Exercise 4.3: The EM algorithm Eq - Main idea is to prove EM-alg. works  $\log p(X; \Theta_{+1}) \ge \log p(X; \Theta_{\ell})$ , for every iteration. ay  $\log p(X|\Theta_{t+1}) - \log p(X|\Theta_t) > 0$ = Zp b) Eqp(z|x,0) [logp(x|0+1)-logp(x|0+)] => As Z does not appear in the difference => the value is [log p(x|0tm) - log p(x|0t)] = 20 c/ log p(x | O++1) - log p(x | O+)  $\log \left( \frac{P(x|\Theta_{t+1}) \cdot P(z|x;\Theta_{t+1})}{P(z|x;\Theta_{t+1})} - \log \left( \frac{P(x|\Theta_t) \cdot P(z|x;\Theta_t)}{P(z|x;\Theta_t)} \right)$ 

Eq [log (P(X|0+1)).P(Z|X;0+1)] - log (P(X|0+).P(Z|X;0+)) 1.p(z|x; 0+) = [P(z|x,0+). |og (P(x|0++1).P(z|x;0+1) - [P(z|x;0+). |og (P(x|0+).P(z|x,0+))] = Ep(z|x; 0). 100 (P(x|0+1). P(z|x; 0+1)) = Ep(z|x; 0+1). 100 (P(x|0+1). P(z|x; 0+1)) = Ep(z|x;0,) logp(z|x;0,+1) - Ep(z|x;0,) (log(p(z|0,+) = Q(0+1 0+) - Ep(z|x;0+)·log p(z|x,0++) = Q(0++1 | 0+)-Ep(z|x;0+). (og p(z|x;0++1) -... (Q(0+10+)- Zp(Z|X;0+)·log p(Z|X;0+)) =Q(0+10+)-Q(0+10+)-Ep(z|x;0+)logp(z|x;0++1)+ Ep(z|x;0) log(p(z|x;0)

taken from last equation a= 2p(z|x; 0+). (log p(z|x; 0+)-log p(z|x; 0++))  $\Rightarrow Using Hint log(a/b) = log(a) - log(b)$   $\Rightarrow Ep(z|x;\theta_t) \cdot log \frac{P(z|x;\theta_t)}{P(z|x;\theta_{t+1})}$ p(2/x;0+) Q(0+10+)-Q(0+10+)+ZP(Z|X;0).log P(Z|X;0++1)  $Q(\theta_{t+1}|\theta_t) - Q(\theta_t|\theta_t) + \sum_{i} p(z|x_i\theta_t) \cdot \log_{i} p(z|x_i\theta_{t+1}) =$ log p(X | 0+1) - log p(X | 0+) 10gp(X10+1)-10gp(X10p)=Q(0+110+)-Q(0+10) log ρ(x|θ+1) + log ρ(x|θf)-Q(θ+1, |θ+)-Q(θ+ |θ+) ≥0

from m-step O++= argmax Q(0 10) Hence = Q(0+10+) = Q(0+10+) > Q(0++10+) - Q(0+10+) ≥0 Therefore, log p(x|0+1) - log p(x|0+) =0