110.  106.
  105.  101.  100.   99.   98.   98.   95.   94.   92.   91.   90.   89.
   89.   88.   87.   85.   84.   83.   83.   82.   80.   78.   78.   76.
   76.   75.   73.   73.   73.   72.   72.   71.   71.   69.   67.   66.
   66.   65.   65.   65.]
```

[52]:
```
nltk.download('stopwords')

#%% Vocabulary pruning
nltkstopwords=nltk.corpus.stopwords.words('english')
pruningdecisions=np.zeros((len(unifiedvocabulary),1))
for k in range(len(unifiedvocabulary)):
    # Rule 1: check the nltk stop word list
    if (unifiedvocabulary[k] in nltkstopwords):
        pruningdecisions[k]=1
    # Rule 2: if the word is in the top 1% of frequent words
```

```
    #if (k in highest_totaloccurrences_indices[\
    #    0:int(np.floor(len(unifiedvocabulary)*0.01))]):
    #    pruningdecisions[k]=1
   # Rule 3: if the word occurs less than 4 times
    if(unifiedvocabulary_totaloccurrencecounts[k] < 4):
        pruningdecisions[k] = 1
    # Rule 4: if the word is too short
    if len(unifiedvocabulary[k])<2:
        pruningdecisions[k]=1
    # Rule 5: if the word is too long
    if len(unifiedvocabulary[k])>20:
        pruningdecisions[k]=1
    # Rule 6: if the word has unwanted characters
    # (here for simplicity only a-z allowed)
    if unifiedvocabulary[k].isalpha()==False:
        pruningdecisions[k]=1
```

```
[nltk_data] Downloading package stopwords to
[nltk_data]     C:\Users\danie\AppData\Roaming\nltk_data…
[nltk_data]   Package stopwords is already up-to-date!
```

[53]:
```
#%% Get indices of documents to remaining words
longestoldtopruned=[]
tempind=-1
for k in range(len(unifiedvocabulary)):
    if pruningdecisions[k]==0:
        tempind=tempind+1
        longestoldtopruned.append(tempind)
    else:
        longestoldtopruned.append(-1)
```

[54]:
```
#%% Create pruned texts
longestparagraph_prunedtexts=[]
longestmyindices_in_prunedvocabulary=[]
for k in range(len(longestpara)):
    print(k)
    temp_newindices=[]
    temp_newdoc=[]
    for l in range(len(longestpara[k])):
        temp_oldindex=myindices_in_unifiedvocabulary[k][l]
        temp_newindex=longestoldtopruned[temp_oldindex]
        if temp_newindex!=-1:
            temp_newindices.append(temp_newindex)
            temp_newdoc.append(unifiedvocabulary[temp_oldindex])
    longestparagraph_prunedtexts.append(temp_newdoc)
    longestmyindices_in_prunedvocabulary.append(temp_newindices)
```

```
0
```

```
␣
↪---------------------------------------------------------------------------

IndexError                                Traceback (most recent call␣
↪last)

<ipython-input-54-c8cd775aa87a> in <module>()
      7     temp_newdoc=[]
      8     for l in range(len(longestpara[k])):
----> 9         temp_oldindex=myindices_in_unifiedvocabulary[k][l]
     10         temp_newindex=longestoldtopruned[temp_oldindex]
     11         if temp_newindex!=-1:


IndexError: index 0 is out of bounds for axis 0 with size 0
```

[55]:
```
print('Top 100 word-list after pruning unified vocabulary:\n')
remaining_indices = np.squeeze(np.where(pruningdecisions==0)[0])
remaining_vocabulary = unifiedvocabulary[remaining_indices]
remainingvocabulary_totaloccurrencecounts =␣
 ↪unifiedvocabulary_totaloccurrencecounts[remaining_indices]
remaining_highest_totaloccurrences_indices = np.
 ↪argsort(-1*remainingvocabulary_totaloccurrencecounts, axis=0)
print(np.
 ↪squeeze(remaining_vocabulary[remaining_highest_totaloccurrences_indices[0:
 ↪100]]))
print(np.
 ↪squeeze(remainingvocabulary_totaloccurrencecounts[remaining_highest_totaloccurrences_indice
 ↪100]]
```

```
  File "<ipython-input-55-aed8dc634baa>", line 7
    print(np.
↪squeeze(remainingvocabulary_totaloccurrencecounts[remaining_highest_totaloccurrences_indices
↪100]]
                                                                          ␣
↪                                    ^
SyntaxError: unexpected EOF while parsing
```

[56]:
```
n_docs=len(longestparagraph_prunedtexts)
n_vocab=len(remaining_vocabulary)
```

```
# Matrix of term frequencies
tfmatrix=scipy.sparse.lil_matrix((n_docs,n_vocab))
# Row vector of document frequencies
dfvector=scipy.sparse.lil_matrix((1,n_vocab))
# Loop over documents
for k in range(n_docs):
    # Row vector of which words occurred in this document
    temp_dfvector=scipy.sparse.lil_matrix((1,n_vocab))
    # Loop over words
    for l in range(len(paragraph_prunedtexts[k])):
        # Add current word to term-frequency count and document-count
        currentword=longestmyindices_in_prunedvocabulary[k][l]
        tfmatrix[k,currentword]=tfmatrix[k,currentword]+1
        temp_dfvector[0,currentword]=1
    # Add which words occurred in this document to overall document counts
    dfvector=dfvector+temp_dfvector

# TF:length-normalized frequency
for i in range(n_docs):
    for j in range(len(tfmatrix.data[i])):
        tfmatrix.data[i][j] = tfmatrix.data[i][j]/len(tfmatrix.data[i])


# smoothed logarithmic idf
idfvector = np.squeeze(np.array(dfvector.todense()))
idfvector = np.log(1 + ((idfvector+1)**-1)*n_docs)

tfidfmatrix = scipy.sparse.lil_matrix((n_docs, n_vocab))
for k in range(n_docs):
    # tf and idf terms
    tfidfmatrix[k,:]=tfmatrix[k,:]*idfvector

# tf-idf matrix
#tfidfmatrix = scipy.sparse.lil_matrix((n_docs, n_vocab))
for k in range(n_docs):
    # find nonzero term frequencies
    tempindices = np.nonzero(tfmatrix[k, :])[1]
    tfterm = np.squeeze(np.array(tfmatrix[k, tempindices].todense()))
    tfidfmatrix[k, tempindices] = tfterm * idfvector[tempindices]
```

[57]:
```
#%% Use the TF-IDF matrix as data to be clustered
X=tfidfmatrix
# Normalize the documents to unit vector norm
tempnorms=np.squeeze(np.array(np.sum(X.multiply(X),axis=1)))
# If any documents have zero norm, avoid dividing them by zero
tempnorms[tempnorms==0]=1
X=scipy.sparse.diags(tempnorms**-0.5).dot(X)
```

```
n_data=np.shape(X)[0]
n_dimensions=np.shape(X)[1]
```

[58]:
```
# Function to initialize the Gaussian mixture model, create component parameters
def initialize_mixturemodel(X,n_components):
    # Create lists of sparse matrices to hold the parameters
    n_dimensions=np.shape(X)[1]
    n_data = np.shape(X)[0]
    mixturemodel_means=scipy.sparse.lil_matrix((n_components,n_dimensions))
    mixturemodel_weights=np.zeros((n_components))
    mixturemodel_covariances=[]
    mixturemodel_inversecovariances=[]
    for k in range(n_components):
        tempcovariance=scipy.sparse.lil_matrix((n_dimensions,n_dimensions))
        mixturemodel_covariances.append(tempcovariance)
        tempinvcovariance=scipy.sparse.lil_matrix((n_dimensions,n_dimensions))
        mixturemodel_inversecovariances.append(tempinvcovariance)
    # Initialize the parameters
    for k in range(n_components):
        mixturemodel_weights[k]=1/n_components
        # Pick a random data point as the initial mean
        tempindex=scipy.stats.randint.rvs(low=0,high=n_data)
        mixturemodel_means[k]=X[tempindex,:].toarray()
        # Initialize the covariance matrix to be spherical
        for l in range(n_dimensions):
            mixturemodel_covariances[k][l,l]=1
            mixturemodel_inversecovariances[k][l,l]=1

    ␣
↪return(mixturemodel_weights,mixturemodel_means,mixturemodel_covariances,mixturemodel_invers
```

[59]:
```
def␣
↪run_estep(X,mixturemodel_means,mixturemodel_covariances,mixturemodel_inversecovariances,mix
↪
    # For each component, compute terms that do not involve data
    meanterms=np.zeros((n_components))
    logdeterminants=np.zeros((n_components))
    logconstantterms=np.zeros((n_components))

    for k in range(n_components):
        # Compute mu_k*inv(Sigma_k)*mu_k
        meanterms[k]=(mixturemodel_means[k,:
↪]*mixturemodel_inversecovariances[k]*mixturemodel_means[k,:].T)[0,0]
        # Compute determinant of Sigma_k. For a diagonal matrix
        # this is just the product of the main diagonal
        logdeterminants[k]=np.sum(np.log(mixturemodel_covariances[k].
↪diagonal(0)))
        # Compute constant term beta_k * 1/(|Sigma_k|^1/2)
```

```
        # Omit the (2pi)^d/2 as it cancels out
        logconstantterms[k]=np.log(mixturemodel_weights[k]) - 0.
↪5*logdeterminants[k]

    print('E-step part2 ')
    # Compute terms that involve distances of data from components
    xnorms=np.zeros((n_data,n_components))
    xtimesmu=np.zeros((n_data,n_components))

    for k in range(n_components):
        #print(k)
        xnorms[:,k]=(X*mixturemodel_inversecovariances[k]*X.T).diagonal(0)
        xtimesmu[:,k]=np.squeeze((X*mixturemodel_inversecovariances[k]*␣
↪mixturemodel_means[k,:].T).toarray())

    xdists=xnorms+np.matlib.repmat(meanterms,n_data,1)-2*xtimesmu
    # Substract maximal term before exponent (cancels out) to maintain␣
↪computational precision
    numeratorterms=logconstantterms-xdists/2
    numeratorterms-=np.matlib.repmat(np.
↪max(numeratorterms,axis=1),n_components,1).T
    numeratorterms=np.exp(numeratorterms)
    mixturemodel_componentmemberships=numeratorterms/np.matlib.repmat(np.
↪sum(numeratorterms,axis=1),n_components,1).T
    return(mixturemodel_componentmemberships)
```

[60]:
```
def run_mstep_sumweights(mixturemodel_componentmemberships):
    # Compute total weight per component
    mixturemodel_weights=np.sum(mixturemodel_componentmemberships,axis=0)
    return(mixturemodel_weights)
```

[61]:
```
def run_mstep_means(X,mixturemodel_componentmemberships,mixturemodel_weights):
    # Update component means
    mixturemodel_means=scipy.sparse.lil_matrix((n_components,n_dimensions))
    for k in range(n_components):
        mixturemodel_means[k,:]=\
            np.sum(scipy.sparse.diags(mixturemodel_componentmemberships[:,k]).
↪dot(X),axis=0)
        mixturemodel_means[k,:]/=mixturemodel_weights[k]
    return(mixturemodel_means)
```

[62]:
```
def␣
↪run_mstep_covariances(X,mixturemodel_componentmemberships,mixturemodel_weights,mixturemodel
↪
    # Update diagonal component covariance matrices
    n_dimensions=np.shape(X)[1]
```

```
    n_components=np.shape(mixturemodel_componentmemberships)[1]
    tempcovariances=np.zeros((n_components,n_dimensions))
    mixturemodel_covariances=[]
    mixturemodel_inversecovariances=[]

    for k in range(n_components):
        tempcovariances[k,:]= np.sum(scipy.sparse.diags(
            mixturemodel_componentmemberships[:,k]).dot(
            X.multiply(X)),axis=0)-mixturemodel_means[k,:].
↪multiply(mixturemodel_means[k,:])*mixturemodel_weights[k]
        tempcovariances[k,:]/=mixturemodel_weights[k]
        # Convert to sparse matrices
        tempepsilon=1e-10
        # Add a small regularization term
        temp_covariance=scipy.sparse.diags(tempcovariances[k,:]+tempepsilon)
        temp_inversecovariance=scipy.sparse.diags((tempcovariances[k,:
↪]+tempepsilon)**-1)
        mixturemodel_covariances.append(temp_covariance)
        mixturemodel_inversecovariances.append(temp_inversecovariance)
    return(mixturemodel_covariances,mixturemodel_inversecovariances)
```

[63]:
```
def run_mstep_normalizeweights(mixturemodel_weights):
    # Update mixture-component prior probabilities
    mixturemodel_weights/=sum(mixturemodel_weights)
    return(mixturemodel_weights)
```

[64]:
```
#%% Perform the EM algorithm iterations
def perform_emalgorithm(X,n_components,n_emiterations):
    mixturemodel_weights,mixturemodel_means,mixturemodel_covariances,\
    mixturemodel_inversecovariances=initialize_mixturemodel(X,n_components)
    for t in range(n_emiterations):
        # ====== E-step: Compute the component membership
        # probabilities of each data point ======
        print('E-step ' + str(t))
        ␣
↪mixturemodel_componentmemberships=run_estep(X,mixturemodel_means,mixturemodel_covariances,\
        mixturemodel_inversecovariances,mixturemodel_weights)
        # ====== M-step: update component parameters======
        print('M-step ' + str(t))
        print('M-step part1 ' + str(t))
        ␣
↪mixturemodel_weights=run_mstep_sumweights(mixturemodel_componentmemberships)
        print('M-step part2 ' + str(t))
        ␣
↪mixturemodel_means=run_mstep_means(X,mixturemodel_componentmemberships,mixturemodel_weights
        print('M-step part3 ' + str(t))
```

```
      ␣
↪mixturemodel_covariances,mixturemodel_inversecovariances=run_mstep_covariances(X,\
      ␣
↪mixturemodel_componentmemberships,mixturemodel_weights,mixturemodel_means)
       print('M-step part4 ' + str(t))
       mixturemodel_weights=run_mstep_normalizeweights(mixturemodel_weights)
   return(mixturemodel_weights,mixturemodel_means,mixturemodel_covariances,\
mixturemodel_inversecovariances)
# Try out the functions we just defined on the data
n_components=10
n_emiterations=100
mixturemodel_weights,mixturemodel_means,mixturemodel_covariances,\
mixturemodel_inversecovariances =␣
↪perform_emalgorithm(X,n_components,n_emiterations)
```

```
      ␣
↪---------------------------------------------------------------------------

       ValueError                                Traceback (most recent call␣
↪last)

       <ipython-input-64-b8e189326e21> in <module>()
        21 n_components=10
        22 n_emiterations=100
   ---> 23␣
↪mixturemodel_weights,mixturemodel_means,mixturemodel_covariances,mixturemodel_inversecovaria
↪= perform_emalgorithm(X,n_components,n_emiterations)


       <ipython-input-64-b8e189326e21> in perform_emalgorithm(X, n_components,␣
↪n_emiterations)
         1 #%% Perform the EM algorithm iterations
         2 def perform_emalgorithm(X,n_components,n_emiterations):
   ----> 3     ␣
↪mixturemodel_weights,mixturemodel_means,mixturemodel_covariances,    ␣
↪mixturemodel_inversecovariances=initialize_mixturemodel(X,n_components)
         4     for t in range(n_emiterations):
         5         # ====== E-step: Compute the component membership


       <ipython-input-58-8993e11d61c8> in initialize_mixturemodel(X,␣
↪n_components)
        17         mixturemodel_weights[k]=1/n_components
        18         # Pick a random data point as the initial mean
   ---> 19         tempindex=scipy.stats.randint.rvs(low=0,high=n_data)
        20         mixturemodel_means[k]=X[tempindex,:].toarray()
```

```
         21         # Initialize the covariance matrix to be spherical


~\Anaconda3\lib\site-packages\scipy\stats\_distn_infrastructure.py in␣
↪rvs(self, *args, **kwargs)
      2807         """
      2808         kwargs['discrete'] = True
   -> 2809         return super(rv_discrete, self).rvs(*args, **kwargs)
      2810
      2811     def pmf(self, k, *args, **kwds):


~\Anaconda3\lib\site-packages\scipy\stats\_distn_infrastructure.py in␣
↪rvs(self, *args, **kwds)
       938         cond = logical_and(self._argcheck(*args), (scale >= 0))
       939         if not np.all(cond):
   --> 940             raise ValueError("Domain error in arguments.")
       941
       942         if np.all(scale == 0):


ValueError: Domain error in arguments.
```

[65]:
```
for k in range(n_components):
    print(k)
    highest_dimensionweight_indices=np.argsort(-np.
 ↪squeeze(mixturemodel_means[k,:].toarray()),axis=0)

    print(' '.join(remaining_vocabulary[highest_dimensionweight_indices[1:10]]))
```

```
0
stranger gravely grateful gown street stretch gold strike glass
1
party pass pat pave perch person pick part piece
2
nearly nearer near mouth monster milkmaid mile midst middle
3
place play pleasant please pleased plenty point pole poor
4
often oil open order others ought palace offer part
5
ought paint party pat patch pave perch pick piece
6
mistake tire tip monster morning mouth move tiny munchkin
7
rock gently gaze gayelette gather roof funny rope fruit
```

```
8
inquire ruler remark dear spider figure five scare inside
9
inquire indeed immediately imagine spider hung humbug spite spot
```

[66]:
```
# Version 2 - Get documents closest to component mean, i.e. highest p(d|k).
# ---The computation of distances here is the same as done in the E-step of␣
 ↪EM---
# For each component, compute terms that do not involve data
meanterms=np.zeros((n_components))
logdeterminants=np.zeros((n_components))
logconstantterms=np.zeros((n_components))
for k in range(n_components):
    # Compute mu_k*inv(Sigma_k)*mu_k
    meanterms[k]=(mixturemodel_means[k,:
 ↪]*mixturemodel_inversecovariances[k]*mixturemodel_means[k,:].T)[0,0]

# Compute terms that involve distances of data from components
xnorms=np.zeros((n_data,n_components))
xtimesmu=np.zeros((n_data,n_components))

for k in range(n_components):
    xnorms[:,k]=(X*mixturemodel_inversecovariances[k]*X.T).diagonal(0)
    xtimesmu[:,k]=np.
 ↪squeeze((X*mixturemodel_inversecovariances[k]*mixturemodel_means[k,:].T).
 ↪toarray())

xdists=xnorms+np.matlib.repmat(meanterms,n_data,1)-2*xtimesmu

for k in range(n_components):
    tempdists=np.array(np.squeeze(xdists[:,k]))
    highest_componentprob_indices=np.argsort(-1*tempdists,axis=0)
    print(k)
    print(highest_componentprob_indices[0:10])
    print(' '.join(paragraph_nltk_texts[highest_componentprob_indices[0]]))
```

```
      ␣
 ↪---------------------------------------------------------------------------

       ValueError                                Traceback (most recent call␣
 ↪last)

       <ipython-input-66-a71fcfb83997> in <module>()
        14
        15 for k in range(n_components):
```

```
   ---> 16     xnorms[:,k]=(X*mixturemodel_inversecovariances[k]*X.T).
↪diagonal(0)
        17     xtimesmu[:,k]=np.
↪squeeze((X*mixturemodel_inversecovariances[k]*mixturemodel_means[k,:].T).
↪toarray())
        18


       ~\Anaconda3\lib\site-packages\scipy\sparse\compressed.py in␣
↪diagonal(self, k)
       517         rows, cols = self.shape
       518         if k <= -rows or k >= cols:
   --> 519             raise ValueError("k exceeds matrix dimensions")
       520         fn = getattr(_sparsetools, self.format + "_diagonal")
       521         y = np.empty(min(rows + min(k, 0), cols - max(k, 0)),


       ValueError: k exceeds matrix dimensions
```

[ ]:

Exercise 4.3: The EM algorithm

- Main idea is to prove EM-alg. works in $\log p(X;\theta_{t+1}) \geq \log p(X;\theta_t)$, for every iteration.

a) $\log p(x|\theta_{t+1}) - \log p(x|\theta_t) > 0$

b) $E_{q_t p(Z|X;\theta_t)}[\log p(x|\theta_{t+1}) - \log p(x|\theta_t)] > 0$

$\Rightarrow$ As Z does not appear in the difference
$\Rightarrow$ the value is $[\log p(x|\theta_{t+1}) - \log p(x|\theta_t)]$

c) $\log p(x|\theta_{t+1}) - \log p(x|\theta_t)$

$\Rightarrow \log\left(\frac{p(x|\theta_{t+1}) \cdot p(z|x;\theta_{t+1})}{p(z|x;\theta_{t+1})}\right) - \log\left(\frac{p(x|\theta_t) \cdot p(z|x;\theta_t)}{p(z|x;\theta_t)}\right)$

$$\mathbb{E}_{q_t}\left[\log\left(\frac{p(x|\theta_{t+1})\cdot p(z|x;\theta_{t+1})}{p(z|x;\theta_{t+1})}\right) - \log\left(\frac{p(x|\theta_t)\cdot p(z|x;\theta_t)}{p(z|x;\theta_t)}\right)\right]$$

$$\| \cdot p(z|x;\theta_t)$$

$$\sum\left(\log\left(\frac{p(x|\theta_{t+1})\cdot p(z|x;\theta_{t+1})}{p(z|x;\theta_{t+1})}\right) - \log\left(\frac{p(x|\theta_t)\cdot p(z|x;\theta_t)}{p(z|x;\theta_t)}\right)\right)\cdot p(z|x;\theta_t)$$

$$= \underbrace{\sum p(z|x;\theta_t)\cdot\log\left(\frac{p(x|\theta_{t+1})\cdot p(z|x;\theta_{t+1})}{p(z|x;\theta_{t+1})}\right) - \sum p(z|x;\theta_t)\cdot\log\left(\frac{p(x|\theta_t)\cdot p(z|x;\theta_t)}{p(z|x;\theta_t)}\right)}_{\geq 0}$$

$$\Rightarrow$$

$$= \sum p(z|x;\theta_t)\cdot\log\left(\frac{p(x|\theta_{t+1})\cdot p(z|x;\theta_{t+1})}{p(z|x;\theta_{t+1})}\right) = \sum p(z|x;\theta_t)\cdot\log\left(\frac{p(x|\theta_t)\cdot p(z|x;\theta_t)}{p(z|x;\theta_t}\right)$$

$$= \sum p(z|x;\theta_t)\log p(z|x;\theta_{t+1}) - \sum p(z|x;\theta_t)(\log(p(z|\theta_t)$$

$$= Q(\theta_{t+1}|\theta_t) - \sum p(z|x;\theta_t)\cdot\log p(z|x,\theta_{t+1})$$

(one side)

$$= Q(\theta_{t+1}|\theta_t) - \sum p(z|x;\theta_t)\cdot\log p(z|x;\theta_{t+1}) - \dots$$

$$\dots\left(Q(\theta_t|\theta_t) - \sum p(z|x;\theta_t)\cdot\log p(z|x;\theta_t)\right)$$

$$= Q(\theta_{t+1}|\theta_t) - Q(\theta_t|\theta_t) - \sum p(z|x;\theta_t)\log p(z|x;\theta_{t+1}) + \dots$$

$$\sum p(z|x;\theta_t)\cdot\log(p(z|x;\theta_t)$$

taken from last equation

a

$$a = \sum p(z|x;\theta_t) \cdot \left(\log p(z|x;\theta_t) - \log p(z|x;\theta_{t+1})\right)$$

$$\Rightarrow \text{Using Hint } \log(a/b) = \log(a) - \log(b)$$

$$\Rightarrow \sum p(z|x;\theta_t) \cdot \log \frac{p(z|x;\theta_t)}{p(z|x;\theta_{t+1})}$$

c)

$$Q(\theta_{t+1}|\theta_t) - Q(\theta_t|\theta_t) + \sum p(z|x;\theta_t) \cdot \log \frac{p(z|x;\theta_t)}{p(z|x;\theta_{t+1})}$$

$$Q(\theta_{t+1}|\theta_t) - Q(\theta_t|\theta_t) + \underbrace{\sum p(z|x;\theta_t) \cdot \log \frac{p(z|x;\theta_t)}{p(z|x;\theta_{t+1})}}_{\geq 0} =$$

$$\log p(x|\theta_{t+1}) - \log p(x|\theta_t)$$

Hence $\Rightarrow$

$$\log p(x|\theta_{t+1}) - \log p(x|\theta_t) = Q(\theta_{t+1}|\theta_t) - Q(\theta_t|\theta_t)$$

$\Rightarrow$

$$\log p(x|\theta_{t+1}) - \log p(x|\theta_t) - Q(\theta_{t+1}|\theta_t) - Q(\theta_t|\theta_t) \geq 0$$

From m-step

$$\theta_{t+1} = \arg\max Q(\theta|\theta_t)$$

Hence

$$\Rightarrow Q(\theta_{t+1}|\theta_t) \geq Q(\theta_t|\theta_t)$$

$$\Rightarrow Q(\theta_{t+1}|\theta_t) - Q(\theta_t|\theta_t) \geq 0$$

Therefore,

$$\log p(x|\theta_{t+1}) - \log p(x|\theta_t) \geq 0$$