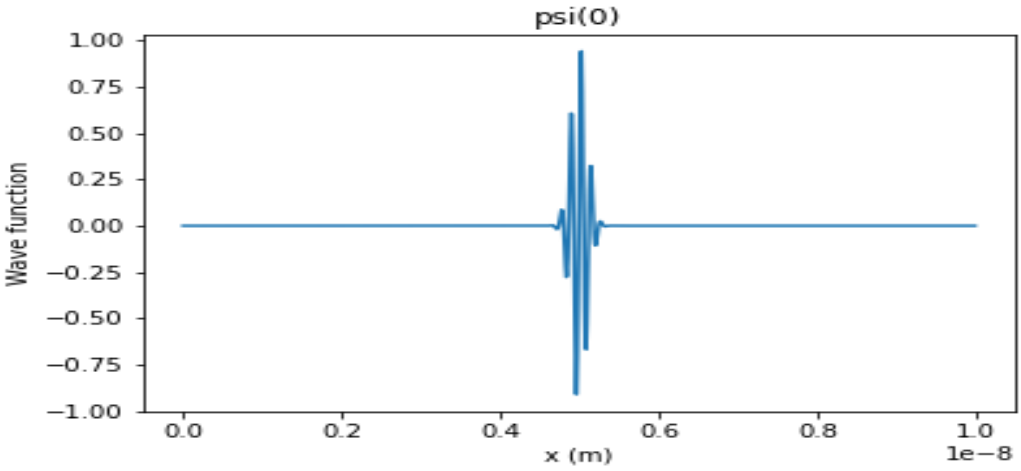
Problem 1

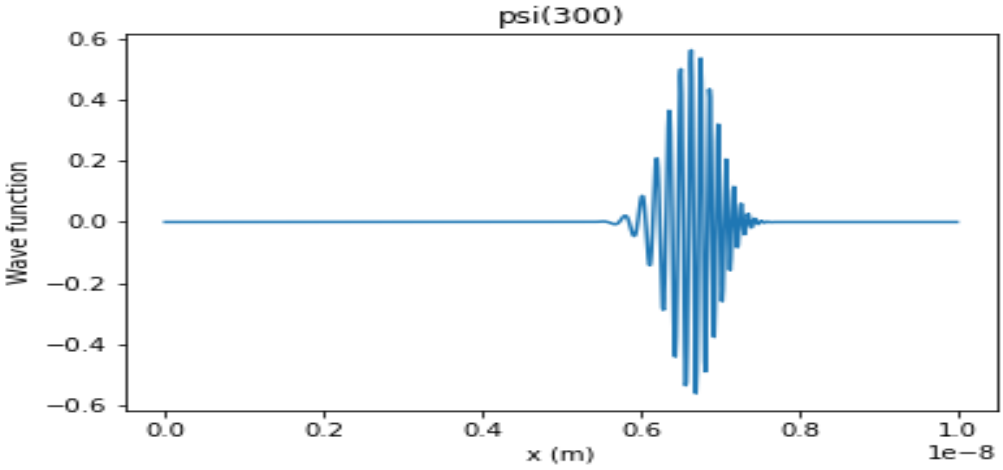
Methods Results

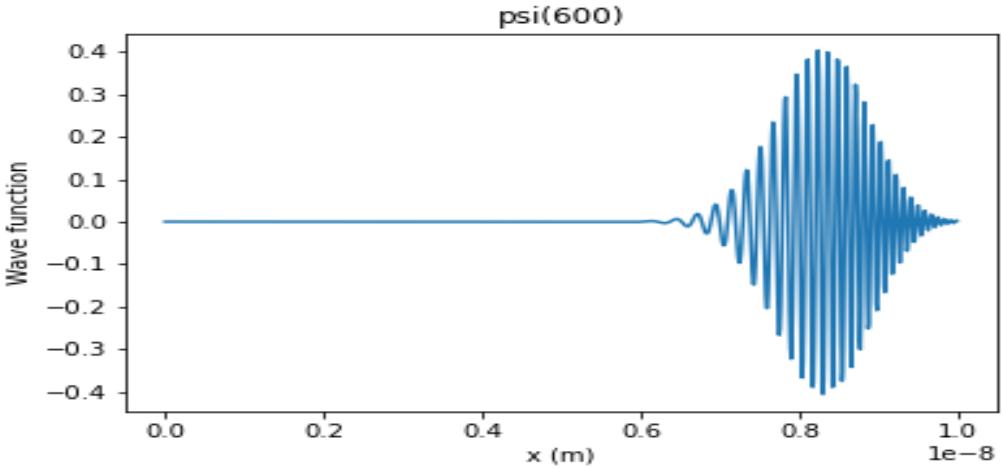
In this exercise we employ the Crank-Nicolson method to solve the time-dependent Schrodinger equation in an infinite square well. To do this, we construct tridiagonal matrices A and B and populate them with the coefficients $a_1 = 1 + h\frac{i\hbar}{2ma^2}$, $a_2 = -h\frac{i\hbar}{4ma^2}$, $b_1 = 1 - h\frac{i\hbar}{2ma^2}$, and $b_2 = h\frac{i\hbar}{4ma^2}$. Expanding the Crank-Nicolson time-step equation, it can be shown that we can find $x = \psi(t+1)$ is the solution to the system $Ax = v$, for $v = B\psi(t)$.

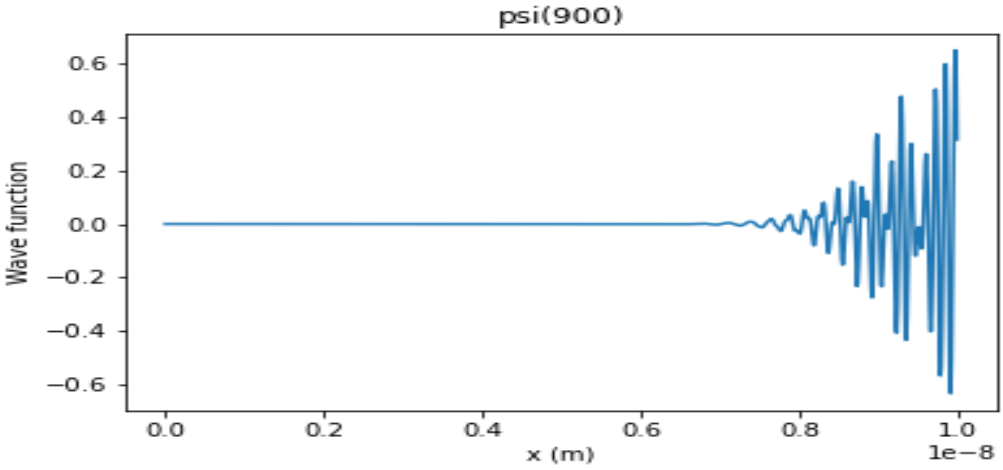
The resulting wave function (whose real part is plotted in the following section) begins localized about $L/2$, then propagates left while spreading out (as expected, since the spatial velocity and phase velocity differ for a quantum mechanical particle). It then bounces off the wall and produces interference patterns.

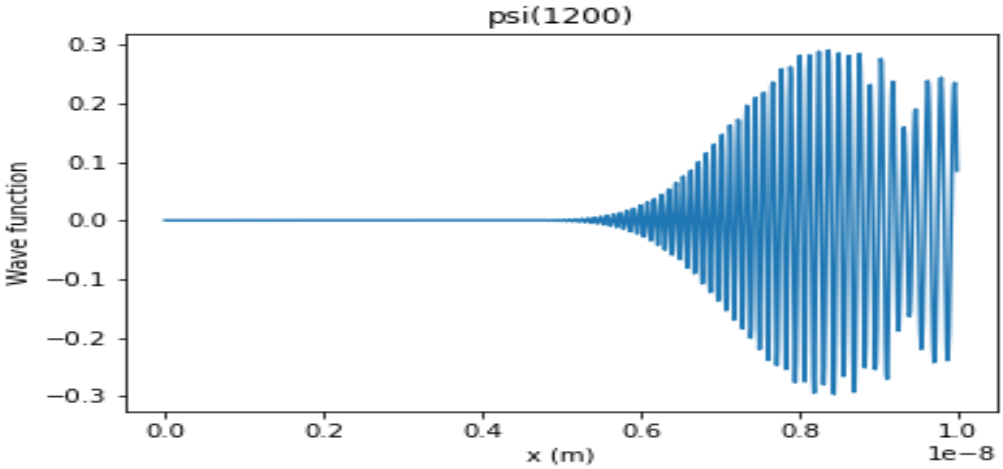
Figures

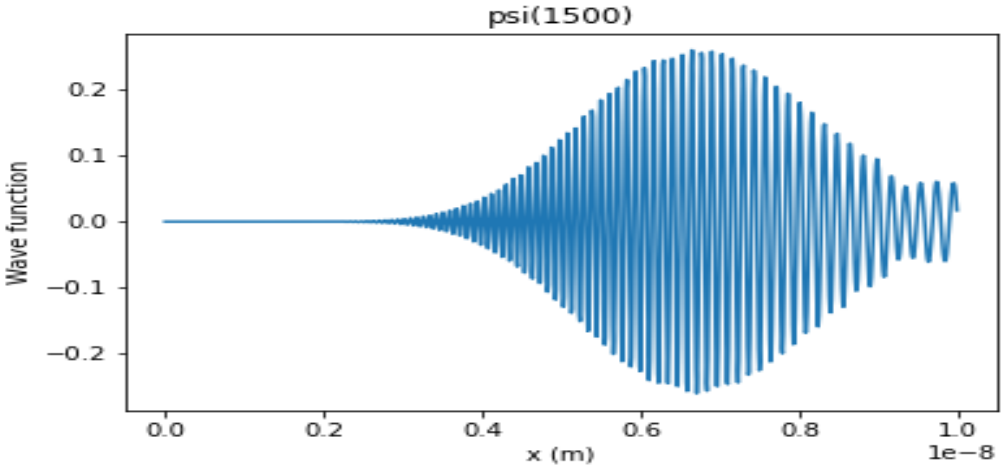


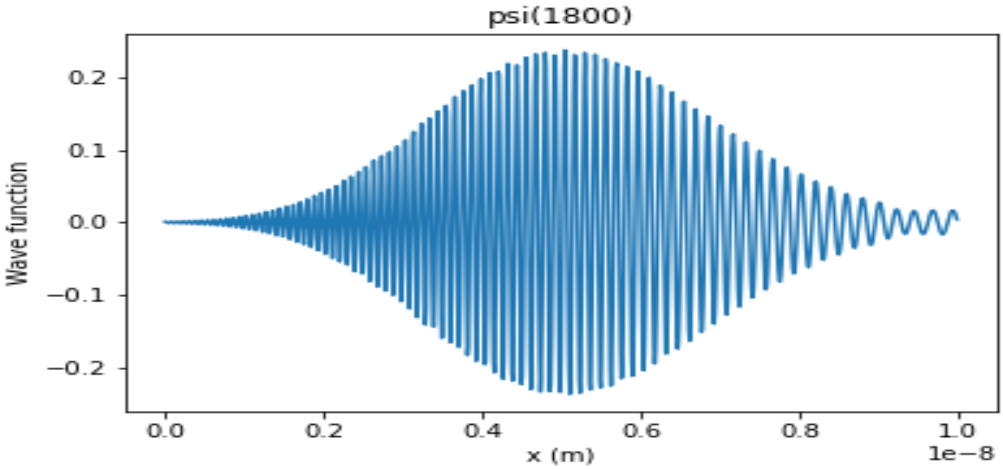

psi(300)
Wave function
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.0
0.2
0.4
0.6
0.8
1.0
1e−8
x (m)

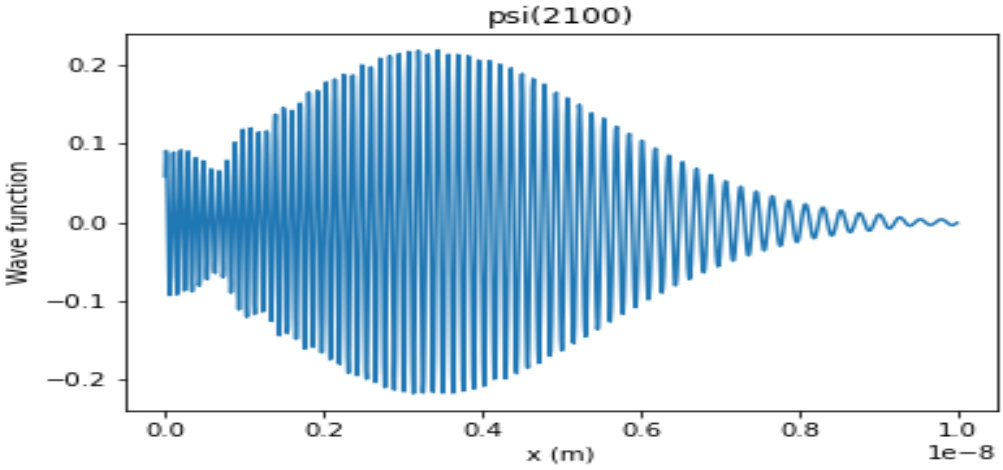

psi(600)
Wave function
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0.0
0.2
0.4
0.6
0.8
1.0
1e−8
x (m)

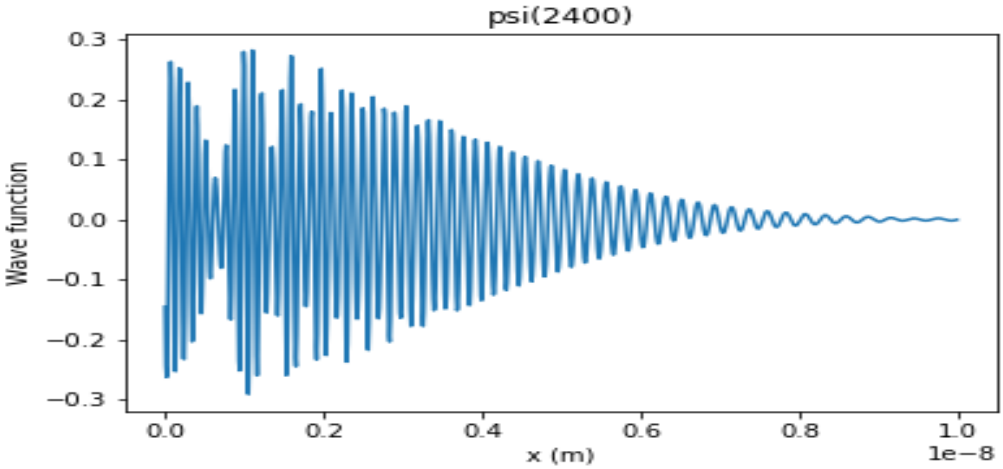

psi(900)
Wave function
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.0
0.2
0.4
0.6
0.8
1.0
1e−8
x (m)


psi(1200)
Wave function
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
0.0
0.2
0.4
0.6
0.8
1.0
1e−8
x (m)


psi(1500)
Wave function
0.2
0.1
0.0
−0.1
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1e−8
x (m)


psi(1800)
Wave function
0.2
0.1
0.0
−0.1
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1e−8
x (m)


psi(2100)
Wave function
0.2
0.1
0.0
−0.1
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1e−8
x (m)


psi(2400)
Wave function
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
0.0
0.2
0.4
0.6
0.8
1.0
1e−8
x (m)

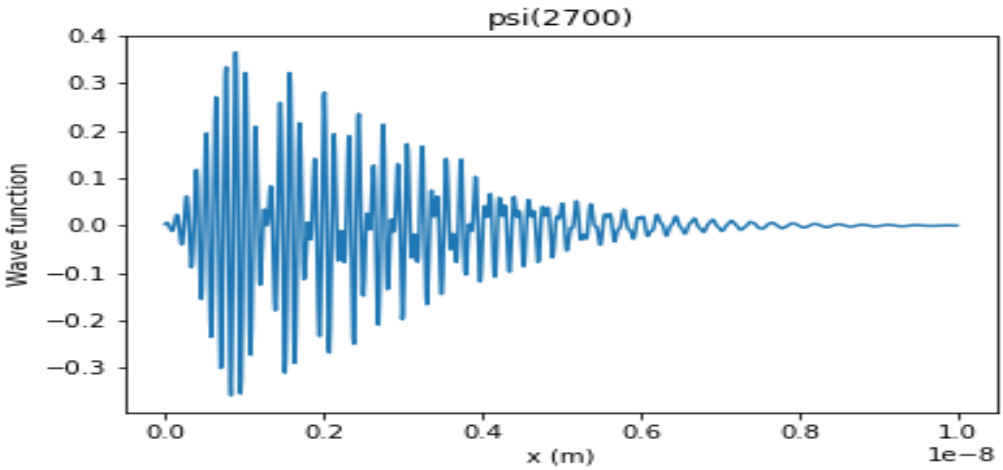


Figure 1: Real part of ψ at different times