#

Simple Linear Regression in Python

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Simple Linear Regression- Closed Form

A Simple Linear Regression fits a line to data points with two dimensions. It does this by defining and then minimizing a cost function. One of the most common methods used is ordinary least squares (OLS), which minimizes the square of the residuals of a line plotted against the data points.

The line of a linear regression is given by the formula, with intercept

$$w_0$$

and slope

$$w_1$$

$$y = w_0 + w_1 x$$

In the case of simple linear regression, these coefficients can be measured by taking the partial derivatives of the cost function with respect to each coefficien, setting them to zero, doing a little algebra to separate the terms, and calculating each coefficient. The full derivation can be found at wikipedia (https://en.wikipedia.org/wiki/Simple_linear_regression#Derivation_of_simple_regression_estimators).

The bottom line is that the closed form solution ends with these two equations:

$$\hat{w}_0 = \frac{\sum_{i=1}^{N} y_i}{N} - \hat{w}_1 \frac{\sum_{i=1}^{N} x_i}{N}$$

$$\hat{w}_1 = \frac{\sum_{i=1}^{N} y_i x_i - \frac{\sum_{i=1}^{N} y_i \sum_{i=1}^{N} x_i}{N}}{\sum_{i=1}^{N} x_i^2 - \frac{\sum_{i=1}^{N} x_i \sum_{i=1}^{N} x_i}{N}}$$

From this point, it's simple to write a python function that calculates the slope and intercept of a simple linear regression.

In [61]:

```
def simple_linear_regression(X, y):
    '''
    Returns slope and intercept for a simple regression line

    inputs- Works best with numpy arrays, though other similar data structures will work fine.
        X - input data
        y - output data

    outputs - floats
    '''
    # initial sums
    n = float(len(X))
    sum_x = X.sum()
    sum_y = y.sum()
    sum_xy = (X*y).sum()
    sum_xx = (X**2).sum()

    # formula for w0
    slope = (sum_xy - (sum_x*sum_y)/n)/(sum_xx - (sum_x*sum_x)/n)

    # formula for w1
    intercept = sum_y/n - slope*(sum_x/n)

    return (intercept, slope)
```

Now let's try it out on some data.

In [84]:

```
import numpy as np
import random
import matplotlib.pyplot as plt
%matplotlib inline
random.seed(199)

# generating some test points
X = np.array(range(10))
y = np.array([random.randint(1, 10) for x in range(10)])
```

A convenient way to check the fit is to make predictions across the range of the input data, then plot them against the original data.

In [85]:

```
intercept, slope = simple_linear_regression(X, y)

print 'Intercept: %.2f, Slope: %.2f' % (intercept, slope)
```

```
Intercept: 3.60, Slope: 0.27
```

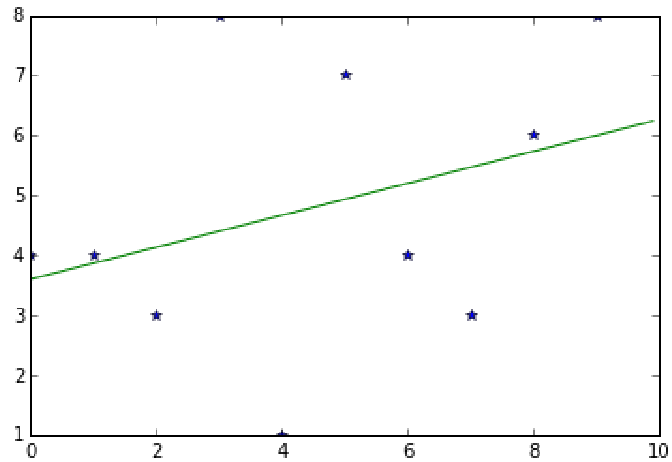
In [86]:

```
def reg_predictions(X, intercept, slope):
    return ((slope*X) + intercept)

line_x = np.array([x/10. for x in range(100)])
line_y = reg_predictions(line_x, intercept, slope)
plt.plot(X, y, '*', line_x, line_y, '-')
```

Out[86]:

```
[<matplotlib.lines.Line2D at 0x107ed97d0>,
 <matplotlib.lines.Line2D at 0x107ed9a50>]
```



The fit isn't very good, as expected for random data, but at least we can see the function in action.

python **regression** **statistics**

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