Integral: Techniques of Integration

Calculus Team
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- **Rasional Reduction (improper fraction)**
- Improper fraction: the degree of numerator is greater than or equal to the degree of denominator
- example:

$$\int \frac{x^2 + 2x + 2}{x^2 + 2} \, dx$$

$$x^2 + 2 \overline{\big)\ x^2 + 2x + 2}$$
$$\ \underline{x^2 + 2}$$
$$\ 2x$$

$$\frac{x^2 + 2x + 2}{x^2 + 2} = 1 + \frac{2x}{x^2 + 2}$$

Exercises :

1. $\displaystyle \int \frac{2x^3}{x^2 - 1} \, dx$

2. $\displaystyle \int \frac{4t^3 - t^2 + 16t}{t^2 + 4} \, dt$

3. $\displaystyle \int \frac{4x^2 - 7}{2x + 3} \, dx$

Rasional Separation

$$\int \frac{x+2}{x^2+1}dx = \int \frac{x}{x^2+1}dx + \int \frac{2}{x^2+1}dx$$

Exercises :

1. $\displaystyle\int \frac{1+\sin x}{\cos^2 x}\,dx$

2. $\displaystyle\int \frac{1-x}{\sqrt{1-x^2}}\,dx$

3. $\displaystyle\int \frac{x+2\sqrt{x-1}}{2x\sqrt{x-1}}\,dx$

Multiplication by 1

$$\int \sec x \, dx$$

solution :

$$\int \sec x \, dx = \int \sec x . \frac{\sec x + \tan x}{\sec x + \tan x} \, dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

example : $\quad u = \sec x + \tan x$

$$du = (\sec^2 x + \sec x \tan x) dx$$

$$\int \sec x \, dx = \int \frac{du}{u} = \ln |u| + C$$

$$= \ln |\sec x + \tan x| + C$$

Exercises

1. $\displaystyle \int \frac{1}{1+\sin x}\, dx$

2. $\displaystyle \int \frac{1}{\csc x + \cot x}\, dx$

3. $\displaystyle \int \frac{1}{1-\sec x}\, dx$

Integration by part

- Integral of multiplication of two functions is not the same with the multiplication of two integrals of those two functions:

- example: $\int f(x).g(x)dx \neq \int f(x)dx.\int g(x)dx$

$$\int x^2 dx \neq \int x\, dx.\int x\, dx$$

Integration by part

- From derivation rules:

$$\frac{d}{dx}\big[f(x).g(x)\big] = \frac{df(x)}{dx}.g(x) + f(x).\frac{dg(x)}{dx}$$

atau :

$$\frac{d}{dx}\big[f(x).g(x)\big] = f'(x)g(x) + f(x)g'(x)$$

$$\int \frac{d}{dx}\big[f(x).g(x)\big]dx = \int \big[f'(x)g(x) + f(x)g'(x)\big]dx$$

$$f(x).g(x) = \int f'(x)g(x)dx + \int f(x)g'(x)dx$$

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

Integration by part

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

Integration by Parts Formula

$$\int u\,dv = uv - \int v\,du$$

Integration by Parts Formula for Definite Integrals

$$\int_a^b f(x)g'(x)\,dx = f(x)g(x)\Big]_a^b - \int_a^b f'(x)g(x)\,dx$$

Example: integration by part

the choice of u and dv

1. $u = 1; dv = x\cos x\, dx$

2. $u = x; dv = \cos x\, dx$

3. $u = x\cos x; dv = dx$

4. $u = \cos x; dv = x\, dx$

$$\int x\cos x\, dx$$

$$example: u = x \qquad dv = \cos x\, dx$$

$$du = dx \qquad v = \sin x$$

$$\int x\cos x\, dx = x\sin x - \int \sin x\, dx = x\sin x + \cos x + C$$

exercises

1. $\displaystyle\int \ln x \, dx$

2. $\displaystyle\int x^2 e^x \, dx$

3. $\displaystyle\int e^x \cos x \, dx$

4. $\displaystyle\int x \ln x \, dx$

Integral by part: tabling

$$\int x^3 \sin x \, dx, \quad f(x) = x^3, \quad g(x) = \sin x$$

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^3	(+)	$\sin x$
$3x^2$	(−)	$-\cos x$
$6x$	(+)	$-\sin x$
6	(−)	$\cos x$
0		$\sin x$

$$\int x^3 \sin x \, dx = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C.$$

example

$$\int x^3 \sin x \, dx$$

- **f(x) = x³ dad g(x)=sin x**

f(x) and derivation		g(x) and integral
x³	+	sin x
3x²	-	- cos x
6x	+	- sin x
6	-	cos x
0		sin x

$$\int x^3 \sin x \, dx = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C$$