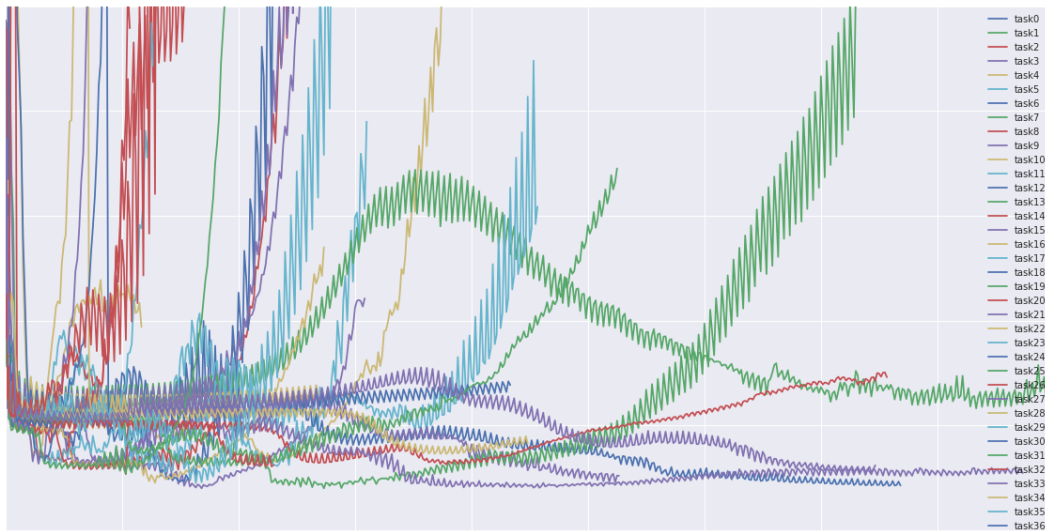≡

19 Jan 2016

An overview of gradient descent optimization algorithms 🐦

Note: If you are looking for a review paper, this blog post is also available as an article on arXiv .

Update 15.06.2017: Added derivations of AdaMax and Nadam .

Update 24.11.2017: Most of the content in this article is now also available as slides .

Update 09.02.2018: Added AMSGrad .

Table of contents:

- Gradient descent variants

- Conclusion
- References

Gradient descent is one of the most popular algorithms to perform optimization and by far the most common way to optimize neural networks. At the same time, every state-of-the-art Deep Learning library contains implementations of various algorithms to optimize gradient descent (e.g. lasagne's , caffe's , and keras' documentation). These algorithms, however, are often used as black-box optimizers, as practical explanations of their strengths and weaknesses are hard to come by.

This blog post aims at providing you with intuitions towards the behaviour of different algorithms for optimizing gradient descent that will help you put them to use. We are first going to look at the different variants of gradient descent. We will then briefly summarize challenges during training. Subsequently, we will introduce the most common optimization algorithms by showing their motivation to resolve these challenges and how this leads to the derivation of their update rules. We will also take a short look at algorithms and architectures to optimize gradient descent in a parallel and distributed setting. Finally, we will consider additional strategies that are helpful for optimizing gradient descent.

Gradient descent is a way to minimize an objective function $J(\theta)$ parameterized by a model's parameters $\theta \in \mathbb{R}^d$ by updating the parameters in the opposite direction of the gradient of the objective function $\nabla_\theta J(\theta)$ w.r.t. to the parameters. The learning rate η determines the size of the steps we take to reach a (local) minimum. In other words, we follow the direction of the slope of the surface created by the objective function downhill until we reach a valley. If you are unfamiliar with gradient descent, you can find a good introduction on optimizing neural networks here .

Gradient descent variants

There are three variants of gradient descent, which differ in how much data we use to compute the gradient of the objective function. Depending on the amount of data, we make a trade-off between the accuracy of the parameter update and the time it takes to perform an update.

Batch gradient descent

Vanilla gradient descent, aka batch gradient descent, computes the gradient of the cost function w.r.t. to the parameters θ for the entire training dataset:

$$\theta = \theta - \eta \cdot \nabla_\theta J(\theta).$$

As we need to calculate the gradients for the whole dataset to perform just *one* update, batch gradient descent can be very slow and is intractable for datasets that don't fit in memory. Batch gradient descent also doesn't allow us to update our model *online*, i.e. with new examples on-the-fly.

In code, batch gradient descent looks something like this:

```
for i in range(nb_epochs):
  params_grad = evaluate_gradient(loss_function, data, para
  params = params - learning_rate * params_grad
```

For a pre-defined number of epochs, we first compute the gradient vector `params_grad` of the loss function for the whole dataset w.r.t. our parameter vector `params`. Note that state-of-the-art deep learning libraries provide automatic differentiation that efficiently computes the gradient w.r.t. some parameters. If you derive the gradients yourself,

then gradient checking is a good idea. (See here for some great tips on how to check gradients properly.)

We then update our parameters in the direction of the gradients with the learning rate determining how big of an update we perform. Batch gradient descent is guaranteed to converge to the global minimum for convex error surfaces and to a local minimum for non-convex surfaces.

Stochastic gradient descent

Stochastic gradient descent (SGD) in contrast performs a parameter update for *each* training example $x^{(i)}$ and label $y^{(i)}$:

$$\theta = \theta - \eta \cdot \nabla_\theta J(\theta; x^{(i)}; y^{(i)}).$$

Batch gradient descent performs redundant computations for large datasets, as it recomputes gradients for similar examples before each parameter update. SGD does away with this redundancy by performing one update at a time. It is therefore usually much faster and can also be used to learn online.

SGD performs frequent updates with a high variance that cause the objective function to fluctuate heavily as in Image 1.
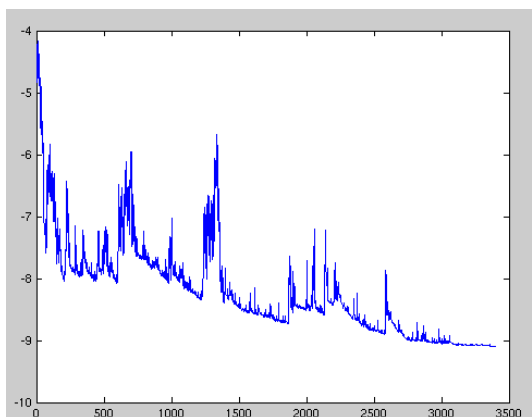

Image 1: SGD fluctuation (Source: Wikipedia)

While batch gradient descent converges to the minimum of the basin the parameters are placed in, SGD's fluctuation, on the one hand, enables it to jump to new and potentially better local minima. On the other hand, this ultimately complicates convergence to the exact

minimum, as SGD will keep overshooting. However, it has been shown that when we slowly decrease the learning rate, SGD shows the same convergence behaviour as batch gradient descent, almost certainly converging to a local or the global minimum for non-convex and convex optimization respectively.

Its code fragment simply adds a loop over the training examples and evaluates the gradient w.r.t. each example. Note that we shuffle the training data at every epoch as explained in this section.

```
for i in range(nb_epochs):
  np.random.shuffle(data)
  for example in data:
    params_grad = evaluate_gradient(loss_function, example,
    params = params - learning_rate * params_grad
```

Mini-batch gradient descent

Mini-batch gradient descent finally takes the best of both worlds and performs an update for every mini-batch of n training examples:

$$\theta = \theta - \eta \cdot \nabla_\theta J(\theta; x^{(i:i+n)}; y^{(i:i+n)}).$$

This way, it *a)* reduces the variance of the parameter updates, which can lead to more stable convergence; and *b)* can make use of highly optimized matrix optimizations common to state-of-the-art deep learning libraries that make computing the gradient w.r.t. a mini-batch very efficient. Common mini-batch sizes range between 50 and 256, but can vary for different applications. Mini-batch gradient descent is typically the algorithm of choice when training a neural network and the term SGD usually is employed also when mini-batches are used. Note: In modifications of SGD in the rest of this post, we leave out the parameters $x^{(i:i+n)}; y^{(i:i+n)}$ for simplicity.

In code, instead of iterating over examples, we now iterate over mini-batches of size 50:

```python
for i in range(nb_epochs):
  np.random.shuffle(data)
  for batch in get_batches(data, batch_size=50):
    params_grad = evaluate_gradient(loss_function, batch, p
    params = params - learning_rate * params_grad
```

Challenges

Vanilla mini-batch gradient descent, however, does not guarantee good convergence, but offers a few challenges that need to be addressed:

- Choosing a proper learning rate can be difficult. A learning rate that is too small leads to painfully slow convergence, while a learning rate that is too large can hinder convergence and cause the loss function to fluctuate around the minimum or even to diverge.

- Learning rate schedules [11] try to adjust the learning rate during training by e.g. annealing, i.e. reducing the learning rate according to a pre-defined schedule or when the change in objective between epochs falls below a threshold. These schedules and thresholds, however, have to be defined in advance and are thus unable to adapt to a dataset's characteristics [10].

- Additionally, the same learning rate applies to all parameter updates. If our data is sparse and our features have very different frequencies, we might not want to update all of them to the same extent, but perform a larger update for rarely occurring features.

- Another key challenge of minimizing highly non-convex error functions common for neural networks is avoiding getting trapped in their numerous suboptimal local minima. Dauphin et al. [19] argue that the difficulty arises in fact not from local minima but from saddle points, i.e. points where one dimension slopes up and another slopes down. These saddle points are usually surrounded by a plateau of the same error, which makes it notoriously hard for SGD to escape, as the gradient is close to zero in all dimensions.

Gradient descent optimization algorithms

In the following, we will outline some algorithms that are widely used by the deep learning community to deal with the aforementioned challenges. We will not discuss algorithms that are infeasible to compute in practice for high-dimensional data sets, e.g. second-order methods such as Newton's method .

Momentum

SGD has trouble navigating ravines, i.e. areas where the surface curves much more steeply in one dimension than in another [1], which are common around local optima. In these scenarios, SGD oscillates across the slopes of the ravine while only making hesitant progress along the bottom towards the local optimum as in Image 2.
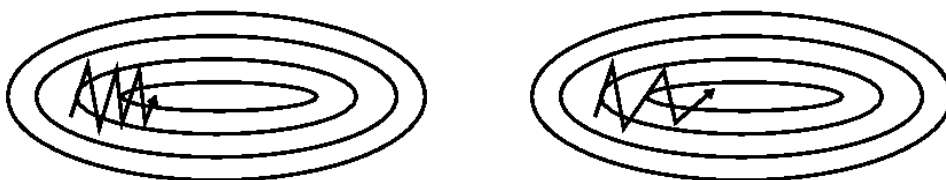

Image 2: SGD without momentum   Image 3: SGD with momentum
Momentum [2] is a method that helps accelerate SGD in the relevant direction and dampens oscillations as can be seen in Image 3. It does

this by adding a fraction γ of the update vector of the past time step to the current update vector:

$$v_t = \gamma v_{t-1} + \eta \nabla_\theta J(\theta)$$
$$\theta = \theta - v_t$$

Note: Some implementations exchange the signs in the equations. The momentum term γ is usually set to 0.9 or a similar value.

Essentially, when using momentum, we push a ball down a hill. The ball accumulates momentum as it rolls downhill, becoming faster and faster on the way (until it reaches its terminal velocity if there is air resistance, i.e. $\gamma < 1$). The same thing happens to our parameter updates: The momentum term increases for dimensions whose gradients point in the same directions and reduces updates for dimensions whose gradients change directions. As a result, we gain faster convergence and reduced oscillation.

Nesterov accelerated gradient

However, a ball that rolls down a hill, blindly following the slope, is highly unsatisfactory. We'd like to have a smarter ball, a ball that has a notion of where it is going so that it knows to slow down before the hill slopes up again.

Nesterov accelerated gradient (NAG) [7] is a way to give our momentum term this kind of prescience. We know that we will use our momentum term γv_{t-1} to move the parameters θ. Computing $\theta - \gamma v_{t-1}$ thus gives us an approximation of the next position of the parameters (the gradient is missing for the full update), a rough idea where our parameters are going to be. We can now effectively look ahead by calculating the gradient not w.r.t. to our current parameters θ but w.r.t. the approximate future position of our parameters:

$$v_t = \gamma v_{t-1} + \eta \nabla_\theta J(\theta - \gamma v_{t-1})$$
$$\theta = \theta - v_t$$

Again, we set the momentum term γ to a value of around 0.9. While Momentum first computes the current gradient (small blue vector in Image 4) and then takes a big jump in the direction of the updated accumulated gradient (big blue vector), NAG first makes a big jump in the direction of the previous accumulated gradient (brown vector), measures the gradient and then makes a correction (red vector), which results in the complete NAG update (green vector). This anticipatory update prevents us from going too fast and results in increased responsiveness, which has significantly increased the performance of RNNs on a number of tasks [8].
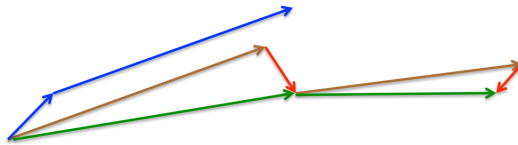

Image 4: Nesterov update (Source: G. Hinton's lecture 6c)

Refer to here for another explanation about the intuitions behind NAG, while Ilya Sutskever gives a more detailed overview in his PhD thesis [9].

Now that we are able to adapt our updates to the slope of our error function and speed up SGD in turn, we would also like to adapt our updates to each individual parameter to perform larger or smaller updates depending on their importance.

Adagrad

Adagrad [3] is an algorithm for gradient-based optimization that does just this: It adapts the learning rate to the parameters, performing smaller updates
(i.e. low learning rates) for parameters associated with frequently occurring features, and larger updates (i.e. high learning rates) for parameters associated with infrequent features. For this reason, it is well-suited for dealing with sparse data. Dean et al. [4] have found that

Adagrad greatly improved the robustness of SGD and used it for training large-scale neural nets at Google, which -- among other things -- learned to recognize cats in Youtube videos . Moreover, Pennington et al. [5] used Adagrad to train GloVe word embeddings, as infrequent words require much larger updates than frequent ones.

Previously, we performed an update for all parameters θ at once as every parameter θ_i used the same learning rate η. As Adagrad uses a different learning rate for every parameter θ_i at every time step t, we first show Adagrad's per-parameter update, which we then vectorize. For brevity, we use g_t to denote the gradient at time step t. $g_{t,i}$ is then the partial derivative of the objective function w.r.t. to the parameter θ_i at time step t:

$$g_{t,i} = \nabla_\theta J(\theta_{t,i}).$$

The SGD update for every parameter θ_i at each time step t then becomes:

$$\theta_{t+1,i} = \theta_{t,i} - \eta \cdot g_{t,i}.$$

In its update rule, Adagrad modifies the general learning rate η at each time step t for every parameter θ_i based on the past gradients that have been computed for θ_i:

$$\theta_{t+1,i} = \theta_{t,i} - \frac{\eta}{\sqrt{G_{t,ii} + \epsilon}} \cdot g_{t,i}.$$

$G_t \in \mathrm{R}^{d \times d}$ here is a diagonal matrix where each diagonal element i, i is the sum of the squares of the gradients w.r.t. θ_i up to time step t [25], while ϵ is a smoothing term that avoids division by zero (usually on the order of $1e - 8$). Interestingly, without the square root operation, the algorithm performs much worse.

As G_t contains the sum of the squares of the past gradients w.r.t. to all parameters θ along its diagonal, we can now vectorize our

implementation by performing a matrix-vector product $\odot$ between G_t and g_t:

$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{G_t + \epsilon}} \odot g_t.$$

One of Adagrad's main benefits is that it eliminates the need to manually tune the learning rate. Most implementations use a default value of 0.01 and leave it at that.

Adagrad's main weakness is its accumulation of the squared gradients in the denominator: Since every added term is positive, the accumulated sum keeps growing during training. This in turn causes the learning rate to shrink and eventually become infinitesimally small, at which point the algorithm is no longer able to acquire additional knowledge. The following algorithms aim to resolve this flaw.

Adadelta

Adadelta [6] is an extension of Adagrad that seeks to reduce its aggressive, monotonically decreasing learning rate. Instead of accumulating all past squared gradients, Adadelta restricts the window of accumulated past gradients to some fixed size w.

Instead of inefficiently storing w previous squared gradients, the sum of gradients is recursively defined as a decaying average of all past squared gradients. The running average $E[g^2]_t$ at time step t then depends (as a fraction γ similarly to the Momentum term) only on the previous average and the current gradient:

$$E[g^2]_t = \gamma E[g^2]_{t-1} + (1 - \gamma)g_t^2.$$

We set γ to a similar value as the momentum term, around 0.9. For clarity, we now rewrite our vanilla SGD update in terms of the parameter update vector $\Delta\theta_t$:

$$\Delta\theta_t = -\eta \cdot g_{t,i}$$
$$\theta_{t+1} = \theta_t + \Delta\theta_t$$

The parameter update vector of Adagrad that we derived previously thus takes the form:

$$\Delta\theta_t = -\frac{\eta}{\sqrt{G_t + \epsilon}} \odot g_t.$$

We now simply replace the diagonal matrix G_t with the decaying average over past squared gradients $E[g^2]_t$:

$$\Delta\theta_t = -\frac{\eta}{\sqrt{E[g^2]_t + \epsilon}} g_t.$$

As the denominator is just the root mean squared (RMS) error criterion of the gradient, we can replace it with the criterion short-hand:

$$\Delta\theta_t = -\frac{\eta}{RMS[g]_t} g_t.$$

The authors note that the units in this update (as well as in SGD, Momentum, or Adagrad) do not match, i.e. the update should have the same hypothetical units as the parameter. To realize this, they first define another exponentially decaying average, this time not of squared gradients but of squared parameter updates:

$$E[\Delta\theta^2]_t = \gamma E[\Delta\theta^2]_{t-1} + (1-\gamma)\Delta\theta_t^2.$$

The root mean squared error of parameter updates is thus:

$$RMS[\Delta\theta]_t = \sqrt{E[\Delta\theta^2]_t + \epsilon}.$$

Since $RMS[\Delta\theta]_t$ is unknown, we approximate it with the RMS of parameter updates until the previous time step. Replacing the learning rate η in the previous update rule with $RMS[\Delta\theta]_{t-1}$ finally yields the Adadelta update rule:

$$\Delta\theta_t = -\frac{RMS[\Delta\theta]_{t-1}}{RMS[g]_t}g_t$$

$$\theta_{t+1} = \theta_t + \Delta\theta_t$$

With Adadelta, we do not even need to set a default learning rate, as it has been eliminated from the update rule.

RMSprop

RMSprop is an unpublished, adaptive learning rate method proposed by Geoff Hinton in Lecture 6e of his Coursera Class .

RMSprop and Adadelta have both been developed independently around the same time stemming from the need to resolve Adagrad's radically diminishing learning rates. RMSprop in fact is identical to the first update vector of Adadelta that we derived above:

$$E[g^2]_t = 0.9E[g^2]_{t-1} + 0.1g_t^2$$

$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{E[g^2]_t + \epsilon}}g_t$$

RMSprop as well divides the learning rate by an exponentially decaying average of squared gradients. Hinton suggests γ to be set to 0.9, while a good default value for the learning rate η is 0.001.

Adam

Adaptive Moment Estimation (Adam) [15] is another method that computes adaptive learning rates for each parameter. In addition to storing an exponentially decaying average of past squared gradients v_t like Adadelta and RMSprop, Adam also keeps an exponentially decaying average of past gradients m_t, similar to momentum. Whereas momentum can be seen as a ball running down a slope, Adam behaves like a heavy ball with friction, which thus prefers flat minima in

the error surface [29]. We compute the decaying averages of past and past squared gradients m_t and v_t respectively as follows:

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t$$
$$v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2$$

m_t and v_t are estimates of the first moment (the mean) and the second moment (the uncentered variance) of the gradients respectively, hence the name of the method. As m_t and v_t are initialized as vectors of 0's, the authors of Adam observe that they are biased towards zero, especially during the initial time steps, and especially when the decay rates are small (i.e. β_1 and β_2 are close to 1).

They counteract these biases by computing bias-corrected first and second moment estimates:

$$\hat{m}_t = \frac{m_t}{1 - \beta_1^t}$$

$$\hat{v}_t = \frac{v_t}{1 - \beta_2^t}$$

They then use these to update the parameters just as we have seen in Adadelta and RMSprop, which yields the Adam update rule:

$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon} \hat{m}_t.$$

The authors propose default values of 0.9 for β_1, 0.999 for β_2, and 10^{-8} for ϵ. They show empirically that Adam works well in practice and compares favorably to other adaptive learning-method algorithms.

AdaMax

The v_t factor in the Adam update rule scales the gradient inversely proportionally to the ℓ_2 norm of the past gradients (via the v_{t-1} term) and current gradient $|g_t|^2$:

$$v_t = \beta_2 v_{t-1} + (1 - \beta_2)|g_t|^2$$

We can generalize this update to the ℓ_p norm. Note that Kingma and Ba also parameterize β_2 as β_2^p:

$$v_t = \beta_2^p v_{t-1} + (1 - \beta_2^p)|g_t|^p$$

Norms for large p values generally become numerically unstable, which is why ℓ_1 and ℓ_2 norms are most common in practice. However, ℓ_∞ also generally exhibits stable behavior. For this reason, the authors propose AdaMax (Kingma and Ba, 2015) and show that v_t with ℓ_∞ converges to the following more stable value. To avoid confusion with Adam, we use u_t to denote the infinity norm-constrained v_t:

$$
\begin{aligned}
u_t &= \beta_2^\infty v_{t-1} + (1 - \beta_2^\infty)|g_t|^\infty \\
&= \max(\beta_2 \cdot v_{t-1}, |g_t|)
\end{aligned}
$$

We can now plug this into the Adam update equation by replacing $\sqrt{\hat{v}_t} + \epsilon$ with u_t to obtain the AdaMax update rule:

$$\theta_{t+1} = \theta_t - \frac{\eta}{u_t}\hat{m}_t$$

Note that as u_t relies on the $\max$ operation, it is not as suggestible to bias towards zero as m_t and v_t in Adam, which is why we do not need to compute a bias correction for u_t. Good default values are again $\eta = 0.002$, $\beta_1 = 0.9$, and $\beta_2 = 0.999$.

Nadam

As we have seen before, Adam can be viewed as a combination of RMSprop and momentum: RMSprop contributes the exponentially decaying average of past squared gradients v_t, while momentum accounts for the exponentially decaying average of past gradients m_t. We have also seen that Nesterov accelerated gradient (NAG) is superior to vanilla momentum.

Nadam (Nesterov-accelerated Adaptive Moment Estimation) [24] thus combines Adam and NAG. In order to incorporate NAG into Adam, we need to modify its momentum term m_t.

First, let us recall the momentum update rule using our current notation :

$$g_t = \nabla_{\theta_t} J(\theta_t)$$
$$m_t = \gamma m_{t-1} + \eta g_t$$
$$\theta_{t+1} = \theta_t - m_t$$

where J is our objective function, γ is the momentum decay term, and η is our step size. Expanding the third equation above yields:

$$\theta_{t+1} = \theta_t - (\gamma m_{t-1} + \eta g_t)$$

This demonstrates again that momentum involves taking a step in the direction of the previous momentum vector and a step in the direction of the current gradient.

NAG then allows us to perform a more accurate step in the gradient direction by updating the parameters with the momentum step *before* computing the gradient. We thus only need to modify the gradient g_t to arrive at NAG:

$$g_t = \nabla_{\theta_t} J(\theta_t - \gamma m_{t-1})$$
$$m_t = \gamma m_{t-1} + \eta g_t$$
$$\theta_{t+1} = \theta_t - m_t$$

Dozat proposes to modify NAG the following way: Rather than applying the momentum step twice -- one time for updating the gradient g_t and a second time for updating the parameters θ_{t+1} -- we now apply the look-ahead momentum vector directly to update the current parameters:

$$g_t = \nabla_{\theta_t} J(\theta_t)$$

$$m_t = \gamma m_{t-1} + \eta g_t$$

$$\theta_{t+1} = \theta_t - (\gamma m_t + \eta g_t)$$

Notice that rather than utilizing the previous momentum vector m_{t-1} as in the equation of the expanded momentum update rule above, we now use the current momentum vector m_t to look ahead. In order to add Nesterov momentum to Adam, we can thus similarly replace the previous momentum vector with the current momentum vector. First, recall that the Adam update rule is the following (note that we do not need to modify $\hat{v}_t$):

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1)g_t$$

$$\hat{m}_t = \frac{m_t}{1 - \beta_1^t}$$

$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon} \hat{m}_t$$

Expanding the second equation with the definitions of $\hat{m}_t$ and m_t in turn gives us:

$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon} \left(\frac{\beta_1 m_{t-1}}{1 - \beta_1^t} + \frac{(1 - \beta_1)g_t}{1 - \beta_1^t} \right)$$

Note that $\dfrac{\beta_1 m_{t-1}}{1 - \beta_1^t}$ is just the bias-corrected estimate of the momentum vector of the previous time step. We can thus replace it with $\hat{m}_{t-1}$:

$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon} \left(\beta_1 \hat{m}_{t-1} + \frac{(1 - \beta_1)g_t}{1 - \beta_1^t} \right)$$

Note that for simplicity, we ignore that the denominator is $1 - \beta_1^t$ and not $1 - \beta_1^{t-1}$ as we will replace the denominator in the next step anyway. This equation again looks very similar to our expanded momentum update rule above. We can now add Nesterov momentum

just as we did previously by simply replacing this bias-corrected estimate of the momentum vector of the previous time step $\hat{m}_{t-1}$ with the bias-corrected estimate of the current momentum vector $\hat{m}_t$, which gives us the Nadam update rule:

$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon}(\beta_1 \hat{m}_t + \frac{(1-\beta_1)g_t}{1-\beta_1^t})$$

AMSGrad

As adaptive learning rate methods have become the norm in training neural networks, practitioners noticed that in some cases, e.g. for object recognition [26] or machine translation [27] they fail to converge to an optimal solution and are outperformed by SGD with momentum.

Reddi et al. (2018) [28] formalize this issue and pinpoint the exponential moving average of past squared gradients as a reason for the poor generalization behaviour of adaptive learning rate methods. Recall that the introduction of the exponential average was well-motivated: It should prevent the learning rates to become infinitesimally small as training progresses, the key flaw of the Adagrad algorithm. However, this short-term memory of the gradients becomes an obstacle in other scenarios.

In settings where Adam converges to a suboptimal solution, it has been observed that some minibatches provide large and informative gradients, but as these minibatches only occur rarely, exponential averaging diminishes their influence, which leads to poor convergence. The authors provide an example for a simple convex optimization problem where the same behaviour can be observed for Adam.

To fix this behaviour, the authors propose a new algorithm, AMSGrad that uses the maximum of past squared gradients v_t rather than the

exponential average to update the parameters. v_t is defined the same as in Adam above:

$$v_t = \beta_2 v_{t-1} + (1 - \beta_2)g_t^2$$

Instead of using v_t (or its bias-corrected version $\hat{v}_t$) directly, we now employ the previous v_{t-1} if it is larger than the current one:

$$\hat{v}_t = \max(\hat{v}_{t-1}, v_t)$$

This way, AMSGrad results in a non-increasing step size, which avoids the problems suffered by Adam. For simplicity, the authors also remove the debiasing step that we have seen in Adam. The full AMSGrad update without bias-corrected estimates can be seen below:

$$m_t = \beta_1 m_{t-1} + (1 - \beta_1)g_t$$
$$v_t = \beta_2 v_{t-1} + (1 - \beta_2)g_t^2$$
$$\hat{v}_t = \max(\hat{v}_{t-1}, v_t)$$
$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon}m_t$$

The authors observe improved performance compared to Adam on small datasets and on CIFAR-10. Other experiments , however, show similar or worse performance than Adam. It remains to be seen whether AMSGrad is able to consistently outperform Adam in practice. For more information about recent advances in Deep Learning optimization, refer to this blog post .

Visualization of algorithms

The following two animations (Image credit: Alec Radford) provide some intuitions towards the optimization behaviour of most of the presented optimization methods. Also have a look here for a description of the same images by Karpathy and another concise overview of the algorithms discussed.

In Image 5, we see their behaviour on the contours of a loss surface (the Beale function) over time. Note that Adagrad, Adadelta, and RMSprop almost immediately head off in the right direction and converge similarly fast, while Momentum and NAG are led off-track, evoking the image of a ball rolling down the hill. NAG, however, is quickly able to correct its course due to its increased responsiveness by looking ahead and heads to the minimum.

Image 6 shows the behaviour of the algorithms at a saddle point, i.e. a point where one dimension has a positive slope, while the other dimension has a negative slope, which pose a difficulty for SGD as we mentioned before. Notice here that SGD, Momentum, and NAG find it difficulty to break symmetry, although the two latter eventually manage to escape the saddle point, while Adagrad, RMSprop, and Adadelta quickly head down the negative slope.
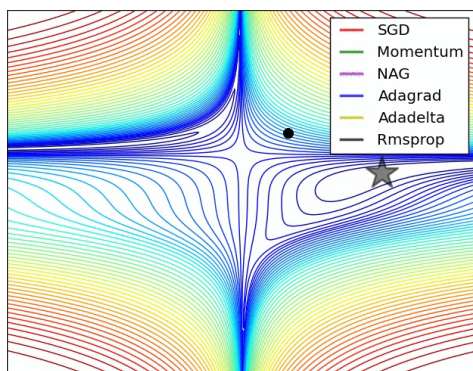

Image 5: SGD optimization on loss surface contours

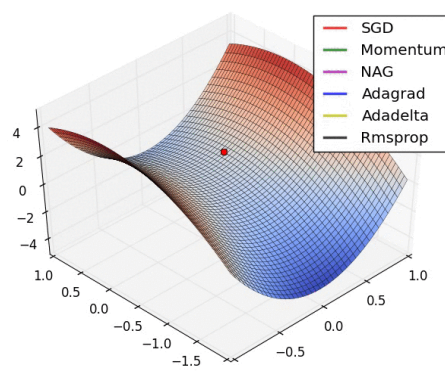

Image 6: SGD optimization on saddle point

As we can see, the adaptive learning-rate methods, i.e. Adagrad, Adadelta, RMSprop, and Adam are most suitable and provide the best convergence for these scenarios.

Note: If you are interested in visualizing these or other optimization algorithms, refer to this useful tutorial .

Which optimizer to use?

So, which optimizer should you now use? If your input data is sparse, then you likely achieve the best results using one of the adaptive learning-rate methods. An additional benefit is that you won't need to

tune the learning rate but likely achieve the best results with the default value.

In summary, RMSprop is an extension of Adagrad that deals with its radically diminishing learning rates. It is identical to Adadelta, except that Adadelta uses the RMS of parameter updates in the numinator update rule. Adam, finally, adds bias-correction and momentum to RMSprop. Insofar, RMSprop, Adadelta, and Adam are very similar algorithms that do well in similar circumstances. Kingma et al. [15] show that its bias-correction helps Adam slightly outperform RMSprop towards the end of optimization as gradients become sparser. Insofar, Adam might be the best overall choice.

Interestingly, many recent papers use vanilla SGD without momentum and a simple learning rate annealing schedule. As has been shown, SGD usually achieves to find a minimum, but it might take significantly longer than with some of the optimizers, is much more reliant on a robust initialization and annealing schedule, and may get stuck in saddle points rather than local minima. Consequently, if you care about fast convergence and train a deep or complex neural network, you should choose one of the adaptive learning rate methods.

Parallelizing and distributing SGD

Given the ubiquity of large-scale data solutions and the availability of low-commodity clusters, distributing SGD to speed it up further is an obvious choice.
SGD by itself is inherently sequential: Step-by-step, we progress further towards the minimum. Running it provides good convergence but can be slow particularly on large datasets. In contrast, running SGD asynchronously is faster, but suboptimal communication between workers can lead to poor convergence. Additionally, we can also parallelize SGD on one machine without the need for a large

computing cluster. The following are algorithms and architectures that have been proposed to optimize parallelized and distributed SGD.

Hogwild!

Niu et al. [23] introduce an update scheme called Hogwild! that allows performing SGD updates in parallel on CPUs. Processors are allowed to access shared memory without locking the parameters. This only works if the input data is sparse, as each update will only modify a fraction of all parameters. They show that in this case, the update scheme achieves almost an optimal rate of convergence, as it is unlikely that processors will overwrite useful information.

Downpour SGD

Downpour SGD is an asynchronous variant of SGD that was used by Dean et al. [4] in their DistBelief framework (predecessor to TensorFlow) at Google. It runs multiple replicas of a model in parallel on subsets of the training data. These models send their updates to a parameter server, which is split across many machines. Each machine is responsible for storing and updating a fraction of the model's parameters. However, as replicas don't communicate with each other e.g. by sharing weights or updates, their parameters are continuously at risk of diverging, hindering convergence.

Delay-tolerant Algorithms for SGD

McMahan and Streeter [12] extend AdaGrad to the parallel setting by developing delay-tolerant algorithms that not only adapt to past gradients, but also to the update delays. This has been shown to work well in practice.

TensorFlow

TensorFlow [13] is Google's recently open-sourced framework for the implementation and deployment of large-scale machine learning models. It is based on their experience with DistBelief and is already used internally to perform computations on a large range of mobile devices as well as on large-scale distributed systems. For distributed execution, a computation graph is split into a subgraph for every device and communication takes place using Send/Receive node pairs. However, the open source version of TensorFlow currently does not support distributed functionality (see here). Update 13.04.16: A distributed version of TensorFlow has been released .

Elastic Averaging SGD

Zhang et al. [14] propose Elastic Averaging SGD (EASGD), which links the parameters of the workers of asynchronous SGD with an elastic force, i.e. a center variable stored by the parameter server. This allows the local variables to fluctuate further from the center variable, which in theory allows for more exploration of the parameter space. They show empirically that this increased capacity for exploration leads to improved performance by finding new local optima.

Additional strategies for optimizing SGD

Finally, we introduce additional strategies that can be used alongside any of the previously mentioned algorithms to further improve the performance of SGD. For a great overview of some other common tricks, refer to [22].

Shuffling and Curriculum Learning

Generally, we want to avoid providing the training examples in a meaningful order to our model as this may bias the optimization

algorithm. Consequently, it is often a good idea to shuffle the training data after every epoch.

On the other hand, for some cases where we aim to solve progressively harder problems, supplying the training examples in a meaningful order may actually lead to improved performance and better convergence. The method for establishing this meaningful order is called Curriculum Learning [16].

Zaremba and Sutskever [17] were only able to train LSTMs to evaluate simple programs using Curriculum Learning and show that a combined or mixed strategy is better than the naive one, which sorts examples by increasing difficulty.

Batch normalization

To facilitate learning, we typically normalize the initial values of our parameters by initializing them with zero mean and unit variance. As training progresses and we update parameters to different extents, we lose this normalization, which slows down training and amplifies changes as the network becomes deeper.

Batch normalization [18] reestablishes these normalizations for every mini-batch and changes are back-propagated through the operation as well. By making normalization part of the model architecture, we are able to use higher learning rates and pay less attention to the initialization parameters. Batch normalization additionally acts as a regularizer, reducing (and sometimes even eliminating) the need for Dropout.

Early stopping

According to Geoff Hinton: "*Early stopping (is) beautiful free lunch*" (NIPS 2015 Tutorial slides , slide 63). You should thus always monitor

error on a validation set during training and stop (with some patience) if your validation error does not improve enough.

Gradient noise

Neelakantan et al. [21] add noise that follows a Gaussian distribution $N(0, \sigma_t^2)$ to each gradient update:

$$g_{t,i} = g_{t,i} + N(0, \sigma_t^2).$$

They anneal the variance according to the following schedule:

$$\sigma_t^2 = \frac{\eta}{(1 + t)^\gamma}.$$

They show that adding this noise makes networks more robust to poor initialization and helps training particularly deep and complex networks. They suspect that the added noise gives the model more chances to escape and find new local minima, which are more frequent for deeper models.

Conclusion

In this blog post, we have initially looked at the three variants of gradient descent, among which mini-batch gradient descent is the most popular. We have then investigated algorithms that are most commonly used for optimizing SGD: Momentum, Nesterov accelerated gradient, Adagrad, Adadelta, RMSprop, Adam, as well as different algorithms to optimize asynchronous SGD. Finally, we've considered other strategies to improve SGD such as shuffling and curriculum learning, batch normalization, and early stopping.

I hope that this blog post was able to provide you with some intuitions towards the motivation and the behaviour of the different optimization algorithms. Are there any obvious algorithms to improve SGD that I've

missed? What tricks are you using yourself to facilitate training with SGD? **Let me know in the comments below.**

Acknowledgements

Thanks to Denny Britz and Cesar Salgado for reading drafts of this post and providing suggestions.

Printable version and citation

This blog post is also available as an article on arXiv , in case you want to refer to it later.

In case you found it helpful, consider citing the corresponding arXiv article as:

Sebastian Ruder (2016). An overview of gradient descent optimisation algorithms. arXiv preprint arXiv:1609.04747.

Translations

This blog post has been translated into the following languages:

- Japanese
- Chinese
- Korean

Update 21.06.16: This post was posted to Hacker News. The discussion provides some interesting pointers to related work and other techniques.

References

1. Sutton, R. S. (1986). Two problems with backpropagation and other steepest-descent learning procedures for networks. Proc. 8th Annual Conf. Cognitive Science Society. ↩

2. Qian, N. (1999). On the momentum term in gradient descent learning algorithms. Neural Networks : The Official Journal of the International Neural Network Society, 12(1), 145–151. http://doi.org/10.1016/S0893-6080(98)00116-6 ↩

3. Duchi, J., Hazan, E., & Singer, Y. (2011). Adaptive Subgradient Methods for Online Learning and Stochastic Optimization. Journal of Machine Learning Research, 12, 2121–2159. Retrieved from http://jmlr.org/papers/v12/duchi11a.html ↩

4. Dean, J., Corrado, G. S., Monga, R., Chen, K., Devin, M., Le, Q. V, … Ng, A. Y. (2012). Large Scale Distributed Deep Networks. NIPS 2012: Neural Information Processing Systems, 1–11. http://papers.nips.cc/paper/4687-large-scale-distributed-deep-networks.pdf ↩

5. Pennington, J., Socher, R., & Manning, C. D. (2014). Glove: Global Vectors for Word Representation. Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing, 1532–1543. http://doi.org/10.3115/v1/D14-1162 ↩

6. Zeiler, M. D. (2012). ADADELTA: An Adaptive Learning Rate Method. Retrieved from http://arxiv.org/abs/1212.5701 ↩

7. Nesterov, Y. (1983). A method for unconstrained convex minimization problem with the rate of convergence o(1/k2). Doklady ANSSSR (translated as Soviet.Math.Docl.), vol. 269, pp. 543– 547. ↩

8. Bengio, Y., Boulanger-Lewandowski, N., & Pascanu, R. (2012). Advances in Optimizing Recurrent Networks. Retrieved from

http://arxiv.org/abs/1212.0901 ↩

9. Sutskever, I. (2013). Training Recurrent neural Networks. PhD Thesis. ↩

10. Darken, C., Chang, J., & Moody, J. (1992). Learning rate schedules for faster stochastic gradient search. Neural Networks for Signal Processing II Proceedings of the 1992 IEEE Workshop, (September), 1–11. http://doi.org/10.1109/NNSP.1992.253713 ↩

11. H. Robinds and S. Monro, "A stochastic approximation method," Annals of Mathematical Statistics, vol. 22, pp. 400–407, 1951. ↩

12. Mcmahan, H. B., & Streeter, M. (2014). Delay-Tolerant Algorithms for Asynchronous Distributed Online Learning. Advances in Neural Information Processing Systems (Proceedings of NIPS), 1–9. Retrieved from http://papers.nips.cc/paper/5242-delay-tolerant-algorithms-for-asynchronous-distributed-online-learning.pdf ↩

13. Abadi, M., Agarwal, A., Barham, P., Brevdo, E., Chen, Z., Citro, C., … Zheng, X. (2015). TensorFlow : Large-Scale Machine Learning on Heterogeneous Distributed Systems. ↩

14. Zhang, S., Choromanska, A., & LeCun, Y. (2015). Deep learning with Elastic Averaging SGD. Neural Information Processing Systems Conference (NIPS 2015), 1–24. Retrieved from http://arxiv.org/abs/1412.6651 ↩

15. Kingma, D. P., & Ba, J. L. (2015). Adam: a Method for Stochastic Optimization. International Conference on Learning Representations, 1–13. ↩

16. Bengio, Y., Louradour, J., Collobert, R., & Weston, J. (2009). Curriculum learning. Proceedings of the 26th Annual International

Conference on Machine Learning, 41–48.

http://doi.org/10.1145/1553374.1553380 ↩

17. Zaremba, W., & Sutskever, I. (2014). Learning to Execute, 1–25.
Retrieved from http://arxiv.org/abs/1410.4615 ↩

18. Ioffe, S., & Szegedy, C. (2015). Batch Normalization : Accelerating
Deep Network Training by Reducing Internal Covariate Shift. arXiv
Preprint arXiv:1502.03167v3. ↩

19. Dauphin, Y., Pascanu, R., Gulcehre, C., Cho, K., Ganguli, S., &
Bengio, Y. (2014). Identifying and attacking the saddle point
problem in high-dimensional non-convex optimization. arXiv, 1–14.
Retrieved from http://arxiv.org/abs/1406.2572 ↩

20. Sutskever, I., & Martens, J. (2013). On the importance of
initialization and momentum in deep learning.
http://doi.org/10.1109/ICASSP.2013.6639346 ↩

21. Neelakantan, A., Vilnis, L., Le, Q. V., Sutskever, I., Kaiser, L., Kurach,
K., & Martens, J. (2015). Adding Gradient Noise Improves Learning
for Very Deep Networks, 1–11. Retrieved from
http://arxiv.org/abs/1511.06807 ↩

22. LeCun, Y., Bottou, L., Orr, G. B., & Müller, K. R. (1998). Efficient
BackProp. Neural Networks: Tricks of the Trade, 1524, 9–50.
http://doi.org/10.1007/3-540-49430-8_2 ↩

23. Niu, F., Recht, B., Christopher, R., & Wright, S. J. (2011). Hogwild! : A
Lock-Free Approach to Parallelizing Stochastic Gradient Descent,
1–22. ↩

24. Dozat, T. (2016). Incorporating Nesterov Momentum into Adam.
ICLR Workshop, (1), 2013–2016. ↩

25. Duchi et al. [3] give this matrix as an alternative to the *full* matrix containing the outer products of all previous gradients, as the computation of the matrix square root is infeasible even for a moderate number of parameters d. ↩

26. Huang, G., Liu, Z., Weinberger, K. Q., & van der Maaten, L. (2017). Densely Connected Convolutional Networks. In Proceedings of CVPR 2017. ↩

27. Johnson, M., Schuster, M., Le, Q. V, Krikun, M., Wu, Y., Chen, Z., … Dean, J. (2016). Google's Multilingual Neural Machine Translation System: Enabling Zero-Shot Translation. arXiv Preprint arXiv:1611.0455. ↩

28. Reddi, Sashank J., Kale, Satyen, & Kumar, Sanjiv. On the Convergence of Adam and Beyond. Proceedings of ICLR 2018. ↩

29. Heusel, M., Ramsauer, H., Unterthiner, T., Nessler, B., & Hochreiter, S. (2017). GANs Trained by a Two Time-Scale Update Rule Converge to a Local Nash Equilibrium. In Advances in Neural Information Processing Systems 30 (NIPS 2017). ↩

Image credit for cover photo: Karpathy's beautiful loss functions tumblr

DJ Strouse • a year ago

Super helpful even a year out! You should've written the optimization section of Goodfellow, Bengio, & Courville. Please write more!

3 ∧ | ∨ • Reply • Share ›

Alexander Chrystopher Harringt ↱ DJ Strouse • a year ago

I too really found this helpful!
It provided some great insights into why I experience certain things when using different optimizers.

Great work!

1 ∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod ↱ DJ Strouse • a year ago

Thanks a lot, DJ! I'm definitely trying to write more -- particularly with such formidable encouragement.

1 ∧ | ∨ • Reply • Share ›

Nitin Bansal • 6 months ago

Thanks Sebastian for great writeup!

A small question though, bit unrelated, How do we actually calculate gradient in our programs/code, do we actually use some numerical methods to calculate differentiator and integration? Do you have any relevant pointer for it.

Thanks Again!

2 ∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod ↱ Nitin Bansal • 5 months ago

Hi Nitin, good question! In practice, unless you are implementing a toy example and are trying to understand how gradient calculation works, you're almost always using the automatic differentiation capabilities of your neural network library. So you can usually get away with calling `tf.gradients` or something similar and care only about how to use the gradients.

∧ | ∨ • Reply • Share ›

Baptiste Wicht • 5 months ago

Hi Sebastian,

Thanks a lot for an awesome collection of information!

Just a question, what tool do you use to generate both a web page and a paper from Latex (I guess it's latex).

1 ∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod → Baptiste Wicht • 5 months ago

Hi Baptiste, glad you like it! At the moment, I'm just writing a blog post in Markdown and then converting it to LaTeX (and making some refinements so it's more like a paper) if it's rigorous enough and there's interest. As I only do that occasionally, the bit of manual work is not a big deal. If I wanted to do this consistently, I'd probably start the other way around with LaTeX and use something like arxiv-vanity (https://www.arxiv-vanity.com/) to convert it to pretty HTML.

∧ | ∨ • Reply • Share ›

Baptiste Wicht → Sebastian Ruder • 5 months ago

Hi Sebastian,

Thank you for the answer!

I will take a look at arxiv-vanity for this.

Best regards

Baptiste

∧ | ∨ • Reply • Share ›

justwantanaccount • a year ago

Thank you so much for such a great article!

1 ∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod → justwantanaccount • a year ago

Thanks a lot! :)

∧ | ∨ • Reply • Share ›

Adam Iou-Jen Liu • a year ago

Very clear explanation, very helpful!
Thank you!

1 ∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod → Adam Iou-Jen Liu • a year ago

Hey Adam,
Thanks! Glad it was helpful! :)

∧ | ∨ • Reply • Share ›

Evrim Efe • a month ago

Hey Sebastian, thanks for the article. In the description of Adagrad, at the end of the second paragraph you have said "we set gt,i to be the gradient of the objective function w.r.t. to the parameter θi at time step t". Did you mean to say "gt,i is the partial of the objective function w.r.t

t". Did you mean to say "g_{t,i} is the partial of the objective function w.r.t the parameter θi at time step t"? Otherwise I'm a little confused about what are numbers and what are vectors. Thanks

∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod → Evrim Efe • 18 days ago

Hey Evrim, you're totally right. I confused that there's no such thing as the gradient of a specific parameter (that's the partial derivative). I've fixed this now. Thanks!

∧ | ∨ • Reply • Share ›

Thomas Unterthiner • 3 months ago

Hi Sebastian, Thanks for the impressive overview, really nice work! :) Sorry that I'm so late to the game, but: I wanted to point out that there is a very intuitive way of understanding Adam, namely as a Heavy Ball with Friction rolling down in the error surface. We've developed that intuition while working on a paper that appeared at NIPS 2017 (https://arxiv.org/abs/1706....). :)

In any case, keep up the great work.

∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod → Thomas Unterthiner • 3 months ago

Hi Thomas, thanks for sharing the paper and your intuition! It's great to be able to provide people with more intuitions, in particular if they're rigorously proven. :) I've added your insight to the section about Adam. Likewise, keep up the great work.

∧ | ∨ • Reply • Share ›

Zaenal Abidin • 3 months ago

This is very awesome, thank you so much Mr Sebastian.

∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod → Zaenal Abidin • 3 months ago

Thank you, Zaenal! :)

∧ | ∨ • Reply • Share ›

markroxor • 3 months ago

"Instead of inefficiently storing w
previous squared gradients, the sum of gradients is recursively defined
as a decaying average of all past squared gradients. The running average E[g2]t at time step t then depends (as a fraction γ similarly to the Momentum term) only on the previous average and the current gradient:"

Can you please explain this in a little more detail? I couldn't find the

significance of `w`? How does this take care of the window of gradients?

∧ | ∨ · Reply · Share ›

Leo Van · 4 months ago

Hi, first it really an great post. But I have a question with the Nadam method. You mean $(\beta_1 * m_{t+1}) / (1 - \beta_1^t)$ is just the bias-corrected estimate of the momentum vector of the previous time step. I think maybe you got mistake in writing, since $\hat{m}_{t-1} = m_{t-1} / (1 - \beta_1^{t-1})$, and $(1 - \beta_1^{t-1}) \approx (1 - \beta_1^t)$, so we replace $(\beta_1 * m_{t+1}) / (1 - \beta_1^t)$ with $\beta_1 * \hat{m}_{t-1}$, is that right?

∧ | ∨ · Reply · Share ›

Paulo Abelha · 4 months ago

Thank you for this post! Great work. Really helpful

∧ | ∨ · Reply · Share ›

> **Sebastian Ruder** Mod ➔ Paulo Abelha · 4 months ago
>
> Hi Paulo, thanks a lot! Glad it was helpful! :)
>
> ∧ | ∨ · Reply · Share ›

Andreas Weinzierl · 4 months ago

Hi Sebastian,

thank you for the overview about the different optimization techniques for gradient descent.

I also have a small question about the update rule of Adagrad. First, you provided an update rule, which describes the update of a single Parameter - Omega_i. Then, you derive a matrix vector based formula from that update rule.

It says you should multiply the learning rate by the inverse of the rooted gradient sum matrix (to which epsilon has been applied). Afterwards you should do element-wise matrix-vector multiplication with the gradients and subtract the result from the current parameters. I do not see how this should work out. Imho, one should do a element-wise matrix-vector multiplication with the learning rate and the inverse. I also would take the root elementwise and do a real matrix vector multiplication with the gradients afterwards to get a good approximation of the single parameter update rule.

The problem is that a element-wise matrix-vector multiplication at the multiplication with the gradients imho yields a matrix instead of a vector which is needed to perform the parameter update.

Unfortunately I really am not yet able to understand the paper which presents Adagrad, so I cannot say anything for sure, but the article of the link, which follows, confirms my thoughts.

http://colinraffel.com/wiki

http://coliniander.com/wiki...

Though, your post helped me a lot. Thank you again for making the effort.

I would be glad if you could resolve this issue.

∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod ➔ Andreas Weinzierl • 4 months ago

Hi Andreas,

I'm not sure I understand your point. We'll do a matrix-vector product in the Adagrad equation, which yields a vector (e.g. see here: https://mathinsight.org/mat...). The post you referred to also mentions that the division is element-wise. Let me know what I'm getting wrong.

∧ | ∨ • Reply • Share ›

Andreas Weinzierl ➔ Sebastian Ruder • 4 months ago

I am OK with the division being elementwise, but you mentioned that the multiplication should be element-wise.

Quotation: "we can now vectorize our implementation by performing an element-wise matrix-vector multiplication ⊙ between Gt and gt"

∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod ➔ Andreas Weinzierl • 4 months ago

Ah, I get your point now. Seems my wording was confusing and ambiguous. I just meant to emphasize that we multiply each row of the matrix with the vector in contrast to what we did before. I'll change the wording to make that clearer. Thanks for pointing it out!

∧ | ∨ • Reply • Share ›

Andreas Weinzierl ➔ Sebastian Ruder • 4 months ago

You are welcome.

∧ | ∨ • Reply • Share ›

saiprasannar • 4 months ago

This paper https://arxiv.org/abs/1506.... (used in fastai library also) has a way to find ideal learning rate for SGD, and it prescribes a cosine annealing. Any idea on does that compare to momentum based + adaptive methods? Jeremy Howard in his course said the learning rate finding can be applied for Adaptive algorithms like adam etc, but not sure how cycling learning rate between max and min found using the lr find works in momentum + adaptive methods.

∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod → saiprasannar • 4 months ago

Hey, I've written here about cyclical learning rates and cosine annealing: http://ruder.io/deep-learni...

∧ | ∨ • Reply • Share ›

Art Linkov • 5 months ago

Thanks for a great summary Sebastian!

Recently (November 2017) a paper was released about fixing weight decay regularization in adam. they made a slight modification to the algorithm. It would be great if you could update it here. (link to paper: https://arxiv.org/abs/1711....

Keep up the good work!

∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod → Art Linkov • 5 months ago

Hi Art, thanks for the reference! I actually wrote about that method in my most recent blog post here: http://ruder.io/deep-learni...

I'm planning to update this one with the latest methods after the ICLR 2018 notification.

∧ | ∨ • Reply • Share ›

Farzad Nozarian • 5 months ago

Hi,

Many thanks for your great post!

Just a note: it seems that the link in reference 4 does not point to the "Large Scale Distributed Deep Networks" paper :)

∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod → Farzad Nozarian • 5 months ago

Thanks for pointing that out! I've fixed the reference now. :)

∧ | ∨ • Reply • Share ›

Igor Aherne • 6 months ago

Thank you so much!

∧ | ∨ • Reply • Share ›

Sebastian Ruder Mod → Igor Aherne • 5 months ago

You're welcome! :)

∧ | ∨ • Reply • Share ›

Sudhanshu Ranjan • 6 months ago

Really great blog post. Thank you for writing it

Really great blog post. Thank you for writing it.

A small doubt. Will the denominators be same in both the cases ?

Note that $\beta_1 m_{t-1} 1 - \beta t 1$ is just the bias-corrected estimate of the momentum vector of the previous time step.

We can thus replace it with : $\hat{m}_{t-1}$

∧ | ∨ • Reply • Share ›

> **Sebastian Ruder** Mod ➔ Sudhanshu Ranjan • 5 months ago
>
> You're totally right. I tried to simplify the derivation but went too far in this case. I've left it it in for now as the alternative is a lot uglier and less intuitive and added a note to point this out. Thanks for spotting, Sudhanshu! :)
>
> ∧ | ∨ • Reply • Share ›

Zheng Qin • 6 months ago

Thanks Sebastian for such a wonderful article. I have a question about Image 6. From the formulations all these method are based on the gradients. In the saddle point case, should the gradient along the x axis be larger than zero and the gradient along the y axis be equal to zero? If so, none of the optimization methods can acquire velocity or update along the y axis, and none of them can break symmetry. So how to explain Image 6? Or am I wrong?

∧ | ∨ • Reply • Share ›

> **Sebastian Ruder** Mod ➔ Zheng Qin • 5 months ago
>
> Hey Zheng, the problem around saddle points is not the direction of the SGD update, but that the step size in order to escape from the saddle point becomes too small. The adaptive models have an easier time escaping as they can adjust the step size. Have a look at [19], Section 4 for more information regarding the dynamics around saddle points.
>
> ∧ | ∨ • Reply • Share ›

Haichao Yu • 6 months ago

Hello Sebastian, could you give some hints on how to generate Fig. 5 and 6? Thanks a lot.

∧ | ∨ • Reply • Share ›

> **Sebastian Ruder** Mod ➔ Haichao Yu • 6 months ago
>
> Hi Haichao, as indicated in the section, those animations were originally created by Alec Radford. For doing them yourself, you could try something as described in this blog post: http://louistiao.me/notes/v...
>
> ∧ | ∨ • Reply • Share ›

Lewen ZHAO • 7 months ago

Great article ! By the way, I'm trying to implement a Reinforcement Learning algorithm called A3C, in the training it uses very small batches (e.g. 5 examples), to learn to play Atari games.

My problem is that the value prediction is first going up normally, then suddenly it falls to 0, this happens if I use SGD and RMSprop. At the head of the network, I see that the output of the convolutional layers turned suddenly to 0. This happens when the activation function is relu.

However, when I use Adam, the network does not improve at all.

Do you have an idea why this is happening ? I guess the optimizer suddenly pushed all the parameters in the conv layer below 0 ?

Thanks !

∧ | ∨   •   Reply   •   Share ›

Sebastian Ruder Mod ➜ Lewen ZHAO • 6 months ago

Hmm, this sounds more like a bug in the model implementation than in the optimizer if it also goes to 0 with SGD and RMSprop.

∧ | ∨   •   Reply   •   Share ›

Lewen ZHAO ➜ Sebastian Ruder • 6 months ago

Yeah, you're right. I later found that it's because I didn't calculate the discounted reward correctly.

∧ | ∨   •   Reply   •   Share ›

Sebastian Ruder Mod ➜ Lewen ZHAO • 6 months ago

Cool. Sorry about the late reply.

∧ | ∨   •   Reply   •   Share ›

Lewen ZHAO ➜ Sebastian Ruder • 6 months ago

No problem, thanks for all your great articles !

∧ | ∨   •   Reply   •   Share ›