Calculus: Integral

Trancendental Functions

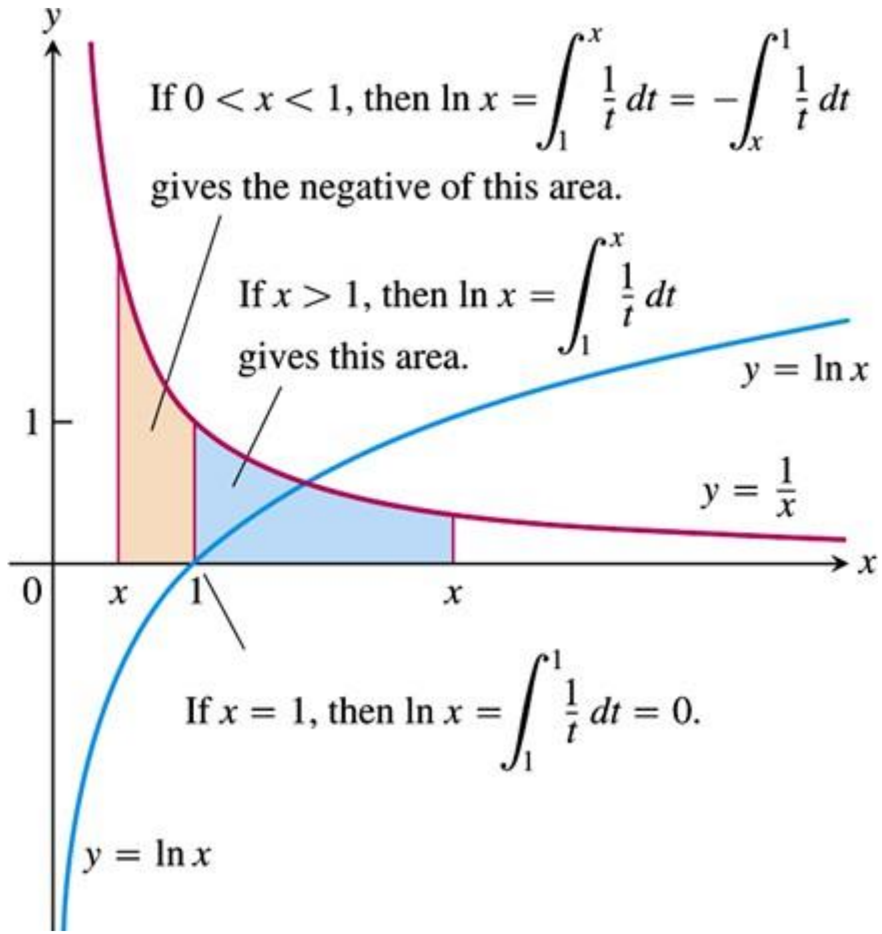
Calculus Team
President University
2025

Definition: Natural Logarithm

DEFINITION **The Natural Logarithm Function**

$$\ln x = \int_1^x \frac{1}{t}\,dt, \qquad x > 0$$

- If x > 1, then ln x is the area under the curve y=1/t; [1,x]
- If 0 < x < 1, then ln x is the negative area from x to 1.



If $0 < x < 1$, then $\ln x = \int_1^x \frac{1}{t}\,dt = -\int_x^1 \frac{1}{t}\,dt$
gives the negative of this area.
If $x > 1$, then $\ln x = \int_1^x \frac{1}{t}\,dt$
gives this area.
If $x = 1$, then $\ln x = \int_1^1 \frac{1}{t}\,dt = 0$.
$y = \ln x$
$y = \frac{1}{x}$
$y = \ln x$
y
x
0
1
x
1
x

$$\ln x = \int_1^x \frac{1}{x}\,dx$$

TABLE 7.1 Typical 2-place values of $\ln x$

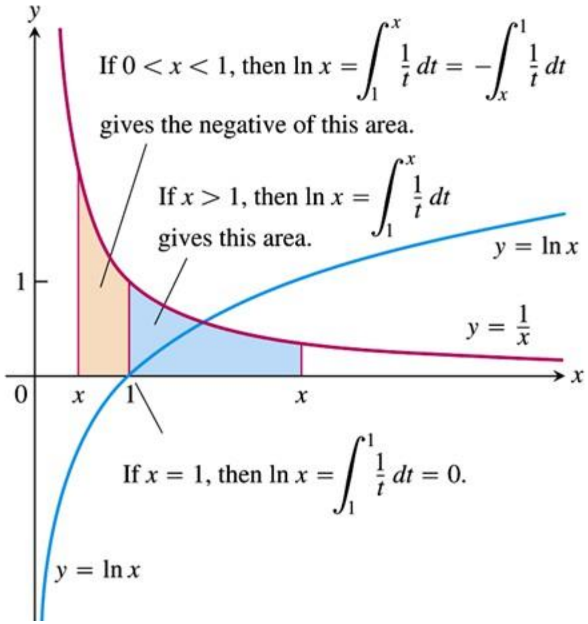
x	$\ln x$
0	undefined
0.05	-3.00
0.5	-0.69
1	0
2	0.69
3	1.10
4	1.39
10	2.30

Definition: number *e*

- Number *e* is a number in teh domain of the natural logarithm which satisfies

  $\ln e = 1$

- The blue area = 1, if x = *e*



Integral

- If u is a differentiable function and never zero, then

$$\int \frac{1}{u}\, du = \ln|u| + C.$$

- Exercises:

1. $\int_0^2 \frac{2x}{x^2-5}\,dx$
2. $\int \tan x \, dx$
3. $\int \cot x \, dx$
4. $\int cot\, 5x \, dx$

Exponential: e^x

$$\int e^u \, du = e^u + C.$$

- Exercises

1. $\displaystyle\int_{\ln 2}^{\ln 3} e^x dx$
2. $\displaystyle\int 8e^{(x+1)} dx$
3. $\displaystyle\int \frac{e^{\sqrt{s}}}{\sqrt{s}} ds$
4. $\displaystyle\int 2e^x \cos e^x dx$

Initial value problem

$$e^{y}\frac{dy}{dx}=2x;\quad x>\sqrt{3};\quad y(2)=0$$

$answer:$

$$\int e^{y}dy=\int 2x\,dx$$

$$e^{y}=x^{2}+C$$

$we\quad use\quad initial\quad value\quad y(2)=0$

$$C=e^{0}-2^{2}=1-4=-3$$

$Solution:$

$$e^{y}=x^{2}-3\quad or\quad y=\ln(x^{2}-3)$$

Example of Initial value problem

- $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}, \mathrm{y}(4) = 0$

1. Solve differential equation

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$y = \sqrt{x} + C$$

2. Evaluate C

$$y(4) = \sqrt{4} + C = 0$$
$$C = -2$$

3. Solution $y = \sqrt{x} - 2$

Exercises initial value problem

1. Find the curve y=f(x) that passes through the point (9,4). The curve has slope at each point is $3\sqrt{x}$

2. $\frac{dy}{dx} = \cos x + \sin x\, , \mathrm{y}(\pi) = 1$

3 $\frac{d^2y}{dx} = \frac{3t}{8}, \frac{dy(4)}{dx} = 3\; and\; y(4) = 4$
.

a^x and $\log_a x$

- Derivation

$$\frac{d}{dx} a^u = a^u \ln a \, \frac{du}{dx}.$$

$$\log_a x = \frac{1}{\ln a} \cdot \ln x = \frac{\ln x}{\ln a}$$

- Integral

$$\int a^u \, du = \frac{a^u}{\ln a} + C.$$

Exercises

1. $\displaystyle\int \frac{\log_3 x}{3x}dx = \int \frac{\ln x}{3x\ln 3}dx$

2. $\displaystyle\int 5^x\,dx$

3. $\displaystyle\int \frac{2^{\ln x}}{x}dx$

4. $\displaystyle\int \frac{\ln 2\log_2 x}{x}dx$

5. $\displaystyle\int \frac{dx}{x\log_8 x}$